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A routing strategy for non-uniform spatial network

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As network scales continue to expand, congestion has emerged as a critical issue in the study of complex networks, particularly in spatial networks such as transportation, aviation, and communication systems, where transmission is the primary function. To address this challenge, various routing strategies have been proposed to alleviate congestion by adjusting transmission paths. However, most of these strategies are based on network models that assume a uniform spatial distribution of nodes, which fails to accurately represent the non-uniform distributions observed in real-world networks. In this paper, we construct a more realistic non-uniform spatial network model and propose a novel routing strategy, termed the FH routing strategy, which integrates distance-based degree and harmonic centrality. Simulation results show that the FH strategy effectively avoids high-load nodes, promotes a more balanced load distribution, and significantly improves traffic throughput compared to traditional routing strategies. These findings provide theoretical support and practical guidance for optimizing information transmission in real-world non-uniform spatial networks.

Keywords: spatial network, routing strategy, network congestion, complex network

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1. Introduction

The concept of complex networks can be traced back to the late 1950s, when the random graph model proposed by Erdös and Rényi[1] laid the theoretical foundation for complex network analysis. Subsequently, the discovery of the small-world phenomenon[2] and the scale-free property[3] sparked widespread interest and led to extensive research into the structural characteristics and dynamic behaviors of complex networks. In recent years, the theory of complex networks has found broad applications across various fields, particularly in transportation networks,[4] aviation networks,[5] communication networks,[6] and wireless sensor networks.[7] These systems can be effectively abstracted as complex networks for analytical purposes, and their transmission performance is closely linked to various aspects of daily life. As a result, improving the transmission performance of such networks has become a key focus in current research on complex networks.

Network congestion remains one of the major challenges in enhancing the transmission performance of networks, being closely associated with both network topology[8] and routing strategies.[9] However, direct modification of network structures is often costly and difficult to implement in practice. Consequently, optimizing routing strategies presents a more practical and effective approach to mitigating network congestion. By designing efficient routing strategies, network throughput can be significantly improved, congestion can be alleviated, and overall transmission efficiency can be enhanced.

In general complex networks, the shortest path (SP) routing strategy is commonly employed due to its technical simplicity and low cost. However, the use of the SP routing strategy often leads to overloading of hub nodes, causing congestion across the entire network. To address this issue, numerous studies have been conducted on routing strategies for complex networks.[10–22] Yan *et al.*[16] noted that nodes with the highest degree are highly vulnerable to traffic congestion and proposed using paths with the smallest sum of node degrees to route packets, which can increase network throughput by a factor of 10. Zhang *et al.*[17] introduced a probabilistic routing strategy that makes more efficient use of hub routers, thereby improving overall network traffic. Jiang *et al.*[18] proposed a dynamic routing strategy based on local information, using betweenness centrality as the link weight to reduce the load on central links and improve network capacity.

These studies primarily investigate routing strategies within topological networks. A topological network is a network model that exclusively captures the connectivity relationships among nodes, without accounting for their physical locations or geometric distances.[23] Typical topological network models include the Erdös–Rényi (ER) random graph,[1] the Watts–Strogatz (WS) small-world network,[2] and the Barabási–Albert (BA) scale-free network.[3] However, many real-world networks are spatially constrained, and such networks are referred to as spatial networks. Unlike topological networks, spatial networks are more representative of practical applications, such as mobile communication base station networks,[24] urban transportation networks,[25] and logistics distribution networks.[26]

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In this context, several studies have explored routing strategies for spatial networks. Guan *et al.*[19] proposed a routing strategy that simultaneously considers the Euclidean distance between nodes and the betweenness centrality of edges to optimize network traffic, effectively balancing these two factors in path decisions and improving network load distribution. Lin *et al.*[20] optimized the use of shortest-path routing by selecting the Euclidean path with the shortest distance for data transmission, significantly improving throughput in spatial networks. Additionally, Lin *et al.*[21] introduced the harmony-degree (HD) routing strategy, which incorporates both harmonic centrality and node degree, effectively identifying highly loaded nodes in spatial networks and enhancing throughput.

Existing spatial network-based routing strategies are primarily developed under the assumption of uniformly distributed node locations. However, real-world network environments often exhibit significant non-uniformity. For instance, in communication networks, the deployment of base stations typically follows a spatial pattern of "dense in the center and sparse at the edges." With the widespread deployment of 5G and ongoing advancements in 6G technology,[27] base station density continues to increase, particularly in urban core areas, to meet growing user demands for higher data rates, lower latency, and greater connectivity. As a result, network infrastructure tends to become highly clustered. In contrast, base stations in suburban or remote regions are sparsely distributed, leading to significant regional disparities in network performance.

While this non-uniform deployment structure enhances communication capacity in certain areas, it may also degrade the overall transmission performance of the network. The central region is prone to resource redundancy due to high node density, whereas peripheral regions may experience increased transmission delays due to limited connectivity paths. Current routing strategies,[10–22] which assume uniform node distributions, often fail to achieve optimal transmission results in such non-uniform spatial networks. Therefore, there is an urgent need to develop a routing strategy that can accommodate non-uniform node distributions, alleviate network congestion, and enhance the overall throughput of these spatial networks.

