


# Interaction of optical solitons under the synergistic effect of dispersion and various orders of nonlinearity

Xin Zhang(张鑫), Xiaofeng Li(李晓凤), Guoli Ma(马国利)
**Citation:**Chin. Phys. B. 35, 094211 (2026). DOI: 10.1088/1674-1056/ae881f | CSTR: 32038.14.CPB.ae881f
http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

**What follows is a list of articles you may be interested in**

---

## Extra-cavity manipulating a bright-gray vector soliton

Yan Zhou(周延), Daqian Tang(唐大千), Junxiao Zhan(占军潇), Yang Wang(汪洋), Weijia Zhao(赵炜佳), Meisong Liao(廖梅松), Yongzheng Fang(房永征)
Chin. Phys. B. 35, 094202 (2026). DOI: 10.1088/1674-1056/ae6d97 | CSTR: 32038.14.CPB.ae6d97

## Nonlinear evolution and formation of cosh–Gaussian breathers in cubic-quintic media possessing hump-shaped quintic nonlinearity

Naveen Gupta, Alex A K, Deepak Kumar
Chin. Phys. B. 35, 084205 (2026). DOI: 10.1088/1674-1056/ae2d3a | CSTR: 32038.14.CPB.ae2d3a

## Mode and orbital angular momentum controlling of nonlocal solitons by anisotropic diffraction

Junying Zhu(朱俊英), Qing Wang(王清), Jianning Wei(魏健宁)
Chin. Phys. B. 35, 044202 (2026). DOI: 10.1088/1674-1056/ae0163 | CSTR: 32038.14.CPB.ae0163

## Stability, bifurcation, chaotic pattern, phase portrait and exact solutions of a class of semi-linear Schrödinger equations with Kudryashov's power law self-phase modulation and multiplicative white noise based on Stratonvich's calculus

Cheng-Qiang Wang(王成强), Xiang-Qing Zhao(赵向青), Yu-Lin Zhang(张玉林), Zhi-Wei Lv(吕志伟)
Chin. Phys. B. 34, 124205 (2025). DOI: 10.1088/1674-1056/ade664 | CSTR: 32038.14.CPB.ade664

## Synchronized dual-wavelength mode-locked laser in normal dispersion regime

Yangrui Shi(史洋瑞), Haojing Zhang(张皓景), Yuxuan Ren(任俞宣), Junsong Peng(彭俊松), Heping Zeng(曾和平)
Chin. Phys. B. 34, 094207 (2025). DOI: 10.1088/1674-1056/add4fe | CSTR: 32038.14.CPB.add4fe

---

**SPECIAL TOPIC — Nonlinear localized waves: Theory, methods and applications**

# Interaction of optical solitons under the synergistic effect of dispersion and various orders of nonlinearity

Xin Zhang(张鑫)[1,†], Xiaofeng Li(李晓凤)[1], and Guoli Ma(马国利)[2,‡]

[1] *School of Mechanical and Electrical Engineering, Weifang University of Science and Technology, Weifang* 262700, *China*
[2] *School of Aviation and Aerospace, Tianjin Sino-German University of Applied Sciences, Tianjin* 300350, *China*

(Received 2 June 2026; revised manuscript received 7 July 2026; accepted manuscript online 9 July 2026)

Studying the influence of various orders of dispersion and nonlinear effects on optical soliton interactions holds crucial theoretical value for optimizing the transmission capacity of optical communication systems, enhancing signal transmission stability, and designing ultrafast photonic devices. However, most studies only consider a single order of dispersion or a specific type of nonlinearity, and research on the coupled effects of multiple factors, such as various orders of dispersion and different types of nonlinearities, is still insufficient, lacking a unified theoretical framework. This paper aims to explore the influence of different orders of dispersion and nonlinear mechanisms on the interaction characteristics of optical solitons. By introducing the bilinear transformation, an analytical two-soliton solution describing the physical process is successfully constructed. The research results reveal that an accurate balance between second-order dispersion and nonlinearity is the core prerequisite for maintaining pulse stability and avoiding morphological degradation. On this basis, third-order dispersion and nonlinear behavior can destroy the symmetrical characteristics of the pulse and excite excess ripples and tails, so suppressing them to near zero is crucial for ensuring transmission quality. Furthermore, although first-order dispersion only changes the timing of optical solitons when acting alone, its dynamic intervention cannot be ignored due to interference from frequency evolution induced by higher-order nonlinearity. When the first-order dispersion effect tends to zero, optimal distortion-free long-distance transmission can be achieved.

