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TOPICAL REVIEW — Biophysical circuits: Modeling & applications in neuroscience

Discrete neuron models and memristive neural network mapping: A comprehensive review

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In recent years, discrete neuron and discrete neural network models have played an important role in the development of neural dynamics. This paper reviews the theoretical advantages of well-known discrete neuron models, some existing discretized continuous neuron models, and discrete neural networks in simulating complex neural dynamics. It places particular emphasis on the importance of memristors in the composition of neural networks, especially their unique memory and nonlinear characteristics. The integration of memristors into discrete neural networks, including Hopfield networks and their fractional-order variants, cellular neural networks and discrete neuron models has enabled the study and construction of various neural models with memory. These models exhibit complex dynamic behaviors, including superchaotic attractors, hidden attractors, multistability, and synchronization transitions. Furthermore, the present paper undertakes an analysis of more complex dynamical properties, including synchronization, speckle patterns, and chimera states in discrete coupled neural networks. This research provides new theoretical foundations and potential applications in the fields of brain-inspired computing, artificial intelligence, image encryption, and biological modeling.

Keywords: discrete neuron, discrete neural network model, memristor, chaotic dynamics, memristive neural network

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1. Introduction

The fundamental components of the human brain are neurons, which are interconnected by synapses to constitute intricate neural networks that possess the capacity to process complex information. Recent findings in the fields of neurodynamics and neurocomputational studies have indicated a robust correlation between cerebral activity and the neuronal networks comprising biological neurons.[1–4] Therefore, the study of neurons is valuable for the study of neural networks. Currently, the most famous continuous neuron models include Hodgkin–Huxley (HH) neuron model,[5] Morris–Lecar (ML) neuron model,[6] Hindmarsh–Rose (HR) neuron model,[7] and Chay neuron model.[8] Some of the most well-known discrete neuron models include the Rulkov neuron model,[9–11] the Chialvo neuron model,[12] the Izhikevich neuron model,[13] and the Courbage neuron model.[14] The nonlinear dynamic properties of these discrete neurons are more likely to induce chaotic behavior than continuous neurons.

However, some neural networks ignore the memory properties of biological nervous systems, and memristors can overcome these shortcomings of neural networks.[15–17] Memristors have had a wide impact in the fields of integrated circuits,[18–21] memristive chaos theory,[22–24] and bioengineering[1,25,26] due to their unique memory functions and nonlinear response properties, and they can be used to influence nonlinear system complexity through the localized activities of different kinds of memristors.[27–30] At the same time, researchers have designed many new types of memristors and introduced them into various neural network systems.[31–38] They have discovered many complex chaotic dynamics phenomena, among which hidden attractors,[39–42] multi-scroll attractors,[43,44] multi-stability of the system,[45–48] phenomenon of synchronization[49] and coexistence of attractors are more significant.[50–52]

At the outset of the research, the emphasis was placed on continuous neural networks, and a substantial corpus of literature was dedicated to this subject.[53–56] However, in recent years, researchers have studied discrete neural networks and discrete memristive neural network models more intensively. On discrete neural networks, in 2023, Lai used a discrete memristor model to simulate electromagnetic radiation effects and synaptic behavior to construct a discrete neural network.[57] In 2023, Al-Husban constructed a 2D fractional discrete Hopfield neural network (HNN) fusing constant and variable ordering.[58] In 2023, Ma *et al.* constructed a nonlinear synaptic chaos model based on a 4D discrete HNN.[59] Subsequently, researchers investigated the integration of discrete neurons with discrete memristor (DM) and

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neural networks to develop a variety of discrete memristive neural network models.[60,61] This study identified numerous novel dynamical phenomena. For example, in 2023, He *et al*. constructed a three-layer discrete memristive BP neural network.[62] In 2024, Li proposed a discrete memristive Rulkov neuron model, then constructed a large-scale discrete memristive Rulkov ring star neural network model, and discovered double-well chaos, asynchrony, multi-cluster behavior, isolation, and synchronization behavior.[63] In 2025,[64] a 3D discrete memristive HNN model was proposed, comprising discrete memristors that simulate self-synapses. This model facilitates the observation of two phenomena: infinite coexistence, including periodic shift behavior, and initial value shift behavior. Subsequent research revealed that discrete neural mapping models hold significant application potential in logic computing and digital circuits. The utilization of discrete dynamic properties facilitates the development of innovative computational methodologies that are particularly well-suited for low-power digital circuits and IoT devices. Reference [19] proposes a discrete memristor model that satisfies memristor characteristics and designs a fully fixed-point digital integrated circuit system based on this model. The mapping proposed by Lai *et al*. has been successfully applied to the design of pseudorandom number generators. This has been implemented in practice through a microcontroller-based digital hardware platform.[65]

This paper provides an overview of discrete neurons and discrete neural network models, as well as amnesic neural network models, from the perspective of chaotic dynamics. In Section 2, concepts related to discrete memristors are presented. Some traditional models of discrete neurons and discretized continuous neurons are summarized in Section 3. In Section 4, the research on nonlinear circuits of discrete neurons and the application research are described, and different kinds of discrete neural network models are summarized in Section 5. In Section 6, the amnesic neural network models are analyzed and discussed. Section 7 elucidates synchronization, pattern formation, and chimera states in neural networks. Finally, in Section 8, a comprehensive overview of the entire paper is provided, along with a detailed exposition of the future research directions presented therein.

2. Memristor models

In 1971, Professor Chua first proposed the concept of the memristor, defining it as the fourth basic circuit element and describing the relationship between charge and magnetic flux based on the theory of circuit variable symmetry.[66] Subsequent research has led to the broadening of the definition of the memristor, a type of electrical device that exhibits specific characteristics. According to the latest findings, a memristor is now considered any double-ended device that fulfills the following two criteria: the first condition is the presentation of a tight hysteresis curve past the origin under periodic voltage or current excitation; the second condition requires that the steady-state voltammetric characteristics satisfy the memory dependence. In 2008, the first physical memristor device was created at HP labs,[67] signifying the transition of memristors from theoretical foundations to practical applications. Subsequent to its initial formulation, the theoretical model of the memristor has undergone a gradual process of refinement, resulting in the development of two distinct memristor models. The symbol for a memristive element is shown in Fig. 1. A typical ideal magnetronic memristor model can be expressed as

$$\begin{cases} i_n = W(\varphi_n)v_n, \\ \varphi_{n+1} = \varphi_n + hv_n. \end{cases} \tag{1}$$

In the magnetically controlled memristor model, i_n denotes the discrete input current, v_n denotes the discrete output voltage, φ_n is the magnetic flux of the memristor, and h represents the internal parameters of the memristor. Another typical memristor model is the charge-controlled memristor model, which can be expressed as follows:

$$\begin{cases} v_{n+1} = M(q_n)i_n, \\ q_{n+1} = q_n + ci_n. \end{cases} \tag{2}$$

In the formula, i_n, q_n, and v_n represent the sampled values of the input $i(t)$, internal state $q(t)$, and output $v(t)$ at the n-th iteration, respectively, c represents the internal parameters of the memristor. The symbol $M(q_n)$ denotes the memristor $M(q)$ at the n-th iteration.