To address the aforementioned challenges, this study first constructs a spatial network model characterized by a non-uniform node distribution. A dual-radius connection mechanism is introduced to enhance the connectivity of peripheral nodes, where the connection radius is dynamically adjusted according to the distance between a node and the network center. In addition, to alleviate congestion in networks, it is effective to avoid routing through nodes with heavy traffic loads.[16] Accordingly, this paper redefines critical nodes in data transmission based on the topology of non-uniform spatial networks. Comprehensive analysis reveals that, in non-uniform spatial networks, the central region does not always carry the highest traffic load. To better capture this phenomenon, a novel metric termed distance-based degree is proposed, which jointly considers node degree and Euclidean distances to neighboring nodes. This metric enables more accurate identification of high-load nodes under non-uniform spatial networks. Based on this metric, a new routing strategy called the FH routing strategy is developed by integrating distance-based degree with harmonic centrality. Simulation results on various non-uniform spatial networks demonstrate that the FH routing strategy significantly improves network throughput and effectively mitigates congestion, outperforming traditional routing methods under non-uniform node distributions.

2. Non-uniform spatial network models

2.1. Model construction

To better simulate the actual network characteristics of a dense center and sparse edges, and to optimize network performance under limited resource conditions, this paper proposes a non-uniform spatial network model. The model not only considers the non-uniform distribution of nodes but also addresses the issue of limited connectivity among peripheral nodes by introducing a dual-radius connection mechanism. Furthermore, in practical networks, node density is typically constrained. Taking wireless communication networks as an example,[28,29] due to the issue of spatial reuse, node density in real wireless networks cannot be excessively high. In wireless networks, when a node transmits to a target node, all nodes within its transmission range must remain silent to avoid interfering with the transmission. If node density is too high, numerous nodes must remain silent during each transmission, thus reducing the overall network capacity. To reflect this constraint, we impose a density limit on the central region of the network by assuming that the node density within a circular area of radius 0.1 does not exceed ten times the average density of the entire network.

We define a non-uniform spatial network as a circular region with a given radius $R = 1$, where the center of the circle represents the network center. All nodes are assumed to be distributed within this region. Consistent with the empirical findings of Wang *et al.*,[30] which are based on mobile-carrier communication records, both the spatial distribution of mobile base stations and their associated user counts in areas with population density below 10000 persons per km^2 follow a power law. Accordingly, we assume that node locations are drawn from a power-law probability density function, expressed as follows:

$$f(\rho,\theta) = \frac{2-\alpha}{2\pi}\rho^{-\alpha}, \tag{1}$$

where α represents the network density parameter, which controls the concentration of nodes in the network. It ranges from 0 to 1. ρ is the distance from a node to the center of the circle, which spans the interval $[0,1]$, while θ is the angle of the node, with its value ranging from $[0,2\pi]$. The specific construction steps of the network model are as follows:

Step 1: Distribute N nodes in space according to the probability density function in Eq. (1) within a circular area of radius 1. By converting to polar coordinates, the positions of all nodes are mapped onto a two-dimensional plane.

Step 2: For each node, an initial small connection radius r_1 is defined. If the Euclidean distance between node i and node j is less than r_1, then node j is considered a candidate node of node i. M denotes the number of candidate nodes for a single node. For each node, $p \times M$ candidate nodes are randomly selected, where $p \in [0,1]$ is the connection ratio. If no candidate node satisfying the distance condition exists for node i, the nearest node to i in terms of Euclidean distance is selected as its candidate node, and a connection is established.

Step 3: To enhance the connectivity of peripheral nodes in a non-uniform spatial network, which are characterized by low node density and large inter-node distances, we introduce a large connection radius r_2 and a distance threshold D. Specifically, we define $r_2 = \beta \times r_1$, where $\beta \geq 1$, and D, the distance threshold from the network center, is used to distinguish between the small connection radius r_1 and the large connection radius r_2. For nodes whose Euclidean distance from the network center is less than D, the connection radius is set to r_1; otherwise, it is set to r_2. This study assumes that both small and large connection radii exist in the network, with $D \in (0,1)$. During the reconnection process, all connections of nodes with a large connection radius are first disconnected, and $p \times M$ candidate nodes are selected for establishing undirected connections. A new undirected connection is established only if the Euclidean distance between two nodes is less than both of their respective connection radii.

Through the above steps, we generate a spatial network model with a non-uniform node distribution, where the average degree of the network can be controlled by adjusting the connection ratio p. Figure 1 illustrates the distribution of network nodes under different conditions. When the parameter $\alpha = 0$, the probability density function of all nodes is uniform, resulting in a spatial network with uniformly distributed nodes. In contrast, when $\alpha \neq 0$, the network becomes a non-uniform spatial network. As α increases, nodes become increasingly concentrated toward the center, and the degree of spatial non-uniformity grows accordingly. By tuning the values of α, β, and D, different non-uniform spatial networks can be generated.

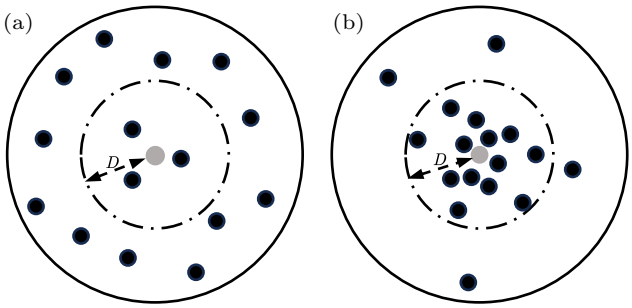


Fig. 1. Distribution of nodes in the network under different values of the parameter α. (a) $\alpha = 0$, corresponding to a uniform spatial network; (b) $\alpha \neq 0$, corresponding to a non-uniform spatial network.