**Keywords:** optical soliton, soliton interaction, soliton solutions, optical communication systems

**PACS:** 42.65.Tg, 05.45.Yv, 42.81.Dp, 42.65.Re

**DOI:** 10.1088/1674-1056/ae881f **CSTR:** 32038.14.CPB.ae881f

## 1. Introduction

As a balanced product of dispersion and nonlinearity, optical solitons have been a core research object in the fields of nonlinear optics and optical communication since their experimental verification in optical fibers.[1,2] In actual fiber optic transmission systems, optical solitons do not exist in isolation, but rather propagate and evolve in the form of pulse sequences.[3,4] Therefore, the study of the interaction between optical solitons has irreplaceable physical significance and engineering value.[5] Soliton interactions directly determine crosstalk and timing jitter in wavelength division multiplexing (WDM) systems, limiting transmission capacity and communication distance.[6] Meanwhile, soliton bound states and their interactions are fundamental to cutting-edge applications such as microcavity soliton frequency combs, all-optical switches, and photonic computing.[7] Therefore, delving into the dynamic mechanism of the interaction between optical solitons is of great guiding significance for breaking through the bottleneck of existing optical information processing.[8,9]

After decades of development, the current research status of optical soliton interactions has formed a relatively complete theoretical framework. Early research was mainly based on the standard nonlinear Schrödinger equation (NLSE), utilizing inverse scattering transformations or perturbation theory to analytically derive the classical conclusions that in-phase solitons attract each other and anti-phase solitons repel each other.[10,11] Based on this, scholars have proposed various methods to suppress interactions, such as introducing initial phase differences, chirp control, amplitude asymmetry, and periodic dispersion management.[12–15] In recent years, with the advancements in numerical simulation and experimental observation methods, studies have found that the interactions between low-order solitons mainly manifest as periodic energy exchange and trajectory shifts, and under specific conditions, stable soliton molecules can be formed. These achievements have greatly promoted the application of soliton theory in fiber optic systems.[16–18]

However, with the evolution of optical communication towards single-channel qubits and the generation of femtosecond-level ultrashort pulses in micro–nano photonics, traditional research based on standard NLSE has gradually exposed significant limitations. When the width of the soliton pulse reaches the femtosecond level, its spectral bandwidth

†Corresponding author. E-mail: phyxin@163.com
‡Corresponding author. E-mail: bz_mgl@163.com

© 2026 Chinese Physical Society and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

increases sharply. At this point, the influence of high-order dispersion and high-order nonlinear effects of optical fibers on the evolution of optical solitons has reached an undeniable level. Standard NLSE cannot accurately describe soliton evolution under such extreme conditions due to its inclusion of only second-order dispersion and Kerr nonlinearity.[19] In addition, the complexity and uncontrollability of the interaction mechanism between optical solitons have become increasingly prominent. In high-power femtosecond lasers, high-order dispersion not only destroys the symmetry of solitons, but also leads to soliton fission and radiation of dispersive waves. These dispersive waves undergo cross-phase modulation with adjacent solitons, causing the originally periodic interactions to evolve into complex non-periodic chaotic behavior, resulting in unpredictable binding and dissociation of soliton molecules.[20] Besides, some higher-order effects are also widely discussed in nonlinear science.[21–23] These higher-order terms do not act in isolation, but rather form a multi-stage synergistic coupling mechanism between them, resulting in rich and complex spatiotemporal dynamics phenomena.[24,25] Based on the existing problems mentioned above, systematically studying the influence of various orders of dispersion and nonlinearity on the interaction of optical solitons is not only a necessary requirement for theoretical improvement, but also an important way to address practical engineering challenges.