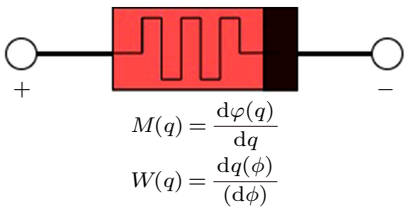


Fig. 1. Symbol for memristive elements.

In recent years, memristors have attracted a great deal of attention and have become widely used in a variety of fields. Due to their bionic properties, memristors are widely used to construct amnesic neural networks. They show great potential in simulating neuronal behavior and synaptic transmission. On the one hand, the flux characteristics of memristors allow them to effectively mimic electromagnetic induction effects in biological nervous systems.[68,69] On the other hand, they can be used to mimic the way synapses work, thereby improving the learning and memory capabilities of neural networks.[70–73] Additionally, the unique dynamic behavior of memristors shows potential for use in image encryption[74–78] and secure communication[38,79] applications.

3. Discrete neuron model

In this section, we will systematically sort out the neuronal models of various types of discrete neurons and common continuous neurons after discrete neurons, which have laid an important theoretical foundation for the development of neurodynamics. By synthesizing and examining these classical models, we can discern their historical contributions and theoretical value to the study of neurodynamics.

3.1. Chialvo neuron model

The Chialvo neuron model was first proposed by Dante R. Chialvo in 1995.[12] The model delineates the electrical activity properties of neurons through a theoretical analysis of their nonlinear dynamics in the form of discrete maps, with particular emphasis on the simulation of chaotic dynamics and discharge behavior. Its mathematical form is described as

$$\begin{cases} x(n+1) = x(n)^2 \exp(y(n) - x(n)) + k, \\ y(n+1) = ay(n) - bx(n) + c. \end{cases} \tag{3}$$

Specifically, the Chialvo model comprises two coupled discrete equations that delineate the neuron's activation variable $x(n)$ and the recovery variable $y(n)$. The parameter k is representative of the external bias input, while a $(a < 1)$ is indicative of the time constant of the recovery variable, b $(b < 1)$ is responsible for characterizing the effect of the activation variable on the recovery process, and c is the offset.

The present study[80] utilizes numerical methods to examine the dynamics of a discrete neuron model (Chialvo map) and explores the effects of two coupling methods: nonlinear function coupling and linear coupling. Research has demonstrated that nonlinear coupling may result in the multiplication of invariant curves and the generation of quasi-periodicity, while linear coupling causes the system to exhibit three-frequency and four-frequency quasi-periodic states. Furthermore, linear coupling gives rise to the formation of resonant two-frequency and four-frequency state bands, which are delineated by the saddle-node bifurcation lines of invariant curves. The introduction of noise has been demonstrated to affect the bifurcation of the invariant circulation and to cause the quasi-periodic state to evolve as the noise intensity increases. In Ref. [81], based on FPGA experiments on a digital circuit platform, we successfully captured uniformly coexisting chaotic and superchaotic attractors, verifying the effectiveness of the superchaotic dynamics and encryption algorithms mapped by the 2D memristive Chialvo neuron.

Chialvo neuron models have been widely used in neurodynamics research,[80,82] chaos encryption and neuromorphic computation due to their simplicity and rich nonlinear features. For instance, its hyperchaotic property can be leveraged to generate pseudo-random sequences, thereby enhancing the security of image encryption and providing a theoretical foundation for the study of synchronization behavior in coupled neuron networks.

3.2. Rulkov neuron model

In 2001, Nikolai F. Rulkov's seminal contribution to the field was the proposal of a 2D discrete-time mapping system as a mathematical model of individual neurons.[9–11] This model was subsequently employed to simulate cluster discharges and chaotic dynamical behavior. The model comprises two variables, and its mathematical formulation is expressed as follows:

$$\begin{cases} x(n+1) = \frac{\gamma}{1+(x(n))^2} + y(n), \\ y(n+1) = y(n) - \mu(x(n) - \sigma). \end{cases} \tag{4}$$

In this model, $x(n)$ is the fast excitation variable, and $y(n)$ is the slow recovery variable. γ and μ are control parameters, and σ represents the external effect. The model's structural simplicity and computational efficiency enable its use in reproducing complex firing patterns of neurons while requiring minimal computational resources. This feature renders the model especially well-suited for simulation studies of large-scale coupled neural networks.

The subsequent step will be to present the relevant characteristics of Ruilov neurons. Reference [83] offers an electronic implementation of a chaos Rulkov neuron model based on maps. The model utilizes a circuit constructed with 18 MOS transistors to efficiently simulate large-scale neural networks, providing a novel approach to analyze the dynamics and synchronization characteristics of large-scale coupled neural networks in a single device. Subsequently, Ref. [84] investigated the impact of random perturbations on a 2D discrete Rulkov neuron model and discovered that noise in the vicinity of the Neimark–Sacker bifurcation point can induce a transition from a stable equilibrium state and minor oscillations to random bursts with substantial spikes. Reference [85] investigates the effect of non-Gaussian multiplicative noise on the discharge behavior of Rulkov neural systems through numerical simulation. It has been determined that there is an optimal noise intensity and correlation time that minimizes coherent resonance. This finding indicates that non-Gaussian multiplicative noise can induce coherent resonance in neurons.

3.3. Izhikevich neuron model

In 2003, Izhikevich proposed a discrete-time neuron model that integrates biological realism with computational efficiency to simulate the cluster firing behavior of neurons.[13] Its mathematical form is described as

$$\begin{cases} x(n+1) = 0.04x(n)^2 + 6x(n) + 140 - y(n) + I, \\ y(n+1) = 0.004x(n) + 0.98y(n), \\ \text{if } x(n) \geq 30, \quad \text{then } \begin{cases} x(n) \leftarrow c, \\ y(n) \leftarrow y(n) + d. \end{cases} \end{cases} \tag{5}$$

In this model, the membrane potential is denoted by the variable $x(n)$, the recovery variable is denoted by $y(n)$, the external input current is denoted by I, and the parameters c and d are used in the model to simulate the reset after a spike. The model displays a range of discharge modes, including pulsed discharges, periodic bursts, and chaotic bursts, indicative of a high degree of nonlinearity and complex topological features. This model is notable for its ability to reproduce the intricate dynamics of neuronal activity through a dual-time-scale nonlinear mapping system. Subsequently, Muni *et al.* contemplated discretizing the Izhikevich neuron network system under the influence of external electromagnetic fields to obtain the new, improved Izhikevich neurons.[86]

The subsequent step will be to present the relevant characteristics of Izhikevich neurons. Reference [87] extends the Izhikevich neural model to explore for the first time the resonant response of two chaotic states (turbulent and intermittent chaos) in chaotic resonance to weak signals under noise. It innovatively finds that intermittent chaos is more sensitive and faster than turbulent chaos in responding to noise signals during resonance. Subsequently, Ref. [88] puts forth a refined Izhikevich neuron model, one that is based on the CORDIC algorithm. This model utilizes adders and shifters to facilitate the implementation of efficient digital hardware. The model accurately simulates the original biological behavior and reveals the transition from periodic to chaotic behavior through numerical analysis.

