

Ultrafast charging quantum battery with cavity-cavity interactions

Zhuoheng Wang(王卓恒), Zefeng Huang(黄泽丰), Dayang Zhang(张大洋), Yu Zhao(赵愈), Youbin Yu(俞友宾), Guangri Jin(金光日), and Aixi Chen(陈爱喜)

Citation: Chin. Phys. B, 2026, 35 (**3**): 034204. DOI: 10.1088/1674-1056/adfef9
http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Quantum-state engineering using enhanced tripartite interactions in atom-photon-phonon hybrid systems

Yaowu Guo(郭耀武), Jiaqiang Zhao(赵加强), Lianzhen Cao(曹连振), Yingde Li(李英德), and Hong-Yan Lu(路红艳)
Chin. Phys. B, 2025, 34 (**2**): 024203. DOI: 10.1088/1674-1056/ad9a5d

Entangling two levitated charged nanospheres through Coulomb interaction

Guoyao Li(李国耀) and Zhangqi Yin(尹璋琦)
Chin. Phys. B, 2024, 33 (**7**): 074205. DOI: 10.1088/1674-1056/ad3229

State transfer and entanglement between two- and four-level atoms in a cavity

Si-Wu Li(李思吾), Tianfeng Feng(冯田峰), Xiao-Long Hu(胡骁龙), and Xiaoqi Zhou(周晓祺)
Chin. Phys. B, 2023, 32 (**10**): 104214. DOI: 10.1088/1674-1056/acf205

Enhanced phase sensitive amplification towards improving noise immunity

Hui Guo(郭辉), Zhi Li(李治), Hengxin Sun(孙恒信), Kui Liu(刘奎), and Jiangrui Gao(郜江瑞)
Chin. Phys. B, 2023, 32 (**5**): 054204. DOI: 10.1088/1674-1056/acbdeb

Quantum properties of nonclassical states generated by an optomechanical system with catalytic quantum scissors

Heng-Mei Li(李恒梅), Bao-Hua Yang(杨保华), Hong-Chun Yuan(袁洪春), and Ye-Jun Xu(许业军)
Chin. Phys. B, 2023, 32 (**1**): 014202. DOI: 10.1088/1674-1056/ac7206

Ultrafast charging quantum battery with cavity–cavity interactions

Zhuoheng Wang(王卓恒), Zefeng Huang(黄泽丰), Dayang Zhang(张大洋), Yu Zhao(赵愈), Youbin Yu(俞友宾)†, Guangri Jin(金光日), and Aixi Chen(陈爱喜)

Zhejiang Key Laboratory of Quantum State Control and Optical Field Manipulation, Department of Physics, Zhejiang Sci-Tech University, Hangzhou 310018, *China*

(Received 14 June 2025; revised manuscript received 8 August 2025; accepted manuscript online 26 August 2025)

We present a quantum battery model comprising two lossless cavities that interact through a controlled photon–hopping mechanism, each housing an isolated two-level atom. This study explores the possibility of influencing the interaction between cavities within the traditional two single–cavity working modes, thereby paving the way for enhanced charging performance. By solving the generalized double Jaynes–Cummings model with cavity–cavity interactions, we demonstrate the positive impact of such interactions on battery charging, enabling the quantum battery to outperform the single–cavity case in terms of charging time and average charging power. Additionally, we investigate the influence of different cavity–cavity interaction strengths on charging efficiency and attempt to explain the underlying mechanisms by analyzing the entanglement within the system.