In this paper, we introduce high-order dispersion and nonlinear terms, and construct a high-order nonlinear Schrödinger equation (HNLSE) model as follows:[26]

$$\begin{aligned} &\mathrm{i}q_t+\gamma_1 q_{xx}+\gamma_2|q|^2 q+\gamma_3 q_{xt}\\ &+\mathrm{i}(\gamma_4 q_{xxx}+\gamma_5|q|^2 q_x)-\mathrm{i}\gamma_6 q_x=0, \end{aligned} \tag{1}$$

where $q$ represents the slowly varying envelope of the optical field, $x$ represents spatial coordinates, and $t$ represents the time coordinate. $\gamma_1$, $\gamma_2$, $\gamma_3$, $\gamma_4$, $\gamma_5$, and $\gamma_6$ respectively represent second-order dispersion, self-phase modulation, spatiotemporal coupling, third-order dispersion, self-steepening, and first-order dispersion coefficients. For Eq. (1), researchers previously focused on suppressing interactions through individual physical parameters such as phase difference and chirp control, which makes it difficult to reveal the synergistic effects of multiple-order terms. Here, we will reveal the asymmetry and stability control mechanisms arising from the synergistic effects of multiple orders of dispersion and nonlinear effects on the interaction between optical solitons, and reconstruct the theory of optical soliton interaction within a complete framework incorporating various orders of dispersion and nonlinear effects.

## 2. Analytical soliton solutions

At first, in order to obtain the soliton solutions of Eq. (1), we assume that

$$q(x,t)=g(x,t)/f(x,t), \tag{2}$$

where $g(x,t)$ and $f(x,t)$ are the complex and real functions, respectively. Substituting expression (2) into Eq. (1), we obtain the bilinear forms of Eq. (1) as follows:

$$\mathrm{i}D_t g\cdot f+\gamma_1 D_x^2 g\cdot f+\mathrm{i}\gamma_4 D_x^3 g\cdot f-\mathrm{i}\gamma_6 D_x g\cdot f=0, \tag{3}$$

$$D_x^2 f\cdot f-2|g|^2=0, \tag{4}$$

with $\gamma_2=2\gamma_1$, $\gamma_3=0$, $\gamma_5=6\gamma_4$.

Secondly, we assume that

$$g(x,t)=\varepsilon g_1(x,t)+\varepsilon^3 g_3(x,t), \tag{5}$$

$$f(x,t)=1+\varepsilon^2 f_2(x,t)+\varepsilon^4 f_4(x,t). \tag{6}$$

In order to obtain the analytical two-soliton solution, we assume

$$g_1(x,t)=\alpha_1\exp[\eta_1(x,t)]+\alpha_2\exp[\eta_2(x,t)], \tag{7}$$

where

$$\begin{aligned} \eta_1(x,t)&=k_1x+w_1t=(k_{11}+\mathrm{i}k_{12})x+(w_{11}+\mathrm{i}w_{12})t,\\ \eta_2(x,t)&=k_2x+w_2t=(k_{21}+\mathrm{i}k_{22})x+(w_{21}+\mathrm{i}w_{22})t. \end{aligned}$$

Substituting Eqs. (5) and (7) into Eqs. (3) and (4), we obtain

$$\begin{aligned} w_{11}&=k_{11}\gamma_6-2k_{11}k_{12}\gamma_1-k_{11}^3\gamma_4+3k_{11}k_{12}^2\gamma_4,\\ w_{12}&=k_{12}\gamma_6+k_{11}^2\gamma_1-k_{12}^2\gamma_1-3k_{11}^2k_{12}\gamma_4+k_{12}^3\gamma_4,\\ w_{21}&=k_{21}\gamma_6-2k_{21}k_{22}\gamma_1-k_{21}^3\gamma_4+3k_{21}k_{22}^2\gamma_4,\\ w_{22}&=k_{22}\gamma_6+k_{21}^2\gamma_1-k_{22}^2\gamma_1-3k_{21}^2k_{22}\gamma_4+k_{22}^3\gamma_4. \end{aligned}$$

