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Two-soliton solutions and rational solutions of wave resonant-interaction equations with higher-order effects

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We focus on the wave resonant-interaction equations incorporating higher-order effects. To explore their exact solution structures and dynamic properties, we employ the Darboux–Dressing transformation to construct two-soliton solutions and rational solutions with quartic polynomials. Furthermore, based on the analytic solutions, proper parameters are chosen to display their evolutions and dynamics.

Keywords: wave resonant-interaction equations, soliton solutions, rational solutions, Darboux–Dressing transformation

PACS: 02.30.Ik, 05.45.Yv, 42.65.Tg

DOI: 10.1088/1674-1056/ae1dee **CSTR:** 32038.14.CPB.ae1dee

1. Introduction

The integrable nonlinear evolution equations (NLEEs) have been extensively recognized as a central mathematical paradigm to investigate the propagation dynamics of nonlinear waves. Their significance is demonstrated by the extensive applications across multiple branches of physics, including but not limited to optics, fluid, and plasma physics.[1–11] Among these equations, the nonlinear Schrödinger (NLS) equation is one typical and foundational model. Specifically designed to describe the behavior of optical pulses propagating in single-mode optical fiber, it comprehensively accounts for critical physical effects such as group velocity dispersion and self-phase modulation.[12] To address more complex physical scenarios, *e.g.*, multi-component wave interactions, coupled NLEE models have been proposed and developed. An important class of such coupled systems arises from resonance phenomena, where two or more waves exchange energy and momentum, resulting in the formation of coupled NLEE systems,[13–16] A well-known example of these coupled models is the vector NLS equations, commonly referred to as the Manakov system,[13]

$$\begin{aligned} &\mathrm{i}q_t + q_{xx} - 2\left(s_1|q|^2 + s_2|p|^2\right)q = 0,\\ &\mathrm{i}p_t + p_{xx} - 2\left(s_1|q|^2 + s_2|p|^2\right)p = 0, \end{aligned} \tag{1}$$

where the subscripts denote partial derivatives with respect to x and t, $q(x,t)$ and $p(x,t)$ represent complex envelopes of two coupled waves, while s_1 and s_2 are real parameters satisfying the integrability condition $s_1^2 = s_2^2 = 1$. The Manakov system is suitable for the weak resonant condition with two quasi-monochromatic waves having the same group velocity.[14] It finds wide applications in physical settings such as pulse propagation in elliptically birefringent optical fibers and weakly nonlinear wave interactions in inviscid fluids.[14]

When transitioning to strong resonant conditions, the three-wave resonant interaction equations become the appropriate model,[14,16] given by

$$\begin{aligned} &p_t = (-c_1 p_x - s_1 w^* q),\\ &q_t = (-c_2 q_x + s_2 w p),\\ &0 = w_x + s_1 s_2 (c_1 - c_2) p^* q, \end{aligned} \tag{2}$$

where $p(x,t)$, $q(x,t)$, and $w(x,t)$ are slowly varying complex envelopes of three mutually coupled waves. The parameters c_1 and c_2 are the real constants associated with group velocities. System (2) applies to strong resonant phenomena in various physical contexts, most notably in fluid dynamics.

In their seminal work, Degasperis and Lombardo[14] constructed particular solutions of both the systems (1) and (2), focusing on the case where the wave numbers of vector plane waves have opposite signs. To further extend these models to account for higher-order physical effects (*e.g.*, third-order dispersion), Sun[15] proposed a generalized three-coupled-wave system that unifies the vector Hirota equations and system (2),

$$\mathrm{i}p_t + p_{xx} - 2\left(s_1|p|^2 + s_2|q|^2\right)p - \mathrm{i}\varepsilon\left[p_{xxx} - 3\left(2s_1|p|^2 + s_2|q|^2\right)p_x - 3s_2 q^* q_x p\right] = \mathrm{i}\beta(-c_1 p_x - s_1 w^* q),$$

