

Unit-probability quantum search for all solutions

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Unit-probability quantum search for all solutions

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The objective of Grover's quantum algorithm is to efficiently identify a single solution within a search space. However, there are search problems that require finding all possible solutions. A direct, repeated application of Grover's algorithm is unsuitable for these problems, as it lacks an internal mechanism to avoid rediscovering previous solutions. Here we introduce a quantum algorithm that efficiently and deterministically discovers all solutions by preparing special initial states of the index register, a step accomplished by leveraging knowledge from solutions obtained during execution. We experimentally demonstrate our algorithm on a superconducting quantum processor. This work provides quantum examples for search scenarios with some knowledge about solutions.

Keywords: Grover's algorithm, finding all solutions, exact quantum search, superconducting quantum processor

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1. Introduction

Quantum computing represents a novel computational paradigm capable of solving certain problems more efficiently than classical computing. The realization of this advantage depends on progress in both hardware platforms and algorithmic development. Many physical systems have been proposed for implementing quantum computing.[1] Superconducting systems are currently considered one of the most promising platforms, and the optimization of their architectures and hardware remains an ongoing endeavor.[2,3] Quantum algorithms provide sophisticated methods for leveraging quantum hardware.[4–6] Grover's quantum search algorithm stands as one of the most prominent quantum algorithms.[7] It achieves a quadratic speedup over classical counterparts for unstructured search problems and serves as a critical technique for achieving computational speedups across diverse applications, including solving vehicle routing problems,[8] discovering binary constant weight codes,[9] path-planning for swarm robotics,[10] and neural network training.[11] In addition to Grover's algorithm, quantum walks can also be used for search problems.[12–14]

The original Grover's algorithm is designed to identify a single solution among numerous candidates. Its core component, the Grover operator, consists of an oracle and a diffuser. The algorithm initializes the index register to a uniform superposition state, applies the Grover operator consecutively, and culminates in a measurement. However, the algorithm does not always guarantee success with unit probability. Several modified versions have been proposed to overcome this limitation. For instance, Brassard *et al.*,[15] Høyer,[16] and Long[17] achieved exact search by tailoring the oracle and diffuser phases. Nevertheless, these methods require modifying the original oracle, which is infeasible in certain practical settings. The D2p scheme circumvented this issue by employing two diffuser phases while keeping the original oracle unchanged,[18] but it needed to solve complex equations. The FXR method improved D2p to arbitrary oracle phases, *i.e.*, the oracle phase can be user-adjustable.[19] One recent approach initializes the quantum register using the information of a known non-solution while preserving the original oracle, thereby achieving unit success probability.[20] This demonstrates that the use of prior knowledge can effectively simplify the quantum search process.

Different from the previously mentioned investigations, this paper focuses on search scenarios aimed at identifying all solutions with unit probability. Similar to the approach in Ref. [20], our method simplifies the quantum search process by incorporating prior knowledge of known solutions while keeping the original oracle unchanged. Specifically, we begin by testing a randomly selected index y to determine whether it is a solution. If y is not a solution, we apply the method from Ref. [20] to find one solution with unit probability. Conversely, if y is a solution, we instead employ the technique from Section 3 to identify a new solution. Once a set of solutions has been identified, the search for the next new solution proceeds by preparing the initial state of the index reg-

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ister in a specialized form that incorporates information from these known solutions. The key factor in obtaining unit success probability is the use of the knowledge of known solutions to adjust the angle between the initial state and the target state. We note that directly applying Grover's algorithm to identify all solutions is inadequate, as its oracle cannot exclude known solutions. As a result, the final superposition before measurement still contains all solutions, including those already identified, which prevents guaranteed discovery of a new solution.

The paper is organized as follows. Section 2 provides a brief review of Grover's algorithm. Section 3 presents the technique for identifying a new solution when one solution is already known. In Section 4, we introduce the complete algorithm for finding all solutions. Finally, Section 5 presents experimental results of the proposed algorithm on a superconducting quantum processor.

