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Jia-Rui Song(宋佳瑞), Chang-Shui Yu(于长水)
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Jensen–Shannon divergence coherence and its quantum speed limit

Jia-Rui Song(宋佳瑞) and Chang-Shui Yu(于长水)†

School of Physics, Dalian University of Technology, Dalian 116024, *China*

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The rate at which quantum coherence is generated or degraded significantly impacts quantum information processing. The minimum time for the coherence-changing process can be well evaluated using the quantum speed limit (QSL), which typically requires taking the maximum across different QSL bounds. In this paper, we derive a coherence QSL bound using the quantum Jensen–Shannon divergence as a coherence measure. It is shown that for any state, there is always a representation in which the dephasing dynamics can saturate the QSL bound. We also numerically demonstrate that our QSL bound can be tighter than previous bounds for some applications. Our results make up for the tightness of the coherence QSL.

Keywords: quantum speed limit, coherence measure, quantum coherence

PACS: 03.65.Ud, 03.67.–a, 03.65.Ta, 03.65.–W

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1. Introduction

Quantum coherence is the most fundamental feature of quantum mechanics; it is the root of the other quantum features and is also a critical quantum resource in many applications.[1–11] To some extent, quantum information processing can be understood as the production and consumption of quantum coherence. Therefore, how fast quantum coherence is produced or consumed, i.e., the quantum speed limit (QSL) based on coherence, is an essential characteristic to assess the potential utility of a quantum system.

QSL originated from the unitary evolution time between two orthogonal pure states, for which Mandelstam and Tamm[12] proposed the well-known MT bound $\tau_{\rm MT} = \pi/(2\Delta E)$, where $(\Delta E)^2 = \langle H^2\rangle - \langle H\rangle^2$ is the variance of the Hamiltonian.[13] QSL has become a fundamental concept in quantum mechanics and is pivotal in understanding the state evolution of a quantum system. QSL has also displayed increasing importance in many fields, such as quantum optimal control,[14–17] quantum metrology,[18,19] quantum information processing,[20,21] quantum battery technology,[22,23] and so on.[22,24–30]

Besides the MT bound, Margolus and Levitin[31] established another important ML-Bound $\tau_{\rm ML} = \pi/2E$ based on the average energy E.[13] The combination of MT and ML bounds $\tau_{\rm MT-ML} = \max\{\tau_{\rm ML}, \tau_{\rm MT}\}$ was suggested as a tight bound.[32] In fact, extensive research on the QSL has been done not only for unitary dynamics,[33–41] but also for non-unitary dynamics of open systems.[42–56] Recently, the QSL of quantum resources has attracted increasing interest, such as coherence[14,18,19,22–28,57–63] and entanglement.[64–66] Evidence shows that a single QSL bound is not enough to give a tight enough evaluation of the minimum evolution time[32,46] because the tightness usually strongly depends on the system of interest. In this sense, it is crucial to explore the different QSL bounds.

In this paper, we present a QSL bound on the coherence variation with a certain degree of tightness. We first prove that the quantum Jensen Shannon divergence (QJSD) can measure coherence with monotonicity. Then, we develop the coherence QSL by considering the production and consumption of coherence based on the new coherence measure. It is shown that the QSL bound is attainable in the sense that, given a qubit state, one can always find a representation in which the dephasing dynamics can saturate the QSL bound. Comparing our coherence QSL with the previous one,[63] it is shown that the two QSLs are equivalent for the states with equal populations. However, our QSL can exhibit stronger tightness in some cases, suggesting that it is necessary to combine all QSL bounds to obtain a reliable estimate of the coherence evolution time. The remainder of this paper is organized as follows. Section 2 presents our QJSD coherence measure and demonstrates its computability. In Section 3, we establish the coherence QSL. In Section 4, we study the attainability and the tightness. In Section 5, we briefly discuss the potential experimental realization and provide a conclusion.

