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Mode and orbital angular momentum controlling of nonlocal solitons by anisotropic diffraction

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This study presents a novel approach to control the mode transform between Laguerre–Gaussian solitons and Hermite–Gaussian solitons by designing anisotropic diffraction in nonlocal medium. The anisotropic diffraction can introduce differential phase shifts and amplitude modulations to the optical field, and then leading to the transform of the soliton mode from one to another, while the accompanying orbital angular momentum exhibits a periodic variation. When the anisotropic diffraction is designed to become isotropic at the desired distance, various mode solitons with different rotation speeds and directions can be generated, which implies that the mode transform process and the change law of orbital angular momentum can be controlled. This finding establishes a versatile platform for nonlinear mode transform, offering new possibilities for applications in optical signal processing, topological photonics, optical micro-manipulation, and so on.

Keywords: anisotropic diffraction, nonlocal solitons, mode transform, orbital angular momentum controlling

PACS: 42.65.–k, 42.50.Md, 42.65.Tg, 42.81.Dp

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1. Introduction

Laguerre–Gaussian (LG) and Hermite–Gaussian (HG) modes are two important families of optical beams that play important roles in modern optics. These modes possess distinct spatial intensity distributions and phase structures, which endow them with unique characteristics and potential applications.[1,2] For instance, LG modes are characterized by a helical phase structure and an azimuthal dependence of the intensity, making them valuable for applications, such as optical trapping, quantum information processing, and orbital angular momentum (OAM) multiplexing in optical communications.[3,4] On the other hand, HG modes have a well-defined rectangular symmetry and are widely used in optical imaging, beam shaping, and laser-based manufacturing.[5] Meanwhile, the transform between different modes of spatial optical beams became a hot point and has great significance, which can expand the functionality of optical systems and enable new applications.[6,7] Up to now, the mode transform are mainly achieved in free space or linear media, and usually through two methods, two cylindrical lenses system[1,2] and adding cross phase to the beam,[8] with the result that it is difficult to control the transform process and the width of the converted beam.

In the realm of nonlinear optics, a spatial optical soliton will be formed when the diffraction effect precisely balances the nonlinear effect in the medium, and enable the beam width to remain constant during propagation.[9–12] In recently decades, the spatial optical solitons have captivated researchers due to their unique properties and potential applications, and achieved a series of significant research results, such as dark solitons,[13] multifrequency solitons,[14] gap solitons,[15] higher-charged vortex solitons,[16] Laguerre–Gaussian solitons,[17] Hermite–Gaussian solitons,[18] rotating optical vortex clusters,[19] and multiple-ring solitons.[20] Furthermore, the interactions of solitons[21] and beam arrays composing of several solitons[22,23] also be investigated.

In recent years, the mode transform between different solitons also been achieved in nonlinear media, and it is revealed that there are more method for realizing the mode transform, such as the by using anisotropic boundaries effect of the media,[24,25] rotating elliptic potential wells,[26] and adding cross phase to the beam,[27] etc.

In this research, we reveal that the anisotropic diffraction also can lead to the periodic inter-transform between HG and LG modes. This work aims to provide a deeper understanding of the underlying physical mechanisms and develop practical methods for efficient mode transform.

2. Theoretical model

The propagation dynamics of an paraxial beam in two dimensions nonlocal nonlinear media can be described as[28–32]

$$\mathrm{i}\frac{\partial\psi}{\partial z}+\frac{1}{2}\left(\delta_x\frac{\partial^2\psi}{\partial x^2}+\delta_y\frac{\partial^2\psi}{\partial y^2}\right)+\psi\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R(x-x',y-y')\left|\psi(x',y',z)\right|^2\mathrm{d}x\mathrm{d}y=0, \quad (1)$$