Thirdly, we assume that

$$\begin{aligned} f_2(x,t)\ =\ &\varphi_1\exp[\eta_1(x,t)+\eta_1^*(x,t)]\\ &+\varphi_2\exp[\eta_2(x,t)+\eta_2^*(x,t)]\\ &+\varphi_3\exp[\eta_1(x,t)+\eta_2^*(x,t)]\\ &+\varphi_4\exp[\eta_2(x,t)+\eta_1^*(x,t)], \end{aligned} \tag{8}$$

and substituting Eq. (8) into Eqs. (3) and (4), we obtain

$$\begin{aligned} \varphi_1&=\frac{\alpha_1\alpha_1^*}{4k_{11}^2},\quad \varphi_2=\frac{\alpha_2\alpha_2^*}{4k_{21}^2},\\ \varphi_3&=\frac{\alpha_1\alpha_2^*}{(k_{11}+\mathrm{i}k_{12}+k_{21}-\mathrm{i}k_{22})^2},\\ \varphi_4&=\frac{\alpha_2\alpha_1^*}{(k_{11}-\mathrm{i}k_{12}+k_{21}+\mathrm{i}k_{22})^2}. \end{aligned}$$

Assuming

$$\begin{aligned} g_3(x,t)\ =\ &\sigma_1\exp[\eta_1(x,t)+\eta_2(x,t)+\eta_1^*(x,t)]\\ &+\sigma_2\exp[\eta_1(x,t)+\eta_2(x,t)+\eta_2^*(x,t)], \end{aligned}$$

we obtain

$$\sigma_1=\frac{(k_1-k_2)^2\alpha_1\alpha_2\alpha_1^*}{(k_1+k_1^*)^2(k_2+k_1^*)^2},\quad \sigma_2=\frac{(k_1-k_2)^2\alpha_1\alpha_2\alpha_2^*}{(k_1+k_2^*)^2(k_2+k_2^*)^2}.$$

According to the above expression, we assume

$$f_4(x,t) = \rho \exp[\eta_1(x,t) + \eta_2(x,t) + \eta_1^*(x,t) + \eta_2^*(x,t)],$$

where the value of $\rho$ can be expressed as

$$\rho = \frac{(k_1-k_2)^2(k_1^*-k_2^*)^2\alpha_1\alpha_2\alpha_1^*\alpha_2^*}{(k_1+k_1^*)^2(k_2+k_1^*)^2(k_1+k_2^*)^2(k_2+k_2^*)^2}.$$

At last, assuming that $\varepsilon = 1$, we can obtain the analytical two-soliton solution as

$$q(x,t) = \frac{g(x,t)}{f(x,t)} = \frac{g_1(x,t)+g_3(x,t)}{1+f_2(x,t)+f_4(x,t)}, \tag{9}$$

where the expressions of $g_1(x,t)$, $g_3(x,t)$, $f_2(x,t)$, and $f_4(x,t)$ are given above.

## 3. Discussion

For the two-soliton solution Eq. (9), there are three physical parameters $\gamma_1$, $\gamma_4$, and $\gamma_6$, and six free parameters $k_{11}$, $k_{12}$, $k_{21}$, $k_{22}$, $\alpha_1$, and $\alpha_2$. In this paper, we mainly discuss the influences of $\gamma_1$, $\gamma_4$, and $\gamma_6$ on the two-soliton interactions. Firstly, we set $k_{11} = 0.1$, $k_{12} = 0.5$, $k_{21} = -0.1$, $k_{22} = -0.4$, $\gamma_4 = 0.3$, $\gamma_6 = -1$, $\alpha_1 = 0.2$, $\alpha_2 = 0.5$, and analyze the effects of second-order dispersion and nonlinear effects on optical soliton interactions in Fig. 1. For the second-order dispersion and nonlinear effects, they should satisfy the following relationship $\gamma_2 = 2\gamma_1$. That is to say, only when the second-order nonlinearity and second-order dispersion satisfy such a relationship can stable optical soliton transmission be achieved. In addition, when $\gamma_1$ takes different values, the interaction phenomena between optical solitons exhibit different characteristics. In Fig. 1(a), $\gamma_1 = 2$, the modulation is relatively small during the interaction process of the two optical solitons. In Figs. 1(b) and 1(c), as the value of $\gamma_1$ decreases, the modulation during the interaction process becomes larger. When $\gamma_1 = 0.2$ in Fig. 1(d), the two optical solitons propagate almost in parallel, and the modulation during their interaction is very intense.