2. Coherence measure based on QJSD

To quantify quantum coherence, we first define the free states and the free operations. Given the basis $\{|i\rangle\}$ in N-dimensional Hilbert space, the free state, i.e., the incoherent

†Corresponding author. E-mail: ycs@dlut.edu.cn

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state, has the density matrix as $\delta = \sum_i \delta_{ii} |i\rangle\langle i|$ with $\sum_i \delta_{ii} = 1, \delta_{ii} > 0$. All the incoherent states form the set of free states denoted by $\mathcal{I}$. For coherence, the free operations are referred to as incoherent operations, which are defined by the Kraus operators $\Phi(\cdot) = \sum_k M_k(\cdot)M_k^\dagger$ with $\Phi(\delta) \in \mathcal{I}$ for any incoherent state δ. A good coherence measure $C(\rho)$ for a state ρ should satisfy the following three conditions:[3,67] (i) $C(\rho) \geq 0$ for all states, and $C(\rho) = 0$ only if ρ is incoherent. (ii) $C(\rho) \geq C(\Phi(\rho))$ for any incoherent operation $\Phi(\cdot)$. (iii) $C(p_1\rho_1 \oplus p_2\rho_2) = p_1C(\rho_1) + p_2C(\rho_2)$ for any $p_1 + p_2 = 1$ and density matrices ρ_1 and ρ_2. Here, we show that the QJSD[68] can induce a coherence measure with monotonicity.

Theorem 1 For an N-dimensional quantum state ρ, the coherence can be quantified by

$$C(\rho) = \min_{\sigma\in\mathcal{I}} J(\rho,\sigma), \tag{1}$$

where

$$J(\rho,\sigma) = \frac{1}{2}S\left(\rho, \frac{\rho+\sigma}{2}\right) + \frac{1}{2}S\left(\sigma, \frac{\rho+\sigma}{2}\right) \tag{2}$$

is the QJSD with $S(A,B) = \mathrm{Tr}A\log A - \mathrm{Tr}A\log B$ denoting the relative entropy for a pair of density matrices A and B.

Proof It is obvious that $J(\rho,\sigma)$ is the combination of relative entropy, which is always positive, so $C_J(\rho) \geq 0$. For $\rho \in \mathcal{I}$, we can select $\sigma = \rho^{\mathrm{diag}} = \rho$ to get $C(\rho) = 0$. So, the condition (i) holds.

To prove condition (ii), let the state ρ undergo an incoherent channel $\Phi(\cdot)$, and σ^* be the optimal incoherent quantum state to achieve $\min_{\sigma\in\mathcal{I}} J(\rho,\sigma)$. Based on the contractivity of relative entropy[69] $S(\tilde{\Phi}(\rho),\tilde{\Phi}(\sigma)) \leq S(\rho,\sigma)$ for any completely positive, trace-preserving maps $\tilde{\Phi}$, one will immediately find

$$\begin{aligned} C_J(\Phi(\rho)) &= \min_{\sigma\in\mathcal{I}} J(\Phi(\rho),\sigma) \\ &\leq \frac{1}{2}\left[S\left(\Phi(\rho), \frac{\Phi(\rho)+\Phi(\sigma^*)}{2}\right)\right. \\ &\quad \left. + S\left(\Phi(\sigma^*), \frac{\Phi(\rho)+\Phi(\sigma^*)}{2}\right)\right] \\ &\leq \frac{1}{2}\left(S\left(\rho, \frac{\rho+\sigma^*}{2}\right) + S\left(\sigma^*, \frac{\rho+\sigma^*}{2}\right)\right) \\ &= J(\rho,\sigma^*) = C_J(\rho), \end{aligned} \tag{3}$$

where the first inequality comes from replacing the optimal incoherent state by a particular incoherent state $\Phi(\sigma^*)$. Thus, we show that the coherence measure $C_J(\rho)$ does not increase under incoherent operations.