2.2. Model parameter setting

In the non-uniform spatial network model, the dual-radius connection mechanism enables the construction of networks with different connection radius ratios by adjusting the parameters β and D, thus accommodating various practical requirements. In real-world networks, the spatial distribution of nodes and the design of connections need to consider factors such as transmission delay and construction cost. It is therefore important to control edge lengths within a reasonable range while limiting the number of transit nodes. These two aspects often involve a trade-off. Specifically, excessively long edge lengths can reduce the number of transit nodes, whereas excessively short edge lengths tend to increase them.[23] To determine suitable parameter values, both edge lengths and the number of transit nodes are analyzed to ensure that they remain within acceptable ranges under the selected parameters. For this purpose, two evaluation metrics are introduced. The average minimum hop count reflects the number of transit nodes, and the average edge length characterizes the edge lengths in the network.

The average minimum hop count is the average of the minimum hop counts between any two nodes in the network and is defined as

$$\overline{H} = \frac{1}{N(N-1)} \sum_{i \neq j} h_{ij}, \tag{2}$$

where h_{ij} represents the shortest hop count between node i and node j, and N represents the total number of nodes in the network. The value of h_{ij} reflects the transmission delay within the network. A smaller value indicates that fewer intermediate nodes are involved in the transmission process, resulting in improved communication efficiency.

The average edge length is the average of the Euclidean distances between connected nodes in the network and is defined as

$$\overline{L} = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}, \tag{3}$$

where d_{ij} represents the Euclidean distance between node i and node j. A shorter average edge length indicates more effi-

cient connectivity among network nodes, resulting in reduced total data transmission time.

Figure 2 illustrates the simulation results under varying values of α, focusing on two key metrics across different D and β values. The results show that as D increases, the average edge length decreases approximately linearly, while the average minimum hop count increases. This trend is primarily due to the reduction in the number of nodes with large connection radii, leading to fewer long-distance edges and shorter overall node distances. Conversely, as D decreases, the number of nodes with small connection radii decreases, resulting in fewer short-range edges and fewer hop counts between nodes. Simulation results further indicate that when $\beta = 3$ and $D = 0.5$, both the average minimum hop count and the average edge length reach relatively low values. Therefore, this set of parameter values is adopted in subsequent routing strategy simulations to construct a non-uniform spatial network characterized by a mixture of nodes with large and small connection radii.

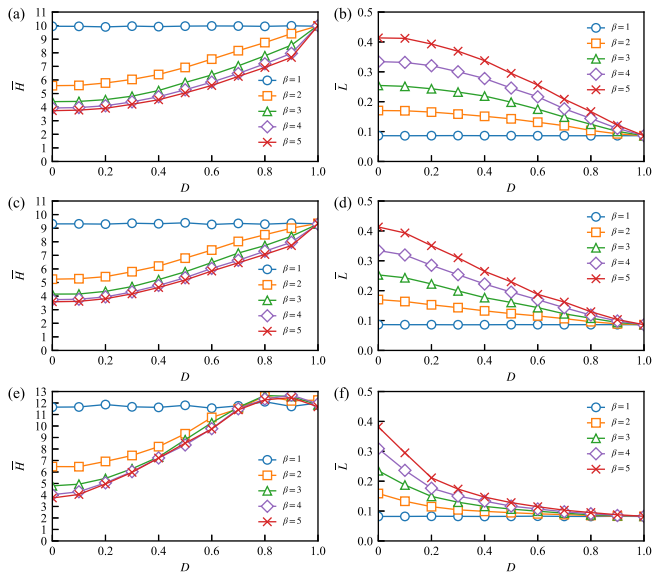


Fig. 2. The effects of parameters D and β on the average minimum hop count ($\overline{H}$) and the average edge length ($\overline{L}$) of the non-uniform spatial network. (a) The average minimum hop count ($\overline{H}$) with $\alpha = 0.1$, (b) The average edge length ($\overline{L}$) with $\alpha = 0.1$, (c) The average minimum hop count ($\overline{H}$) with $\alpha = 0.5$, (d) The average edge length ($\overline{L}$) with $\alpha = 0.5$, (e) The average minimum hop count ($\overline{H}$) with $\alpha = 1.0$, and (f) The average edge length ($\overline{L}$) with $\alpha = 1.0$. The simulation parameters are set as $N = 1000$, $r_1 = 0.13$, and $\langle k \rangle = 8$, with results averaged over 20 independent runs.

3. Traffic dynamics modelling

At each time step, each node can process at most C units of load, while the system as a whole generates R units of load. The corresponding source and target nodes are randomly selected. For each unit of load, a path from the source to the target is determined. Once the load reaches its designated target, it is immediately removed from the system. To evaluate the performance of the network, we introduce the order parameter $\eta(R)$[31] as follows:

$$\eta(R) = \lim_{t\to\infty} \frac{C\langle W(t+\Delta t) - W(t)\rangle}{R\Delta t}, \tag{4}$$

where $W(t)$ represents the network load at time t, and $\langle\cdot\rangle$ represents the average value within the sliding window Δt. We refer to R_c as the maximum network throughput. When $R < R_c$, the input and output flows at each node are balanced, and no congestion occurs in the network, so $\eta(R) = 0$. When $R > R_c$, the network starts to experience congestion, and $\eta(R)$ increases. Thus, maximizing R_c is essential for enhancing the overall performance and efficiency of the network.