2. A brief review of Grover's algorithm

Consider a search space consisting of N distinct items. Each item is uniquely identified by an index x, where $x \in \{0,1,2,\ldots,N-1\}$. This search space is unstructured, meaning there is no underlying order or classification that can be exploited to guide the search. The access to the solution criterion is provided by an oracle, which acts as a black-box function f. The objective of a typical search problem is to find a solution with a high probability of success.

Grover's quantum algorithm achieves a quadratic reduction in oracle queries compared to classical search. The core of it is the Grover operator G, which consists of two parts: $G = WO$. Here O is the quantum oracle and W is the diffuser. The oracle O recognizes and marks solutions by phase-shifting them. Assume the search space has dimension $N = 2^n$; the action of an oracle call on computational basis $|x\rangle$ is written as[21]

$$|x\rangle \xrightarrow{O} (-1)^{f(x)}|x\rangle, \tag{1}$$

where $|x\rangle$ is the n-qubit index register state, and f is the oracle function that takes index x as input satisfying $f(x) = 1$ if x is a solution, and $f(x) = 0$ otherwise. The diffusion operator is defined as

$$W = 2|\psi\rangle\langle\psi| - I, \tag{2}$$

where $|\psi\rangle = H^{\otimes n}|0\rangle^{\otimes n}$ is the uniform superposition of all computational basis states, and H denotes the Hadamard gate, whose matrix representation is

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{3}$$

Grover's algorithm begins with $|\psi\rangle$ as an initial state of the n-qubit index register. Adopt the convention that $\sum_x'$ indicates a sum over all x which are solutions to the search problem, and $\sum_x''$ indicates a sum over all x which are not solutions to the search problem. Two normalized states are defined as follows:

$$|l\rangle = \frac{1}{\sqrt{N-M}}\sum_x''|x\rangle, \tag{4}$$

$$|t\rangle = \frac{1}{\sqrt{M}}\sum_x'|x\rangle, \tag{5}$$

where M is the number of solutions. The initial state $|\psi\rangle$ can be re-expressed as

$$|\psi\rangle = \cos\frac{\theta}{2}|l\rangle + \sin\frac{\theta}{2}|t\rangle, \tag{6}$$

where $\theta = 2\arcsin\left(\sqrt{M/N}\right)$. In the geometric representation of Grover's algorithm as shown in Fig. 1, $|l\rangle$ and $|t\rangle$ are orthogonal vectors, with $|\psi\rangle$ represented as a vector between them at an angle $\theta/2$ from $|l\rangle$. Each application of G rotates the state by an angle θ away from $|l\rangle$. It can be shown that after R iterations the index register will be in the state

$$G^R|\psi\rangle = \cos\left(\frac{2R+1}{2}\theta\right)|l\rangle + \sin\left(\frac{2R+1}{2}\theta\right)|t\rangle. \tag{7}$$

The algorithm concludes with measurement of the register after a proper number of iterations.

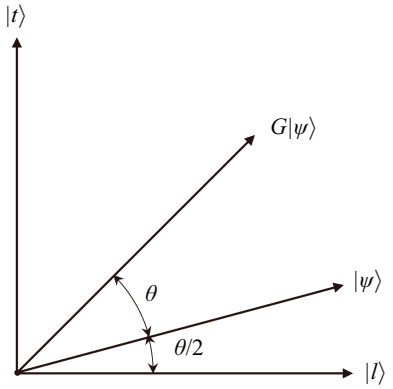


Fig. 1. Geometric representation of Grover's algorithm.