To show condition (iii), the subspace coherence additivity, we first show that this condition is satisfied by the relative entropy. Let us consider a state given as the direct sum of two states as $\rho = p_1\rho_1 \oplus p_2\rho_2$ with $p_1 + p_2 = 1$ and divide the incoherent state σ as $\sigma = q_1\sigma_1 \oplus q_2\sigma_2$ with $q_1 + q_2 = 1$. Substituting them into the relative entropy, one can easily obtain

$$S(\rho,\sigma) = p_1S(\rho_1,\sigma_1) + p_2S(\rho_2,\sigma_2),$$

which directly leads to

$$C_J(\rho) = p_1J(\rho_1,\sigma_1) + p_2J(\rho_2,\sigma_2). \tag{4}$$

It is obvious the above two equations don't hold for general $p \neq q$. So C_J could not satisfy the strong monotonicity.

We would like to emphasize that the QJSD can be viewed as an alternative to the quantum relative entropy. When used as a distance, QJSD can effectively address the problem of support of input states. Strictly speaking, quantum relative entropy is not a reasonable distance due to its asymmetry, nor is the QSJD due to its violation of the triangle inequality. Unlike the coherence measure based on relative entropy, the QJSD coherence measure cannot be analytically calculated in the general case. Still, it has its own advantages in terms of the QSL's tightness. In fact, there are other coherence measures based on different types of entropies, such as the square root of the QSJD[68] and the Tsallis relative α-entropy,[70,71] but we still employ the QJSD because of the relatively simple form of the QSL. Next, we present analytically calculable examples that can help demonstrate the tightness of our following QSL.

Theorem 2 For any qubit state $\rho = \frac{\mathbb{I}_2}{2} + a\sigma_x + b\sigma_y$ with real a, b, the coherence is given by $C_J(\rho) = J(\rho, \frac{\mathbb{I}_2}{2})$.

Proof Let us define the incoherent state as $\sigma = \mathrm{diag}\{x, 1-x\}$ with $0 \leq x \leq 1$. Suppose the eigendecomposition of $(\rho+\sigma)/2$ is

$$\frac{\rho+\sigma}{2} = \Psi \begin{bmatrix} \lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} \Psi^\dagger, \tag{5}$$

where Ψ is the eigenvector unitary matrix and $\lambda = (1+\sqrt{x^2-x+a^2+b^2+1/4})/2$ is the eigenvalue. Then, $C_J(\rho)$ can be rewritten as

$$\begin{aligned} C_J(\rho) &= \min_x J(\rho,\sigma) \\ &= \frac{1}{2}\mathrm{Tr}(\rho\log\rho + \sigma\log\sigma) - \mathrm{Tr}\frac{\rho+\sigma}{2}\log\frac{\rho+\sigma}{2} \\ &= \frac{1}{2}x\log x + \frac{1}{2}(1-x)\log(1-x) \\ &\quad -\lambda\log\lambda - (1-\lambda)\log(1-\lambda) + \frac{1}{2}\mathrm{Tr}\rho\log\rho. \end{aligned} \tag{6}$$

Taking the derivative of J with respect to x yields

$$\frac{\partial J(\rho,\sigma)}{\partial x} = \frac{1}{2}\log\frac{x}{1-x} - \frac{\partial\lambda}{\partial x}\log\frac{\lambda}{1-\lambda}. \tag{7}$$

One can find that only for $x = 1/2$, $\partial J(\rho,\sigma)/\partial x = 0$, which implies $\sigma = \rho^{\mathrm{diag}} = \mathbb{I}_2/2$.

Theorem 2 only gives the analytically calculable case, but for the general case, one can use MATLAB's numerical optimization, which is straightforward. We have compared the numerical and analytical results in Fig. 1(a), where we consider a state as Eq. (14) with $\rho_{01} = 0.5$, which undergoes a dephasing channel defined in Eq. (15) with $\gamma = 0.1$. It is shown

that the analytical results are well consistent with the numerical solutions, which verifies the correctness of Theorem 2. Of course, one can also see that the coherence of the initial state monotonically decreases as the dephasing time increases, which is consistent with our physics. Simultaneously, we also give general numerical results for randomly generated density matrices. We randomly generated 1000 density matrices with dimensions ranging from 3 to 30 and calculated their coherences. It is shown that, as the dimension increases, the MATLAB built-in tools can stably calculate coherence, demonstrating the usability of the numerical process.