Figure 2 is the waveform diagram at the highest amplitude point in Fig. 1. For Fig. 2(a), $x = 3.67$, which means that during the interaction process of optical solitons, the amplitude value $|q| = 0.186$ is at its maximum at $t = 2.92$. In Fig. 2(b), the maximum amplitude is $|q| = 0.186$ at $x = 4.69$ and $t = 3.99$. In Fig. 2(c), the maximum amplitude is $|q| = 0.192$ at $x = 13.85$ and $t = 13.69$. In Fig. 2(d), the maximum amplitude is $|q| = 0.199$ at $x = 33.93$ and $t = 35.12$. We can find that as the value of $\gamma_1$ decreases, the interaction between optical solitons becomes more intense. This is primarily due to the condition that when the second-order dispersion matches the second-order nonlinearity, the weaker the dispersion, the more likely it is for issues such as pulse splitting and waveform distortion to occur. On the contrary, if the dispersion is too strong, it cannot form a soliton and cannot effectively undergo nonlinear shaping. Therefore, we set the value of $\gamma_1$ to 2 in Fig. 1(a).

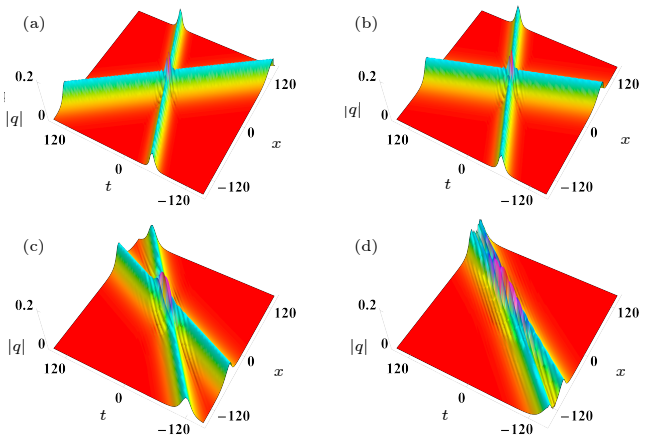



**Fig. 1.** Analysis of the effects of second-order dispersion and nonlinear effects on optical soliton interactions. The related parameters are $k_{11} = 0.1$, $k_{12} = 0.5$, $k_{21} = -0.1$, $k_{22} = -0.4$, $\gamma_4 = 0.3$, $\gamma_6 = -1$, $\alpha_1 = 0.2$, and $\alpha_2 = 0.5$, with (a) $\gamma_1 = 2$, (b) $\gamma_1 = 1.5$, (c) $\gamma_1 = 0.5$, and (d) $\gamma_1 = 0.2$.

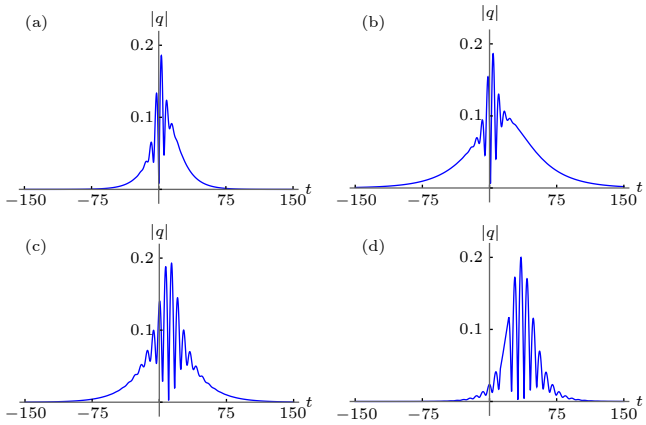



**Fig. 2.** The waveform diagrams at the highest amplitude points in Fig. 1. The $x$-values at these highest points are, respectively: (a) $x = 3.67$, (b) $x = 4.69$, (c) $x = 13.85$, and (d) $x = 33.93$.