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where x and y are the transverse coordinates, z is the longitudinal coordinate, which are the propagation direction of the optical beams ψ. δ_x and δ_y describe the coefficients of isotropic linear diffraction, which determined by the material property. Up to now, most of research have mainly focused on the special case of $\delta_x = \delta_y = 1$, which indicates the isotropic diffraction. Whereas, the anisotropic diffraction ($\delta_x \neq \delta_y$) is the common case in the actual medium. Hence, its effect on the propagation of higher-order beams has become a task of great importance. $R(x,y)$ is the nonlocal response function, and here it was taken as the Gaussian-shaped[17,18]

$$R(x,y) = \frac{1}{\pi\sigma^2}\exp\left[-\frac{x^2+y^2}{\sigma^2}\right] \tag{2}$$

with σ represents the characteristic length of nonlocality. Although the Gaussian-shaped response is merely a theoretical model, it has been widely applied in the study of solitons in nonlocal media, and always provides excellent predictions for the general properties of nonlocal solitons. Just as the HG and LG solitons were initially derived in the phenomenological nonlocal medium in the form of analytical solutions,[17,18] this achievement provides guidance for the study of higher-order solitons in more complex actual nonlocal media (including the lead glass and nematic liquid crystals). The distribution of HG and LG profiles can be expressed in the following form[17,18]

$$\psi^{\mathrm{HG}}_{m,n}(x,y,0) = A_{\mathrm{HG}}\exp\left(-\frac{x^2+y^2}{2w^2}\right)\mathrm{H}_m\left(\frac{x}{w}\right)\mathrm{H}_n\left(\frac{y}{w}\right), \tag{3}$$

$$\psi^{\mathrm{LG}}_{\pm l,p}(x,y,0) = A_{\mathrm{LG}}\exp\left(-\frac{x^2+y^2}{2w^2}\right) \times\left(\frac{x\pm \mathrm{i}y}{w}\right)^l \mathrm{L}^l_p\left(\frac{x^2+y^2}{w^2}\right), \tag{4}$$

where L_{lp} and $\mathrm{H}_{m(n)}$ denote the famous generalized Laguerre and Hermite polynomials, respectively, with w is the beam width. $A_{\mathrm{LG}} = [p!P/(\pi(l+p)!w^2)]^{1/2}$ and $A_{\mathrm{HG}} = [P/(\pi 2^{n+m}n!m!w^2)]^{1/2}$ are normalized coefficient which obtained by the beam power $P = \iint|\psi|^2\mathrm{d}x\mathrm{d}y$.

The OAM per power of the beam is an important concepts, and can be obtained as[28]

$$\frac{M}{P} = \frac{\mathrm{Im}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi^*\left(x\partial\psi/\partial y - y\partial\psi/\partial x\right)\mathrm{d}x\mathrm{d}y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|\psi|^2\mathrm{d}x\mathrm{d}y}, \tag{5}$$

we will show that the anisotropic diffraction can bring in the periodic change law of the OAM, which accompanying the modes transform between HG and LG modes. Then the propagation of the HG and LG beams in nonlocal media are simulated by the split-step Fourier method, and the stability of these solutions is evaluated through the addition of 5% random noise to the amplitude. In addition, we fix $\delta_x = 1$ and change δ_y in the following numerical simulations for discussing the anisotropy conveniently.

3. Mode transform between HG and LG solitons

In the isotropic ($\delta_x = \delta_y = 1$) nonlocal media, the authors in Ref. [16] reported that the LG solitons can be obtained when the beam power equals to the critical value, as shown in Fig. 1(a), which show that the beam maintains its width and pattern very well. This is because that the diffraction effect precisely balances the nonlinear effect induced by the beam. However, when $\delta_x \neq \delta_y$, the LG_{20} modes will convert to the HG_{02} modes at $z = 32$ and 16 for $\delta_y = 0.9$ and 0.8, respectively, as shown in Figs. 1(b) and 1(c). Though the LG_{20} mode is selected as the initial beam, the change law is also valid for the other LG modes. Figure 2 demonstrates the change law of the M/P for different δ_y. Since the action of anisotropic

