SPECIAL TOPIC — A 40th Anniversary Celebration: Physics Research and Innovation at Ningbo University

Symmetries and critical dimensions of tensionless branes

Bin Chen(陈斌)[1,2,3,†] and Zezhou Hu(胡泽州)[2,‡]

[1] *Institute of Fundamental Physics and Quantum Technology, School of Physical Science and Technology, Ningbo University, Ningbo* 315211, *China*
[2] *School of Physics, Peking University, Beijing* 100871, *China*
[3] *Center for High Energy Physics, Peking University, Beijing* 100871, *China*

(Received 18 April 2026; revised manuscript received 27 June 2026; accepted manuscript online 7 July 2026)

We investigate the worldsheet symmetry of bosonic brane theories and its quantum consistency in the tensionless limit. We find that the residual worldsheet symmetry after specific gauge fixing is generated by a novel algebra, denoted as $g^{(p)}_{\lambda}$. To achieve full quantization of the tensionless brane, we introduce a *bc* ghost system and derive the overall BRST charge. Moreover, we calculate the quantum anomaly of the $g^{(p)}_{\lambda}$ algebra for general parameters p and λ within the framework of canonical quantization. After demanding that this quantum anomaly vanishes, we derive the critical dimensions of the bosonic brane theories. In particular, we obtain nontrivial solutions for the tensionless string: $D = 14$ spacetime dimensions when $\lambda = -1$ and $D = 26$ spacetime dimensions when $\lambda = 1$. We also note that the quantum anomaly automatically vanishes after Riemann zeta function regularization for $p > 1$. This is consistent with the mathematical fact that there is no scalar central extension of the algebra $g^{(p)}_{\lambda}$ for $p > 1$.

Keywords: tensionless brane, $g^{(p)}_{\lambda}$ algebra, BRST quantization, quantum anomaly, critical dimension, Carrollian symmetry

PACS: 02.20.Sv, 11.25.–w, 11.25.Hf

DOI: 10.1088/1674-1056/ae86d5 **CSTR:** 32038.14.CPB.ae86d5

1. Introduction

For a theory of p-brane (string) living in a flat background with Minkowski metric $\eta_{\mu\nu}$, there is a Nambu–Goto action

$$S = T_p \int \mathrm{d}^d\sigma \sqrt{|G_{\alpha\beta}|}, \tag{1}$$

where $G_{\alpha\beta}$ is the induced metric

$$G_{\alpha\beta} = \eta_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu, \tag{2}$$

and the worldsheet coordinates are given by $\sigma^\alpha = (t, \boldsymbol{\sigma})$ and $\boldsymbol{\sigma} = \{\sigma^i | i = 1, 2, \ldots, p\}$. (In this paper, the Greek letters μ, ν varying from 0 to $D-1$ with D being the spacetime dimension. And the Greek letters $\alpha, \beta, \gamma, \ldots$ vary from 0 to p, and the Latin letters $i, j, k, \ldots$ vary from 1 to p. The total dimension of the p-brane worldsheet is $d = p+1$. In the main body of this paper, we sometimes express the spacetime vector as $\boldsymbol{X} \equiv X^\mu \partial_\mu$ and the worldsheet vector as $\hat{\boldsymbol{\varepsilon}} \equiv \varepsilon^\alpha \partial_\alpha$ for simplicity.) Furthermore, this action is classically equivalent to the Polyakov action

$$\begin{aligned} S = &-\frac{T_p}{2}\int \mathrm{d}^d\sigma \sqrt{-h} h^{\alpha\beta}\partial_\alpha X \cdot \partial_\beta X \\ &+ \frac{(p-1)T_p}{2}\int \mathrm{d}^d\sigma \sqrt{-h}, \end{aligned} \tag{3}$$

with $h_{\alpha\beta}$ being the auxiliary metric on the worldsheet. It is usually easier to quantize the theory from the Polyakov action. In the $p = 1$ case of string theory,[1] the metric $h_{\alpha\beta}$ has three independent components. None of them are dynamical variables because of the two-dimensional worldsheet diffeomorphism and the one-dimensional Weyl invariance of the action, which renders the equations of motion (EoM) of string theory fully linear. After classical covariant gauge fixing, the residual worldsheet symmetry is generated by the Virasoro algebra. Then we can obtain the critical dimension by requiring the quantum anomaly of the Virasoro symmetry to vanish. However, such a strategy does not work for $p > 1$ brane theory due to the intrinsic non-linearity. There, the auxiliary metric is a symmetric $(p+1) \times (p+1)$ tensor, and there is only a $(p+1)$-dimensional diffeomorphism such that $h_{\alpha\beta}$ has nontrivial dynamics, and the total system is highly non-linear.

There has been long-standing interest in studying $p > 1$ brane theory. In the mid-1980s, by including a Wess–Zumino–Witten term in the membrane action, it was shown that supermembranes[2,3] should live in the 11-dimensional background.[4–7] Still, it is very difficult to deal directly with the quantum consistency of worldsheet diffeomorphism and fermionic Siegel symmetry, and to the best of our knowledge, one can only work under the light-cone gauge.[8] Efforts

†Corresponding author. E-mail: chenbin1@nbu.edu.cn
‡Corresponding author. E-mail: z.z.hu@pku.edu.cn

© 2026 Chinese Physical Society and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

CPB Online In-Press

to perform quantization of the supermembrane include.[9–11] Later on, researchers turned to explore the non-perturbative aspects of supermembranes by using matrix models[12] and found that the supermembrane is unstable.[13] In the 1990s, the discovery of mysterious M-theory[14] rekindled interest in membranes,[15] matrix models,[16] and M5-branes.[17]

In this work, we would like to investigate higher-dimensional branes in the tensionless limit, and we start from the bosonic case. In the tensionless limit, the theory gets linearized: the second term in Eq. (3) effectively vanishes, and the first term can be well described by an ILST-type action,[18] which is linear. For the $p=1$ case, the tensionless (super)string[19–23] has already been explored for some years; see Ref. [24] for a nice review. It has been shown[25] that the residual worldsheet symmetry of the tensile string gets modified from the Virasoro algebra to $\mathfrak{bms}_3$ (or $\mathfrak{g}_1^{(1)}$ in our language). Its T-duality partner, the Carrollian string, has also been studied broadly.[26–29] For higher-dimensional cases, the quantization of the null brane was studied in Ref. [30,31] and was recently explored in the light-cone gauge in Ref. [32]. Another recent work[33] studying brane models demonstrates an enhancement of symmetry algebras in the tensionless limit. Our study will focus on the quantum consistency of the worldvolume symmetry.