3.4. Courbage model

The Courbage model, proposed by Courbage *et al.* in 2007,[14] is a seminal example of discrete neural dynamics, mathematically expressed as follows:

$$\begin{cases} x(n+1) = x(n) + F(x(n)) - \gamma H(x(n) - a) - y(n), \\ y(n+1) = y(n) + \varepsilon(x(n) - J), \end{cases} \tag{6}$$

where $x(n)$ represents the neuronal membrane potential, and $y(n)$ represents the recovery variable. γ, a, J, ξ are system parameters. $F(x) = x(n)(x(n) - a)(1 - x(n))$, $0 < a < 1$, and $H(x)$ is a piecewise function $H(x) = \begin{cases} 1, x \geq 0, \\ 0, x \leq 0. \end{cases}$

Subsequently, Zhu *et al.* proposed a novel discrete neural model based on the Courbage model.[89] This model synchronizes through coupled nodes and provides new dynamical properties and innovative simulation circuits for implementing neural networks. Reference [90] utilizes digital simulation to examine the impact of Gaussian white noise on the nonlinear dynamic characteristics of a two-dimensional neural network model. The study's findings indicate that noise can induce action potentials and random resonance in neurons with subthreshold input signals. The generation of random resonance is closely related to noise intensity and input signal frequency.

3.5. Discretized FitzHugh–Nagumo neuron model

In 1961, FitzHugh and Nagumo *et al.* proposed a simplified version of the FitzHugh–Nagumo (FHN) neuron model[91,92] based on the HH neuron model.[5] The following mathematical expression represents the FHN neuron model:

$$\begin{cases} \dot{x} = x - \dfrac{x^3}{3} - y + I, \\ \dot{y} = c(x + a - by), \end{cases} \tag{7}$$

where $x(n)$ represents the membrane potential, $y(n)$ represents the ionic current, I is the external stimulus, and constants a, b, c are the model parameters. Zhan *et al.* proposed Enon-like maps originating from the discrete-time FitzHugh–Nagumo model to analyze the underlying dynamical behavior.[93] The discrete mathematical model of the FHN neuron is as follows:

$$\begin{cases} x(n+1) = x(n) + h(x(n) - \dfrac{x(n)^3}{3} - y(n) + I), \\ y(n+1) = y(n) + h \cdot c(x(n) + a - by(n)), \end{cases} \tag{8}$$

where h is the iteration step.

The subsequent step will be to present the relevant characteristics of FHN neurons. Reference [94] uses the FHN model to reveal the phenomenon of noise-driven stochastic resonance. The model reproduces the statistical phase locking and the multimodal inter-spike interval histogram (ISIH) of neurons. As noise intensity increases, the ISIH peak exhibits stochastic resonance characteristics, altering the phase-locking phenomenon via limit cycles and underscoring the role of noise in neural coding. Subsequently, Ref. [95] introduces a basic dynamic model of neuronal activity with a focus on the behavioral characteristics of the FHN model. The FHN model reveals the characteristics and mechanisms of multi-threshold excitation in neurons. Reference [96] studies the dynamic behavior of coupled FHN neural systems using discrete, implicit mapping methods. It also verifies the theoretical results of neural network discharge and synchronization mechanisms through FPGA circuit implementation.

3.6. Discrete-time Hindmarsh–Rose neuron model

In 1984, Hindmarsh and Rose proposed a powerful model of HR neurons.[7] The model demonstrates both computational efficiency and the capacity to simulate a range of typical neuronal discharge patterns, including resting state, spiking discharge, and periodic bursting discharge. This renders it a valuable instrument for research in neuronal dynamics. The mathematical expressions are as follows:

$$\begin{cases} \dot{x} = x - \dfrac{x^3}{3} - y + I, \\ \dot{y} = c(x + a - by), \end{cases} \tag{9}$$

$$\begin{cases} \dot{x} = y - ax^3 + bx^2 + I - z, \\ \dot{y} = c - dx^2 - y, \\ \dot{z} = r[s(x - x_0) - z]. \end{cases} \tag{10}$$

Here, x, y, z represent dimensionless dynamic variables, and a, b, c, d, l, r, s, as well as x_0 are parameters. Li *et al.* examined the dynamics of a 2D discrete Hindmarsh–Rose model.[97] The model's 2D HR model expression is as follows:

$$\begin{cases} x(n+1) = x(n) + h(y(n) - ax(n)^3 + bx(n)^2 + I), \\ y(n+1) = y(n) + h(c - dx(n)^2 - y(n)). \end{cases} \tag{11}$$

Felicio *et al.* derived a discrete-time Hindmarsh–Rose neuron model by employing the forward Eulerian approach to the 3D maps derived from the continuous-time Hindmarsh–Rose neuron model.[98] The 3D expression is shown below:

$$\begin{cases} x(n+1) = x(n) + \delta(y(n) - ax(n)^3 + bx(n)^2 + I - z(n)), \\ y(n+1) = y(n) + \delta(c - dx(n)^2 - y(n)), \\ z(n+1) = z(n) + \delta r[s(x(n) - x(0)) - z(n)], \end{cases} \tag{12}$$

where $x(n)$ represents the membrane potential, $y(n)$ is the recovery variable associated with the fast current, and $z(n)$ represents the slowly varying adaptation current. a, b, c are the model parameters, and δ is an additional parameter.

The conversion of continuous models into mapping models can be achieved through the implementation of appropriate discretization methods, such as the Euler method. However, by refining the discrete time step size, mapping models can also approximate the behavior of continuous models. When the Euler algorithm is applied to the oscillator model, a new bifurcation parameter is introduced: the time step. In order to mitigate this effect, it is possible to reduce the time step, Δt, thus decreasing the discretization error and thereby minimizing the occurrence of bifurcation phenomena caused by the time step. In comparison with continuous neuron models, discrete neuron models are better suited to digital computing environments, improving computational efficiency and stability. For instance, the discrete processing of HR neurons has been demonstrated to reduce computational complexity, enhance the model's feasibility in actual hardware, and improve simulation and optimization flexibility.