Keywords: cavity–cavity quantum battery, energy transportation, entanglement

PACS: 42.50.−p, 37.30.+i, 64.70.qd

DOI: 10.1088/1674-1056/adfef9 **CSTR:** 32038.14.CPB.adfef9

1. Introduction

As quantum technology continues to advance, techniques for precisely manipulating microscopic systems at the single-atom and single-photon levels have reached a high degree of maturity.[1,2] The concepts of heat and work are not only fundamental in classical thermodynamics but are now being actively extended to the quantum domain. This extension has spurred the emergence of "quantum thermodynamics", a vibrant and rapidly evolving research field. Quantum thermodynamics effectively accounts for the significant role of quantum effects in energy storage mechanisms. By incorporating these quantum-level phenomena, it unlocks substantial development potential and offers new perspectives for the design of advanced energy-storage systems.[3–5] Researchers worldwide are actively leveraging quantum systems to develop novel batteries. These next-generation batteries are envisioned to be micro-scale yet exhibit remarkable capabilities: high-density energy storage, ultra-rapid charging, and extended operational lifetimes.

Alicki and Fannes were pioneers in introducing the concept of quantum batteries (QBs).[6] A QB, by definition, is a quantum system designed for storing and supplying energy. Its emergence has opened new research directions in the quest for high-performance energy-storage solutions. When leveraging quantum resources such as quantum coherence and entanglement, QBs exhibit multiple advantages over their classical counterparts. First, they can achieve significantly shorter charging durations. In addition, the charging process is characterized by higher efficiency, enabled by the unique properties of these quantum states.[6–11]

Currently, implementation strategies for QBs predominantly rely on two-level systems. Charging the individual units of these batteries can be accomplished through several distinct operations. For instance, it can be realized by precisely and timely controlling the direct spin-like interaction between different units, as reported in Refs. [12–15]. Additionally, charging may be achieved by harnessing the effective interaction between two-level systems induced by their coupling with external cavity radiation.[16–19] Current research demonstrates that the implementation of QBs spans multiple physical platforms, including nuclear magnetic resonance,[20] superconducting circuits,[21] semiconductor quantum dots,[22] and photonic systems.[23] Dou *et al.*[24] revealed that both atomic interactions and external driving fields can accelerate the charging process and enhance the energy storage capacity of QBs. Jiang *et al.*[25] found that when a QB is charged using a combination of harmonic and electrostatic fields, its charging efficiency surpasses that achieved with only a harmonic field. Zhang *et al.*[26] showed that collective charging in the presence of interactions results in greater stability.

Liu *et al.*[27] revealed a nonmonotonic relationship between the battery-charger size and the energy transportation efficiency. Yu *et al.*[28] demonstrated that the quantum coherence of the bath can serve as a "fuel" for QBs to improve charging performance. Yang *et al.*[29] identified the optimal driving field for a multipartite quantum battery coupled to a common thermal bath. Building on these findings, an interesting question emerges: can the interaction formed by photon

†Corresponding author. E-mail: ybyu@zstu.edu.cn

© 2026 Chinese Physical Society and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

exchange between lossless cavities housing atoms positively influence the charging rate of QBs?

In this study, we present a charging model depicted in Fig. 1. The model comprises two lossless cavities that interact via a controlled photon–hopping mechanism. Each cavity encloses an isolated two-level atom, enabling clear observation of the inter-cavity interaction. We investigate the performance of the QB throughout the charging process. Our analysis focuses on key parameters such as the stored energy and the associated charging time. In addition, we take into account the average charging power and the fluctuations in stored energy. Through comprehensive calculations and theoretical analysis, we systematically characterize these performance metrics.

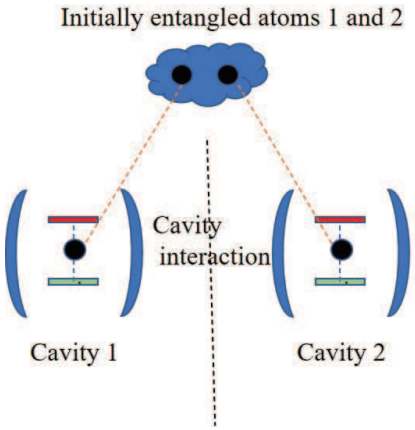


Fig. 1. Schematic illustration of the double JC model consisting of two spatially separated atoms with initial entanglement, each located in a cavity. The atom is treated as a two-level system, and the cavity is represented in the Fock basis. The cavity–cavity interaction discussed here allows the cavities to exchange photons during the system evolution.