The improved betweenness centrality of a node serves as a key metric for characterizing its load within the network. It is defined as follows:

$$b(n) = \sum_{s\neq t} \frac{\sigma_{st}(n)}{\sigma_{st}}, \tag{5}$$

where σ_{st} represents the total number of routing paths from node s to node t under a given routing strategy, and $\sigma_{st}(n)$ represents the number of those paths that pass through node n. Traffic congestion initially occurs at the node with the maximum improved betweenness centrality.

Furthermore, the congestion of the network can be evaluated using the following expression:[16]

$$R_c = \frac{N(N-1)C}{b_{\max}}, \tag{6}$$

where $b_{\max}$ represents the maximum improved betweenness centrality among all nodes. To enhance the capacity of the network, the value of $b_{\max}$ should be minimized.

4. Routing strategies

4.1. Traditional routing strategies

4.1.1. Shortest path routing strategy

For any pair of nodes (s,t), the path between them is defined as follows:

$$P(s\to t) : s = x^{(0)}, x^{(1)}, \ldots, x^{(n)} = t. \tag{7}$$

The shortest path (SP) routing strategy, as a classical routing strategy, selects the path with the minimum number of hops between the source and destination nodes to transmit the load. While this routing strategy enables rapid delivery of loads, it often results in congestion at central nodes, thereby reducing the overall network throughput.

4.1.2. Efficient routing strategy

The efficient routing (ER) strategy[16] is based on the observation that nodes with higher degrees are more susceptible to congestion. To address this issue, the routing strategy selects transmission paths that minimize the sum of the degrees of the nodes along the path, thereby effectively balancing traffic and alleviating congestion at central nodes. For any path

from node i to node j, denoted as $P(i \to j)$, the path cost is defined as

$$L(P(i \to j);\beta) = \sum_{i=0}^{n-1} k_i^{\beta}, \tag{8}$$

where k_i represents the degree of node i along the path. The ER routing strategy determines the optimal path from node i to node j by minimizing $L(P(i \to j);\beta)$.

4.1.3. Degree-location routing strategy

The degree-location (DL) routing strategy[22] integrates both node degree and spatial location to optimize data transmission paths and enhance network throughput. Compared with other routing strategies, the DL strategy effectively distributes traffic load, avoids congestion at critical nodes, and thereby improves overall network performance. Simulation results demonstrate that the DL strategy achieves higher throughput in spatial networks.

The degree of a node is normalized as k_i', which is defined as

$$k_i' = \frac{k_i - \min(k)}{\max(k) - \min(k)}, \tag{9}$$

where $\max(k)$ represents the maximum degree of nodes in the network, and $\min(k)$ represents the minimum degree of nodes in the network.

The distance from node i to the center of the region is defined as L_i, given by

$$L_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}, \tag{10}$$

where (x_i, y_i) represents the two-dimensional coordinates of node i, and (x_c, y_c) represents the two-dimensional coordinates of the center of the region. Similarly, we define the normalized distance L_i' as

$$L_i' = 1 - \frac{L_i - \min(L)}{\max(L) - \min(L)}. \tag{11}$$

The DL routing strategy combines the degree of nodes and their spatial positions in the network, and the weight of node i is defined as

$$V_i = k_i'^{\gamma} + L_i'^{\delta}, \tag{12}$$

where γ and δ are tuning parameters used to adjust the importance of the degree and the spatial position.

The DL routing strategy tends to avoid nodes with high V values and selects the path with the minimum cumulative V value, given by

$$G_V(P(s \to t)) = \min \sum_{m=0}^{n} V_m. \tag{13}$$

4.1.4. Harmony-degree routing strategy

The HD routing strategy[21] redefines key nodes in the transmission process by incorporating both harmonic centrality and node degree. This routing strategy has been shown to outperform other routing strategies in terms of network throughput in both homogeneous and heterogeneous spatial networks.

Harmonic centrality quantifies the connectivity efficiency of a node with respect to all other nodes in the network and is defined as

$$H_i = \sum_{j \neq i} \frac{1}{d(i,j)}, \tag{14}$$

where $d(i,j)$ represents the number of hops from node i to node j. In a network, if node i has higher harmonic centrality, it means that this node establishes connections with other nodes through shorter paths. Therefore, the node experiences a higher load. To eliminate the impact of network scale and topology on metrics, we normalize the metrics as follows:

$$H_i' = \frac{H_i - \min(H)}{\max(H) - \min(H)}, \tag{15}$$

where $\max(H)$ represents the largest value of harmonic centrality in the network, and $\min(H)$ represents the smallest value of harmonic centrality in the network.

The HD routing strategy combines the harmonic centrality and degree of a node to define the weight of node i as

$$Q_i = H_i'^{\gamma} + k_i'^{\delta}, \tag{16}$$

where γ and δ are adjustment parameters used to control the importance of harmonic centrality and node degree.