The subsequent step will be to present the relevant characteristics of HR neurons. Reference [99] examines how noise affects coherent resonance and synchronization in the HR neuron model. The study finds that, above a critical noise intensity, decoupled and weakly coupled neurons completely synchronize. Additionally, the study finds that bursting neurons are more susceptible to noise-induced synchronization than pulsing neurons. These findings highlight the enhancing effect of noise on the synchronization sensitivity of neurons. Reference [100] implemented the HR neuron model via circuit simulation, verified its physical feasibility with Multisim, and demonstrated favorable projection synchronization effects.

Table 1. Analysis of the characteristics of different discrete neurons.

Neuron	Chialvo neuron	Rulkov neuron	Izhikevich neuron
Physical properties	x: membrane potential y: recovery variable $x^2\,e^{y-x}$ simulates the activation k: spike input current a, b, c: parameters of ion channel dynamics	x: fast variable y: slow variable $\frac{2}{1+x(n)^2}$: s-shaped saturation function σ: firing threshold voltage μ: calcium pump rate	x: membrane potential y: recovery variable $0.04x^2$: the quadratic activation function I: direct current input c, d: simulate the reset after a spike
Statistical properties of signals	typical firing patterns: chaotic firing, periodic peak firing ISI characteristics: multi-peak distribution, single-peak narrowband	typical firing patterns: bursting, chaos oscillation ISI characteristics: dual-peak distribution	typical firing patterns: impulse firing, clustered firing, chaotic cluster firing ISI characteristics: bimodal distribution, multimodal index distribution
Digital circuit implementation	core modules: exponential function, multiplier	core modules: square calculator, state controller	core modules: bitwise logic, multiplication tree

Table 2. Analysis of the characteristics of different discrete neurons.

Neuron	Courbage neuron	Discretized FHN neuron	Discrete-time Hindmarsh–Rose neuron
Physical properties	x: membrane potential y: slow recovery variable γ, a, J, ξ: system parameters $F(x)$: cubic nonlinear term $H(x)$: piecewise function	x: fast variable y: slow variable h: dissipation step size a, b, c: model parameters	x: membrane potential y: fast current z: slow current r: calcium ion accumulation rate s: calcium sensitivity a, b, c: model parameters I: direct current input
Statistical properties of signals	typical firing patterns: periodic bursting, chaotic bursting ISI characteristics: bimodal distribution	typical firing patterns: single peak discharge, periodic bursting ISI characteristics: narrowband Gaussian distribution, bimodal distribution	typical firing patterns: complex bursting, chaotic rhythm ISI characteristics: narrowband Gaussian distribution, bimodal distribution
Digital circuit implementation	core modules: cubic operation, random generator	core modules: cubic operation, multiplier-accumulator cascade	core modules: 3D state machine

To better demonstrate and analyze the characteristics of different discrete neurons, we conduct the analysis from three aspects (physical properties, statistical properties of signals, and digital circuit implementation), as shown in Tables 1 and 2.

4. Research and applications of nonlinearity in discrete neurons

The theoretical value of discrete neurons lies in their ability to explain the complexity of intricate neural systems. Beyond theoretical applications, the field of nonlinear circuits and their practical applications in discrete neurons is gaining increasing attention.

4.1. Research on discrete neuron nonlinear circuits

At present, there are numerous studies on discrete neurons in nonlinear circuits. Discrete-time neuron circuits represent a pivotal research trajectory at the nexus of neural networks and signal processing. Reference [101] primarily introduces the discrete-time dynamic K-winners-take-all (DTDKWTA) neural circuit. The dynamic offset mechanism of DTDKWTA adjusts the global threshold iteratively through difference equations to select the top K largest inputs. The output satisfies the KWTA characteristic of 'the first K are positive, and the rest are negative'. The objective is to identify the top K largest inputs through the dynamic offset mechanism. The specific method is as follows: first, unify the input shift to obtain $c_n = a_n - A_{\min}$, here, a_n represents the n-th input value, $A_{\min}$ is the minimum value among all inputs, and c_n is the input after translation; then, in each iterative step, use the difference equation $x^{(k+1)} = x^{(k)} - A \cdot \alpha^k \cdot \mathrm{sgn}(R(x^{(k)}))$ to continuously adjust the global threshold value $x^{(k)}$, where $R(x^{(k)})$ is the residual function, and α is the decay coefficient. The final output is $b_n = c_n - x$, here, b_n is the output of the n-th neuron, and x is the stable threshold value after convergence. If $b_n > 0$, then the corresponding input belongs to the top K largest (winners); otherwise, it is a loser. On this basis, multiple KWTA circuits can run in parallel, gradually generating a sorting matrix, and ultimately completing the overall sorting with a complexity of $O(N)$, which is much better than the traditional $O(N^2)$ algorithm. Reference [102] proposes a parallel sorting system, the theoretical framework for which is the discrete-time KWTA neural circuit model. The system does not require information about the input range and converges solely based on the minimum input difference. This offers advantages such as fast convergence, moderate complexity, and arbitrary finite resolution. In Ref. [103], the authors studied and implemented the discrete HNN (DHNN) using the superconductor single flux quantum (SFQ) circuit. The article designed a DHNN circuit using two modes, each with eight elements, enabling the circuit to operate at a clock frequency above 50 GHz.

4.2. Research on the applications of discrete neurons

The distinctive value and potential exhibited by discrete neurons in neural network modelling, image processing, cryptography and hardware implementation constitute a significant characteristic. The ensuing discourse will address these domains sequentially.

4.2.1. Model units in neural networks

Discrete neurons are considered to be the fundamental building blocks of neural networks. In structures such as multilayer perceptrons, convolutional neural networks, and recurrent neural networks, neurons operate in a discrete manner. The subsequent procedure involves the implementation of weighted summation on the inputs, followed by the application of nonlinear transformation through activation functions. Reference [104] introduces two identical Rulkov neurons to form a new discrete neural network and explores the dynamic behavior of the neural network. Reference [105] constructs two types of neuron models, including the single discrete memristive synaptic neuron model and the double discrete memristive synaptic neuron model, and analyzes their complex dynamical behaviors.

4.2.2. Image processing and encryption

The fundamental objective of image encryption is to render encrypted images difficult to decipher through statistical analysis and visual inspection by employing obfuscation and diffusion techniques. Conventional methods frequently depend on chaotic systems. Discrete neuron models, on the other hand, are mathematical neural models capable of generating complex nonlinear dynamic behavior, serving as viable alternatives or enhancements to chaotic mappings. Reference [106] puts forward a new discrete memristor-based heterogeneous dual-neuron network (MHDNN) and a new encryption scheme for multi-scenario police images based on MHDNN. Reference [107] proposes a novel parallel color image encryption algorithm based on a two-dimensional logistic-Rulkov neural network mapping. This algorithm enhances security and optimizes computational efficiency.