When $G^R|\psi\rangle$ is close to $|t\rangle$ (the solution superposition), measurement of the index register yields a solution with high probability. The optimal number of iterations R is

$$R_{\text{opt}} = \left\lfloor \frac{\pi}{4\arcsin\sqrt{\frac{M}{N}}} - \frac{1}{2} \right\rceil, \tag{8}$$

where $\lfloor x \rceil$ denotes the integer closest to the real number x and we round halves down, *e.g.*, $\lfloor 3.2 \rceil = 3$, $\lfloor 3.5 \rceil = 3$, and $\lfloor 3.6 \rceil = 4$. After R_{opt} iterations of G, measuring the register will yield a solution with the maximal success probability

$$P = \left|\sin((2R_{\text{opt}}+1)\theta/2)\right|^2. \tag{9}$$

Note that P is generally not unity.

3. Finding a new solution given one known solution

In this section, we address search problems where one solution y is known a priori, leaving $M-1$ solutions among $N-1$ candidates. The objective is to identify a new solution with unit probability. We will show how to achieve the objective through modifying Grover's algorithm.

The following quantum algorithm achieves this objective by leveraging the prior knowledge that index y is a solution. The index register is initialized to the state:

$$|\psi_y\rangle = \varepsilon|y\rangle + \sqrt{1-\varepsilon^2}|y_\perp\rangle, \tag{10}$$

where the positive ε depends on N and M (its value will be specified later), and $|y_\perp\rangle$ is a uniform superposition of all index states except $|y\rangle$:

$$|y_\perp\rangle = \frac{1}{\sqrt{N-1}}\sum_{x\neq y}|x\rangle. \tag{11}$$

The algorithm employs the oracle $O_y = P_y O$ with

$$P_y = I - 2|y\rangle\langle y|, \tag{12}$$

and the diffusion operator

$$W_y = 2|\psi_y\rangle\langle\psi_y| - I. \tag{13}$$

Given $O|y\rangle = -|y\rangle$ and $P_y|y\rangle = -|y\rangle$, we have $O_y|y\rangle = |y\rangle$. Since O_y preserves the phase of $|y\rangle$, the state $|y\rangle$ is not identified as a solution by the Grover operator $G_y = W_y O_y$.

Similar to Grover's original algorithm, this algorithm ends with a measurement on the index register after applying the Grover operator G_y many times to the initial state. The initial state $|\psi_y\rangle$ of the index register can be divided into two parts: one is the superposition state of the $M-1$ unknown solutions that will be denoted by the normalized state $|t_y\rangle$, which has the expression

$$|t_y\rangle = \frac{1}{\sqrt{M-1}}\sum_{x\neq y}{}'|x\rangle. \tag{14}$$

Given that

$$\langle t_y||\psi_y\rangle = \sqrt{1-\varepsilon^2}\sqrt{\frac{M-1}{N-1}}, \tag{15}$$

we can express the initial state as

$$|\psi_y\rangle = \cos\left(\frac{\theta_y}{2}\right)|l_y\rangle + \sin\left(\frac{\theta_y}{2}\right)|t_y\rangle, \tag{16}$$

where

$$\theta_y = 2\arcsin\left(\sqrt{1-\varepsilon^2}\sqrt{\frac{M-1}{N-1}}\right), \tag{17}$$

and $|l_y\rangle$ is a superposition of the non-solution states and $|y\rangle$, making it orthogonal to $|t_y\rangle$. After applying R iterations of the Grover operator G_y to $|\psi_y\rangle$, the index register state becomes

$$G_y^R|\psi_y\rangle = \cos\left(\frac{2R+1}{2}\theta_y\right)|l_y\rangle + \sin\left(\frac{2R+1}{2}\theta_y\right)|t_y\rangle. \tag{18}$$