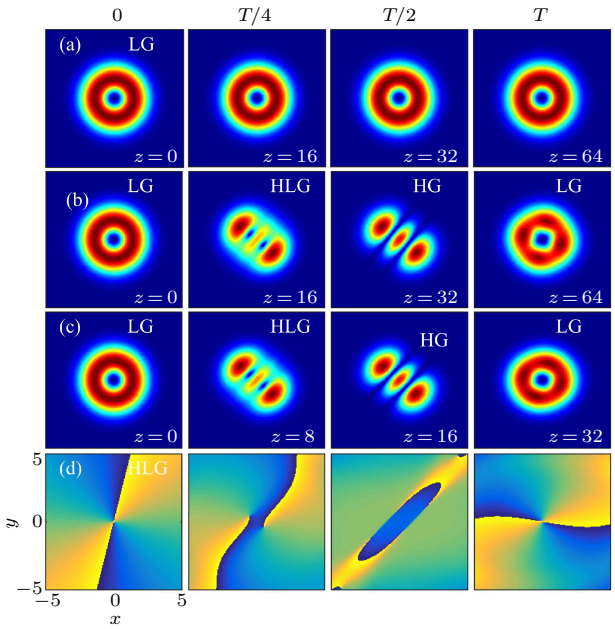


Fig. 1. LG_{20} mode soliton ($P = 17200$) keep unchanged (a), or transform into the HG_{02} mode solitons at $z = 32$ (b) and $z = 16$ (c) for $\delta_y = 1$, 0.9, and 0.8, respectively; (d) the corresponding phase dynamics of panel (c).

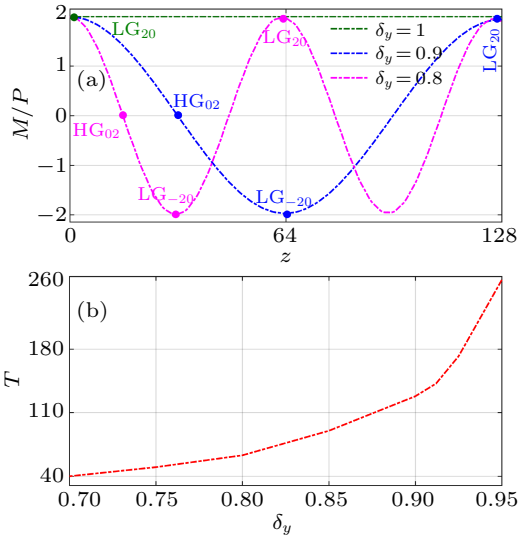


Fig. 2. (a) The change law of M/P: green, blue, and pink lines correspond to the mode transforms of Figs. 1(a), 1(b), and 1(c), respectively. (b) The the transform distance of unit period for different δ_y.

diffraction, the input LG_{20} mode soliton changes into the HG_{02} mode, then to the LG_{-20} mode, and finally recovers to the initial LG_{20} mode at the propagation distance of $z = 64$ (pink lines for $\delta_y = 0.8$) and 128 (blue lines for $\delta_y = 0.9$), respectively. Hence, $z = 64$ and 128 can be regard as the transform distance of unit period for LG_{20} mode soliton when $\delta_y = 0.8$ and 0.9, respectively. Then the relation of the transform distance versus δ_y are shown in Fig. 2(b).

The process of HG mode beam transform into the LG ones are shown in Fig. 3. Similarly, the parameter value of δ_y also has a significant impact on the transition. Such as figure 3(a) demonstrates that when $\delta_x = \delta_y$ the beam is in the soliton state with the critical value, as described in Ref. [17]. When $\delta_y = 0.9$ and 0.8, figures 3(b) and 3(c) respectively show that the HG modes can be converted into the LG modes at $z = 32$ and 16, respectively. Then they will restored to the HG states, whereas, the direction of long axis of the HG beam is different from that of the initial ones. In fact, the beam undergoes a period and completely returns to their original state at $z = 64$ (pink lines for $\delta_y = 0.8$) and 128 (blue lines for $\delta_y = 0.9$), as shown in Fig. 4(a), and the the transform distance of unit period for different δ_y are presented in Fig. 4(b). Figure 3(d) shows the change of phase which corresponds to Fig. 3(c). The comparisons of Figs. 1 and 3 show that the transform between different soliton modes for the same δ_y displays the same period. Furthermore, the inter-transform comply with the R law between the LG and HG modes.[1–3] The R law can be described as $2p + |l| = m + n$, with $l = n - m$ and $p = \min[m, n]$.

