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Manipulations of atomic below-threshold harmonic generation by electrostatic fields

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We have studied the harmonic spectra of hydrogen atoms subjected to different combined laser fields with increasing electrostatic fields, which are based on the numerical solutions of the time-dependent Schrödinger equation. It is found that, as the electrostatic field increases, a hump structure in the below-threshold harmonic spectra will show up. The reasons are that the below-threshold harmonic generation is closely related to the excited states, and the probabilities of the excited states 2s and 2p evidently increase with the increasing electrostatic fields.

Keywords: below-threshold harmonic, electrostatic fields, B-splines

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1. Introduction

High-order harmonic generation (HHG) is one of the most fundamental phenomena in the interaction of intense laser fields with atoms, ions or molecules, which has become one of the hottest topics over the last three decades due to its scientific applications, such as the generation of table-top extreme ultraviolet (XUV) light sources and attosecond pulses, the detection of ultrafast dynamics inside atoms and molecules, and tomographic imaging of molecular orbitals.[1–11] The generation of these high-order harmonics can be well understood by a semiclassical three-step model: Firstly, the bound electron is freed by tunneling ionization, then accelerated by the driving laser field, and finally recombines with the parent ion, emitting the harmonics.[12,13]

Most previous studies have focused on the harmonic generation above the ionization threshold. Recently the HHG below the ionization threshold has attracted considerable attention, since it has been demonstrated that below-threshold harmonic emission can provide a potential way to generate novel high-repetition-rate and high-intensity light sources in the vacuum-ultraviolet band due to the high conversion efficiency.[14–20] Some pioneering works about below-threshold harmonic generation have been carried out both experimentally and theoretically. For instance, Power *et al*. and Yost *et al*. show that long trajectories have important impacts on the generation of the below-threshold harmonics due to the influence of the atomic potential.[21,22] Chini *et al*. have found that the phase-matched below-threshold harmonics are produced around the atomic resonance structures.[23] Li *et al*. investigated the dynamical origin of below-threshold harmonic generation for the Cs atom.[24] Xiong *et al*. showed the vital role of excited states in the new below-threshold harmonics.[25]

The influence of an additional electric field on the atomic and molecular HHG has gained great attention in the past three decades.[26–30] For instance, Milošević and coworkers have systematically studied the intensity dependence of the plateau in laser-assisted x-ray — atom scattering both with and without an additional static electric field.[31,32] Borca *et al*. have analytically and numerically presented two static-electric-field-induced effects: elliptic dichroism and elliptical polarization of harmonics generated by a linearly polarized field. Subsequently, it is shown that the cutoff position of atomic HHG can be extended up to a maximum energy ($I_p + 42U_p$) by combining a chirped linearly polarized laser with a static electric field,[33] where U_p and I_p denote the ponderomotive energy and the ionization energy, respectively. In addition to extending the HHG cutoff, the long quantum trajectories in HHG generation can be modulated by the additional static electric field for molecules in the presence of combined laser fields.[34]

As stated above, atomic and molecular below-threshold HHG has become a research focus for experimental and theoretical studies in the past few years owing to its significant scientific applications. Moreover, an additional static electric field has an important impact on the extension of harmonic spectra, while the study of the modulation of atomic below-threshold harmonic spectra by a static electric field, in a de-

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tailed manner based on a quantitative analysis, is still missing. Therefore, the role of an additional static electric field in atomic below-threshold harmonic spectra is an intriguing problem to be explored. In the present work, we have studied the effect of the static field on the below-threshold harmonic spectra of hydrogen atoms subjected to combined laser fields by numerically solving the full-dimensional time-dependent Schrödinger equation. It is shown that the intensities of below-threshold harmonics become stronger with the enhanced electrostatic fields, which are ascribed to the dynamics of excited states 2s and 2p.