The parameter ε in the initial state $|\psi_y\rangle$ controls the value of θ_y (see Eq. (17)). In the geometric representation of Grover's algorithm, the angle between the initial state $|\psi_y\rangle$ and the target state $|t_y\rangle$ is $\pi/2-\theta_y/2$, where a single application of G_y rotates the state by θ_y toward $|t_y\rangle$. The parameter ε is chosen such that $R_y = (\pi/2-\theta_y/2)/\theta_y$ is an integer, so that after R_y applications of G_y to $|\psi_y\rangle$, the index register state becomes exactly the $|t_y\rangle$, *i.e.*, $G_y^{R_y}|\psi_y\rangle = |t_y\rangle$. Therefore, under such an ε and R_y a final measurement will output a solution different from y with unit probability. The smallest ε satisfying that $R_y = (\pi/2-\theta_y/2)/\theta_y$ is an integer is

$$\varepsilon = \sqrt{1-\frac{N-1}{M-1}\sin^2\left(\frac{\pi}{4R_y+2}\right)}, \tag{19}$$

with

$$R_y = \left\lceil \frac{\pi}{4\arcsin\sqrt{\dfrac{M-1}{N-1}}} - \frac{1}{2} \right\rceil, \tag{20}$$

where $\lceil x\rceil$ denotes the smallest integer greater than or equal to x. We use the above values of ε and R_y in the algorithm, as this R_y is minimal.

In Fig. 2, we show the growth of success probability for finding a new solution as the number of Grover operator G_y iterations increases. It can be seen that the algorithm achieves unit success probability after R_y applications of G_y. The key to achieving this result lies in preparing an initial state $|\psi_y\rangle$ that forms a specific angle with the target state $|t_y\rangle$. Such preparation utilizes the prior knowledge that y is a solution.

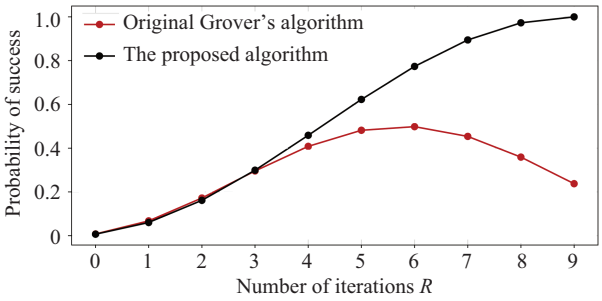


Fig. 2. Success probability *versus* the number of Grover operator G_y iterations for finding a new solution under the condition there is a pre-known solution (black curve). This case has $M=2$ solutions among $N=128$ candidates, so that there is $\varepsilon = 0.3660$ and $R_y = 9$ according to Eqs. (19) and (20). The original Grover's algorithm (red curve) achieves a 49.83% maximum success probability for finding a solution distinct from the known one after 6 iterations. By contrast, the algorithm (black curve) in Section 3 achieves unit success probability after 9 iterations.

4. Algorithm for finding all solutions

Here we consider the search problem of finding all M solutions among N candidates with unit probability. We assume $M \leq N/2$ for simplicity. The algorithm's first step is to classically test an arbitrary index y to determine whether $f(y)=1$. The subsequent step depends on whether y is a solution to the search problem.

4.1. The index y is a solution

If index y is a solution (i.e., $f(y)=1$), $M-1$ solutions remain to be found. We will find these unknown solutions one by one. Suppose we have found m solutions ($1 \le m < M$, including y), and we aim to find a new solution. We use a modified version of the previous section's technique. The index register is first prepared in the initial state

$$|\psi_a\rangle = \varepsilon_m |y\rangle + \sqrt{1-\varepsilon_m^2}\,|f_m\rangle, \tag{21}$$

where $|f_m\rangle$ is a normalized equal superposition of all index states excluding the m found solutions. The oracle used is $O_a = P_y O$, identical to that in the previous section. The diffusion operator is

$$W_a = 2|\psi_a\rangle\langle\psi_a| - I. \tag{22}$$

Since oracle O_a does not mark y as a solution, we decompose $|\psi_a\rangle$ into two parts: the superposition of $(M-m)$ unknown solutions (denoted by normalized state $|t_a\rangle$) and the orthogonal component. Explicitly:

$$|\psi_a\rangle = \cos\frac{\theta_a}{2}|l_a\rangle + \sin\frac{\theta_a}{2}|t_a\rangle, \tag{23}$$