2. Model

We consider a modification of the Hamiltonian for two isolated atom–cavity pairs. This is achieved by incorporating a cavity–cavity interaction term into the classical Jaynes–Cummings (JC) model. The relevant Hamiltonian can be expressed as[27] (for the subsequent discussions we set the reduced Planck constant $\hbar = 1$)

$$\begin{aligned}\hat{H} \;=\; & \omega(\hat{\sigma}_z^a + \hat{\sigma}_z^b) + g(\hat{a}^\dagger \hat{\sigma}_-^a + \hat{a}\hat{\sigma}_+^a) + g(\hat{b}^\dagger \hat{\sigma}_-^b + \hat{b}\hat{\sigma}_+^b) \\ & + \nu(\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}) + \kappa(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger), \end{aligned} \tag{1}$$

where ω is the transition frequency between the atomic levels $|e\rangle$ and $|g\rangle$. ν is the frequency of the harmonic oscillator mode of the cavity. g is the atom–cavity coupling constant, also known as the single-photon Rabi frequency. κ is the cavity–cavity coupling constant.[28,29] a (b) is the annihilation operator of the cavity radiation field, which destroys a photon in the corresponding cavity. Conversely, $a^\dagger$ ($b^\dagger$) is the creation operator of the cavity radiation field, responsible for creating a photon in the cavity. $\hat{\sigma}_+$ ($\hat{\sigma}_-$) are the atomic raising (lowering) operators and $\hat{\sigma}_z^a$ ($\hat{\sigma}_z^b$) are the atomic inversion operators given by the Pauli spin operator along the z direction. Here, we assume $\omega_a = \omega_b = \omega$ and $g_a = g_b = g$. This simplifying assumption implies that the two atom–cavity systems share identical atomic transition frequencies and atom–cavity coupling strengths.

3. Effect of cavity interaction on battery performance

One effective approach to studying the dynamics of the simplified double JC model is to exploit symmetry. Since the atom–cavity pair constitutes a closed system, the total energy of the system is a conserved quantity. Therefore, for the maximum double excitation of atomic states, the Hamiltonian of the system can be expressed in terms of the following basis states ($|v_i\rangle$) for the entire atom–cavity system: $\{|\uparrow\uparrow 00\rangle, |\uparrow\downarrow 10\rangle, |\uparrow\downarrow 01\rangle, |\downarrow\uparrow 10\rangle, |\downarrow\uparrow 01\rangle, |\downarrow\downarrow 20\rangle, |\downarrow\downarrow 11\rangle, |\downarrow\downarrow 02\rangle, |\downarrow\downarrow 00\rangle\}$, where $\{|0\rangle, |1\rangle, |2\rangle, \ldots, |n\rangle\}$ are the basis states in the Fock space and $|\uparrow\rangle$ ($|\downarrow\rangle$) represent the atomic excited (ground) state. The matrix form of the reduced-basis Hamiltonian is

$$\hat{H} = \begin{pmatrix} -\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega & \kappa & 0 & \sqrt{2}g & 0 & 0 & 0 & 0 \\ 0 & \kappa & \omega & \kappa & 0 & g & g & 0 & 0 \\ 0 & 0 & \kappa & \omega & 0 & 0 & 0 & \sqrt{2}g & 0 \\ 0 & \sqrt{2}g & 0 & 0 & \omega & \kappa & 0 & 0 & 0 \\ 0 & 0 & g & 0 & \kappa & \omega & 0 & 0 & g \\ 0 & 0 & g & 0 & 0 & 0 & \omega & \kappa & g \\ 0 & 0 & 0 & \sqrt{2}g & 0 & 0 & \kappa & \omega & 0 \\ 0 & 0 & 0 & 0 & 0 & g & g & 0 & \omega \end{pmatrix}. \tag{2}$$