The remaining parts of this note are organized as follows. In Section 2, we show that after some gauge fixing, the residual worldvolume symmetry is generated by the algebra $\mathfrak{g}_\lambda^{(p)} \cong \mathrm{Vect}(T^p) \ltimes_\lambda C^\infty(T^p)$, where λ is a free parameter and different λs correspond to different gauge transformations. Furthermore, we construct the generators of the algebra $\mathfrak{g}_\lambda^{(p)}$ in terms of the modes of matter fields in the formalism of canonical quantization. In Section 3, we consider path-integral quantization of the tensionless brane, and thereafter introduce a bc ghost system. We obtain the BRST charge of the ghost system and the $\mathfrak{g}_\lambda^{(p)}$ generators for the ghost fields. In Section 4, we calculate the quantum anomaly of the algebra $\mathfrak{g}_\lambda^{(p)}$ for general p and λ. Finally, we obtain the critical dimensions of the tensionless branes in Section 5. We also give some comments on the conclusions and future directions concerning the novel algebra $\mathfrak{g}_\lambda^{(p)}$.

2. The worldsheet symmetry of the tensionless brane: $\mathfrak{g}_\lambda^{(p)}$

The direct tensionless limit of the Nambu-Goto action (1) is ill-defined. The action of the tensionless brane (string) is actually given by the ILST action,[18] shown as

$$S=\left(\frac{1}{2\pi}\right)^p\int \mathrm{d}^d\sigma\frac{1}{2}V^\alpha V^\beta\partial_\alpha \boldsymbol{X}\cdot\partial_\beta\boldsymbol{X},\quad p\geq 1. \tag{4}$$

The action has a reparameterization invariance (diffeomorphism) given by

$$\begin{aligned}\delta_{\hat\varepsilon}\boldsymbol{X}&=-\varepsilon^\alpha\partial_\alpha\boldsymbol{X},\\ \delta_{\hat\varepsilon}V^\alpha&=-\varepsilon^\beta\partial_\beta V^\alpha+V^\beta\partial_\beta\varepsilon^\alpha-\frac{1}{2}\partial_\beta\varepsilon^\beta V^\alpha,\end{aligned} \tag{5}$$

and a local Weyl-like invariance given by

$$\delta_\chi\boldsymbol{X}=-\frac{\Delta}{d}\chi\boldsymbol{X},\quad \delta_\chi V^\alpha=+\frac{\Delta}{d}\chi V^\alpha, \tag{6}$$

with $\chi=\chi(\sigma)$ satisfying $V^\beta\partial_\beta\chi=0$. If the diffeomorphism $\hat\varepsilon$ also satisfies

$$V^\beta\partial_\beta(\partial_\alpha\varepsilon^\alpha)=0, \tag{7}$$

then we can linearly combine the two local invariances above by setting $\chi=\partial_\beta\varepsilon^\beta$. Under such a combined local transformation, the fields transform as

$$\begin{aligned}\delta_{\hat\varepsilon}\boldsymbol{X}&=-\varepsilon^\alpha\partial_\alpha\boldsymbol{X}-\frac{\Delta}{d}\partial_\alpha\varepsilon^\alpha\boldsymbol{X},\\ \delta_{\hat\varepsilon}V^\alpha&=-\varepsilon^\beta\partial_\beta V^\alpha+V^\beta\partial_\beta\varepsilon^\alpha-\left(\frac{1}{2}-\frac{\Delta}{d}\right)\partial_\beta\varepsilon^\beta V^\alpha,\end{aligned} \tag{8}$$

where the parameter Δ labeling the conformal dimension of the matter field X can be chosen arbitrarily due to the condition Eq. (7).

The vielbein V^α serves as the geometric structure of the tensionless worldsheet. We can choose a gauge $\hat V=\hat V_{(\hat 0)}\equiv(1,\boldsymbol{0})$. Under this gauge, we find a solution (but not the most general solution) to the condition Eq. (7)

$$\hat\varepsilon=f^i(\boldsymbol{\sigma})\partial_i+(h(\boldsymbol{\sigma})t+g(\boldsymbol{\sigma}))\,\partial_0, \tag{9}$$

which is a subset of the Carrollian diffeomorphism.* The diffeomorphism that preserves the gauge $\hat V_{(\hat 0)}\equiv(1,\boldsymbol{0})$ is the associated global symmetry, which is

$$\delta_{\hat\xi}V^\alpha|_{\hat V=\hat V_{(\hat 0)}}=0. \tag{10}$$

Its general solution is

$$\hat\xi=f^i(\boldsymbol{\sigma})\partial_i+\Big(\lambda\partial_k f^k(\boldsymbol{\sigma})t+g(\boldsymbol{\sigma})\Big)\,\partial_0,\quad \lambda=\frac{\frac{1}{2}-\frac{\Delta}{d}}{\frac{1}{2}+\frac{\Delta}{d}}. \tag{11}$$

It is a special case of Eq. (9) with $h(\boldsymbol{\sigma})$ set to $\lambda\partial_k f^k(\boldsymbol{\sigma})$. For simplicity, we consider the spatial manifold of the tensionless worldsheet to be a torus T^p. Then the functions $f^i(\boldsymbol{\sigma}),g(\boldsymbol{\sigma})$ can be expanded in modes $\mathrm{e}^{-\mathrm{i}m_k\sigma^k}$. Finally, we can rewrite the global symmetry denoted by $\hat\xi$ as $-i\hat l^i_{\{m\}}$ and $-i\hat m_{\{m\}}$

$$\begin{aligned}\hat l^i_{\{m\}}&=-i\mathrm{e}^{-\mathrm{i}m_k\sigma^k}\partial_i-\lambda m_i t\,\mathrm{e}^{-\mathrm{i}m_k\sigma^k}\partial_0,\\ \hat m_{\{m\}}&=-i\mathrm{e}^{-\mathrm{i}m_k\sigma^k}\partial_0,\end{aligned} \tag{12}$$