where

$$\theta_a = 2\arcsin\left(\sqrt{1-\varepsilon_m^2}\sqrt{\frac{M-m}{N-m}}\right),$$

and $|l_a\rangle$ is a superposition of non-solution states and $|y\rangle$. Similar to the previous section's algorithm, the minimum number of Grover operator $G_a = W_a O_a$ applications required to output a new solution with unit probability is

$$R_m = \left\lceil \frac{\pi}{4\arcsin\sqrt{\dfrac{M-m}{N-m}}} - \frac{1}{2} \right\rceil, \tag{24}$$

with the specific parameter choice

$$\varepsilon_m = \sqrt{1 - \frac{N-m}{M-m}\sin^2\left(\frac{\pi}{4R_m+2}\right)}. \tag{25}$$

In other words, there is $G_a^{R_m}|\psi_a\rangle = |t_a\rangle$ and a final measurement will output a new solution. All solutions will be found by repeating the procedure.

Thus, when y is a solution, the total oracle calls N_a to find all $M-1$ remaining solutions is

$$N_a = \sum_{m=1}^{M-1} R_m, \tag{26}$$

which quantifies the algorithm's resource requirements. We analyze the computational complexity of the total oracle calls N_a. Since $\sin(x) < x$ for $x \in (0, \pi/2)$, we can show that R_m is in the class of $O\left(\sqrt{N/(M-m)}\right)$. There is

$$\sum_{m=1}^{M-1}\sqrt{\frac{N}{M-m}} = \sqrt{N}\sum_{m=1}^{M-1}\frac{1}{\sqrt{m}} \tag{27}$$

$$< \sqrt{N}\int_0^M \frac{1}{\sqrt{x}}\,\mathrm{d}x = 2\sqrt{MN}, \tag{28}$$

which implies N_a is in the class of $O(\sqrt{NM})$. This means there exists a constant c (independent of N and M) such that $N_a < c\sqrt{MN}$.

4.2. The index y is not a solution

If index y is not a solution (i.e., $f(y)=0$), all M solutions remain to be found. We will repeat the following procedure to find all solutions one by one. Assume m solutions have been found ($0 \le m < M$), a new solution is located as follows. The index register is first prepared in the initial state

$$|\psi_b\rangle = \varepsilon_m |y\rangle + \sqrt{1-\varepsilon_m^2}\,|f_m\rangle, \tag{29}$$

where $|f_m\rangle$ is a normalized equal superposition of all index states excluding the m found solutions and y. The oracle $O_b = O$ is the same as the original Grover's scheme, while the diffusion operator is

$$W_b = 2|\psi_b\rangle\langle\psi_b| - I. \tag{30}$$

We decompose $|\psi_b\rangle$ into two components: the superposition of $(M-m)$ unknown solutions (denoted by normalized state $|t_b\rangle$) and an orthogonal complement. This yields

$$|\psi_b\rangle = \cos\frac{\theta_b}{2}|l_b\rangle + \sin\frac{\theta_b}{2}|t_b\rangle, \tag{31}$$

where

$$\theta_b = 2\arcsin\left(\sqrt{1-\varepsilon_m^2}\sqrt{\frac{M-m}{N-m-1}}\right),$$

and $|l_b\rangle$ consists of $|y\rangle$ and the $(N-M-1)$ non-solution states in $|f_m\rangle$. As in the previous case, the minimum number of Grover operator $G_b = W_b O_b$ applications required for unit success probability is

$$R_m = \left\lceil \frac{\pi}{4\arcsin\sqrt{\dfrac{M-m}{N-m-1}}} - \frac{1}{2} \right\rceil, \tag{32}$$

with the specific parameter

$$\varepsilon_m = \sqrt{1 - \frac{N-m-1}{M-m}\sin^2\left(\frac{\pi}{4R_m+2}\right)}. \tag{33}$$

Thus, when y is not a solution, the total oracle calls to find all M solutions is

$$N_b = \sum_{m=0}^{M-1} R_m, \tag{34}$$

which is also in the complexity class of $O\left(\sqrt{NM}\right)$.