Consequently, our computational scope is restricted to this reduced Hilbert space. This space is spanned by the basis states described above, which streamlines the analysis within the context of our study. The Hamiltonian described above can be diagonalized exactly. This exact diagonalization yields the following spectrum of eigenvalues: $E_1 = -\omega$, $E_2 = E_3 = \omega$, $E_4 = \omega - \sqrt{2g^2 + \kappa^2}$, $E_5 = \omega + \sqrt{2g^2 + \kappa^2}$, $E_6 = \omega + \lambda_+$, $E_7 = \omega - \lambda_+$, $E_8 = \omega + \lambda_-$, $E_9 = \omega - \lambda_-$, where $\lambda_\pm$ are given by

$$\lambda_\pm = \sqrt{\frac{6g^2 + 3\kappa^2 \pm \sqrt{4g^4 + 4\left(5 + 4\sqrt{2}\right) g^2\kappa^2 + \kappa^4}}{2}}.$$

We begin by characterizing the initial state of the atomic–cavity system. To facilitate the study of entanglement, we define the initial state as

$$|\psi(0)\rangle = (\cos\alpha\,|\uparrow\uparrow 00\rangle + \sin\alpha\,|\downarrow\downarrow 00\rangle). \tag{3}$$

Due to the time-evolution operator, the entangled state $|\psi(0)\rangle$ evolves into the state $|\psi(t)\rangle$. This state can be expressed in terms of the reduced basis discussed above as

$|\psi(t)\rangle = \sum_i c_i |v_i\rangle$, where the coefficients c_i depend on the system parameters ω, g, ν, and κ. The detailed derivations of these functions are provided in Ref. [30].

3.1. Stored energy and average power

The energy stored in the QB at time t is[31–33]

$$\begin{aligned}\Delta E(t) &= E(t) - E(0) \\ &= \langle\psi(t)|\hat{H}_{\rm QB}|\psi(t)\rangle - \langle\psi(0)|\hat{H}_{\rm QB}|\psi(0)\rangle, \end{aligned} \tag{4}$$

where $\hat{H}_{\rm QB}$ is the battery-related part of the total Hamiltonian in Eq. (1), which can be written as

$$\hat{H}_{\rm QB} = \omega\left(\hat{\sigma}_z^a + \hat{\sigma}_z^b\right). \tag{5}$$

From the preceding analysis we can derive the energy stored in the system at time t, which is given by

$$\Delta E(t) = 2\omega(c_1^2 - c_6^2 - c_7^2 - c_8^2 - c_9^2 - \cos^2\alpha + \sin^2\alpha). \tag{6}$$

An important task for QBs is to store the maximum energy in the shortest possible time. From this perspective, the maximum stored energy can be defined as[15,32]

$$\Delta E_{\rm max} = \max_t [\Delta E(t)]. \tag{7}$$

In Fig. 2, we show the evolution of the stored energy $\Delta E(t)$ over time for two different atom–cavity coupling strengths and three different cavity–cavity coupling strengths. The unit is ω and the average photon number is $N = 1$. In Fig. 2(a), when $g = 0.5$ and $\kappa = 0.01$, the maximum stored energy can approach 1, but the time required for the battery to reach this maximum is relatively long. When $\kappa = 0.5$, although the maximum stored energy reaches only about 0.9, the battery reaches this state much faster. In Fig. 2(b), when $g = 0.2$ and $\kappa = 0.01$ or $\kappa = 0.5$, the maximum stored energy is approximately 0.9, and the battery reaches this value rapidly. When $\kappa = 0.1$, the battery can almost reach the theoretical optimal stored energy of 1, but it takes the longest time and oscillation period to reach this maximum. These results indicate that the influence of cavity–cavity interaction on battery performance also depends on the strength of the atom–cavity coupling. Specifically, when both the atom–cavity and cavity–cavity interactions are strong and of comparable magnitude, the charging speed becomes faster, whereas weaker cavity–cavity interactions allow the stored energy to approach its optimal value. Conversely, when the atom–cavity interaction is relatively weak, a cavity–cavity interaction of comparable strength may theoretically achieve the optimal stored energy, but smaller cavity–cavity interactions help maintain a favorable charging speed. When both the atom–cavity and cavity–cavity interactions are sufficiently strong, the resulting increase in entanglement can enhance energy transfer efficiency and accelerate the charging process. However, excessively strong interactions may lead to stronger decoherence of the quantum states, causing energy loss and limiting the maximum stored energy. When the atom–cavity interaction is weak, weaker cavity–cavity interactions may enhance the cooperative effect between the two cavities and thus accelerate the charging process. As the cavity–cavity interaction becomes stronger, this cooperative effect may weaken and hinder energy transfer.