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$$
\begin{aligned}
&\mathrm{i}q_t + q_{xx} - 2\left(s_1|p|^2 + s_2|q|^2\right)q - \mathrm{i}\varepsilon\left[q_{xxx} - 3\left(s_1|p|^2 + 2s_2|q|^2\right)q_x - 3s_2p^*p_xq\right] = \mathrm{i}\beta(-c_2q_x + s_2wp),\\
&0 = \beta[w_x + s_1s_2(c_1 - c_2)p^*q],
\end{aligned}
\tag{3}
$$

where ε is a small real parameter characterizing the strength of higher-order effects, and β is a coupling coefficient that modulates the intensity of interactions between the vector Hirota subsystem and the three-wave resonant subsystem. Based on the Darboux–Dressing transformation (DDT),[17,18] rational and semi-rational vector solutions of system (2) in the presence of higher-order effects, as well as vector solutions on mixed backgrounds, have been derived.[15] In this paper, we will construct the two-soliton solutions and rational vector solutions with quartic polynomials of system (3). Besides, we will demonstrate that vector solutions with quartic polynomials can be reduced to the corresponding results in Ref. [15].

2. Two-soliton solutions

According to Ref. [15], the Lax pair of system (3) can be given as

$$
\begin{aligned}
&\Psi_x = [\mathrm{i}k\sigma + Q(x,t)]\Psi,\\
&\Psi_t = [T_h(x,t,k) + \beta T_w(x,t,k)]\Psi,
\end{aligned}
\tag{4}
$$

where Ψ is the eigenfunction, and

$$
\begin{aligned}
&T_h(x,t,k) = -4\mathrm{i}\varepsilon k^3\sigma + k^2(2\mathrm{i}\sigma - 4\varepsilon Q) - 2kV_1 - V_2,\\
&T_w(x,t,k) = 2\mathrm{i}kC - \sigma W + \sigma[C,Q],\\
&\sigma = \begin{pmatrix} 1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 0\\ 0 & c_1 & 0\\ 0 & 0 & c_2 \end{pmatrix},\\
&Q = \begin{pmatrix} 0 & s_1p^* & s_2q^*\\ p & 0 & 0\\ q & 0 & 0 \end{pmatrix}, \quad W = \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & -s_1w^*\\ 0 & s_2w & 0 \end{pmatrix},
\end{aligned}
$$

$$
\begin{aligned}
&V_1 = \begin{pmatrix} \mathrm{i}\varepsilon(s_1|p|^2 + s_2|q|^2) & -s_1p^* - \mathrm{i}\varepsilon s_1p_x^* & -s_2q^* - \mathrm{i}\varepsilon s_2q_x^*\\ -p + \mathrm{i}\varepsilon p_x & -\mathrm{i}\varepsilon s_1|p|^2 & -\mathrm{i}\varepsilon s_2pq^*\\ -q + \mathrm{i}\varepsilon q_x & -\mathrm{i}\varepsilon s_1p^*q & -\mathrm{i}\varepsilon s_2|q|^2 \end{pmatrix},\\
&V_2 = \begin{pmatrix} v_{11} & v_{12} & v_{13}\\ v_{21} & v_{22} & v_{23}\\ v_{31} & v_{32} & v_{33} \end{pmatrix},\\
&v_{11} = -s_1(\mathrm{i}|p|^2 + \varepsilon p_xp^* - \varepsilon p_x^*p) - s_2(\mathrm{i}|q|^2 + \varepsilon q_xq^* - \varepsilon q_x^*q),\\
&v_{12} = \mathrm{i}s_1p_x^* - \varepsilon s_1p_{xx}^* + 2\varepsilon s_1p^*(s_1|p|^2 + s_2|q|^2),\\
&v_{13} = \mathrm{i}s_2q_x^* - \varepsilon s_2q_{xx}^* + 2\varepsilon s_2q^*(s_1|p|^2 + s_2|q|^2),\\
&v_{21} = -\mathrm{i}p_x - \varepsilon p_{xx} + 2\varepsilon p(s_1|p|^2 + s_2|q|^2),\\
&v_{22} = \varepsilon s_1(p_xp^* - p_x^*p) + \mathrm{i}s_1|p|^2,\\
&v_{23} = \varepsilon s_2(p_xq^* - q_x^*p) + \mathrm{i}s_2pq,\\
&v_{31} = -\mathrm{i}q_x - \varepsilon q_{xx} + 2\varepsilon q(s_1|p|^2 + s_2|q|^2),\\
&v_{32} = \varepsilon s_1(q_xp^* - p_x^*q) + \mathrm{i}s_2qp^*,\\
&v_{33} = \varepsilon s_2(q_xq^* - q_x^*q) + \mathrm{i}s_2|q|^2.
\end{aligned}
$$