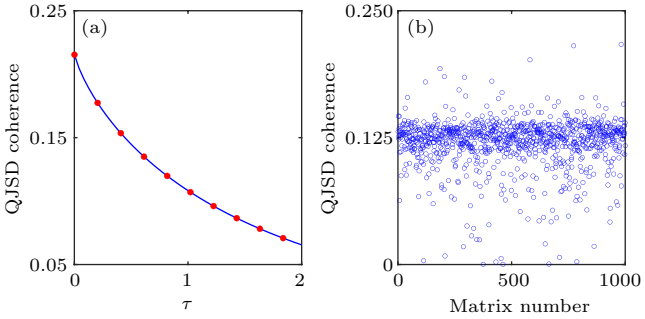


Fig. 1. (a) The QJSD coherence of a qubit state versus the evolution time τ subject to a dephasing dynamics. The dotted line is generated by MATLAB numerical optimization, and the solid line is the analytical result in Theorem 2. (b) The QJSD coherence versus the number of randomly generated density matrices.

3. Quantum speed limits

With the above coherence measure, we can establish the coherence QSL. Considering a dynamical evolution given in the general form of a master equation as $\dot{\rho}_t = \mathcal{L}_t(\rho_t)$, the coherence of the state ρ_t will usually be changed. The variation of coherence can be characterized as follows.

Theorem 3 Considering the evolution from the state ρ_0 to ρ_τ, the change of the coherence is denoted by ΔC_J. The required time τ is lower bounded by $\tau \geq T_{\mathrm{C}}(\rho_0,\rho_\tau)$ with

$$T_{\mathrm{C}}(\rho_0,\rho_\tau) = \frac{|\Delta C_J|}{\frac{1}{2}\langle|\mathrm{Tr}\mathcal{L}_t(\rho_t)\log(\frac{\rho_t+\sigma_x^{\blacktriangledown}}{2})+\dot{S}|\rangle_\tau}, \tag{8}$$

where $x=0$ for $\Delta C_J \geq 0$, and $x=\tau$ for $\Delta C_J \leq 0$ and $\langle f_t\rangle_\tau \equiv \frac{1}{\tau}\int_0^\tau f_t\,\mathrm{d}t$ corresponds to the time-average of f_t along the trajectory.

Proof The increment of the coherence reads

$$\begin{aligned}\Delta C_J &= J(\rho_\tau,\sigma_\tau^{\blacktriangledown}) - J(\rho_0,\sigma_0^{\blacktriangledown}) \\ &\leq J(\rho_\tau,\sigma_0^{\blacktriangledown}) - J(\rho_0,\sigma_0^{\blacktriangledown}) \\ &= -\frac{1}{2}\Delta S + S\left(\frac{\rho_\tau+\sigma_0^{\blacktriangledown}}{2}\right) - S\left(\frac{\rho_0+\sigma_0^{\blacktriangledown}}{2}\right),\end{aligned} \tag{9}$$

where $\Delta S = S(\rho_\tau) - S(\rho_0)$, $\sigma_0^{\blacktriangledown}$ and $\sigma_\tau^{\blacktriangledown}$ are the optimal incoherent states for $C_J(\rho_0)$ and $C_J(\rho_\tau)$. Equation (9) can be rewritten in the integral form as

$$\begin{aligned}\Delta C_J &\leq -\frac{1}{2}\int_0^\tau \mathrm{d}t\dot{S}(\rho_t) + \int_0^\tau \mathrm{d}t\dot{S}\left(\frac{\rho_t+\sigma_0^{\blacktriangledown}}{2}\right) \\ &= -\frac{1}{2}\int_0^\tau \mathrm{d}t\left[\dot{S}(\rho_t) + \mathrm{Tr}\mathcal{L}_t(\rho_t)\log\frac{\rho_t+\sigma_0^{\blacktriangledown}}{2}\right].\end{aligned} \tag{10}$$