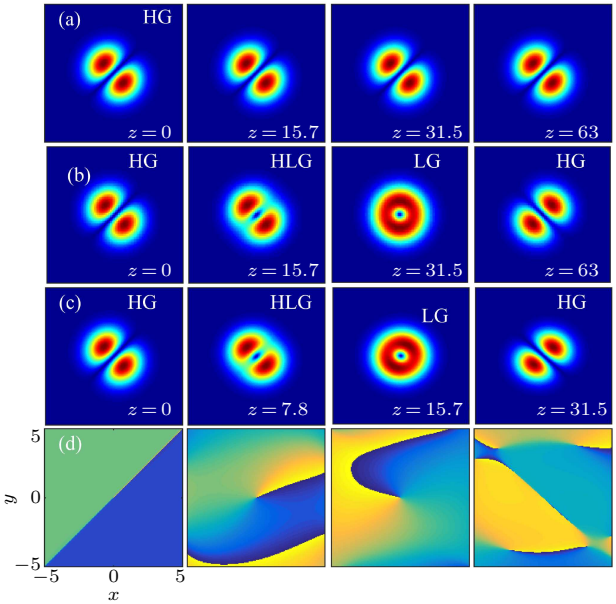


Fig. 3. HG_{01} mode beam ($P = 17300$) keep unchanged (a), or transform into the LG_{-10} mode beams at $z = 32$ (b) and $z = 16$ (c) for $\delta_y = 1$, 0.9, and 0.8, respectively; (d) the corresponding phase dynamics of panel (c).

In Fig. 3 the degrees of the angle (β) between the long axis of the HG beam and the y coordinate is 45. When the degrees taken different value, the change law of the beam also displays completely different behaviors, which is presented in Fig. 5. For $\beta = 0$, the HG mode beam can remain the pattern in the anisotropic media as shown in Fig. 3(a). Whereas, for nonzero β, the HG modes can achieve a transform to the HLG modes, as shown in Figs. 5(b) and 5(c) for $\beta = 15$ and 30, respectively, or the LG modes in Fig. 5(d) for $\beta = 45$. Although the HG_{02} beam is selected as the initial beam, the change law is also applicable to the other HG modes. Figure 6 presents the change law of the M/P *versus* β. Obviously, when $\beta = 0$, the value of M/P is zero. However, for other values of β, the value undergoes periodic variations and has a minimun value. As shown by the red line in Fig. 6(a), when $\beta = 45$, the minimum value of M/P can reach -2, which indicates that the HG

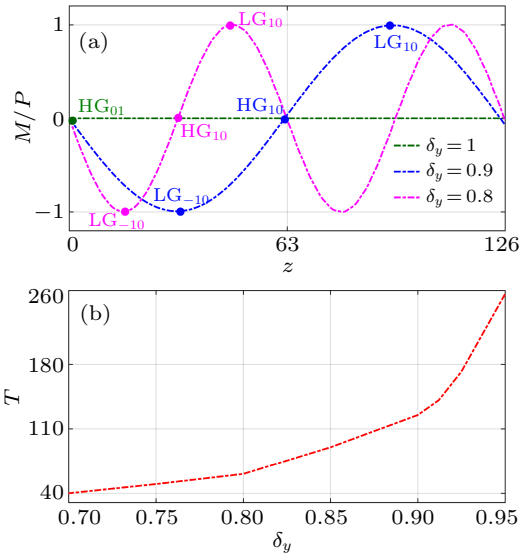


Fig. 4. (a) The change law of M/P: green, blue, and pink lines correspond to the mode transforms of Figs. 1(a), 1(b), and 1(c), respectively. (b) The the transform distance of unit period for different δ_y.