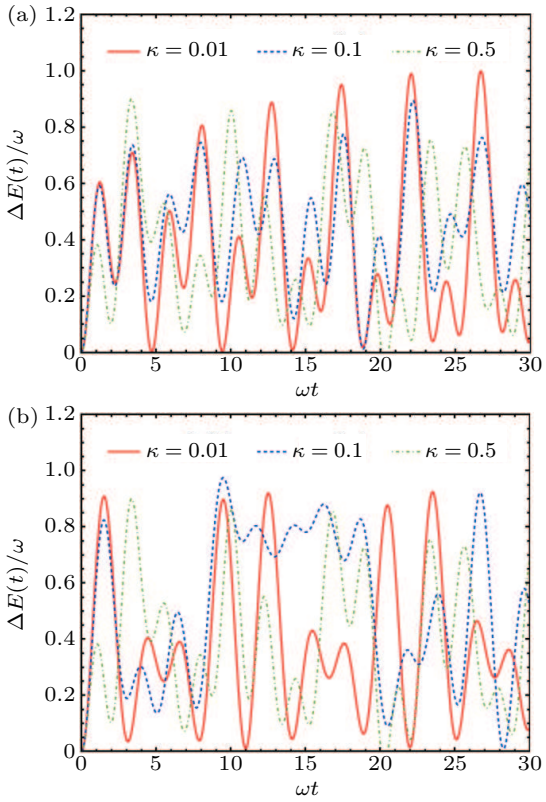


Fig. 2. Time evolution of the stored energy $\Delta E(t)$ of the QB with cavity–cavity interaction for (a) $g = 0.5$ and (b) $g = 0.2$, with $\alpha = \pi/4$, for three values of the cavity–cavity coupling parameter κ: 0.01, 0.1, and 0.5, respectively.

Another related performance parameter is the average charging power, which is defined as[8,34]

$$P(t) = \frac{\Delta E(t)}{t}. \tag{8}$$

We aim to achieve the maximum charging power in the shortest possible time. Therefore, the maximum charging power is defined as[34,35]

$$P_{\rm max} = \max_t [P(t)]. \tag{9}$$

Figure 3 shows the time evolution of the average power $P(t)$ for the cavity–cavity QB and the single-cavity JC model QB under two different atom–cavity coupling strengths and three different cavity–cavity coupling strengths. The unit is ω, and the average photon number is $N = 1$. In Fig. 3(a), the maximum power can reach $0.55\,\omega$ at $g = 0.01$ and $\kappa = 0.01$. When the coupling coefficient increases to 0.5, the maximum power decreases to about $0.4\,\omega$. In Fig. 3(b), the maximum power can reach nearly $0.7\,\omega$ at $g = 0.01$ and $\kappa = 0.01$. These

results indicate that an increase in the cavity–cavity coupling strength reduces the maximum power, whereas weaker atom–cavity interactions lead to an increase in the maximum power. This behavior may arise because the cavity–cavity interaction affects the charging dynamics, and when the interaction becomes sufficiently strong, it may degrade the battery performance. Weaker atom–cavity interactions may enhance the cooperative effect between the two cavities, leading to an increase in the maximum power. A more detailed discussion of this phenomenon will be provided in the following sections.