If $\Delta C_J \leq 0$, we have

$$\begin{aligned}-\Delta C_J &= J(\rho_0,\sigma_0^{\blacktriangledown}) - J(\rho_\tau,\sigma_\tau^{\blacktriangledown}) \\ &\leq J(\rho_0,\sigma_\tau^{\blacktriangledown}) - J(\rho_\tau,\sigma_\tau^{\blacktriangledown}) \\ &= \frac{1}{2}\Delta S + S\left(\frac{\sigma_\tau^{\blacktriangledown}+\rho_0}{2}\right) - S\left(\frac{\rho_\tau+\sigma_\tau^{\blacktriangledown}}{2}\right).\end{aligned} \tag{11}$$

Analogous to Eq. (10), one can arrive at

$$\begin{aligned}-\Delta C_J &\leq \frac{1}{2}\int_0^\tau \mathrm{d}t\dot{S}(\rho_t) - \int_0^\tau \mathrm{d}t\dot{S}\left(\frac{\rho_t+\sigma_\tau^{\blacktriangledown}}{2}\right) \\ &= \frac{1}{2}\int_0^\tau \mathrm{d}t\left[\dot{S}(\rho_t) + \mathrm{Tr}\mathcal{L}_t(\rho_t)\log\frac{\rho_t+\sigma_\tau^{\blacktriangledown}}{2}\right].\end{aligned} \tag{12}$$

Combining Eqs. (10) and (12), one can immediately arrive at Eq. (8).

Based on the above proof process, one can find that the coherence QSL can be given in another form as follows.

Theorem 4 The required time τ for the coherence change ΔC_J is lower bounded by $\tau \geq T_{\mathrm{CS}}(\rho_0,\rho_\tau)$ with

$$T_{\mathrm{CS}}(\rho_0,\rho_\tau) = \frac{|2\Delta C_J + \Delta S|}{\langle|\mathrm{Tr}\mathcal{L}_t(\rho_t)\log\left(\frac{\rho_t+\sigma_x^{\blacktriangledown}}{2}\right)|\rangle_\tau}. \tag{13}$$

Proof The proof is straightforward by combining Eqs. (10) and (12). The proof is completed.

From Theorem 3, one can find that the average speed is the combination of the entropy change rates of two states, the state ρ_t and the state $(\rho_t+\sigma_x^{\blacktriangledown})/2$. When the state undergoes the unitary evolution, the entropy is invariant, so $\dot{S}(\rho_t)=0$. The average speed will be reduced to the entropy change rate of the state $(\rho_t+\sigma_x^{\blacktriangledown})/2$. In this sense, Theorem 3 and Theorem 4 are equivalent to each other. Our results are parallel to those in Ref. [63]; however, from the following applications, it can be seen that our current results can compensate for the tightness of the previous coherence QSL.

4. Applications and attainability

Let us consider a qubit state to show the attainability and the applications. For any qubit state, one can always find a representation in which the state can be given with equal populations as

$$\rho_0 = \begin{pmatrix} 1/2 & \rho_{01} \\ \rho_{01}^* & 1/2 \end{pmatrix}. \tag{14}$$

In this representation, let us consider a dephasing dynamics as

$$\dot{\rho}_t = \mathcal{L}_t(\rho_t) = \gamma(\sigma_z\rho_t\sigma_z - \rho_t), \tag{15}$$

where σ_z is the Pauli matrix and γ is the decay rate. Solving the dynamics, one can directly obtain the density matrix as

$$\rho_t = \begin{pmatrix} 1/2 & \mathrm{e}^{-2\gamma t}\rho_{01} \\ \mathrm{e}^{-2\gamma t}\rho_{01}^* & 1/2 \end{pmatrix}. \tag{16}$$