2. Theoretical calculations

Within the dipole approximation and the length gauge, the three-dimensional time-dependent Schrödinger equation (TDSE) describing the interaction of a hydrogen atom with the combined laser field $E(t)$ is given by (atomic units (a.u.) are employed unless otherwise indicated.)

$$\mathrm{i}\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t)=\left[-\frac{1}{2}\nabla^2-\frac{1}{r}+\boldsymbol{E}(t)\cdot\boldsymbol{r}\right]\Psi(\boldsymbol{r},t). \tag{1}$$

The combined laser field $\boldsymbol{E}(t)$ is defined by $\boldsymbol{E}(t)=-\partial\boldsymbol{A}(t)/\partial t$ with the vector potential $\boldsymbol{A}(t)=E_0\sin^2(\pi t/t_{\max})\cos(\omega t)/\omega\varepsilon+\alpha E_0 t\varepsilon$. Here E_0, $t_{\max}$, ω, and ε indicate the amplitude of the electric field, the duration, the frequency of the laser pulse, and the unit vector, respectively. α denotes the parameter which modulates the electrostatic field strength. The laser field is assumed to be linearly polarized along the z axis in the present work.

The time-dependent wave function is expanded by B-splines and spherical harmonics as follows:[35]

$$\Psi(\boldsymbol{r},t)=\sum_{\mu lm}C_{\mu lm}(t)\frac{B_\mu^k(r)}{r}Y_{lm}(\theta,\phi). \tag{2}$$

Here $k=7$ represents the order of B-splines.[36,37] The initial state is 1s, and the magnetic quantum number is a good quantum number in the calculation, so we take the magnetic quantum number $m=0$. The above time-dependent wave function is numerically solved by implementing the Crank–Nicolson scheme.[36,37] The probabilities of excited states are obtained by projecting the wave function onto the corresponding field-free eigenstates after the conclusion of the strong laser pulse, and the field-free eigenstates are calculated by a diagonalization scheme for the field-free Hamiltonian matrix.[36]

The harmonic spectra are obtained Fourier transform of the time-dependent induced dipole in the acceleration form

$$S(\omega)\propto\left|\int\boldsymbol{D}(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t\right|^2. \tag{3}$$

Here, the induced dipole is calculated by $\boldsymbol{D}(t)=\langle\Psi(\boldsymbol{r},t)|-\frac{\partial V(r)}{\partial r}-E(t)|\Psi(\boldsymbol{r},t)\rangle$ with $V(r)=-1/r$.

In our calculations, the truncated radius is $r_{\max}=125$ a.u., 150 B-splines are employed for the radial wave function, and the maximum number of partial waves is $L_{\max}=20$. The hydrogen atom (1s) is exposed to laser pulses of the frequency $\omega=0.057$ a.u. (800 nm) and three optical cycles. In the TDSE calculations, we employ the time step $\delta t=0.05$ a.u. Convergence of numerical calculations is reached with the above settings.

3. Results and discussion

Figure 1(a) displays the harmonic spectra of hydrogen atoms subject to laser pulses of intensity 30 TW/cm^2 and different additional electrostatic fields. It is clearly shown that a hump structure in the below-threshold harmonic spectra gradually appears around the 7th harmonic as electrostatic field strength increases for the parameter α from 0 to 0.4. Figure 1(b) presents the harmonic spectra of hydrogen atoms in laser fields of intensity 50 TW/cm^2 and different additional electrostatic fields, and a hump structure in the harmonic spectra also appears about the 7th harmonic as a function of the electrostatic fields.

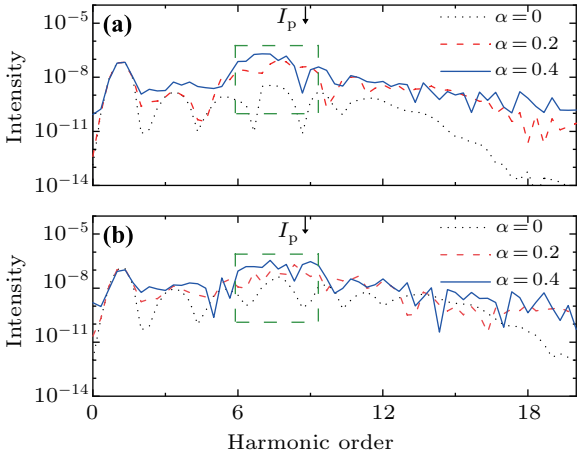


Fig. 1. Harmonic spectra of hydrogen atoms exposed to combined laser fields with different static electric fields. The peak laser intensities are 30 TW/cm^2 (a) and 50 TW/cm^2 (b), respectively. Here α denotes a parameter that controls the strength of the static electric field. The hydrogen atom is subjected to laser pulses of the wavelength 800 nm and three optical cycles.