The HD routing strategy tends to avoid nodes with high Q values and selects the path with the minimum cumulative Q value, defined as

$$G_Q(P(s \to t)) = \min \sum_{m=0}^{n} Q_m. \tag{17}$$

4.2. Routing strategy based on non-uniform spatial networks

4.2.1. Load distribution of the network

Network congestion is often caused by non-uniform load distribution among nodes. In uniform spatial networks, all nodes have the same connectivity radius and can only establish links with nearby neighbors, resulting in the absence of long-range connections. Consequently, cross-regional traffic must pass through the central area, imposing a heavier load on central nodes and making them potential bottlenecks.[22] In contrast, non-uniform spatial networks feature varying connectivity radii, and the central region is not necessarily the area with the highest load.

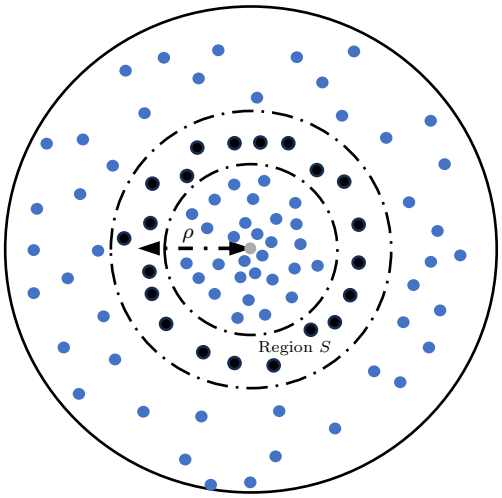


Fig. 3. Demonstration of Region S with concentric annuli. Nodes are non-uniformly distributed within a circle of radius $R = 1$, where ρ denotes the distance of each node from the center. The average load is calculated for nodes within the distance interval $[\rho - 0.05, \rho + 0.05]$ from the center.

Betweenness centrality is an important indicator of a node's load. Therefore, we use the betweenness centrality of each node as a direct measure of load. As illustrated in Fig. 3, all nodes are placed within a circle with radius $R = 1$, with the circle center serving as the origin. The circle is divided into concentric annuli, referred to as Region S, with a radial width of 0.1, extending outward to cover the entire domain. We then compute the average load within each annulus with a width of 0.1.

Figure 4 shows the relationship between the average load $\bar{b}$ and the distance ρ from the node to the center of the circle under the SP routing strategy in different non-uniform spatial networks with $\alpha = 1.0$. The results clearly show that the region with the highest average load appears just outside the boundary where the large and small radii meet. This area is referred to as the transition region of the network.

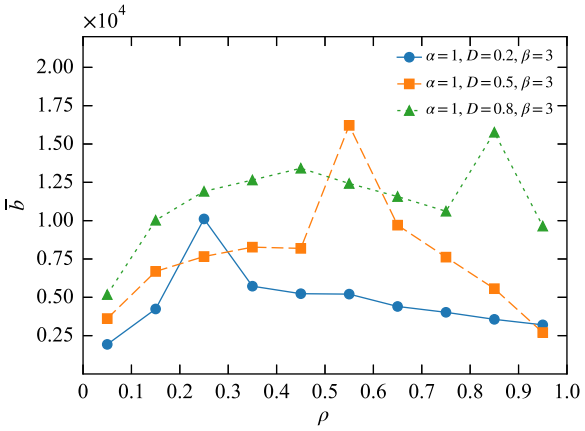


Fig. 4. The relationship between the average load $\bar{b}$ and the node distance ρ from the center under the *SP* routing strategy in various non-uniform spatial networks with $\alpha = 1.0$. The simulation parameters are set as $N = 1000$, $r_1 = 0.13$, and $\langle k \rangle = 8$, with results averaged over 50 independent runs.

In non-uniform spatial networks, nodes can have different connection radii, allowing the coexistence of long and short connecting edges. Nodes with long connecting edges can bypass the center and transmit directly, thereby reducing the load on the central region. Nodes in the central region are limited by the short connectivity radius and cannot directly establish connections with peripheral regions. Traffic from the periphery to the center must be forwarded through intermediate nodes in the transition region. In addition, some nodes in the transition region have both long and short connecting edges. These nodes communicate with the central region through short connecting edges and connect to farther peripheral nodes through long connecting edges. Therefore, in non-uniform spatial networks, the transition region often becomes a hotspot for traffic convergence and carries a higher load.

4.2.2. Routing strategy

In the design of routing strategies, Yan *et al.*[16] proposed the ER strategy for topological networks, which aims to bypass critical nodes as much as possible. In such networks, nodes with high degrees are typically regarded as critical because they tend to bear heavier traffic loads. Following this principle, it is essential to avoid high-load nodes when designing routing strategies for non-uniform spatial networks. Therefore, to mitigate congestion, routing strategies in non-uniform spatial networks should preferentially circumvent high-load nodes located in transition regions. Specifically, we redefine critical nodes in traffic transmission by considering the topological characteristics of non-uniform spatial networks.

In topological networks and uniform spatial networks, node degree is commonly considered an important local metric reflecting its traffic load-bearing capacity. A higher node degree typically indicates more neighboring connections and, thus, a greater likelihood of carrying heavier traffic loads during transmission. However, in non-uniform spatial networks, the non-uniform distribution of node locations can significantly alter the correlation between degree distribution and actual traffic load.

Specifically, in non-uniform spatial networks, nodes with higher traffic loads are often not those with the highest degrees. This phenomenon primarily arises from the higher node density in central regions, where the spatial distances between nodes are shorter, leading to higher node degrees. However, due to strong spatial clustering and the abundance of alternative paths in these regions, traffic exhibits high path selectivity. As a result, the transmission load is more evenly distributed, and high-degree nodes in the central region tend to carry relatively lower traffic loads. Therefore, in non-uniform spatial networks, node degree is no longer a reliable indicator of traffic load-bearing capacity.