*The Carrollian diffeomorphism is defined in Ref. [34,35] as $t\to t'(t,\boldsymbol{x})$ and $\boldsymbol{x}\to\boldsymbol{x}'(\boldsymbol{x})$. The associated infinitesimal diffeomorphism $x^\alpha\to x^\alpha+\varepsilon^\alpha(x)$ is $\hat\varepsilon=f^i(\boldsymbol{\sigma})\partial_i+g(t,\boldsymbol{\sigma})\partial_0$. Under such a diffeomorphism, the Carrollian structure $\{\partial_0, g_{ij}\mathrm{d}x^i\mathrm{d}x^j\}$ remains intact.

CPB Online In-Press

which forms the $\mathfrak{g}_\lambda^{(p)}$ algebra

$$[\hat{l}^i_{\{m\}},\hat{l}^j_{\{n\}}] = m_j\hat{l}^i_{\{m+n\}} - n_i\hat{l}^j_{\{m+n\}},$$
$$[\hat{l}^i_{\{m\}},\hat{m}_{\{n\}}] = (\lambda m_i - n_i)\hat{m}_{\{m+n\}}, \tag{13}$$

where the set $\{m\}$ is a short notation for $\{m_1,m_2,\ldots,m_p\}$ and $\{m+n\}$ is defined to be $\{m_1+n_1,m_2+n_2,\ldots,m_p+n_p\}$. For $\lambda=1$, this algebra $g^{(p)}_{\lambda=1}$ and its possible central extension have been discussed in Ref. [30,31]. For $p=1,\lambda=1$, it is just the $\mathfrak{bms}_3$ algebra that appears in the usual tensionless string theory. In fact, the parameter λ is responsible for the anisotropic scaling symmetry in the Electric Carrollian free scalar. The isotropic scaling symmetry is the special case $\lambda=\frac{1}{p}$. For $p=1$, the algebra $\mathfrak{g}^{(1)}_\lambda$ with general λ has already been discussed in Ref. [36] with slightly different notations. For $p=1$ and $p=2$, similar algebras involving s-supertranslations in Ref. [37] could be related to $\mathfrak{g}^{(p)}_\lambda$ by $\lambda\sim\frac{s}{p}$. For $p>1$ and $\lambda=\frac{1}{p}$, one can consider the subalgebra within Eq. (38)

$$[L^i_m,L^i_n] = (m-n)L^i_{m+n},$$
$$[L^i_m,L^j_n] = 0,\quad (i\neq j),$$
$$[L^i_m,M_n] = \left(\frac{m}{p}-n\right)M_{m+n}, \tag{14}$$

where

$$L^i_m \equiv L^i_{\{m_i=m,m_{j\neq i}=0\}}. \tag{15}$$

This is an extension of BMS_{p+2} symmetry.

Next, we perform canonical quantization of the fields $\boldsymbol{X}$ under the gauge $\hat{V}_{(\hat{0})}\equiv(1,\boldsymbol{0})$. Solving the equations of motion, we have the mode expansions

$$\boldsymbol{X} = \sum_{\{m\}}(i\boldsymbol{A}_{\{m\}} + \boldsymbol{B}_{\{m\}}t)\,\mathrm{e}^{\mathrm{i}m_k\sigma^k}. \tag{16}$$

Its canonical momentum is

$$\boldsymbol{\Pi} = \left(\frac{1}{2\pi}\right)^p\partial_t\boldsymbol{X} = \left(\frac{1}{2\pi}\right)^p\sum_{\{m\}}\boldsymbol{B}_{\{m\}}\,\mathrm{e}^{\mathrm{i}m_k\sigma^k}. \tag{17}$$

From the canonical commutation relation

$$[X^\mu(t,\boldsymbol{\sigma}),\Pi^\nu(t,\boldsymbol{\sigma}')] = i\eta^{\mu\nu}\delta^p(\boldsymbol{\sigma}-\boldsymbol{\sigma}'), \tag{18}$$

we have

$$[A^\mu_{\{m\}},B^\nu_{\{n\}}]$$
$$= \eta^{\mu\nu}\delta_{\{m+n\}} \equiv \eta^{\mu\nu}\delta_{m_1+n_1,0}\delta_{m_2+n_2,0}\ldots\delta_{m_p+n_p,0}. \tag{19}$$

The Noether current associated with the global symmetry is

$$j^\alpha = \frac{\partial L}{\partial\partial_\alpha\boldsymbol{X}}\cdot\delta_{\hat{\xi}}\boldsymbol{X} - L\xi^\alpha. \tag{20}$$

By substituting $\xi^\alpha\partial_\alpha$ with $-i\hat{l}^i_{\{m\}}$ and $-i\hat{m}_{\{m\}}$, the Noether charge $Q=\int\mathrm{d}^p\boldsymbol{\sigma}j^0$ can be expressed as

$$L^{i,(X)}_{\{m\}} = \sum_{\{n\}}\left(\frac{1+\lambda}{2}m_i - n_i\right):\boldsymbol{A}_{\{m-n\}}\cdot\boldsymbol{B}_{\{n\}}:,$$
$$M^{(X)}_{\{m\}} = -\frac{1}{2}\sum_{\{n\}}\boldsymbol{B}_{\{m-n\}}\cdot\boldsymbol{B}_{\{n\}}. \tag{21}$$