It is well-known that the atomic below-threshold harmonic generation is closely related to the dynamics of excited states,[17,25] so we also calculate the time-dependent probabilities of excited states 2s and 2p in Fig. 2, in which the laser intensities are 30 TW/cm^2 and 50 TW/cm^2 with the increase of the electrostatic fields from $\alpha=0$ to $\alpha=0.4$. Figure 2(a) displays the time-varying probabilities of 2s state for a hydrogen atom exposed to the laser fields of intensity 30 TW/cm^2 and different electrostatic fields. It is found that the time-dependent probabilities of 2s state are obviously increased with increasing electrostatic fields. For instance, the maximum probability of 2s state is less than 1.5×10^{-4} for $\alpha=0$

around the time $1.3T$ ($T = 2\pi/\omega$), while the peak probability of 2s state is close to 6×10^{-4} for $\alpha = 0.4$ about the time $1.3T$. Figure 2(b) shows the time-dependent probabilities of 2p state for a hydrogen atom in the laser fields of intensity 30 TW/cm^2 and various electrostatic field strengths. It is also demonstrated that the time-varying probabilities of 2p state are evidently raised with increasing electrostatic field strengths. For example, the maximum probability of 2p state is less than 4×10^{-4} for $\alpha = 0$ about the time $1.3T$, while the peak probability of 2p state is near 8×10^{-4} for $\alpha = 0.4$ around the time $1.3T$. Figures 2(c) and 2(d) display the time-dependent probabilities of 2s and 2p states for hydrogen atoms, respectively, and the laser fields correspond to the intensity 50 TW/cm^2 and various electrostatic field strengths. The time-varying probabilities of 2s and 2p states in Figs. 2(c) and 2(d) show similar results with increasing electrostatic field strengths compared with those in Figs. 2(a) and 2(b). It is worthwhile to mention that the probabilities of 2s and 2p states are much larger than those of other excited states, so the time-dependent probabilities of other excited states are not plotted in Fig. 2.

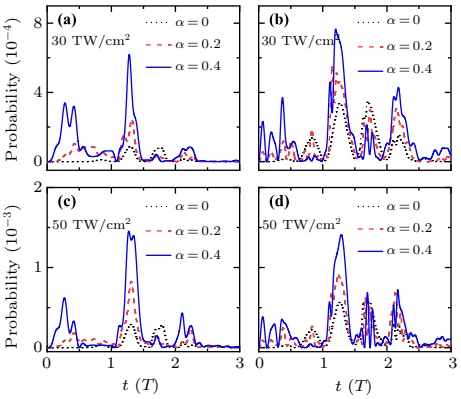


Fig. 2. Time-dependent probabilities of the 2s and 2p states under different static electric field intensities. Panels (a) and (b) show the probabilities of the 2s and 2p states as a function of time at 30 TW/cm^2, respectively. Panels (c) and (d) show the time-varying probabilities of the 2s and 2p states at 50 TW/cm^2, respectively. α is a parameter used to regulate the strength of the static electric field. The hydrogen atom is exposed to laser pulses of the wavelength 800 nm and three optical cycles.

The time-dependent probabilities of 2s and 2p states show a strong dependence on the additional electrostatic fields in Fig. 2, and atomic below-threshold harmonic generation is closely linked with the dynamics of excited states, so we will remove the roles of 2s and 2p states in the TDSE calculations to explain the distinct phenomena in Fig. 1. Figure 3 shows the harmonic spectra of hydrogen atoms subjected to the laser fields analogous to those in Fig. 1, but the roles of 2s and 2p states are not included in the TDSE calculations. It is found that the harmonic intensities around the 7th order harmonic remain nearly unchanged with the increasing electrostatic fields in Fig. 3, while the harmonic intensities around the 7th harmonic gradually become stronger as the electrostatic field strengths α increase from 0 to 0.4 in Fig. 1. Compared with results in Fig. 1, the hump structures in harmonic spectra around the 7th order disappear in Fig. 3 by removing the roles of 2s and 2p states in the TDSE calculations for electrostatic field strengths $\alpha = 0$–0.4. Consequently, the hump structures in harmonic spectra around the 7th order are closely related to the dynamics of 2s and 2p states. The reasons are that the probabilities of excited states 2s and 2p gradually increase with increasing electrostatic fields in Fig. 2, the energy difference between the excited states (2s and 2p) and the ground state (1s) is around seven photon energies, and the transitions from the excited states 2s and 2p to the ground state 1s give rise to the distinct hump structures in below-threshold harmonics with increasing electrostatic fields in Fig. 1. It is also worthwhile to mention that laser pulse shapes have been employed to control and enhance below-threshold harmonic generation through the excited-state resonance structures for multielectron molecules CO and N_2 subjected to different types of laser pulses,[16] while additional electrostatic fields are adopted to control the atomic below-threshold harmonic generation in this work.