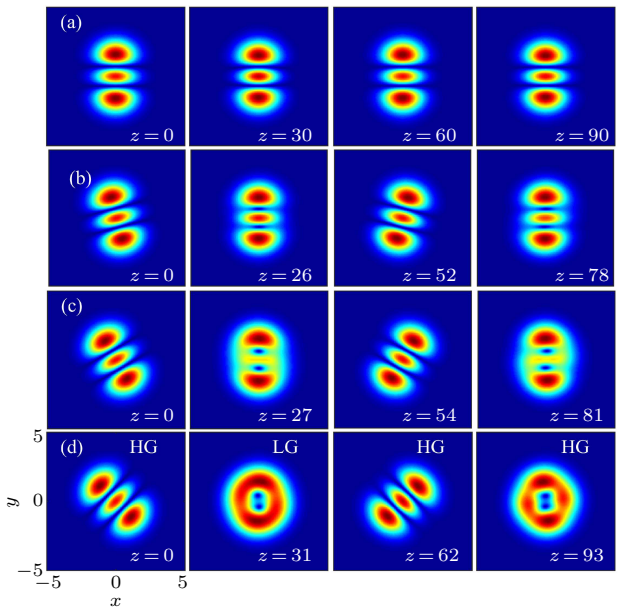


Fig. 5. HG_{02} mode ($P = 17500$) keep unchanged (a), or transform into the HLG (b), (c), and LG_{-20} mode beams (d) for $\beta = 0$, 15, 30, and 45 respectively, in the nonlocal media with anisotropic diffraction of $\delta_y = 0.9$.

mode has been converted to the LG mode. The minimum value of M/P that can reached, and the transform distance of unit period versus β are shown in Figs. 6(b) and 6(c), respectively.

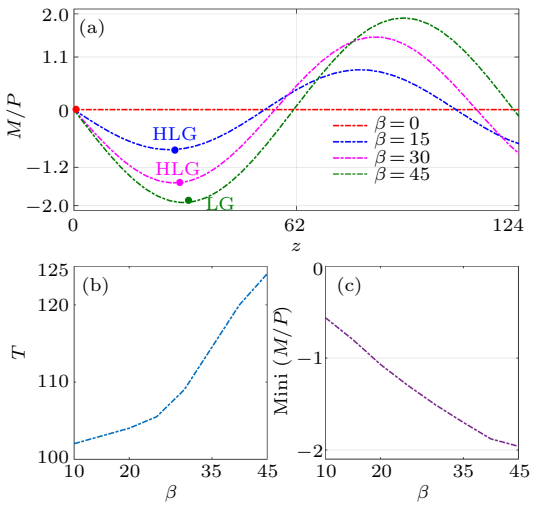


Fig. 6. (a) The change law of M/P: red, blue, pink, and green lines correspond to the mode transforms of Figs. 1(a), 1(b), 1(c), and 1(d), respectively. (b) The the transform distance of unit period and (c) the minimum M/P of the beam can be achieved for different β.

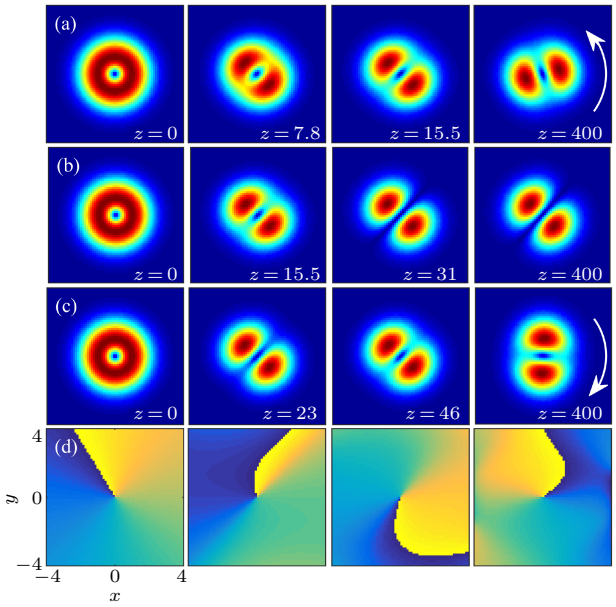


Fig. 7. LG_{10} mode soliton ($P=17100$) transform into panels (a), (c) HLG, and (b) HG_{01} mode solitons with $\delta_y=0.9$ existing in the range of (a) $0\le z\le 7.8$, (b) $0\le z\le 15.6$, (c) $0\le z\le 23.4$; (d) the corresponding phase dynamics of panel (c).