Here we introduce the normal-ordering operator ::, placing all the creation operators to the left of the annihilation operators. The exact meaning of its definition depends on the choice of vacuum. We will address this issue in Section 4. From the commutation relations Eq. (19), we can calculate the algebra generated by these conserved charges. The result, regardless of the quantum anomaly, is

$$[L^{i,(X)}_{\{m\}},L^{j,(X)}_{\{n\}}] = m_jL^{i,(X)}_{\{m+n\}} - n_iL^{j,(X)}_{\{m+n\}},$$
$$[L^{i,(X)}_{\{m\}},M^{(X)}_{\{n\}}] = (\lambda m_i - n_i)M^{(X)}_{\{m+n\}}. \tag{22}$$

3. The *bc* ghost system

To fully quantize the tensionless brane, we need to consider the partition function in terms of the path integral

$$Z = \int\mathrm{D}\hat{V}\mathrm{D}\boldsymbol{X}\,\mathrm{e}^{\mathrm{i}S(\hat{V},\boldsymbol{X})}$$
$$= \int\mathrm{D}\hat{V}\mathrm{D}\boldsymbol{X}\,\mathrm{e}^{\mathrm{i}S(\hat{V},\boldsymbol{X})}\int\mathrm{D}\hat{\varepsilon}\,\delta(\hat{V}_{(\hat{\varepsilon})}-\hat{V}_{(\hat{0})})\det(\delta_{\hat{\varepsilon}}\hat{V}_{(\hat{\varepsilon})}). \tag{23}$$

If there is no anomaly for the Carrollian diffeomorphism, we have $\mathrm{D}\hat{V}_{(\hat{\varepsilon})}\mathrm{D}\boldsymbol{X}_{(\hat{\varepsilon})} = \mathrm{D}\hat{V}\mathrm{D}\boldsymbol{X}$. As a result

$$Z \to Z\Big/\int\mathrm{D}\hat{\varepsilon}$$
$$= \int\mathrm{D}\hat{V}\mathrm{D}\boldsymbol{X}\,\delta(\hat{V}-\hat{V}_{(\hat{0})})\det(\delta_{\hat{\varepsilon}}\hat{V})\,\mathrm{e}^{\mathrm{i}S(\hat{V},\boldsymbol{X})}$$
$$= \int\mathrm{D}\boldsymbol{X}\det(\delta_{\hat{\varepsilon}}\hat{V})\,\mathrm{e}^{\mathrm{i}S(\hat{V},\boldsymbol{X})}|_{\hat{V}=\hat{V}_{(\hat{0})}}. \tag{24}$$

The determinant $\det(\delta_{\hat{\varepsilon}}\hat{V})$ in the path integral can be expressed as the path integral of a *bc* ghost system

$$\det(\delta_{\hat{\varepsilon}}\hat{V})|_{\hat{V}=\hat{V}_{(\hat{0})}} = \int\mathrm{D}\hat{b}\mathrm{D}\hat{c}\,\mathrm{e}^{\mathrm{i}S(\hat{b},\hat{c})}, \tag{25}$$

with

$$S(\hat{b},\hat{c}) = i\left(\frac{1}{2\pi}\right)^p\int\mathrm{d}^d\boldsymbol{\sigma}c^\alpha\Big[-\delta^\beta_\alpha V^\gamma\partial_\gamma+\partial_\alpha V^\beta$$
$$+\left(\frac{1}{2}-\frac{\Delta}{d}\right)V^\beta\partial_\alpha\Big]b_\beta\Big|_{\hat{V}=\hat{V}_{(\hat{0})}}$$
$$= i\left(\frac{1}{2\pi}\right)^p\int\mathrm{d}^d\boldsymbol{\sigma}\Big[\left(\frac{1}{2}+\frac{\Delta}{d}\right)c^0\partial_0b_0+c^i\partial_0b_i$$
$$-\left(\frac{1}{2}-\frac{\Delta}{d}\right)c^i\partial_ib_0\Big].$$

For convenience, we rescale the component b_0 to be $\left(\frac{1}{2}+\frac{\Delta}{d}\right)^{-1}b_0$. Then the action of *bc* ghost fields becomes

$$S(\hat{b},\hat{c}) = i\left(\frac{1}{2\pi}\right)^p\int\mathrm{d}^d\boldsymbol{\sigma}\left(c^0\partial_0b_0+c^i\partial_0b_i-\lambda c^i\partial_ib_0\right). \tag{26}$$

CPB Online In-Press

Solving the equations of motion, we have the mode expansions

$$c^0 = \lambda\partial_i c^i t + \sum_{\{m\}} c^0_{\{m\}}\, \mathrm{e}^{\mathrm{i}m_k\sigma^k},\ \ c^i = \sum_{\{m\}} c^i_{\{m\}}\, \mathrm{e}^{\mathrm{i}m_k\sigma^k}, \tag{27}$$

$$b_0 = \sum_{\{m\}} b_{0,\{m\}}\, \mathrm{e}^{\mathrm{i}m_k\sigma^k},\ \ b_i = \lambda\partial_i b_0 t + \sum_{\{m\}} b_{i,\{m\}}\, \mathrm{e}^{\mathrm{i}m_k\sigma^k}. \tag{28}$$

Performing canonical quantization gives

$$\begin{aligned}&\{c^\alpha(t,\boldsymbol{\sigma}), b_\beta(t,\boldsymbol{\sigma}')\} = (2\pi)^p\delta^\alpha_\beta\delta^p(\boldsymbol{\sigma}-\boldsymbol{\sigma}'),\\ &\{c^\alpha_{\{m\}}, b_{\beta,\{n\}}\} = \delta^\alpha_\beta\delta_{\{m+n\}}.\end{aligned} \tag{29}$$

In the rest of this section, we are going to discuss the contribution to the $\mathfrak{g}^{(p)}_\lambda$ generators from the ghost system using the BRST method. We first determine the BRST charge of the whole system. The ghost field c^α takes the place of the diffeomorphism parameter under the BRST transformation

$$\begin{aligned}&\delta_B c^\alpha = i\{c^\alpha, Q_B\} = c^\beta\partial_\beta c^\alpha,\\ &\delta_B \boldsymbol{X} = -i[\boldsymbol{X}, Q_B] = c^\alpha\partial_\alpha\boldsymbol{X} + \frac{\Delta}{d}\partial_\alpha c^\alpha\boldsymbol{X}.\end{aligned} \tag{30}$$