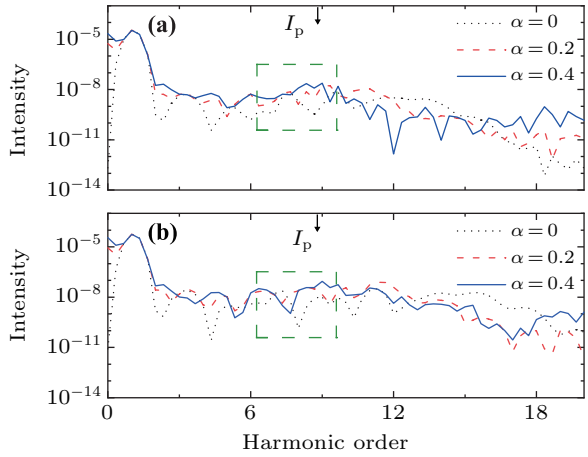


Fig. 3. Similar to Fig. 1, but the harmonic spectra are obtained by excluding the roles of 2s and 2p states in TDSE calculations. Laser intensities are 30 TW/cm^2 and 50 TW/cm^2 in panels (a) and (b), respectively. Here α indicates a parameter that controls the strength of the static electric field. The hydrogen atom is subjected the laser pulses with the wavelength 800 nm and three optical cycles.

4. Conclusion

In summary, we have investigated the manipulation of atomic below-threshold harmonic generation in laser fields by adding static electric fields, in which TDSE calculations are carried out numerically. It is clearly shown that, as the additional static electric fields become stronger, the intensities of below-threshold harmonics are obviously enhanced. In addition, as the static electric fields are enhanced, the intensity change of below-threshold harmonics is negligible without including the roles of 2s and 2p states in TDSE calculations. Consequently, the probabilities of excited states 2s and 2p increase with increasing electrostatic fields, and the transitions from the excited states 2s and 2p to the ground state 1s lead to

stronger below-threshold harmonics with increasing electrostatic fields. Our work indicates that the dynamics of excited states play important roles in below-threshold harmonic generation, and static electric fields provide useful tools to manipulate atomic below-threshold harmonic generation by changing the dynamics of excited states. It is worthwhile to mention that the excited states 2s and 2p play a critical role in the above-threshold HHG.[38]