In view of the limitations of traditional node degree in characterizing traffic load in non-uniform spatial networks, we

propose a novel metric, termed the distance-based degree F_i, which is defined as follows:

$$F_i = \sum_{j \in N_i} \left(\frac{d_{ij}}{k_j} \right)^{\alpha}, \tag{18}$$

where N_i denotes the set of neighboring nodes of node i, d_{ij} represents the Euclidean distance between node i and its neighboring node j, and α is a parameter that characterizes the non-uniformity of the network. The proposed distance-based degree F_i considers both the number of neighboring nodes and the spatial distances to these neighbors. This metric can effectively guide traffic to preferentially traverse regions with a higher density of short-distance connections, i.e., regions with a greater concentration of nodes and tighter connectivity.

In non-uniform spatial networks, nodes located in the transition region typically possess more long-distance connections due to their larger external connectivity radius. Compared with central-region nodes, these nodes have longer connecting edges. Compared with outer-region nodes, they generally have more neighbors. As a result, their distance-based degree values tend to be relatively higher. The routing strategy seeks the path with the minimum total weight, and during transmission through the boundary region between large and small radii, traffic tends to avoid nodes in the transition region as much as possible, thereby alleviating congestion in this region. Moreover, the parameter α in F_i increases with the degree of non-uniformity in the network, thereby amplifying the influence of the distance-based degree during path selection.

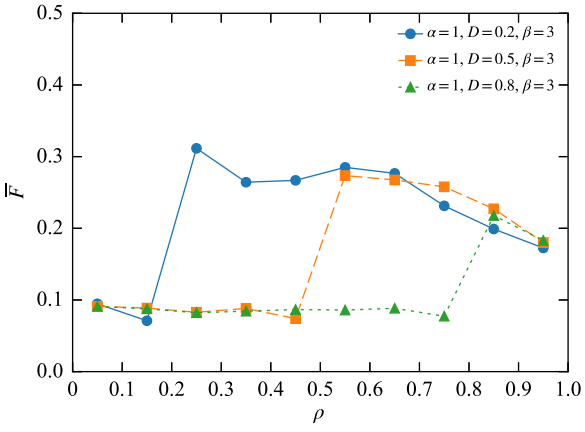


Fig. 5. The relationship between the average distance-based degree $\overline{F}$ and the node distance ρ from the center in various non-uniform spatial networks with $\alpha = 1.0$. The simulation parameters are set as $N = 1000$, $r_1 = 0.13$, and $\langle k \rangle = 8$, with results averaged over 50 independent runs.

To further investigate the effectiveness of the distance-based degree metric, we extend the network load analysis presented in Fig. 4 by examining the behavior of the distance-based degree in non-uniform spatial networks. Figure 5 illustrates the average distance-based degree $\overline{F}$ of nodes as a function of their distance from the center of the circular region under the condition $\alpha = 1.0$. As shown in the figure, the region exhibiting the highest average distance-based degree generally coincides with the region of maximum network load. This observation further supports the validity of employing the distance-based degree as an indicator for identifying high-load nodes within non-uniform spatial networks.

The distance-based degree is normalized, and the specific formula is as follows:

$$F_i' = \frac{F_i - \min(F)}{\max(F) - \min(F)}. \tag{19}$$

Finally, we combine the distance-based degree F_i and the harmonic centrality H_i and propose a more suitable routing strategy for non-uniform spatial networks, named the FH routing strategy. The weight of node i is defined as

$$w_i = F_i'^{\gamma} + H_i'^{\delta}, \tag{20}$$

where γ and δ are the two control parameters. By applying this formula, the overall weight w_i of a node can be adjusted dynamically according to the required conditions. The goal of the FH routing strategy is to find the path with the minimum weight w, as given by

$$G_w(P(s \to t)) = \min \sum_{m=0}^{n} w_m. \tag{21}$$

5. Simulation results

To evaluate the impact of various network parameters on the performance of the proposed routing strategy, we adopt the network throughput R_c as a key performance indicator. A larger value of R_c indicates a higher capacity before congestion occurs, implying better overall network performance. In the following simulations, we set the delivery capacity per node to $C = 1$ for generality.

To ensure a fair comparison between the FH routing strategy and other routing strategies, we first examine all routing strategies under a general setting, i.e., with the routing strategy parameters $\delta = \gamma = 1$. Figure 6 presents the variation of network throughput R_c with respect to the non-uniformity parameter α for five different routing strategies. Simulation results indicate that the network throughput of the FH routing strategy consistently outperforms that of the other four routing strategies under all values of α. Moreover, the throughput of the FH routing strategy initially increases and then decreases as the non-uniformity parameter α increases, whereas the throughput of the other four routing strategies exhibits a monotonic decrease with increasing α.

This phenomenon can be attributed to the fact that, for small values of α, the distance-based degree, whose value is close to that of the node degree, and the routing parameters γ and δ have not yet reached their optimal values, resulting in the critical throughput R_c of the FH routing strategy being

relatively close to that of the HD routing strategy. Additionally, as the degree of non-uniformity increases, the topological disparities among different network regions become more pronounced, leading to an increasingly non-uniform load distribution across nodes. Consequently, the network throughput of all routing strategies tends to decline with increasing α.