4.2.3. Analog circuit implementation and FPGA hardware implementation

It is crucial to recognize that neuron models are inherently nonlinear systems, and their discrete forms display dynamic characteristics such as chaos and bifurcation during the iterative process. The field of analog circuits has seen significant advancements. Reference [108] introduces a discrete mapping with intricate neuronal dynamics, realized through analog circuits and experimentally confirmed, thus highlighting its potential for chaotic and hyperchaotic behavior as well as synchronization properties. The implementation of most

discrete neurons and systems is commonly achieved using analog circuits or digital signal processors (DSPs). However, these systems can encounter various issues, including limitations in accuracy, concerns about portability, and extended development cycles. In contrast, field programmable gate arrays (FPGAs) offer flexibility, high parallelism, and high integration, facilitating more efficient and convenient hardware implementations. Reference [109] examines the dynamics in discrete neural networks affected by memristor-based delayed feedback, successfully implementing the model on an FPGA platform. The experimental results obtained have substantiated the hypothesis that the proposed approach is viable and effective. Reference [110] employed discrete mapping techniques to clarify bifurcation and discharge characteristics in the three-dimensional Hindmarsh–Rose model under periodic excitation. The use of FPGA circuit implementation to verify these findings showed a high degree of alignment with theoretical results.

5. Neural network model

In this section, three distinct neural network models will be introduced, followed by a presentation of articles based on their respective models.

5.1. DRNN neural network model

Chen *et al*. deduced the chaotic structure of a discrete-time recurrent neural network (DRNN)[111] by applying Marotto's theorem and provided the following description of the DRNN:

$$\begin{cases} y_i(n+1) = k y_i(n) + \sum_{j=1}^{t} w_{ij} x_j(n) + a_i - w_{ii}(n) a_{0i}, \\ x_i(n) = f(y_i(n)) = \frac{1}{1+\mathrm{e}^{-y_i(n)/\varepsilon}}. \end{cases} \tag{13}$$

Among the variables of interest, the random variable $x_i(n)$ corresponds to the output of the i-th neuron at time step n, while the deterministic variable $y_i(n)$ represents the internal state of the i-th neuron. The dynamic evolution of the output is determined by the internal state of the neuron. w_{ij} represents the connection weight from neuron j to i. a_i represents the external input bias, providing constant excitation or inhibition; while a_{0i} indicates the self-loop bias, used to adjust the strength of the neuron's own feedback. k represents the neural membrane amplification coefficient, controlling the degree of retention of the internal state; ε is the steepness parameter of the sigmoid function, determining the non-linearity strength of the activation function. Lai *et al*. constructed discrete neural network models[57] with integrated discrete memristors to simulate the effects of electromagnetic radiation on neurons and synaptic dynamics, revealing new chaotic behaviors and heterogeneous attractor phenomena. Cheng *et al*. developed two distinct models of discrete amnestic synaptic neurons,[112] thereby elucidating the intricate dynamic behavior of the coupling strength parameter and the nerve membrane amplification coefficient.

5.2. Discrete Hopfield neural network model

Recent findings in the field of neuroscience have revealed that the human brain's nervous system comprises a vast network of interconnected neurons,[113] with estimates suggesting that there are tens of billions of these cells. These neurons are linked through a complex network of synapses, facilitating an intricate and dynamic interaction that underlies various cognitive functions.[114,115] Among them, the most prominent is the HNN neural network. HNN is a single-layer recurrent neural network, and its most significant feature is the ability to simulate and implement associative memory, acting like an intelligent memory that can recall complete content from "fuzzy fragments." The following are three forms of HNNs.

5.2.1. Single integer order hopfield model

The dynamic equation for a single discrete-time Hopfield neuron can be expressed as follows:

$$C_i \Delta x_i(n) = -\frac{x_i(n)}{R_i} + \sum_{j=1}^{m} w_{ij} G(x_j(n)) + I_i. \tag{14}$$

Here, $\Delta x_i(n)$ is the forward difference of the voltage $x_i(n)$ across the capacitor C_i at the current time of dispersion, R_i and I_i are the membrane resistance and input bias current, respectively. The activation function of a neuron, typically represented by the hyperbolic tangent, is denoted by $G(x_j(n))$, and the synaptic weight, which indicates the influence of other neurons on the current neuron, is denoted by w_{ij}. A substantial corpus of literature exists on this subject. For example, Ma *et al*. proposed a novel pixel-relative bit-plane chaotic random diffusion algorithm based on a self-feedback DHNN that exhibits high diffusion capacity, security, and enhanced attack resistance.[59] Bao *et al*. advanced a 2D discrete model of a two-neuron HNN with a sinusoidal activation function. This model revealed the complex coexistence dynamics behavior and was used to design a pseudo-random number generator with high stochasticity and strong anti-chaos degradation properties.[116]

5.2.2. Single-fractional order Hopfield model

Fractional-order discrete Hopfield networks overcome the limitations of traditional integer-order models by introducing non-integer-order calculus, which offers clear advantages in terms of dynamical complexity and stability. These networks are particularly well-suited for application scenarios that necessitate high-dimensional dynamics with long-term dependence modeling, such as chaotic encryption and neuromorphic computing. The following DHNN model has fractional order

constants:

$$\begin{cases} {}^{\mathrm{C}}\Delta_{\theta}^{\eta} y_k(s) = -v_k y_k(s+\eta-1) \\ \qquad\qquad +\sum_{l=1}^{n} E_{kl}\phi_l(y_l(s+\eta-1)) + X_k, \\ \forall s \in \mathbb{N}_{\theta+1-\eta}, \forall k = 1,2,\ldots,n. \end{cases} \tag{15}$$

The starting point is $\theta \in \mathbb{R}$, and $E = (E_{kl})_{n\times n}$ represents the weight matrix connecting the specified neurons. $v_k > 0$ is the parameter of the self-regulating neuron. X_k is the external input. The fractional difference operator ${}^{\mathrm{C}}\Delta_{\theta}^{\eta}$ is used for the η-Caputo fractional difference operator. The following is the definition of ${}^{\mathrm{C}}\Delta_{\theta}^{\eta}$:

$$\begin{aligned} {}^{\mathrm{C}}\Delta_{\theta}^{\eta} g(s) &= \Delta_{\theta}^{-(j-\eta)} \Delta^{j} g(s) \\ &= \frac{1}{\Gamma(j-\eta)} \sum_{k=\theta}^{s-(j-\eta)} \frac{\Gamma(s-k)}{\Gamma(s-k-j-\eta+1)} \Delta^{j} g(k). \end{aligned} \tag{16}$$

Wang *et al.* were the first to apply the truncation method to Caputo fractional order operators.[73] They also constructed a fractional-order DHNN based on a memristor and verified the attractor coexistence characteristics with FPGA hardware. The first 2D fractional-order DHNN fusing constant-order and variable-order fractional calculus was constructed by Al-Husban *et al.*[58] He *et al.* constructed for the first time a 4D fractional-order DHNN model,[117] fusing fractional-order calculus with nonlinear self-synaptic weight functions, to reveal the mechanism of chaos-steady state dynamic transition under external stimuli.