It is not hard to check that $\delta_B^2 c^\alpha = \delta_B^2 X = 0$. To satisfy these BRST transformations, the BRST charge must be of the form

$$\begin{aligned}Q_B = \int \mathrm{d}^p\sigma\Bigg[&\left(\frac{(2\pi)^p}{2}c^0\boldsymbol{\Pi}^2 + c^i\partial_i\boldsymbol{X}\cdot\boldsymbol{\Pi} + \frac{1-\lambda}{2}\partial_i c^i\boldsymbol{X}\cdot\boldsymbol{\Pi}\right)\\ &-\frac{i}{(2\pi)^p}(c^\alpha\partial_\alpha c^\beta b_\beta)\Bigg].\end{aligned} \tag{31}$$

In terms of the mode expansions, the BRST charge Q_B can be expanded as

$$\begin{aligned}Q_B =\ &-\sum_{\{m\}}\left(c^0_{\{-m\}}M^{(X)}_{\{m\}} + c^i_{\{-m\}}L^{i,(X)}_{\{m\}}\right)\\ &+\sum_{\{m\},\{n\}}(m_k-n_k)\Big(\lambda c^0_{\{n\}}c^k_{\{m-n\}}b_{0,\{-m\}}\\ &+c^k_{\{n\}}c^0_{\{m-n\}}b_{0,\{-m\}} + c^k_{\{n\}}c^j_{\{m-n\}}b_{j,\{-m\}}\Big).\end{aligned} \tag{32}$$

Another way to derive this expression is to write the matter part of the BRST charge first, and the ghost part is determined uniquely by the nilpotency of the BRST charge, regardless of the quantum anomaly, $Q_B^2 = 0$. In BRST quantization, the generators of the $\mathfrak{g}^{(p)}_\lambda$ algebra can be defined through the anticommutation relations between the antighost modes and the BRST charge

$$M_{\{m\}} = -\left\{b_{0,\{m\}}, Q_B\right\},\quad L^i_{\{m\}} = -\left\{b_{i,\{m\}}, Q_B\right\}, \tag{33}$$

where $M_{\{m\}} = M^{(X)}_{\{m\}} + M^{(\mathrm{gh})}_{\{m\}}$ and $L^i_{\{m\}} = L^{i,(X)}_{\{m\}} + L^{i,(\mathrm{gh})}_{\{m\}}$ are the total generators. Under this relation, the ghost parts of the generators are given by

$$\begin{aligned}L^{i,(\mathrm{gh})}_{\{m\}} =\ &-\sum_{\{n\}} n_i : c^\alpha_{\{n\}}b_{\alpha,\{m-n\}} : -\sum_{\{n\}} m_k : c^k_{\{n\}}b_{i,\{m-n\}} :\\ &-\lambda\sum_{\{n\}} m_i : c^0_{\{n\}}b_{0,\{m-n\}} :,\\ M^{(\mathrm{gh})}_{\{m\}} =\ &-\sum_{\{n\}}(\lambda n_k + m_k)c^k_{\{n\}}b_{0,\{m-n\}}.\end{aligned}$$

4. Quantum anomaly of $\mathfrak{g}^{(p)}_\lambda$

In this section, we calculate the quantum anomaly of the algebra $\mathfrak{g}^{(p)}_\lambda$ for the matter fields $\boldsymbol{X}$ and the bc ghost fields for general p and λ with respect to a set of vacua $|\{h\}\rangle = |h_0, h_1, h_2\rangle, \{h\}\subset\boldsymbol{Z}$. The vacuum $|\{h\}\rangle$ is defined by

$$\begin{aligned}&\boldsymbol{A}_{\{m\}}|\{h\}\rangle = 0 \text{ with } \sum_i m_i \geq 1-h_0,\\ &\boldsymbol{B}_{\{m\}}|\{h\}\rangle = 0 \text{ with } \sum_i m_i \geq h_0,\\ &c^0_{\{m\}}|\{h\}\rangle = 0 \text{ with } \sum_i m_i \geq 1-h_1,\\ &b_{0,\{m\}}|\{h\}\rangle = 0 \text{ with } \sum_i m_i \geq h_1,\\ &c^i_{\{m\}}|\{h\}\rangle = 0 \text{ with } \sum_j m_j \geq 1-h_2,\\ &b_{i,\{m\}}|\{h\}\rangle = 0 \text{ with } \sum_j m_j \geq h_2.\end{aligned} \tag{34}$$

It is natural to choose $h_0 = 0$ since $\boldsymbol{B}_{\{0\}}$ is the momentum operator and $\boldsymbol{A}_{\{0\}}$ is the position operator. But we have no reason to set h_1, h_2 in advance. Any two different vacua with the same h_0 but different h_1, h_2 can be related to each other. For example, the vacuum $|\{h'\}\rangle$ with $h'_1 = 0$ is related to the vacuum $|\{h\}\rangle$ with $h_1 = 1$ by

$$|\{h'\}\rangle = \prod_{\{m\}\in\mathrm{Set}} b_{0,\{m\}}|\{h\}\rangle,\ \ \mathrm{Set} = \left\{\{m\}\middle|\sum_i m_i = 0\right\}. \tag{35}$$

This relation is realized by the nilpotency of the fermionic-mode operators, i.e. $b^2_{0,\{m\}} = \{b_{0,\{m\}}, b_{0,\{m\}}\}/2 = 0$. In string theory ($p = 1$), the right-hand side of Eq. (35) is composed of a finite number of mode operators acting on the vacuum. Thus, $|\{h'\}\rangle$ and $|\{h\}\rangle$ belong to the same module of a highest-weight representation. But when $p > 1$, the expression above involves an infinite product of mode operators, and its convergence is not guaranteed, implying that they may belong to different modules of the highest-weight representations of $\mathfrak{g}^{(p)}_\lambda$.