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References

[1] L'Huillier A and Balcou P 1993 *Phys. Rev. Lett.* **70** 774
[2] Corkum P and Krausz F 2007 *Nat. Phys.* **3** 381
[3] Goulielmakis E, Schultze M, Hofstetter M, Yakovlev V S, Gagnon J, Uiberacker M, Aquila A L, Gullikson E, Attwood D T, Kienberger R, Krausz F and Kleineberg U 2008 *Science* **320** 1614
[4] Krausz F and Ivanov M 2009 *Rev. Mod. Phys.* **81** 163
[5] Itatani J, Levesque J, Zeidler D, Niikura H, Pépin H, Kieffer J C, Corkum P B and Villeneuve D M 2004 *Nature* **432** 867
[6] Lan P F, Ruhmann M, He L X, Zhai C Y, Wang F, Zhu X S, Zhang Q B, Zhou Y M, Li M, Lein M and Lu P X 2017 *Phys. Rev. Lett.* **119** 033201
[7] Peng P, Marceau C and Villeneuve D M 2019 *Nat. Rev. Phys.* **1** 144
[8] Shu Z, Liang H J, Wang Y C, Hu S L, Chen S, Xu H F, Ma R, Ding D J and Chen J 2022 *Phys. Rev. Lett.* **128** 183202
[9] He L X, Yuen C H, He Y Q, Sun S Q, Goetz E, Le A T, Deng Y, Xu C Q, Lan P F, Lu P X and Lin C D 2024 *Phys. Rev. Lett.* **133** 023201
[10] Wang X W, Xiao F, Wang J C, Wang L, Zhang B, Liu J L, Zhao J and Zhao Z X 2024 *Ultrafast Sci.* **4** 0080
[11] Wen T Y, Yin Z M, Guo Z Y, Chen Z H, Yang Z Q and Jin C 2026 *Phys. Rev. A* **113** 013122
[12] Corkum P B 1993 *Phys. Rev. Lett.* **71** 1994
[13] Schafer K J, Yang B R, DiMauro L I and Kulander K C 1993 *Phys. Rev. Lett.* **70** 1599
[14] Hostetter J A, Tate J L, Schafer K J and Gaarde M B 2010 *Phys. Rev. A* **82** 023401
[15] Henkel J, Witting T, Fabris D, Lein M, Knight P L, Tisch J W G and Marangos J P 2013 *Phys. Rev. A* **87** 043818
[16] Heslar J, Telnov D A and Chu S I 2016 *Phys. Rev. A* **93** 063401
[17] Xiong W H, Peng L Y and Gong Q H 2017 *J. Phys. B: At. Mol. Opt. Phys.* **50** 032001
[18] Li P C, Sheu Y L and Chu S I 2020 *Phys. Rev. A* **101** 011401
[19] Zuffi A V F, Junior N D V and Samad R E 2022 *Phys. Rev. A* **105** 023112
[20] Chen S Q and He F 2025 *Acta Phys. Sin.* **74** 133202
[21] Power E P, March A M, Catoire F, Sistrunk E, Krushelnick K, Agostini P and DiMauro L F 2010 *Nat. Photonics* **4** 352
[22] Yost D C, Schibli T R, Ye J, Tate J L, Hostetter J, Gaarde M B and Schafer K J 2009 *Nat. Phys.* **5** 815
[23] Chini M, Wang X W, Cheng Y, Wang H, Wu Y, Cunningham E, Li P C, Heslar J, Telnov D A, Chu S I and Chang Z H 2014 *Nat. Photonics* **8** 437
[24] Li P C, Sheu Y L, Laughlin C and Chu S I 2015 *Nat. Commun.* **6** 7178
[25] Xiong W H, Geng J W, Tang J Y, Peng L Y and Gong Q 2014 *Phys. Rev. Lett.* **112** 233001
[26] Wang B B, Li X F and Fu P M 1999 *Phys. Rev. A* **59** 2894
[27] Odžak S and Miloević D B 2005 *Phys. Rev. A* **72** 033407
[28] Xia C L, Zhang G T, Wu J and Liu X S 2010 *Phys. Rev. A* **81** 043420
[29] Serrat C 2011 *Phys. Rev. A* **84** 061803
[30] Yuan K J and Bandrauk A D 2011 *Phys. Rev. A* **83** 063422
[31] Miloević D B and Starace A F 1998 *Phys. Rev. Lett.* **81** 5097
[32] Miloević D B and Starace A F 1999 *Phys. Rev. A* **60** 3943
[33] Xiang Y, Niu Y P and Gong S Q 2009 *Phys. Rev. A* **79** 053419
[34] Koushki A M and Sarikhani S 2021 *Chem. Phys.* **541** 111020
[35] Hu S L, Yi X P, Guo L, Bi C R and Chen J 2023 *Phys. Rev. A* **107** 033104
[36] Hu S L, Zhao Z X and Shi T Y 2014 *Int. J. Quantum Chem.* **114** 441
[37] Bachau H, Cormier E, Decleva P, Hansen J E and Martín F 2001 *Rep. Prog. Phys.* **64** 1815
[38] Fang Y, Tian D W, Cao Y, Hao X L and Shu Z 2025 *Chin. Phys. B* **34** 103201