Then, via the DDT, solutions of system (3) can be expressed as

$$
\begin{aligned}
&\begin{pmatrix} p(x,t)\\ q(x,t) \end{pmatrix} = \begin{pmatrix} p_0(x,t)\\ q_0(x,t) \end{pmatrix} + \frac{2\mathrm{i}(k^* - k)\tau^*}{|\tau|^2 - s_1|r_1|^2 - s_2|r_2|^2}\begin{pmatrix} r_1\\ r_2 \end{pmatrix},\\
&w(x,t) = w_0(x,t) - \frac{2\mathrm{i}s_1s_2(c_1 - c_2)(k^* - k)r_1^*r_2}{|\tau|^2 - s_1|r_1|^2 - s_2|r_2|^2},
\end{aligned}
\tag{5}
$$

where τ, r_1, and r_2 are solutions of the Lax pair (4) and $p_0(x,t)$, $q_0(x,t)$, and $w_0(x,t)$ are seed solutions.

Taking the zero seed solutions and using the iteration of DDT, two-soliton solutions of system (3) with a choice of proper parameters can be obtained as

$$
\begin{aligned}
&p(x,t) = -\frac{M_p}{N}, \quad q(x,t) = 80\mathrm{e}^{[(4/5+4\mathrm{i}/5)t + (2-2\mathrm{i})x]}\frac{M_q}{N}, \quad w(x,t) = -8\mathrm{e}^{[4x + (18/5+2\mathrm{i})t]}\frac{M_w}{N},\\
&M_p = 80\mathrm{e}^{(22/5-6\mathrm{i}/5)t + (6-2\mathrm{i})x}\left[(80+32\mathrm{i})t^2 + 40xt + 2(5+4\mathrm{i})t + 10x - 5\right]\\
&\quad + 80\mathrm{e}^{(42/5-6\mathrm{i}/5)t + (6-2\mathrm{i})x}[(30+8\mathrm{i})t + 10x - 5] + 320\mathrm{e}^{(14/5-6\mathrm{i}/5)t + (2-2\mathrm{i})x}[-(30-8\mathrm{i})t - 10x - 5],\\
&M_q = \mathrm{e}^{(28t/5+4x)}\left[(120+32\mathrm{i})t^2 + 40tx + 8(-5-\mathrm{i})t - 10x + 5\right] - \mathrm{e}^{(8t/5+4x)}[(20+8\mathrm{i})t + 10x - 5]\\
&\quad + 4[(20-8\mathrm{i})t + 10x + 5],\\
&M_w = (-5312+640\mathrm{i})t^2 - 1000t(4x+1) + 25(2t-1)\mathrm{e}^{(28t/5+4x)} - 25(2t+1)\mathrm{e}^{(8t/5+4x)}\\
&\quad - 100\left(8x^2 + 4x + 1\right),\\
&N = 8\mathrm{e}^{(8t/5+4x)}\left(928t^2 + 800tx + 200x^2 + 25\right) + 8\mathrm{e}^{(28t/5+4x)}\left(1928t^2 + 1200tx + 200x^2 + 25\right)\\
&\quad + 50\left(8t^2 + 1\right)\mathrm{e}^{(36t/5+8x)} + 25\mathrm{e}^{(16t/5+8x)} + 25\mathrm{e}^{(56t/5+8x)} + 400.
\end{aligned}
\tag{6}
$$

The dynamics of the two solitons are shown in Fig. 1.

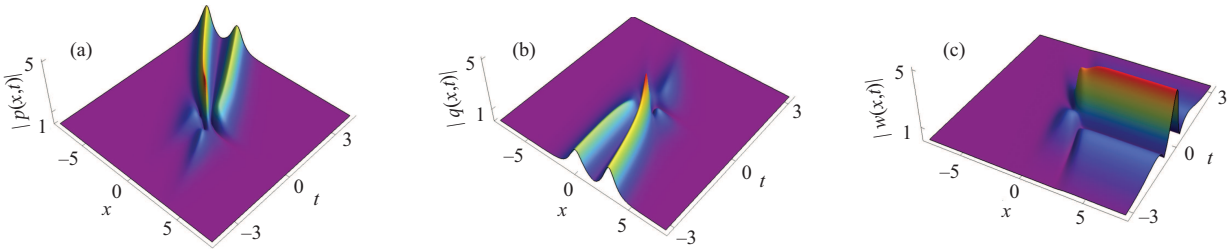


Fig. 1. Two solitons of system (3) with $\varepsilon = 1/5$, $\beta = 1$, $s_1 = s_2 = -1$, $c_1 = 1$, $c_2 = 2$ via solutions (6).