The quantum anomaly arises from the normal ordering associated with these vacua. For $p = 1$, the $\mathfrak{g}^{(1)}_\lambda$ algebra

$$\begin{aligned}&[L_m, L_n] = (m-n)L_{m+n} + A(m)\delta_{m+n,0},\\ &[L_m, M_n] = (\lambda m - n)M_{m+n},\end{aligned} \tag{36}$$

has a finite anomalous term

$$\begin{aligned}A^{(X)}(m) =\ &\frac{D}{6}\left[\frac{3\lambda^2-1}{2}m^2 - 6h_0(h_0-1) - 1\right]m,\\ A^{(\mathrm{gh})}(m) =\ &-\frac{1}{6}\Big[(6\lambda^2+6\lambda+14)m^2\end{aligned}$$

CPB Online In-Press

$$-6h_1(h_1-1)-6h_2(h_2-1)-2\Big]m. \quad (37)$$

The algebra $\mathfrak{g}_\lambda^{(p)}$ for $p>1$ must be of the form

$$\begin{aligned}
[L^i_{\{m\}},L^i_{\{n\}}] &= (m_i-n_i)L^i_{\{m+n\}}+A^i_1(\{m\})\delta_{\{m+n\},0},\\
[L^i_{\{m\}},L^j_{\{n\}}] &= m_jL^i_{\{m+n\}}-n_iL^j_{\{m+n\}}\\
&\quad +A^{ij}_2(\{m\})\delta_{\{m+n\},0}\quad (i\neq j),\\
[L^i_{\{m\}},M_{\{n\}}] &= (\lambda m_i-n_i)M_{\{m+n\}}. \quad (38)
\end{aligned}$$

Thus, we may calculate the anomalous terms $A^i_1(\{m\}),A^{ij}_2(\{m\})$ by

$$\begin{aligned}
\left[L^i_{\{m\}},L^i_{\{-m\}}\right]|\{h\}\rangle &= A^i_1(\{m\})|\{h\}\rangle\\
\left[L^i_{\{m\}},L^j_{\{-m\}}\right]|\{h\}\rangle &= A^{ij}_2(\{m\})|\{h\}\rangle \ \text{ for } j\neq i.
\end{aligned}$$

The anomalous terms involve some infinite sums. For example, the anomaly from the fields X is

$$\begin{aligned}
A^{i,(X)}_1(\{m\}) &= \sum_{\{n\}\in \mathrm{Set}(h_0)}\left(\frac{1+\lambda}{2}m_i-m_i-n_i\right)\\
&\quad\times\left(\frac{1+\lambda}{2}m_i+n_i\right), \quad (39)
\end{aligned}$$

with

$$\mathrm{Set}(h_0)=\left\{\{n\}\Big|-\sum_i m_i+h_0\le\sum_i n_i\le -1+h_0\right\}. \quad (40)$$

Denote $\sum_i n_i=N$; the range of the sum can be written as a finite sum times some infinite sums

$$\sum_{\mathrm{Set}(h_0)}=\sum_{N=-\sum_i m_i+h_0}^{-1+h_0}\sum_{n_1=-\infty}^{\infty}\cdots\sum_{n_{p-1}=-\infty}^{\infty}\Big|_{n_p=N-\sum_{i=1}^{p-1}n_i}. \quad (41)$$

For example, assuming $i\neq p$ without loss of generality, the anomalous terms Eq. (39) can be written as

$$\begin{aligned}
&A^{i,(X)}_1(\{m\})\\
&=\sum_{n_i=-\infty}^{\infty}\left(\frac{1+\lambda}{2}m_i-m_i-n_i\right)\left(\frac{1+\lambda}{2}m_i+n_i\right)\\
&\quad\times\sum_{N=-\sum_i m_i+h_0}^{-1+h_0}\prod_{j\neq i}^{p-1}\sum_{n_j=-\infty}^{\infty}1\\
&=\left(\sum_i m_i\right)\left(\sum_{n=-\infty}^{\infty}1\right)^{p-2}\\
&\quad\times\sum_{n_i=-\infty}^{\infty}\left(\frac{1+\lambda}{2}m_i-m_i-n_i\right)\left(\frac{1+\lambda}{2}m_i+n_i\right). \quad (42)
\end{aligned}$$

Note that Eq. (41) only holds for $p>1$. If $p=1$, it is a finite sum, and we have

$$A^{(X)}(m)=\sum_{n=-m+h_0}^{-1+h_0}\left(\frac{1+\lambda}{2}m-m-n\right)\left(\frac{1+\lambda}{2}m+n\right), \quad (43)$$

which exactly gives the first result of Eq. (37).

Thus, when $p>1$, all the anomalous terms can finally be translated into expressions including at least one factor of the form

$$S(l)=\sum_{n=-\infty}^{\infty}n^l. \quad (l \text{ is some non-negative integer}) \quad (44)$$

Exploiting the Riemann Zeta function $\xi(s)\equiv\sum_{n=1}^{\infty}1/n^s$, we can regularize the expression above. If l is an odd positive integer, $S(l)=0$ by parity. And if l is an even non-negative integer, $S(l)=\delta_{l,0}+2\xi(-l)=\delta_{l,0}+2\delta_{l,0}\xi(0)=0$. Anyway, the divergent anomalous terms in the form of $S(l)$ are zero after the Riemann Zeta function regularization.

To make this point more explicit, we illustrate with two representative cases. For $p>2$, the factorized expression for $A^{i,(X)}_1$ contains an overall factor $(\sum_{n=-\infty}^{\infty}1)^{p-2}=[S(0)]^{p-2}$, which is already zero under the zeta regularization, so the entire anomalous term vanishes irrespective of the remaining factor. For $p=2$, there is no such divergent overall factor; however, the single sum $\sum_{n_i=-\infty}^{\infty}(\ldots)$ is a polynomial in n_i. Expanding this polynomial yields a linear combination of $S(l)$ with $l\ge 0$, and each $S(l)$ vanishes under the same regularization. The same reasoning extends to the cross-terms arising from the multiple sums in the decomposition Eq. (41): every term can be written as a product of factors of the form $S(l)$, and no finite residual contribution survives the zeta regularization.