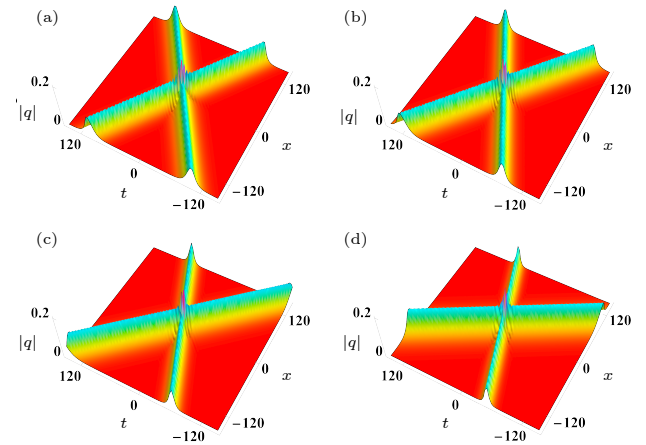



**Fig. 3.** Analysis of the effects of third-order dispersion and nonlinear effects on optical soliton interactions. The values of $k_{11}$, $k_{12}$, $k_{21}$, $k_{22}$, $\gamma_6$, $\alpha_1$, and $\alpha_2$ are the same as those in Fig. 1. The other parameter is $\gamma_1 = 2$, with (a) $\gamma_4 = 2$, (b) $\gamma_4 = 1.5$, (c) $\gamma_4 = 0.8$, and (d) $\gamma_4 = 0.001$.

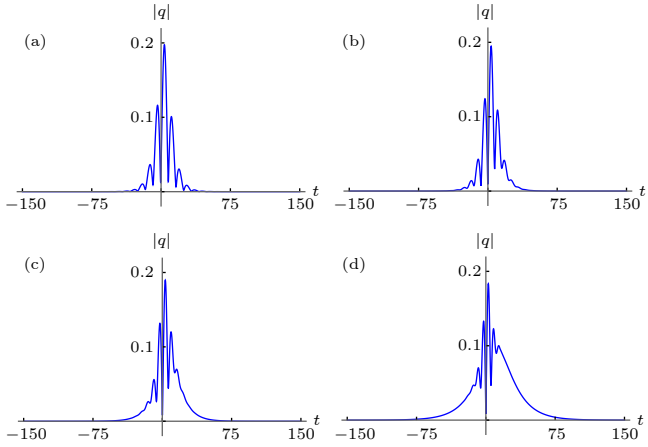



**Fig. 4.** The waveform diagrams at the highest amplitude points in Fig. 3. The $x$-values at these highest points are, respectively: (a) $x = 3.14$, (b) $x = 3.45$, (c) $x = 3.69$, and (d) $x = 3.56$.

In Figs. 1 and 2, we discussed the influence of second-order dispersion and second-order nonlinearity on the interaction process of optical solitons. In Figs. 3 and 4, we primarily discuss the impact of third-order dispersion and third-order nonlinear effects on the interaction of optical solitons with the value of second-order dispersion $\gamma_1 = 2$ under the condition that $\gamma_5 = 6\gamma_4$. In Fig. 3, the interaction between optical solitons becomes weaker as the value of $\gamma_4$ decreases. The amplitude of the optical solitons is approximately 0.19. In Fig. 4, the maximum amplitude occurs at $x = 3.14$, 3.45, 3.69, and 3.56, respectively. In Figs. 3(a) and 4(a), third-order dispersion and third-order nonlinear effects significantly alter the interaction between optical solitons, induce asymmetric deformation of optical solitons, and generate sidelobes. In Figs. 4(c) and 4(d), the pulse has asymmetric front and trailing edges, exhibiting time-domain skew and trailing dispersion waves. From the comparison between Figs. 4(a) and 4(d), it can be seen that the greater the third-order dispersion, the stronger the sidelobes, and the more chaotic the modulation of the interaction. Thus, under the premise that the second-order dispersion and second-order nonlinearity have been perfectly matched to form standard solitons, neither third-order dispersion nor third-order nonlinearity is necessarily better at greater or lesser values. Instead, they are preferably weaker, and ideally, they should approach 0 as much as possible. Any increase in these values will deteriorate the morphology of optical solitons and the stability of their interactions.