3. Rational solutions with quartic polynomials

In order to construct the rational solutions, seed solutions of system (3) are chosen as

$$p_0(x,t) = \mathrm{e}^{\mathrm{i}(29t/5+x)}, \quad q_0(x,t) = \mathrm{e}^{\mathrm{i}(2x-2t/5)}, \quad w_0(x,t) = -\mathrm{i}\,\mathrm{e}^{\mathrm{i}(x-31t/5)}.$$

Based on the DDT, we obtain the rational solutions under $k = (3/4)\mathrm{i}\left(\mathrm{i}-\sqrt{3}\right)$, $s_1 = s_2 = -1$, $c_2 = 2c_1 = 2$,

$$\begin{aligned}
p(x,t) &= \mathrm{e}^{(29\mathrm{i}t/5+\mathrm{i}x)} - \frac{3\sqrt{3}\tau^*(x,t)r_1(x,t)}{|\tau(x,t)|^2+|r_1(x,t)|^2+|r_2(x,t)|^2},\\
q(x,t) &= \mathrm{e}^{(-2/5)\mathrm{i}(t-5x)} - \frac{3\sqrt{3}\tau^*(x,t)r_2(x,t)}{|\tau(x,t)|^2+|r_1(x,t)|^2+|r_2(x,t)|^2},\\
w(x,t) &= -\mathrm{i}\,\mathrm{e}^{\mathrm{i}(x-31t/5)} - \frac{3\sqrt{3}r_1^*(x,t)r_2(x,t)}{|\tau(x,t)|^2+|r_1(x,t)|^2+|r_2(x,t)|^2},
\end{aligned} \tag{7}$$

with

$$\begin{aligned}
\tau(x,t) &= \frac{1}{100}\mathrm{e}^{\left\{(3/20)\left(15\sqrt{3}-11\mathrm{i}\right)t-(1/4)\left(\sqrt{3}+3\mathrm{i}\right)x\right\}}[2\alpha_1(x,t)z_1+\alpha_2(x,t)z_2+\alpha_3(x,t)z_3],\\
r_1(x,t) &= \frac{1}{100}\mathrm{e}^{\left[(1/20)\left(45\sqrt{3}+83\mathrm{i}\right)t-(1/4)\left(\sqrt{3}-\mathrm{i}\right)x\right]}[\beta_1(x,t)z_1+\beta_2(x,t)z_2+2\beta_3(x,t)z_3],\\
r_2(x,t) &= \frac{1}{100}\mathrm{e}^{\left[(1/20)\left(45\sqrt{3}-41\mathrm{i}\right)t-(1/4)\left(\sqrt{3}-5\mathrm{i}\right)x\right]}[\gamma_1(x,t)z_1+2\gamma_2(x,t)z_2+\gamma_3(x,t)z_3],\\
\alpha_1(x,t) &= 294\left(1+\mathrm{i}\sqrt{3}\right)t^2-5\mathrm{i}t\left[14\left(\sqrt{3}-3\mathrm{i}\right)x-39\mathrm{i}\sqrt{3}+71\right]+25\left(x^2+2\sqrt{3}x+2\right),\\
\alpha_2(x,t) &= -1176\mathrm{i}t^2+10t\left[14\left(\sqrt{3}+3\mathrm{i}\right)x+27\mathrm{i}\sqrt{3}+23\right]-25x\left[4+\left(\sqrt{3}+\mathrm{i}\right)x\right],\\
\alpha_3(x,t) &= -588\left(\sqrt{3}+\mathrm{i}\right)t^2+20t\left(14\sqrt{3}x+15\mathrm{i}\sqrt{3}+26\right)-25x\left[4+\left(\sqrt{3}-\mathrm{i}\right)x\right],\\
\beta_1(x,t) &= 1176\mathrm{i}t^2-10t\left[14\left(\sqrt{3}+3\mathrm{i}\right)x+27\mathrm{i}\sqrt{3}+23\right]+25x\left[4+\left(\sqrt{3}+\mathrm{i}\right)x\right],\\
\beta_2(x,t) &= 588\left(1-\mathrm{i}\sqrt{3}\right)t^2+20t\left(14\mathrm{i}\sqrt{3}x+5\mathrm{i}+6\sqrt{3}\right)+25\left[\left(-1-\mathrm{i}\sqrt{3}\right)x^2-2\left(\sqrt{3}-\mathrm{i}\right)x+4\right],\\
\beta_3(x,t) &= -294\mathrm{i}\left(\sqrt{3}-\mathrm{i}\right)t^2+5t\left(42x+14\mathrm{i}\sqrt{3}x+29\mathrm{i}-3\sqrt{3}\right)-25x^2,\\
\gamma_1(x,t) &= 588\left(\sqrt{3}+\mathrm{i}\right)t^2-20t\left(14\sqrt{3}x+15\mathrm{i}\sqrt{3}+26\right)+25x\left[4+\left(\sqrt{3}-\mathrm{i}\right)x\right],\\
\gamma_2(x,t) &= -294\mathrm{i}\left(\sqrt{3}-\mathrm{i}\right)t^2+5t\left(42x+14\mathrm{i}\sqrt{3}x+29\mathrm{i}-3\sqrt{3}\right)-25x^2,\\
\gamma_3(x,t) &= -1176t^2+10t\left[14\left(3-\mathrm{i}\sqrt{3}\right)x+61\mathrm{i}+27\sqrt{3}\right]+25\left[\mathrm{i}\left(\sqrt{3}+\mathrm{i}\right)x^2-2\left(\sqrt{3}+\mathrm{i}\right)x+4\right].
\end{aligned}$$