This vanishing of the anomalous terms in the $p>1$ case is in accordance with the fact that the Gelfand-Fuchs cohomology $H^2(\mathrm{Vect}(T^p),\mathbb{C})$ is trivial,[38] and there is no scalar central extension of the algebra involving $L^i_{\{m\}}$. We also expect this for the algebra $\mathfrak{g}_\lambda^{(p)}\cong\mathrm{Vect}(T^p)\ltimes_\lambda C^\infty(T^p)$ by Proposition 1 in Section 3 of the papers.[39,40] A generalized Abelian extension is given in Ref. [41], but that is beyond the scope of our interest.

5. Discussion and conclusion

For general p and the chosen vacua, we obtain the quantum anomaly with respect to the algebra $\mathfrak{g}_\lambda^{(p)}$. The consistency of a quantum gauge system then requires the total central charge to vanish. When $p=1$, the leading term gives the constraint

$$\frac{3\lambda^2-1}{2}D-[6(2+\lambda+\lambda^2)+2]=0, \quad (45)$$

which has two nontrivial and sensible solutions

$$D=14 \quad \text{when} \quad \lambda=-1, \quad (46)$$

$$D=26 \quad \text{when} \quad \lambda=1. \quad (47)$$

The lower-order term in the quantum anomaly can be absorbed into the normal-ordering constant (or, equivalently, by redefining $L_0\to L_0-a_L$).

CPB Online In Press

For the $p > 1$ cases, the quantum anomaly vanishes automatically after Riemann Zeta function regularization. Thus, there is no constraint on the critical dimension for the brane theories.

We emphasize that this result only implies that the quantum consistency of the $\mathfrak{g}^{(p)}_{\lambda}$ algebra does not restrict the spacetime dimension; other potential quantum instabilities of the brane theory, such as negative-norm states, global anomalies, or dynamical instabilities, still need to be examined separately.

In string theory, the origin of the parameter λ traces back to different Ultra-relativistic (UR) limits. The action for the bc ghost returns to the inhomogeneous case of Ref. [22] when $\lambda = 1$ and to the homogeneous case when $\lambda = 0$. In Ref. [22], the authors claimed that the homogeneous bc ghost does not admit a BRST charge and therefore does not make sense. This claim is not correct, as they did not find the correct symmetry algebra $\mathfrak{g}^{(p)}_{\lambda=0}$ there. As shown in this work, the homogeneous bc ghost corresponds to the Carrollian conformal symmetry with $\lambda = 0$ rather than with $\lambda = 1$, the symmetry generators L_m, M_m should be modified for both the matter sector and the ghost sector, and the correct BRST charge is constructed as Eq. (31) in this manuscript. We also note that the result in Ref. [22] for the bosonic tensionless string is exactly the special case $p = 1, \lambda = 1$ in this work.

A subtle point concerns the critical dimension $D = 14$ obtained at $\lambda = -1$. Recall that λ is not a free parameter but follows from the conformal dimension Δ through $\lambda = (\frac{1}{2} - \frac{\Delta}{d})/(\frac{1}{2} + \frac{\Delta}{d})$; solving $\lambda = -1$ requires $|\Delta|/d \to \infty$, a limit inaccessible for any finite Δ. Nevertheless, the quantities entering the quantum theory all remain finite in this limit thanks to a systematic $0 \cdot \infty$ cancellation. From the gauge-preserving condition one finds $\varepsilon^0 = \lambda \partial_i f^i t + g$, giving $\partial_\alpha \varepsilon^\alpha = (1+\lambda)\partial_i f^i$, which vanishes as $\lambda \to -1$ and cancels the divergent factor Δ/d in the combined gauge transformation of X. The ghost field inherits the same structure $c^0 = \lambda \partial_i c^i t + \ldots$, so $\partial_\alpha c^\alpha = (1+\lambda)\partial_i c^i \to 0$. The BRST charge Eq. (31) contains the coefficient $\frac{1-\lambda}{2}$ multiplying $\partial_i c^i X \cdot \Pi$; this coefficient is not singular because of the identity $\frac{1-\lambda}{2} = \frac{\Delta}{d}(1+\lambda)$, which tends to 1 as $|\Delta|/d \to \infty$. Consequently, the BRST charge, the algebra $\mathfrak{g}^{(1)}_{-1}$, and the anomaly calculation in Section 4 are all perfectly regular when evaluated at $\lambda = -1$, and $D = 14$ follows as a mathematically consistent critical dimension. The obstruction is not algebraic but lies in the Faddeev–Popov derivation of the ghost action: before the rescaling $b_0 \to (\frac{1}{2} + \frac{\Delta}{d})^{-1} b_0$, the ghost kinetic coefficients $\frac{1}{2} \pm \frac{\Delta}{d}$ diverge, and the rescaling itself becomes an infinite field redefinition whose Jacobian in the path integral measure is not controlled. Whether $\lambda = -1$ can be physically realized — for instance through a different gauge-fixing prescription or by treating λ as an intrinsic parameter of the symmetry algebra — remains an open question.

We note that the algebra $\mathfrak{g}^{(1)}_{\lambda}$ can be understood as a linear combination of the standard BMS_3 algebra and the Carroll-Weyl symmetry generators; a detailed analysis of this connection will be presented in a forthcoming work.

If we consider the tensionless super-brane, we expect some SUSY $\mathfrak{g}^{(p)}_{\lambda}$ algebra as in the string case.[42,43] We hope to address this in the future. Since we have obtained the BRST charge for the tensionless brane, discussing its spectrum through BRST quantization[29] is also a very interesting question.