In a real search scenario, y is most likely not a solution to the search problem. Figure 3 demonstrates N_b as a function of M for $N = 64$.

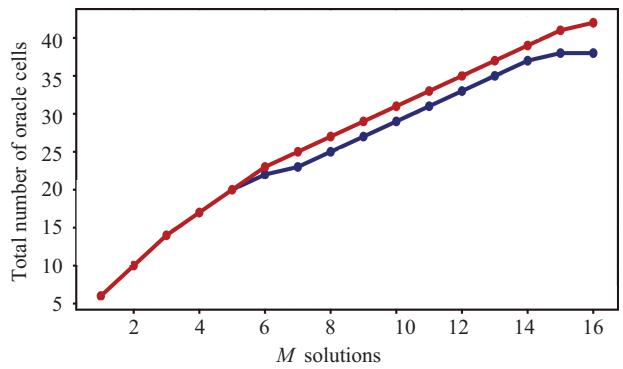


Fig. 3. The total oracle calls N_b in Eq. (34) as a function of M with $N = 64$ (blue curve). As a comparison, the red curve shows the total oracle calls $\tilde{N}_b$ in Eq. (36) as a function of M. There is always $\tilde{N}_b \geq N_b$.

4.3. Comparisons

Our described algorithm represents a specific method of finding all M solutions with unit probability, which utilizes the information of known solutions to prepare initial states for identifying new ones. It is worth noting that this is one of various possible methods.

We can modify any unit-probability Grover-based algorithm[15–19] (initialized with a uniform superposition) to find all solutions with unit probability. In such algorithms, the oracle O_s applies a phase shift of $\mathrm{e}^{\mathrm{i}\theta}$ to solution states, where θ is not necessarily π since the oracle is usually modified. To find all solutions with unit probability, we can replace the oracle O_s by $M_k O_s$ for the k-th solution search, where M_k applies a phase shift of $\mathrm{e}^{-\mathrm{i}\theta}$ to all $k-1$ known solution states, thus ensuring that $M_k O_s$ identifies only the yet undiscovered solutions. To analyze the complexity, suppose m solutions have already been found ($0 \leq m < M$). The minimal number of oracle calls $\tilde{R}_m$ required to find the next solution in this modified setting is given by

$$\tilde{R}_m = \left\lceil \frac{\pi}{4 \arcsin\sqrt{\dfrac{M-m}{N}}} - \frac{1}{2} \right\rceil . \tag{35}$$

The total number of oracle calls to identify all M solutions is then

$$\tilde{N}_b = \sum_{m=0}^{M-1} \tilde{R}_m . \tag{36}$$

A comparison between $\tilde{N}_b$ and the quantity N_b in Eq. (34) is given in Fig. 3. It reveals a key insight: although $\tilde{N}_b$ shares the same computational complexity of $O\left(\sqrt{NM}\right)$ as N_b, it holds that there is $\tilde{N}_b \geq N_b$ as demonstrated in Fig. 3. This is because each term $\tilde{R}_m$ in Eq. (35) is greater than or equal to its counterpart R_m in Eq. (32).

5. Experimental demonstration

We present experimental results for the algorithm designed to identify all solutions, as implemented on OriginQ's "Wukong" superconducting quantum processor (we conducted the experiments on the OriginQ Cloud platform using the graphical programming tool, with the OriginQ Wukong 102 superconducting quantum processor). In our setup, the dimension of the search space is $N = 4$ and there are $M = 2$ solutions, so the index register consists of 2 qubits, which are denoted by qubit q_0 and q_1 (see Fig. 4). As a demonstration we set the index '10' and '11' as solutions. So the quantum oracle is $O = Z_0$, which is Pauli-Z gate on the qubit q_0. According to the algorithm in Section 4 we begin by testing a randomly selected index y to determine whether it is a solution.