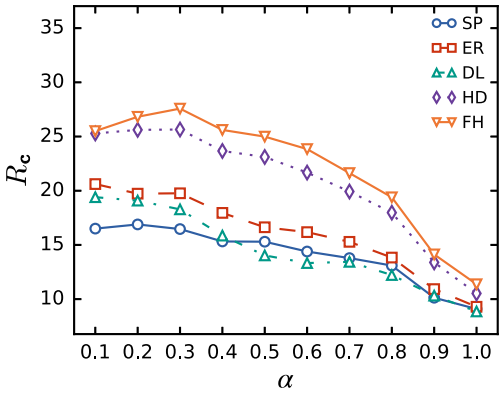


Fig. 6. The variation of R_c with α for the five routing strategies in the general case. The simulation parameters are set as $N = 1000$, $r_1 = 0.13$, $\langle k \rangle = 8$, $\delta = \gamma = 1$, with results averaged over 50 independent runs.

The DL, HD, and FH routing strategies can be improved by optimizing the parameters δ and γ. Therefore, it is necessary to select the optimal values of δ and γ for each routing strategy. Taking the FH routing strategy as an example, Fig. 7 shows the network throughput under different combinations of δ and γ.

For the FH routing strategy with $\alpha = 0.1$, the optimal parameters are $\gamma = 0.2$ and $\delta = 2.0$. When $\alpha = 0.5$, the optimal parameters are $\gamma = 0.4$ and $\delta = 1.5$. When α increases to 1.0, the optimal parameters adjust to $\gamma = 1.3$ and $\delta = 1.5$. Similarly, the optimal parameters for the other two routing strategies can also be determined in the same way by identifying the maximum values of R_c.

To further evaluate the performance of the FH routing strategy across different non-uniform spatial networks, we analyze the critical packet generation rate R_c under varying network sizes and average degrees. For comparison, all routing strategies are configured with their optimal parameter settings. Figure 8 illustrates the variation of R_c with network size N under different routing strategies. As the network size increases, the R_c of all routing strategies also increases, indicating that larger networks are generally capable of handling higher traffic loads before congestion occurs.

When the network exhibits a nearly uniform node distribution (i.e., $\alpha = 0.1$), the FH strategy achieves a R_c comparable to that of the HD strategy while outperforming the other three routing schemes. In contrast, under more heterogeneous spatial distributions, the FH strategy consistently demonstrates superior performance compared with all other routing strategies.

Figure 9 illustrates the variation of the network capacity R_c with the average degree $\langle k \rangle$ under different routing strategies when the weighting parameters are set to their optimal values. The network capacity R_c increases almost linearly with $\langle k \rangle$. As the average degree $\langle k \rangle$ increases, the network becomes more densely connected, providing more routing path options during transmission. As a result, the load distribution becomes more balanced, which directly leads to an increase in R_c. Moreover, the proposed FH routing strategy consistently achieves higher values of R_c than other routing strategies across networks with different average degrees.

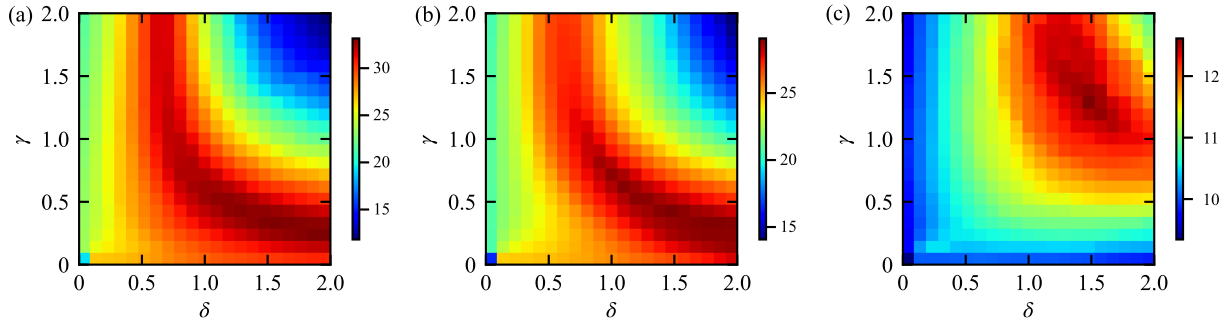


Fig. 7. The influence of the FH routing strategy parameters γ and δ on R_c. (a) $\alpha = 0.1$, (b) $\alpha = 0.5$, and (c) $\alpha = 1.0$. The simulation parameters are set as $N = 1000$, $r_1 = 0.13$, and $\langle k \rangle = 8$, with results averaged over 10 independent runs.

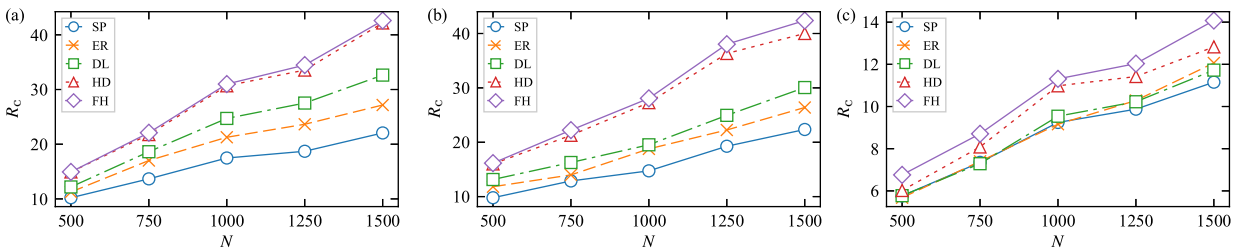


Fig. 8. R_c of different routing strategies on non-uniform spatial networks with different network sizes N. (a) $\alpha = 0.1$, (b) $\alpha = 0.5$, and (c) $\alpha = 1.0$. The simulation parameters are set as $r_1 = 0.13$ and $\langle k \rangle = 8$. The routing strategy parameters γ and δ are set to their respective optimal values. Each result is averaged over 20 independent simulation runs.