In the string case ($p = 1$), the action of the Carrollian string contains magnetic sectors of Carrollian conformal scalar theory,[44] which are T-dual to the electric sectors in the action of the tensionless string. From this point of view, the $\mathfrak{g}^{(1)}_{\lambda}$ algebra also appears in the magnetic sector when $p = 1$. For the higher-dimensional case ($p > 1$), the action of the tensionless brane is also composed of the electric sectors of Carrollian conformal scalar theory, but there is no evidence supporting the T-duality so far. Thus, whether the magnetic sector or other models exhibit the $\mathfrak{g}^{(p)}_{\lambda}$ algebra is another interesting question.

Acknowledgments

We thank Yu-Fan Zheng for meaningful discussions. In particular, he provides insight from point-particle models. This work was supported by the National Natural Science Foundation of China (Grant Nos. 12275004 and 12588101).

References

[1] Green M B, Schwarz J H and Witten E 2012 *Superstring Theory Vol. 1: 25th Anniversary Edition* (Cambridge University Press)
[2] Hughes J, Liu J and Polchinski J 1986 *Phys. Lett. B* **180** 370
[3] Achucarro A, Evans J M, Townsend P K and Wiltshire D L 1987 *Phys. Lett. B* **198** 441
[4] Duff M J, Howe P S, Inami T and Stelle K S 1987 *Phys. Lett. B* **191** 70
[5] Bergshoeff E, Sezgin E and Townsend P K 1987 *Phys. Lett. B* **189** 75
[6] Bergshoeff E, Duff M J, Pope C N and Sezgin E 1987 *Phys. Lett. B* **199** 69
[7] Bars I, Pope C N and Sezgin E 1987 *Phys. Lett. B* **198** 455
[8] Hoppe J 1982 *Quantum Theory of a Massless Relativistic Surface and a Two-Dimensional Bound State Problem* (PhD Thesis) (Cambridge: Massachusetts Institute of Technology)
[9] Duff M J, Inami T, Pope C N, Sezgin E and Stelle K S 1988 *Nucl. Phys. B* **297** 515
[10] Fujikawa K and Kubo J 1987 *Phys. Lett. B* **199** 75
[11] Bars I, Pope C N and Sezgin E 1988 *Phys. Lett. B* **210** 85
[12] de Wit B, Hoppe J and Nicolai H 1988 *Nucl. Phys. B* **305** 545
[13] de Wit B, Luscher M and Nicolai H 1989 *Nucl. Phys. B* **320** 135
[14] Witten E 1995 *Nucl. Phys. B* **443** 85
[15] Townsend P K 1995 *Phys. Lett. B* **350** 184
[16] Banks T, Fischler W, Shenker S H and Susskind L 1997 *Phys. Rev. D* **55** 5112
[17] Aganagic M, Park J, Popescu C and Schwarz J H 1997 *Nucl. Phys. B* **496** 191
[18] Isberg J, Lindstrom U, Sundborg B and Theodoridis G 1994 *Nucl. Phys. B* **411** 122
[19] Bagchi A, Chakrabortty S and Parekh P 2016 *J. High Energy Phys.* **10** 113
[20] Bagchi A, Banerjee A, Chakrabortty S and Parekh P 2018 *J. High Energy Phys.* **02** 065
[21] Bagchi A, Banerjee A, Chakrabortty S, Dutta S and Parekh P 2020 *J. High Energy Phys.* **04** 061
[22] Chen B, Hu Z, Yu Z F and Zheng Y F 2023 *J. High Energy Phys.* **08** 133

CPB Online In-Press

[23] Bagchi A, Chakraborty P, Chakrabortty S, Fredenhagen S, Grumiller D and Pandit P 2025 *Phys. Rev. Lett.* **134** 071604
[24] Bagchi A, Banerjee A, Chatterjee R and Pandit P 2024 *Phys. Rept.* **1185** 1
[25] Demulder S, Driezen S, Knighton B, Oling G, Retore A L, Seibold F K, Sfondrini A and Yan Z 2024 *J. Phys. A* **57** 423001
[26] Gomis J and Yan Z 2024 *J. High Energy Phys.* **07** 102
[27] Bagchi A, Banerjee A, Hartong J, Have E and Kolekar K S 2024 *J. High Energy Phys.* **11** 024
[28] Chen B and Hu Z 2025 *Phys. Rev. D* **112** 046005
[29] Figueroa-O'Farrill J, Have E and Obers N A 2026 *J. High Energy Phys.* **07** 098
[30] Saltsidis P 1997 *Phys. Lett. B* **401** 21
[31] Bozhilov P 1998 *Mod. Phys. Lett. A* **13** 2571
[32] Dutta S, Rasulian I M, Sheikh-Jabbari M M and Yavartanoo H 2025 *J. High Energy Phys.* **05** 029
[33] Borsten L, Galdeano M and Kim H 2026 *J. High Energy Phys.* **07** 132
[34] Ciambelli L, Marteau C, Petkou A C, Petropoulos P M and Siampos K 2018 *Class. Quantum Grav.* **35** 165001
[35] Ciambelli L, Leigh R G, Marteau C and Petropoulos P M 2019 *Phys. Rev. D* **100** 046010
[36] Figueroa-O'Farrill J M and Vishwa G S 2025 *J. Math. Phys.* **66** 042303
[37] Grumiller D, Pérez A, Sheikh-Jabbari M M, Troncoso R and Zwikel C 2020 *Phys. Rev. Lett.* **124** 041601
[38] Fuchs D B 1986 *Cohomology of Infinite-Dimensional Lie Algebras* (Consultants Bureau)
[39] Gao S, Liu D and Pei Y 2016 *Rep. Math. Phys.* **78** 107
[40] Buzaglo L and Vishwa G S 2025 *J. Lie Theory* **35** 455
[41] Berman S and Moody R V 1994 *Commun. Math. Phys.* **159** 239
[42] Bagchi A, Grumiller D and Nandi P 2022 *J. High Energy Phys.* **05** 044
[43] Bagchi A, Chakrabortty S, Chakraborty P, Chatterjee R and Pandit P 2025 *J. High Energy Phys.* **12** 146
[44] Chen B, Sun H and Zheng Y F 2024 *Phys. Rev. D* **110** 125010

CPB Online In-Press