From solutions (7), it is obvious that quartic polynomials are included. With $z_1 = 2$, $z_2 = 1$, $z_3 = 0$ taken in the above solutions, double-peak vector rogue waves appear as shown in Fig. 2.

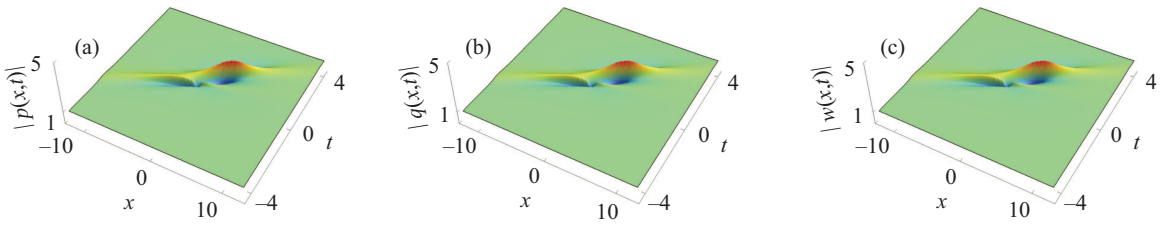


Fig. 2. Rational solutions of system (3) with $\varepsilon = 1/5$, $\beta = 1$ via solutions (7).

If the dressing parameters $z_1 = 2$, $z_2 = 1$, and z_3 satisfy $z_3 = \left[(3-\mathrm{i}\sqrt{3})z_1 - 2\sqrt{3}z_2\right] / \left(\sqrt{3}-3\mathrm{i}\right)$, quartic terms in solutions (7) vanish and the corresponding solutions are given as

$$p(x,t) = \mathrm{e}^{(29\mathrm{i}t/5+\mathrm{i}x)}\frac{F_1}{2D}, \quad q(x,t) = \mathrm{e}^{[(-2/5)\mathrm{i}t+2\mathrm{i}x]}\frac{F_2}{2D}, \quad w(x,t) = -\mathrm{i}\,\mathrm{e}^{[\mathrm{i}x-(31\mathrm{i}t/5)]}\frac{F_3}{2D}, \tag{8}$$