5.2.3. Variable order Hopfield model

Hioual *et al.* proposed a variable order fractional discrete neural network model,[118] which is also referred to as the following variable order Hopfield model:

$$\begin{cases} {}_{h}^{\mathrm{C}}\Delta_{t_0}^{v_0} x(t) = -Ax(t+v_0 h) + Bf(t; x(t+v_0 h)), & t \in \{t_0 + (1-v_0)h, \ldots, t_l - v_0 h\}, \\ {}_{h}^{\mathrm{C}}\Delta_{t_l}^{v_1} x(t) = -Ax(t+v_1 h) + Bf(t; x(t+v_1 h)), & t \in \{t_l + (1-v_1)h, \ldots, t_{2l} - v_1 h\}, \\ \vdots \\ {}_{h}^{\mathrm{C}}\Delta_{t_{kl}}^{v_k} x(t) = -Ax(t+v_k h) + Bf(t; x(t+v_k h)), & t \in \{t_{kl} + (1-v_k)h, \ldots, t_{(k+1)l} - v_k h\}. \end{cases} \tag{17}$$

In this context, t falls within the interval $(hN)_{t_0+(1-v_k)h}$, where $0 < v_k \le 1$ for $k = 0,\ldots,m-1$, and m denotes the number of intervals. The term ${}_{h}^{\mathrm{C}}\Delta_{t_{kl}}^{v_k}$ signifies the variable-order fractional step Caputo h-difference operator with order v_k. The state vector $x(t) = (x_1(t),\ldots,x_p(t))^{\mathrm{T}}$ resides in $\mathbb{R}^p$, representing the state at time t, with p being the dimensionality. The matrix $A = \mathrm{diag}(-a_1,\ldots,-a_p)$ is defined with $a_i > 0$, indicating that when neurons are disconnected from the network, their membrane potentials are reset to the resting state. The matrix $B \in \mathbb{R}^{p\times p}$ represents the connection weights. Lastly, $f(t,x(t))$ belongs to $C\left((hN)_{t_0+(1-v_k)h}, \mathbb{R}^p\right)$, which is the activation function.

5.3. Discrete memristive cellular neural networks

Cellular neural networks (CNNs) are locally coupled dynamical systems consisting of interconnected cellular units that interact only with neighboring units.[119] Unlike traditional artificial neural networks, which are fully connected, CNNs' parallel computing capability enables them to demonstrate unique advantages in image processing, signal analysis, and complex system models. Grassi designed discrete-time CNNs with globally asymptotically stable equilibrium points.[120] The model is as follows:

$$\begin{cases} x_{ij}(n+1) = \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l) v_{kl}(n) + \sum_{C(k,l)\in N_u(i,j)} B(i,j;k,l) u_{kl}(n) + I, \\ v_{ij}(n) = f(x_{ij}(n)) = \frac{1}{2}\left(\left|x_{ij}(n)+1\right| - \left|x_{ij}(n)-1\right|\right), \quad i = 1,\ldots,\bar{M};\ j = 1,\ldots,\bar{N}. \end{cases} \tag{18}$$

It is imperative to note that A and B are designated as feedback operators and control operators, respectively. Feedback operator A describes the interaction between a cell $C(i,j)$ and the output v_{kl} of each cell $C(k,l)$ that belongs to the neighborhood $N_r(i,j)$. Similarly, control operator B describes the interaction between the cell $C(i,j)$ and the input u_{kl} of each $C(k,l)$ within the neighborhood $N_g(i,j)$. Here, u represents the input, v represents the output, x denotes the state, and I is the cell bias. Wang *et al.* introduced heterogeneous activation functions into discrete CNNs for the first time and proposed a discrete memristive CNN based on sinusoidal function memristors.[121] Select an M by N CNN, where the cell at the i-th row and j-th column is denoted as $C(i,j)$. Each cell connects with its neighboring cells. $N(i,j)$ represents the set of neighboring cells to cell $C(i,j)$, and r denotes the radius of the neighborhood, defined as follows:

$$\begin{aligned} N_r(i,j) = \{&C(k,l) \mid \max(|k-i|, |l-j|) \le r\}, \\ &1 \le k \le M;\ 1 \le l \le N. \end{aligned} \tag{19}$$

6. Discrete memristive neural network

Ordinary neural network systems typically study the dynamics of the system by regulating the synaptic strength or external input current. However, these neural networks do not

take into account the memory properties of biological neural systems. Memristors, with their non-volatile and low-power characteristics, have the potential to address the limitations of neural networks. Previously, Lin *et al*. described a continuous-type memristive neural network model.[122] In this article, we will present three discrete memristive neural network models.

6.1. Discrete memristive autosynaptic model

Memristors are able to efficiently model synapses by introducing them into a traditional discrete single-neuron model.[123] Within the framework of the single neuron model, a memristor-based self-synaptic neural network is built.[124] Detailed analyses are given below:

$$\begin{cases} x(n+1) = f(x(n), y(n), \ldots) - I_{\text{aut}}, \\ y(n+1) = f(x(n), y(n), \ldots), \\ \qquad \vdots \\ \varphi(n+1) = f(\varphi(n), x(n)). \end{cases} \tag{20}$$

where $x(n)$ represents the membrane potential of the neuron, $W(\varphi(n))$ represents the memristor's conductance, ρ represents the feedback of the self-recovery current, and $\varphi(n)$ represents the internal state of the memristor, memristive self-hysteresis current $I_{\text{aut}} = \rho x(n) W(\varphi(n))$. The conceptual diagram of the discrete memristive autosynaptic model is shown in Fig. 2. Recently, Wang *et al*. proposed a 2D discrete adaptive synaptic neuron model based on Eulerian discretization,[125] demonstrating complex nonlinear dynamical behavior. A number of articles have been published on the subject of such models.[126,127]

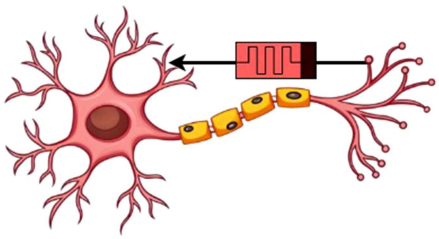


Fig. 2. Conceptual diagram of the discrete memristive autosynaptic model.

The neuron map model can be defined as an energy function similar to that of an oscillator model. In the memristor model, the internal state $\varphi(n)$ is analogous to the displacement or state variable in the oscillator model, while the conductivity of the memristor $W(\varphi(n))$ is similar to the force or potential energy in the oscillator. By introducing the potential energy term, an energy function can be constructed, where the internal state of the memristor and the feedback mechanism together determine the system's 'energy'. The stability of the system is described by the function f and combined with the energy function to ensure that the energy changes comply with physical laws, maintaining the system's rational behavior and stability.