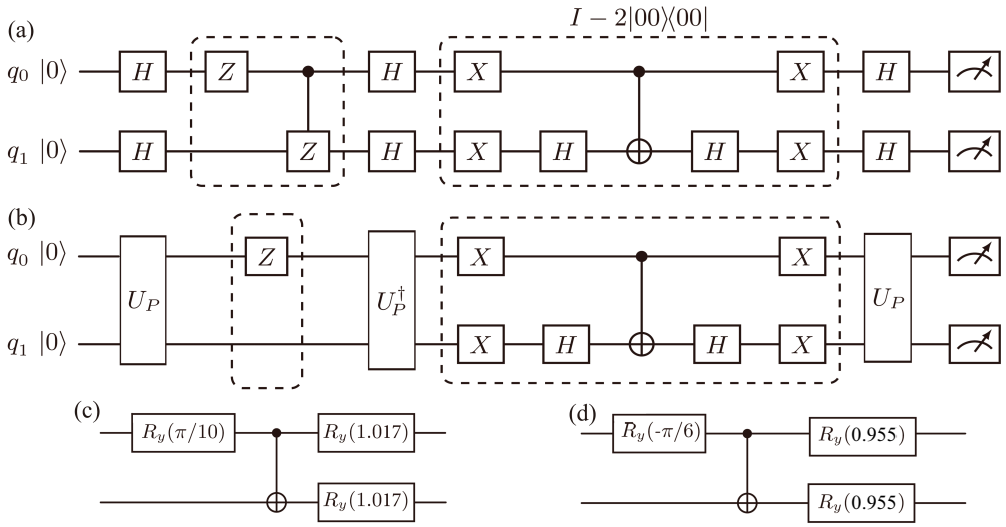


Fig. 4. Experimental circuits for identifying all solutions '11' and '10' from four possible candidates. The algorithm starts by testing a randomly selected index y to determine if it is a solution. (a) Circuit for finding the second solution '10' when the initial testing index is $y =$'11'. (b) Circuit for the case where a non-solution, $y =$'00', is selected as the testing index. The corresponding U_P gate for finding the first solution is shown in panel (c), while panel (d) shows the U_P gate for finding the second solution, under the assumption that '11' has been identified first.

5.1. The tested y is a solution

Assume we select $y =$'11' as the testing index. Fortunately it is a solution, and there is one solution to be found. According to the algorithm in Section 4, the oracle used in this case is

$$O_a = P_y O = (I - 2|11\rangle\langle 11|)Z_0 = CZ_{01}Z_0, \tag{37}$$

where CZ_{01} is the controlled Z gate with q_0 being the controlling qubit and q_1 being the target qubit. According to Eqs. (24) and (25) with $m = 1$, there are

$$R_1 = 1, \quad \varepsilon_1 = \frac{1}{2}. \tag{38}$$

So according to Eq. (21) the initial state in our demonstration is an equal superposition of all index states, *i.e.*,

$$|\psi_a\rangle = H|0\rangle \otimes H|0\rangle. \tag{39}$$

The diffusion operator $W_a = 2|\psi_a\rangle\langle\psi_a| - I$ used in the algorithm is

$$W_a = (H \otimes H)(2|00\rangle\langle 00| - I)(H \otimes H). \tag{40}$$

In Fig. 4(a), we show the experimental circuit, where the first two H gates are used to prepare the index register in the initial state $|\psi_a\rangle$. After preparing the index register in the state $|\psi_a\rangle$, the Grover operator $G_a = W_a O_a$ is applied for $R_1 = 1$ times (actually what we applied is $e^{i\pi}G_a$ but the global phase $e^{i\pi}$ does not matter), then the circuit ends with a measurement on the two register qubits.