with

$$\begin{aligned}
F_1 &= 1764\mathrm{i}\left(-6+5\sqrt{3}-5\mathrm{i}+2\sqrt{3}\mathrm{i}\right)t^2 - 630\mathrm{i}t\left[\left(-6+5\sqrt{3}-5\mathrm{i}+2\sqrt{3}\mathrm{i}\right)x - \mathrm{i}\sqrt{3}+3\right] \\
&\quad + 75\mathrm{i}\left(-6+5\sqrt{3}-5\mathrm{i}+2\sqrt{3}\mathrm{i}\right)x^2 + 25\left(-12+17\sqrt{3}-9\mathrm{i}+12\sqrt{3}\mathrm{i}\right)x + 100\left(4\sqrt{3}-5\right), \\
F_2 &= -1764\left(-5+2\sqrt{3}-6\mathrm{i}+5\sqrt{3}\mathrm{i}\right)t^2 + 630\left(-5+2\sqrt{3}-6\mathrm{i}+5\sqrt{3}\mathrm{i}\right)xt \\
&\quad + 630(-6+2\mathrm{i})\left(\sqrt{3}-1\right)t - 75\left(-5+2\sqrt{3}-6\mathrm{i}+5\sqrt{3}\mathrm{i}\right)x^2 \\
&\quad + 25\left[(12-9\mathrm{i})-(17-12\mathrm{i})\sqrt{3}\right]x + 50\left[(-8+9\mathrm{i})+(1-6\mathrm{i})\sqrt{3}\right], \\
F_3 &= 1764\left(-6+5\sqrt{3}-5\mathrm{i}+2\sqrt{3}\mathrm{i}\right)t^2 - 630\left(-6+5\sqrt{3}-5\mathrm{i}+2\sqrt{3}\mathrm{i}\right)xt \\
&\quad + 630t\left[(8-6\mathrm{i})-(2-4\mathrm{i})\sqrt{3}\right] + 75\left(-6+5\sqrt{3}-5\mathrm{i}+2\sqrt{3}\mathrm{i}\right)x^2 \\
&\quad + 25\left[(36-42\mathrm{i})-(6-28\mathrm{i})\sqrt{3}\right]x + 25\left[(18+11\mathrm{i})-(3-2\mathrm{i})\sqrt{3}\right], \\
D &= 1764\left(2\sqrt{3}-5\right)t^2 - 630\left(2\sqrt{3}-5\right)tx + 630\sqrt{3}t + 75\left(2\sqrt{3}-5\right)x^2 \\
&\quad + 630t\left[(8-6\mathrm{i})-(2-4\mathrm{i})\sqrt{3}\right] + 75\left(-6+5\sqrt{3}-5\mathrm{i}+2\sqrt{3}\mathrm{i}\right)x^2 - 50\left(\sqrt{3}+3\right)x + 50\left(\sqrt{3}-5\right).
\end{aligned}$$

Solutions (8) result in the single-peak rogue waves as shown in Fig. 3.

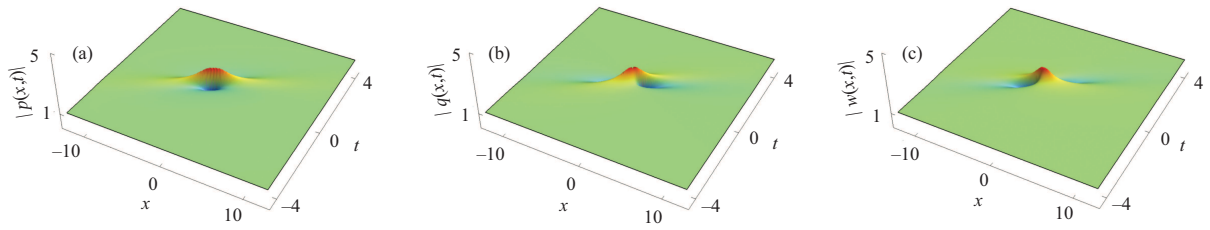


Fig. 3. Rational solutions of system (3) with $\varepsilon = 1/5$, $\beta = 1$ via solutions (8).

4. Conclusions

In this paper, we concentrate on exact solution structures of the wave resonant-interaction equations with higher-order effects, *i.e.*, system (3). With the DDT adopted, we derive the expressions of two types of physically significant solutions: two-soliton solutions and vector rogue wave solutions. Evolutions of the two solitons are shown in Fig. 1. One of the pivotal results in our research is the transformation of vector rogue wave structures governed by dressing parameters: when the dressing parameters satisfy the specific condition (detailed in the main text), the double-peak vector rogue waves can be converted into single-peak vector rogue waves. This structural transition directly reflects the tunability of extreme wave behaviors via parameter adjustment, which demonstrates theoretical and practical implications for controlling rogue wave generation in practical systems. Figures 2 and 3 clearly demonstrate the morphological evolution from double-peak to single-peak vector rogue waves under the parameter condition,

providing intuitive evidence for our theoretical results.

In the coming study, higher-order rational solutions could be obtained by the DDT to reveal more interesting phenomena of rogue waves.

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