More interesting, the LG_{10} beam can generates three different solitons in the anisotropic diffraction nonlocal medium with different lengths in the z-coordinate, as shown in Figs. 7(a)–7(c). Then, these converted solitons can stably rotate and propagate in different directions and at different speeds in the medium with isotropic diffraction. For example, the ones in Figs. 7(a) and 7(c) rotate counterclockwise and clockwise, respectively. And the corresponding values of M/P are shown in Fig. 8, which are greater than 0 and less than 0, respectively. Meanwhile, when the diffraction becomes isotropic, the deformation of OAM carried by solitons stops as well and keep unchanged over many distance. Hence, this paper reveals a novel manner to produce static or rotating solitons by the anisotropic diffraction of nonlocal media.

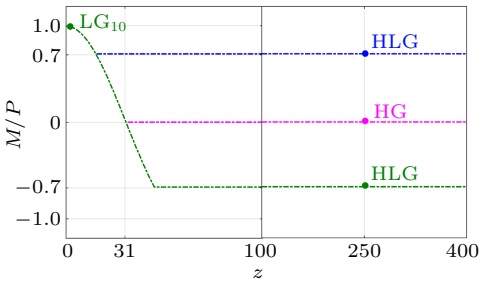


Fig. 8. Pink, green, and blue lines represent the change law of M/P, accompanying the soliton transforms in Figs. 5(a), 5(b), and 5(c), respectively.

The mode transform between higher-ordered HG and LG solitons can also be received by this means. Such as the HG_{11} mode soliton in Figs. 9(a) and 9(b) transforms periodically to double-ring LG_{02} mode. Similarly, where one could clearly identify the periodic appearance between the LG_{11} and HG_{12} solitons states from Figs. 9(c) and 9(d). In addition, the OAM accompanying the soliton transform also vary and can be controlled.

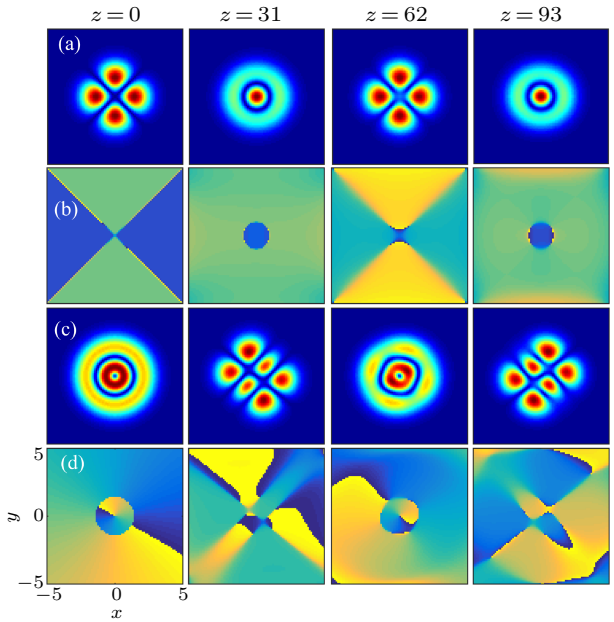


Fig. 9. (a) HG_{11} mode ($P=17400$) transforms into the LG_{02} one, (c) LG_{11} mode ($P=17500$) transforms into the HG_{12} one in nonlocal media with $\delta_y=0.9$; (b) and (d) the change law of phase patterns corresponding to panels (a) and (c), respectively.

4. Conclusion

This study numerically investigates the propagation dynamics of optics beam in nonlocal media with anisotropic diffraction and finds that the anisotropic diffraction introduces two interesting phenomena. First, the anisotropic diffraction can induce differential phase shifts and amplitude modulations to the optical field, and enable the soliton converts from one

mode to another. Next, the beam carrying OAM exhibits periodic variation simultaneously. An interesting consequence is that we can design the diffraction of media from anisotropic to isotropic at predetermined propagation distances, then the LG, HG, and HLG modes soliton with different rotating velocities and directions can be generated. A series of interesting examples demonstrate that the inter-transform of higher-order solitons also can be achieved very well.

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