If there were no error in the quantum processor, the final measurement would output the index '10' perfectly. In our experiment, we repeat the computation for 1000 times and the results are presented in the row denoted 'Case A' of Table 1. The experiment only outputs the index '10' with probability 0.985, which is due to the imperfection of the quantum processor.

Table 1. Experimental results.

	Output states and probability			
	'00'	'01'	'10'	'11'
Case A	0.005	0	0.985	0.010
Case B_0	0	0.003	0.586	0.411
Case B_1	0	0.007	0.985	0.008

5.2. The tested y is not a solution

Assume, unfortunately, that we select $y =$'00' as the testing index and it turns out not to be a solution. According to the algorithm described in Section 4, the oracle used in this case is the original oracle O and there are two solutions to be found.

To find the first solution, based on Eqs. (32) and (33) with $m = 0$, simple calculation shows that

$$R_0 = 1, \quad \varepsilon_0 = \sqrt{\frac{5}{8}}, \tag{41}$$

thus the initial state $|\psi_b\rangle$ is

$$|\psi_{b0}\rangle = \varepsilon_0|00\rangle + \frac{\sqrt{1-\varepsilon_0^2}}{\sqrt{3}}(|01\rangle + |10\rangle + |11\rangle). \tag{42}$$

This initial state can be prepared by applying a gate U_P to $|00\rangle$, *i.e.*, $|\psi_{b0}\rangle = U_P|00\rangle$, and the circuit for U_P is shown in Fig. 4(c).[22] The diffusion operator $W_{b0} = 2|\psi_{b0}\rangle\langle\psi_{b0}| - I$ used in the experiment is

$$W_{b0} = U_P(2|00\rangle\langle 00| - I)U_P^\dagger. \tag{43}$$

The total circuit for finding the first solution is shown in Fig. 4(b) where U_P is in Fig. 4(c). Theoretically, the state prior to the final measurement is an equal superposition of $|10\rangle$ and $|11\rangle$. Consequently, the measurement will project the state into either $|10\rangle$ or $|11\rangle$ with equal probability. We repeat the computation for 1000 times and the result is shown in Table 1 'Case B_0' line.

Without loss of generality, we assume the first solution found is '11'. To locate the second solution, we now proceed as follows. Based on Eqs. (32) and (33) with $m = 1$, there is

$$R_1 = 1, \quad \varepsilon_1 = \frac{1}{\sqrt{2}}, \tag{44}$$

and the required initial state is

$$|\psi_{b1}\rangle = \varepsilon_1|00\rangle + \frac{\sqrt{1-\varepsilon_1^2}}{\sqrt{2}}(|01\rangle + |10\rangle), \tag{45}$$

which can be prepared by applying a gate U_P shown in Fig. 4(d) to $|00\rangle$. The diffusion operator $W_{b1} = 2|\psi_{b1}\rangle\langle\psi_{b1}| - I$ in the experiment is

$$W_{b1} = U_P(2|00\rangle\langle 00| - I)U_P^\dagger. \tag{46}$$

The total circuit for finding the second solution is shown in Fig. 4(b) where U_P is in Fig. 4(d). The ideal measurement is to output '10' certainly. The number of experimental repetitions remained 1000 and the result is shown in Table 1 'Case B_1' line. Despite minor experimental uncertainties, the results are in good agreement with theoretical expectations.

6. Conclusion

Grover's algorithm provides a powerful quantum approach for identifying a single solution from numerous candidates with a high probability of success. This work extends its capability by introducing an algorithm that deterministically

finds all solutions with unit probability. Key to our method is the iterative use of information from previously identified solutions to prepare specialized initial states for subsequent searches. Rather than modifying the oracle's phase, our approach adjusts the angles between the initial states and the equal superposition of remaining target states, ensuring deterministic identification of new solutions. The algorithm has been successfully executed on OriginQ's "Wukong" superconducting quantum processor, confirming its validity. Our results advance quantum search techniques for scenarios where partial information about solution spaces is available.

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