6.2. Memristive synaptic coupling dual neuron model

Neurons are interconnected via synapses, and the two neuron models form a feedback coupling via memristors. These devices mimic the adjustable synaptic weights property, thereby achieving dynamic connections analogous to those observed in biological neural networks. The present paper will analyze the following memristive synaptic coupling dual neuron model in great detail:

$$\begin{cases} x_1(n+1) = f(x_1(n), y_1(n), \ldots, I_{\text{ext1}}) - I_{\text{syn}}, \\ y_1(n+1) = f(x_1(n), y_1(n), \ldots), \\ \qquad \vdots \\ x_2(n+1) = f(x_2(n), y_2(n), \ldots, I_{\text{ext2}}) + I_{\text{syn}}, \\ y_2(n+1) = f(x_2(n), y_2(n), \ldots), \\ \qquad \vdots \\ \varphi(n+1) = f\big(\varphi(n), (x_1(n) - x_2(n))\big). \end{cases} \tag{21}$$

Here, $x_1(n)$ and $x_2(n)$ represent the membrane potentials of neuron 1 and neuron 2, respectively. $W(\varphi(n))$ represents the memristor conductance. ρ represents the feedback gain of the self-recovery current. When two neurons are coupled via a memristor, the memristive synapse is able to modulate interneuronal signaling and generate nonlinear synaptic currents. Based on the nonlinear and memory properties of the memristor, this synaptic current can be expressed as $I_{\text{syn}} = \rho(x_1(n) - x_2(n)) W(\varphi(n))$. The coupled structure can simulate the process of information transmission from one neuron (N1) to another neuron (N2), as shown in Fig. 3. Based on this type of memristive neuron model, it is possible to study many complex chaotic dynamics.[128–131] For example, discmemristoristor-coupled heterogeneous neural networks are constructed by coupling KTZ neurons and contraindicated learning neurons to locally active discrete memristors.[125] Furthermore, designing a discrete memristor-coupled Chialvo neuron map yields complex dynamical phenomena such as chaos, hyperchaos, and heterogeneous multistability.[132]

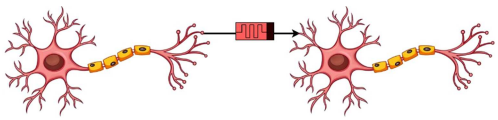


Fig. 3. Conceptual diagram of the memristive synapse-coupled two-neuron model.

The neuron map model may be formulated through an energy function that parallels the structure of an oscillator model. In this model, the neuronal membrane potentials $x_1(n)$ and $x_2(n)$ are analogous to the displacement or state variables in an oscillator, while the conductivity of the memristor $W(\varphi(n))$ and the feedback mechanism ρ correspond to the force or potential energy in an oscillator. The energy function can be

constructed by combining potential energy terms and kinetic energy terms, with the feedback mechanism influencing the energy flow of the system.

6.3. Discrete neuron model under electromagnetic radiation

In an industrial context full of electromagnetic radiation, numerous physical and biological experiments have confirmed that the activity of biological nervous systems is frequently significantly impacted by internal transmembrane voltage differences and external electromagnetic radiation.[122,133,134] To characterize the modulation of neural dynamics by electromagnetic fields, magnetically controlled memristors are introduced into conventional discrete neuron models to describe electromagnetically induced currents.[135,136] When a neuron is in an electromagnetic field, the magnetic flux distribution and density around its cell membrane change. This alters the neuron's electrical activity characteristics. A detailed description of the discrete neuron model under electromagnetic radiation is given below:

$$\begin{cases} x(n+1) = f(x(n), y(n), \ldots, I_{\text{ext}}) - I_{\text{ER}}, \\ y(n+1) = f(x(n), y(n), \ldots), \\ \qquad \vdots \\ \varphi(n+1) = f(\varphi(n), x(n), \ldots). \end{cases} \tag{22}$$

Here, φ describes the magnetic flux on the neuron membrane and k represents the coupling strength between the membrane potential and magnetic flux, electromagnetic induction current $I_{\text{ER}} = kW(\varphi)x(n)$. The simplified diagram of the discrete neuron (N1) model under electromagnetic radiation is shown in Fig. 4. Current researchers have conducted more studies in this area.[137–139] For example, introducing a novel Rulkov neuron mapping in combination with a charge-controlled memristor reveals phenomena such as mixed-mode discharges, burst discharges, and coexisting discharges, and simulates the effects of electromagnetic radiation.[140]

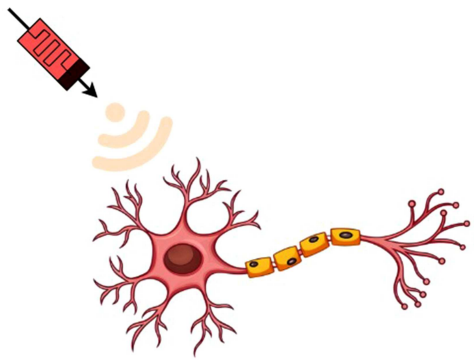


Fig. 4. Conceptual diagram of discrete neuron model under electromagnetic radiation.

An energy function, comparable to that used in oscillator models, can be employed to define the neuron map model. In this model, the neuronal membrane potentials $x_1(n)$ and $x_2(n)$ are analogous to the displacement or state variables in an oscillator, while the conductivity of the memristor $W(\varphi(n))$ and the feedback mechanism ρ correspond to the force or potential energy in an oscillator. The energy function can be constructed by combining potential energy terms and kinetic energy terms, with the feedback mechanism influencing the energy flow of the system.

In order to analyze the physical reliability of the aforementioned discrete neuron mapping model, the following three aspects must be considered.

(1) The physical reliability of the discrete neuron mapping model is validated by comparing its simulation results, such as bifurcation diagrams, phase diagrams and time series, with experimental data from real physical systems. To illustrate this, consider a qualitative comparison between the chaotic attractor structure simulated by the model and the phase diagram observed on an oscilloscope for an actual circuit based on memristors. When the geometric patterns and bifurcation sequences of these structures exhibit a high degree of consistency, the model provides strong evidence that it accurately reflects the dynamical behavior of the physical system.

(2) The reliability of a model is evidenced by its adherence to fundamental physical constraints, particularly energy conservation laws. As a system incorporating passive memristive elements, the model must exhibit energy dissipation, meaning its constructed discrete energy function should decay over time or remain bounded. This ensures that state variables do not diverge to infinity without physical justification, thereby conforming to the real-world principles of energy conservation and dissipation.

(3) The reliability of the model is validated through sensitivity tests on parameters and initial conditions. Within stable parameter regions, minor perturbations of parameters should only induce smooth variations in outputs. Conversely, within chaotic regions, the model must exhibit extreme sensitivity to initial conditions. This behavior aligns with the characteristics of genuine chaotic systems, thereby demonstrating that the model's responses conform to physical intuition.