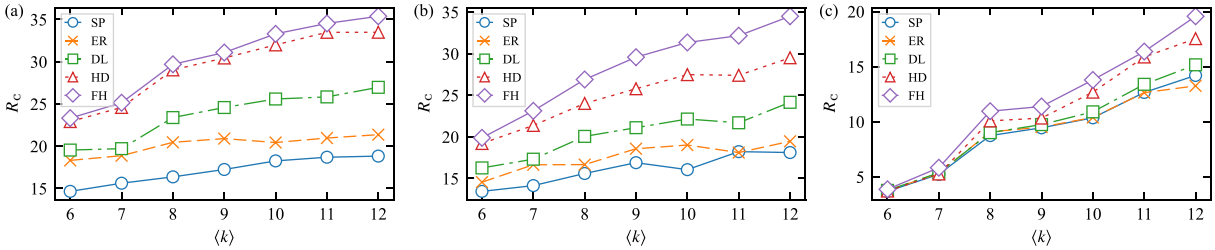


Fig. 9. R_c of different routing strategies on non-uniform spatial networks with different average degrees $\langle k \rangle$. (a) $\alpha = 0.1$, (b) $\alpha = 0.5$, and (c) $\alpha = 1.0$. The simulation parameters are set as $N = 1000$ and $r_1 = 0.13$. The routing strategy parameters γ and δ are set to their respective optimal values. Each result is averaged over 50 independent simulation runs.

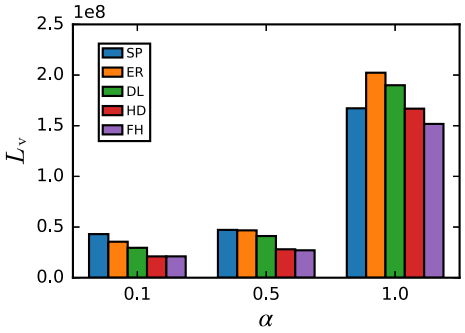


Fig. 10. The load variance L_v of different routing strategies under different α conditions. The simulation parameters are set as $N = 1000$, $r_1 = 0.13$, and $\langle k \rangle = 8$. The routing strategy parameters γ and δ are set to their respective optimal values. Each result is averaged over 20 independent simulation runs.

To investigate the underlying reason why the FH routing strategy achieves superior network throughput in nonuniform spatial networks, we introduce the metric of load variance to analyze the traffic load distribution across the network under different routing strategies.

In spatial networks, the load of a node is typically proportional to its improved betweenness centrality. Accordingly, we define the network load variance as the statistical variance of node loads, which quantifies the degree of load imbalance in the system. The load variance is defined as

$$L_v = \frac{1}{N}\sum_{i=1}^{N}(b(i) - \langle b \rangle)^2, \tag{22}$$

where $b(i)$ represents the load of node i, corresponding to its improved betweenness centrality, and $\langle b \rangle$ represents the average load across all nodes.

Figure 10 shows the load variance under different routing strategies with optimized parameter settings. When α is relatively small, the load variance of the FH strategy is nearly identical to that of the HD strategy due to the similarity between the distance-based degree and node degree in this case. Nonetheless, the FH strategy still achieves a lower load variance than the other three routing strategies. As α increases, the FH strategy consistently maintains the lowest load variance, with the difference becoming particularly pronounced at $\alpha = 1.0$, where its variance is significantly smaller than those of all other strategies.

This behavior indicates that the FH strategy is more effective in balancing traffic loads across the network. A more uniform load distribution enables data flows to be more evenly allocated among nodes and links, thereby reducing the risk of localized congestion. In contrast, when the load is non-uniformly distributed, certain nodes may become bottlenecks, resulting in increased packet queuing, longer transmission delays, and even packet loss, which collectively degrade network throughput. By mitigating load concentration, the FH strategy reduces the probability of congestion and enhances both the data handling capacity and transmission efficiency of the network. These findings provide a clear explanation for the superior throughput performance of the FH routing strategy in non-uniform spatial networks.

6. Conclusion

In this paper, we address the optimization problem of routing strategies in spatial network models with non-uniform distributions. Given the prevalent dense-center and sparse-periphery phenomenon observed in real-world networks, we propose a double-radius non-uniform spatial network model that captures this feature by dynamically adjusting the connection radius. We examine the limitations of the traditional shortest path routing strategy and compare it with strategies based on harmonic centrality and node degree. Additionally, we introduce a novel metric, termed the distance-based degree, and combine it with harmonic centrality to develop a composite routing strategy. This strategy enables adaptive adjustment of node weights, thereby significantly improving network throughput in non-uniform spatial networks. The proposed approach provides valuable theoretical insights and practical guidance for enhancing information transmission efficiency in complex real-world networks.

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