Substituting ρ_t into Theorem 3, one will arrive at

$$\begin{aligned}&\frac{1}{2}\int_0^\tau \mathrm{Tr}\,\mathcal{L}_t(\rho_t)\log\frac{\rho_t+\sigma_\tau^{\blacktriangledown}}{2}\\&=\gamma\int_0^\tau \mathrm{e}^{-2\gamma t}\,|\rho_{01}|\left(\log\frac{1-|\rho_{01}|\,\mathrm{e}^{-2\gamma t}}{2}\right.\\&\left.-\log\frac{1+|\rho_{01}|\,\mathrm{e}^{-2\gamma t}}{2}\right).\end{aligned}\tag{17}$$

Calculating the integral, one can find that the right-hand side (rhs.) yields

$$\begin{aligned}\text{rhs.}=&\left(1+\frac{|\rho_{01}|\,\mathrm{e}^{-2\gamma t}}{2}\right)\log\frac{1+|\rho_{01}|\,\mathrm{e}^{-2\gamma t}}{2}\Bigg|_0^\tau\\&+\left(1-\frac{|\rho_{01}|\,\mathrm{e}^{-2\gamma t}}{2}\right)\log\frac{1-|\rho_{01}|\,\mathrm{e}^{-2\gamma t}}{2}\Bigg|_0^\tau,\end{aligned}\tag{18}$$

which, adding $\frac{1}{2}\Delta S$, equals ΔC_J. This means that our QSL is attainable.

To give an intuitive illustration of the attainability, we plot our QSL in Fig. 2. One can easily see that our coherence QSL matches the practical evolution time of the qubit state under dephasing dynamics in Fig. 2(a). In addition, we also consider another numerical example, namely, a qubit undergoes a depolarizing dynamics with the Liouvillian operator given by $\mathcal{L}_t(\rho_t)=\gamma(\frac{\mathbb{I}_2}{2}-\rho_0)$ and the initial state given by $\rho_0=\begin{pmatrix}1/2 & 0.2\\ 0.2 & 1/2\end{pmatrix}$. Numerical results (in Fig. 2(b)) show that our coherence QSL is also attainable. In fact, by solving the evolution of the aforementioned depolarizing dynamics, one finds that the off-diagonal elements of ρ_0 decay at some rate, corresponding to a dephasing process.

To show the tightness of the coherence QSL, we compare our current QSLs to our previous QSL T_{M} in Ref. [63] in Fig. 3. Figure 3(a) shows that our current QSL T_{C}, T_{CS}, and our previous QSL time T_{M}, $\tilde{T}_{\mathrm{M}}$ can all be saturated for the pure qubit state with maximal coherence undergoing the dephasing dynamics. In Fig. 3(b), we consider a state $\rho_0=\begin{pmatrix}0.02 & 0.2\\ 0.2 & 0.98\end{pmatrix}$ undergoing a dynamics with the Liouvillian operator given by $\mathcal{L}_t(\rho_t)=\gamma(\sigma_x\rho_t\sigma_x-\rho_t)$. In Fig. 3(c), we study a state $\rho_0=\begin{pmatrix}0.1 & 0.22\\ 0.22 & 0.9\end{pmatrix}$ undergoing an amplitude-damping dynamics with the Liouvillian operator given by $\mathcal{L}_t(\rho_t)=\gamma(\sigma_-\rho_t\sigma_+-\frac{1}{2}\{\sigma_+\sigma_-,\rho_t\})$. Figure 3(b) shows that T_{C} is less than T_{M}, but T_{CS} is larger than $\tilde{T}_{\mathrm{M}}$, and T_{CS} is always larger than the other bounds. Figure 3(c) shows that T_{C} is larger than T_{M} in some range, and T_{CS} is larger than $\tilde{T}_{\mathrm{M}}$, but T_{CS} is always larger than the other bounds. These figures first show that our current coherence QSL is tight in many cases, but also indicate that, to evaluate the coherence evolution time with relatively high precision, one must combine all potential bounds to determine the maximum value. In this sense, our current QSL provides a significant supplement to the quantum coherence QSL.