In Figs. 5 and 6, we primarily discuss the impact of first-order dispersion effects on the interaction of optical solitons. $\gamma_1 = 2$ indicates that the second-order dispersion and second-order nonlinear effects have been precisely matched. $\gamma_4 = 0.05$ indicates the presence of a relatively weak third-order effect. First-order dispersion $\gamma_6$ itself does not alter the soliton waveform, disrupt the soliton equilibrium, or change the inherent interaction rules between optical solitons; however, it only produces an overall time shift. Due to the presence of third-order nonlinearity, the optical solitons themselves will undergo spectral drift, and the first-order dispersion will vary dynamically with frequency. Therefore, this will in turn modulate the interaction between optical solitons. In Fig. 5, the smaller the absolute value of first-order dispersion, the weaker the interaction between optical solitons. The maximum amplitude of optical solitons is 0.18 in Fig. 6. Besides, in Fig. 6(d), we can observe that due to $\gamma_6 \to 0$, the optical solitons exhibit almost no modulation during their interaction process, demonstrating excellent waveform quality. Therefore, there is no need to deliberately increase or retain a small amount of $\gamma_6$. When $\gamma_6 \to 0$, it is most beneficial for the stability of soliton timing sequence and conformal transmission over long distances.

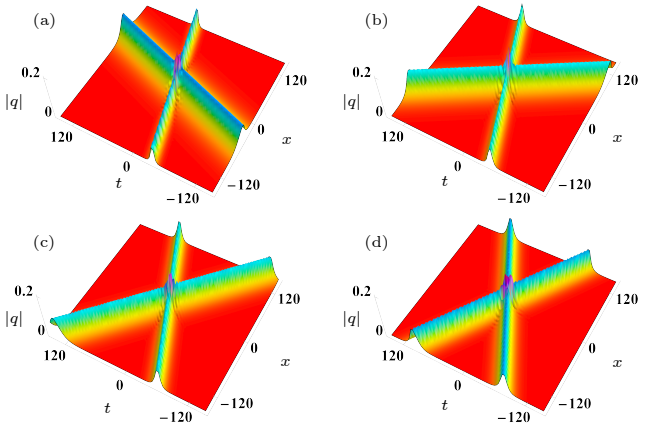



**Fig. 5.** Analysis of the effects of first-order dispersion and nonlinear effects on optical soliton interactions. The values of $k_{11}$, $k_{12}$, $k_{21}$, $k_{22}$, $\alpha_1$ and $\alpha_2$ are the same as those in Fig. 1. The other parameters are $\gamma_1 = 2$ and $\gamma_4 = 0.05$, with (a) $\gamma_6 = -2$, (b) $\gamma_6 = -1$, (c) $\gamma_6 = -0.5$, and (d) $\gamma_6 = -0.01$.

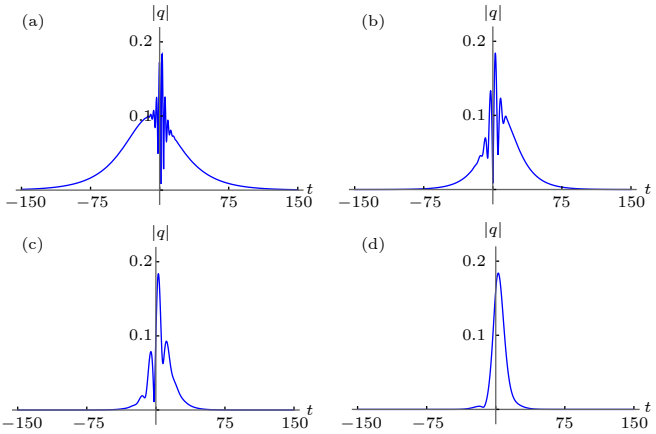



**Fig. 6.** The waveform diagram at the highest amplitude point in Fig. 5. The $x$-values at these highest points are, respectively: (a) $x = 6.31$, (b) $x = 3.59$, (c) $x = 2.23$, and (d) $x = 0.89$.