To better demonstrate the chaotic dynamics of discrete memristive neural networks, we analyze the dynamic phenomena under different conditions, as shown in Table 3. These networks exhibit more complex dynamic phenomena, particularly chaotic dynamics such as chaotic attractors, superchaotic attractors, coexisting attractors, hidden attractors, extreme multistability, and synchronism. Therefore, researching memristor neural networks is significant for advancing computational neurodynamics.

Table 3. Comparison of different discrete neural networks regarding different dynamic phenomena.

References	Chaos	Hyperchaos	Periodic	Multistability	Coexisting attractors	Hidden attractors	Extreme multistability	Synchronization
[141]	Yes	Yes	Yes	Yes	Yes	Yes	No	No
[142]	Yes	Yes	Yes	Yes	Yes	No	No	No
[143]	Yes	No	Yes	No	No	No	No	Yes
[144]	Yes	Yes	Yes	Yes	Yes	Yes	No	No
[145]	Yes	No	No	Yes	Yes	No	No	No
[146]	Yes	Yes	Yes	No	No	No	No	Yes
[147]	No	No	No	No	No	No	No	Yes
[125]	Yes	Yes	Yes	Yes	Yes	No	Yes	No
[126]	Yes	Yes	Yes	Yes	Yes	No	Yes	No
[127]	Yes	Yes	Yes	Yes	Yes	No	Yes	No
[128]	Yes	Yes	Yes	Yes	Yes	No	No	No
[129]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
[130]	Yes	No	Yes	No	No	No	No	Yes
[131]	Yes	Yes	Yes	Yes	Yes	No	No	No
[135]	Yes	No	Yes	Yes	Yes	No	No	No
[148]	Yes	Yes	Yes	Yes	No	No	No	No

7. Synchronization, speckle patterns, and chimera states in discrete coupled neural networks

Neuronal synchronization is defined as the phenomenon in which a group of neurons discharge or activate in a temporally coordinated manner. In the context of a neural network, neuronal synchronization is defined as the occurrence of synchronized discharge patterns among multiple neurons under specific conditions. Reference [149] investigates the synchronization phenomenon in coupled neural networks and finds that the network topology, synaptic delay, and synaptic type significantly influence the synchronization of neural networks. Appropriate synaptic delay has been demonstrated to promote the synchronization of neural networks with different types of neurons, especially under the regulation of synaptic delay, membrane area, and synaptic type. In such cases, the synchronization of neural networks gradually increases and stabilizes. Reference [131] investigates the synchronization behavior of coupled neurons by establishing a coupling between two Rulkov maps and discrete memristors. The model demonstrates the capacity to generate complete and delayed synchronous discharges, and it is evident that the synchronization transition is substantially influenced by the memristor coupling strength and the initial state. There are also many papers on synchronization research, as shown in Table 3.

Neural network patterns are defined as the specific structures or patterns generated by neural networks during the processing of images, signals, or data through their weights, activation functions, and other computational methods. Reference [150] investigates the firing behavior of neurons and the evolution of spatiotemporal spotty patterns in ring–lattice coupled neural networks. The findings indicate that the firing behavior and spotty pattern evolution are predominantly influenced by the coupling strength between the two networks.

The state of chimera is a complex dynamic phenomenon that typically occurs in systems composed of multiple interconnected spins, neurons, or oscillators. This term refers to a situation in which certain components of a system exhibit synchronous behavior, while others exhibit asynchronous behavior. Reference [151] investigates the chimera states generated by FHN oscillators in a double-layer network, elucidating the various types of chimera states and their multistability characteristics. Reference [152] identifies chimera states in 2D and 3D HR oscillator networks under different coupling schemes, providing substantial evidence to support the application of chimera phenomena in neuroscience and verifying the existence of chimera states in coupled bistable elements.

8. Conclusions and outlook

This paper reviews existing discrete neuron models and the theoretical advantages of discrete neural networks in simulating complex neural dynamics, and provides a systematic summary of their relevant characteristics. It also highlights the key role of memristors in introducing biological memory mechanisms due to their mnemonic and non-linear characteristics. By incorporating memristors into discrete neural networks, including HNNs and their fractional-order variants, CNNs, etc. and discrete neurons, multiple classes of amnestic neural models have been constructed. The present study provides evidence of complex dynamic behaviors, including hidden attractors, multistability, and extreme multistability. Furthermore, it conducts an in-depth analysis of more complex dynamical phenomena, such as synchronization phenomena, spotted patterns, and singular states in discrete coupled neural networks. The focus of future research will be on the collaborative innovation of devices, architectures, and algorithms. The aim of this research will be to transform discrete memristor neural networks from laboratory prototypes into practical hardware platforms, these platforms are set to propel the development of edge artificial intelligence and autonomous sys-

tems. Concurrently, the enhancement of the nonlinearity of memristors will precipitate an escalation in the complexity of memristor neural networks, thereby engendering the support of low-power edge devices and the realization of more efficient dynamic real-time processing and learning capabilities.

The following section outlines the prospects that have been formulated on the basis of the three primary research components of this paper.

(1) Research on discrete neurons and their applications. Discrete neurons still have great potential in understanding and simulating complex neural dynamics. Their advantage lies in the simplicity and efficiency of computation, making them particularly suitable for applications that require rapid response and low power consumption. However, the current models still fall short in exploring the unknown characteristics and details of biological neurons and often fail to truly reproduce complex neural firing patterns. In the future, if the model design can be closer to biological mechanisms and deeply integrated with new hardware platforms, it is believed that discrete neurons will not only play a greater role in neural simulation research, but also show more prominent application value in the fields of biomedical and artificial intelligence.

(2) Research on discrete neural networks. Discrete neural networks, with their high degree of nonlinearity and multistable dynamics arising from the interaction of multiple neurons, offer a promising avenue for pattern recognition and the control of complex systems. However, in the course of in-depth research and application, they also face challenges that cannot be ignored: on the one hand, the scalability of network structures remains limited; on the other hand, the interpretability of complex dynamical processes still falls short. In anticipation of future developments, it is recommended that research efforts be concentrated on two primary domains. First, the enhancement of structural flexibility and efficiency of networks is achieved through the leveraging of topological optimization. Second, the enhancement of analytical comprehension of intricate dynamics is imperative to optimize the theoretical interpretability of models.

(3) Research on discrete memristive neural networks. The integration of memristors within discrete neural networks confers upon them memory characteristics inspired by biological systems, in addition to the prospect of hardware implementation. This integration renders the networks well-suited for applications such as low-power edge intelligence and real-time learning, which are of significant potential. Nevertheless, difficulties persist in the precise control of the nonlinear properties of memristors. It is recommended that future research focus on the exploration of more effective nonlinear control and adjustment mechanisms. Through in-depth investigation of discrete memristive neural networks, it is expected that they will demonstrate broader application prospects in the field of edge artificial intelligence.

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