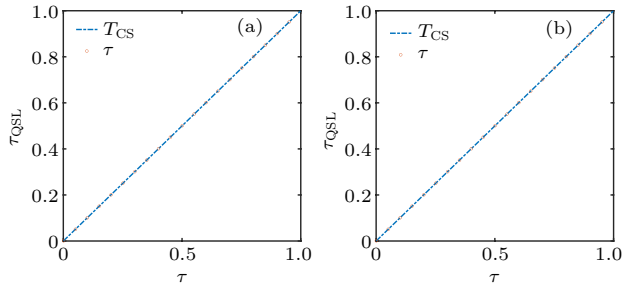


Fig. 2. QSL time τ_{QSL} versus the practical evolution time τ: (a) shows the evolution in a dephasing dynamics, and (b) illustrates the evolution in a depolarizing dynamics. Both figures show our QSL time is attainable.

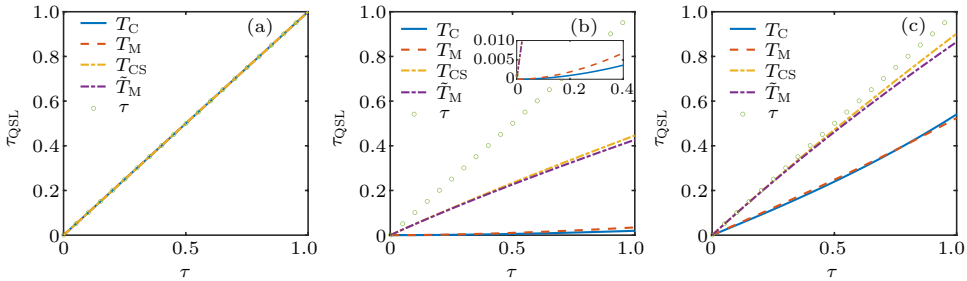


Fig. 3. QSL time τ_{QSL} versus the practical evolution time τ in different dynamics: (a) shows the case $T_{\mathrm{M}}=T_{\mathrm{C}}=\tilde{T}_{\mathrm{M}}=T_{\mathrm{CS}}=\tau$, (b) shows that $T_{\mathrm{CS}}>\tilde{T}_{\mathrm{M}}$ and $T_{\mathrm{C}}<T_{\mathrm{M}}$, but (c) shows $T_{\mathrm{CS}}>\tilde{T}_{\mathrm{M}}$ and especially $T_{\mathrm{C}}>T_{\mathrm{M}}$ in some time range.

5. Discussion and conclusion

Before the end, we would like to briefly discuss the experimental verification of our coherence QSL, similar to Refs. [72,73]. The pure dephasing dynamics have been well treated in the literature, such as in the nitrogen-vacancy centre in high-purity diamond at room temperature.[74] One can prepare a superposed state for simplicity, then allow it to evolve under dephasing dynamics for a duration τ. Both the initial and final states can be measured experimentally, and the dephasing rate can be determined by the dephasing time $T_2^\star$. One can substitute the states and the dephasing rate into our QSL to calculate the QSL time, and then compare it with the practical evolution time τ. If the initial state is the maximally superposed state, the two times should be the same; otherwise, the QSL time should be less than τ.

We have presented a coherence QSL based on the QJSD coherence measure. The coherence measure satisfies all the requirements but the strong monotonicity. We develop a new QSL concerning coherence variation based on the new coherence measure. For any qubit state, one can always find a rep-

resentation where the dephasing dynamics can be analytically demonstrated to saturate our coherence QSL bound. Additionally, numerical results indicate the attainability of other dynamics. We compare our QSL and the previous QSL[63] and find that our coherence QSL is sometimes tighter than the previous one. This implies that one should consider all potential QSLs to characterize the minimum time required to produce or consume quantum coherence effectively. Besides the coherence QSL, we would like to emphasize that the QJSD coherence measure cannot be analytically calculated for a general density matrix; however, an analytical expression can be obtained for the qubit state with equal diagonal entries. In addition, numerical calculations are quite convenient using MATLAB's built-in functions. However, this coherence measure helps derive the coherence QSL. Whether it can be analytically calculated for more and especially high-dimensional states, and whether it can be shown that it is useful for more applications, deserve further research.

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