## 4. Conclusion

In summary, based on the bilinear method, this paper derives analytical two-soliton solutions and systematically explores the dynamical mechanism of optical soliton interaction under the synergistic effects of dispersion and nonlinearities of

various orders. The results show that second-order dispersion and second-order nonlinearity must satisfy a strict matching relationship $\gamma_2 = 2\gamma_1$ to achieve stable transmission of optical solitons, and the degree of their matching directly modulates the interaction strength between optical solitons. On this basis, third-order dispersion and third-order nonlinearity with $\gamma_5 = 6\gamma_4$ can disrupt soliton equilibrium, induce asymmetric waveform distortion, sidelobes, and dispersive waves, thereby deteriorating the stability of the interaction. Theoretically, their parameters should approach zero. Furthermore, due to the coupled influence of spectral drift caused by third-order nonlinearity, first-order dispersion dynamically modulates the interaction between optical solitons. The smaller its absolute value, the more beneficial it is for suppressing waveform perturbations. When it is zero, optimal stability and long-distance stable transmission of optical solitons can be achieved. The conclusions of this paper establish clear parameter regulation criteria for suppressing interference in optical soliton interactions and achieving stable transmission over long distances in optical communication systems.

## Acknowledgment

Project supported by the Scientific Research Foundation of Weifang University of Science and Technology (Grant No. KJRC2023035).

## References

[1] Mitschke F M and Mollenauer L F 1987 *Opt. Lett.* **12** 355
[2] Cui X N, Zhang Z Y, Liu M W, Qi F H and Liu W J 2025 *Chin. Phys. Lett.* **42** 100402
[3] Zhou H S, Zhang Z Y, Liu M W, Qi F H and Liu W J 2025 *Chin. Phys. Lett.* **42** 070001
[4] Wang H T, Qi F H and Liu W J 2025 *Appl. Math. Lett.* **171** 109691
[5] Yi H H, Zhang X, Shi L X, Yao Y L, Wang S B and Ma G L 2024 *Chin. Phys. Lett.* **41** 044204
[6] Agrawal G P 2019 *Nonlinear Fiber Optics* 6th ed. (Cambridge: Academic Press) 251
[7] Kippenberg T J, Gaeta A L, Lipson M and Gorodetsky M L 2018 *Science* **361** eaan8083
[8] Deng B J, Guo R and Zhang J W 2026 *Phys. Rev. E* **113** 054205
[9] Wang S B, Zhang X, Ma G L and Zhu D Y 2023 *Chin. Phys. B* **32** 030506
[10] Zakharov V E and Gelash A A 2013 *Phys. Rev. Lett.* **111** 054101
[11] Zhao Y, Huang Q H, Gong T X, Xu S L, Li Z P and Malomed B A 2024 *Chaos Solitons Fractals* **187** 115329
[12] Yuan W X and Guo R 2024 *Phys. Rev. E* **110** 065307
[13] Huang Y H and Guo R 2024 *Chaos* **34** 103103
[14] Belyaeva T L and Serkin V N 2024 *Chin. Phys. Lett.* **41** 080501
[15] Wang X M, Li J and Hu X X 2025 *Chin. Phys. B* **34** 110502
[16] Chen S S 2025 *Appl. Math. Lett.* **160** 109307
[17] Korobko D A, Ribenek V A, Itrin P A, Tertyshnikova G V and Fotiadi A A 2025 *J. Lightwave Technol.* **43** 10667
[18] Jia Y H, Si Z Z, Ju Z T, Feng H Y, Zhang J H, Yan X and Dai C Q 2025 *Sci. China Phys. Mech. Astron.* **68** 284211
[19] Hasegawa A 2022 *Front. Phys.* **10** 1044845
[20] Yan Z W, Yan Y Y, Liu M W and Liu W J 2024 *Appl. Math. Lett.* **157** 109164
[21] Lan Z Z 2024 *Chin. Phys. Lett.* **41** 090501
[22] Lan Z Z 2024 *Chin. Phys. B* **33** 060201
[23] Lan Z Z 2024 *Appl. Math. Lett.* **158** 109239
[24] Yi H H, Li X F, Zhang J L, Zhang X and Ma G L 2024 *Chin. Phys. Lett.* **41** 074204
[25] Raza N, Arshed S and Alhussain Z A 2024 *Opt. Quantum Electron.* **56** 451
[26] Hang Z and Lu T 2026 *Jpn. J. Ind. Appl. Math.* **43** 32