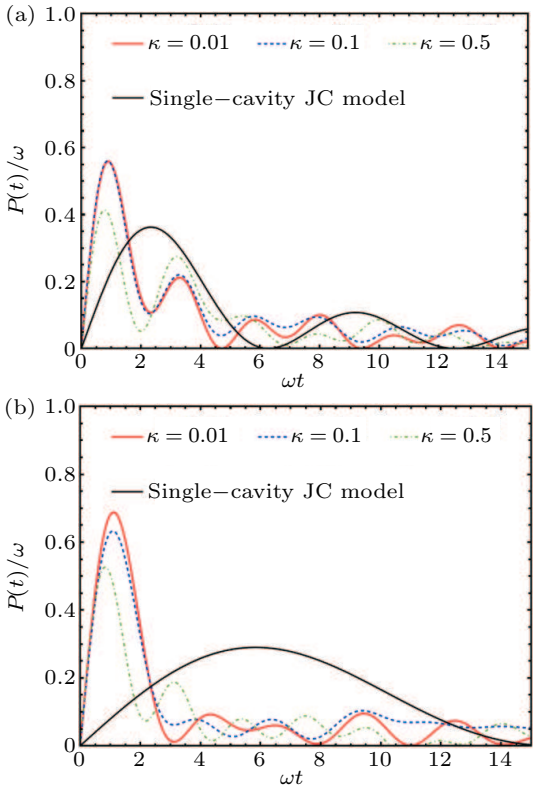


Fig. 3. Behavior of the average power $P(t)$ over time. The cavity–cavity coupling coefficients are 0.01 (red solid line), 0.1 (blue dot–dashed line), and 0.5 (green dashed line), respectively. The atom–cavity coupling coefficient is (a) $g = 0.5$ and (b) $g = 0.2$, and the average photon number is $N = 1$ for the single-cavity JC model QB.

In order to investigate the impact of cavity–cavity interactions on QB performance more intuitively, we introduce a single-cavity JC model QB that includes a cavity and an atom for comparison. The Hamiltonian of the classical JC model is

$$\hat{H} = \omega\hat{\sigma}_z + v\hat{a}^\dagger\hat{a} + g\left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-\right). \tag{10}$$

As depicted in Fig. 3, the maximum power of the single-cavity JC model-based QB is observed to approach approximately $0.35\,\omega$ and $0.3\,\omega$, respectively. Notably, when the quantum battery incorporates weak cavity–cavity interactions, its charging speed is enhanced to 1.5 times and 2.3 times that of the classical JC model-based QB. Additionally, the time required to reach the peak charging power is reduced by factors of three and four. Evidently, the introduction of the cavity–cavity interaction term leads to a substantial improvement in the average charging power of the QB, underscoring the significant impact of this interaction on enhancing the performance of the QB system.

3.2. Energy fluctuations

In order to obtain a complete description of the performance of the cavity–cavity QB, we also need to evaluate the quantum fluctuations related to the stored energy, because they can have a negative impact on the charging performance. We can discuss the stability of the charging process based on the fluctuation of the stored energy within the same time interval, which can be expressed as[35–37]

$$\Xi(t) = \sqrt{\langle\psi(t)|\hat{H}_{\rm QB}^2|\psi(t)\rangle - \left\{\langle\psi(t)|\hat{H}_{\rm QB}|\psi(t)\rangle^2\right\}}, \tag{11}$$

where $\hat{H}$ is the abbreviation of $\hat{H}_{\rm QB}$, which represents the battery-related part of the total Hamiltonian. $\Xi(t)$ is related to the reciprocal of the reverse quantum speed limit, which can be used to characterize the discharge of the QB.

In Fig. 4 we report the energy quantum fluctuations over time for three different cavity–cavity coupling terms. It can be seen from the figure that when the cavity–cavity coupling is either strong or weak, the quantum fluctuations of the battery energy are relatively large. When the coupling coefficient is 0.1, the quantum fluctuations of the energy are relatively small, and at this time a quantum battery with better performance can be obtained.

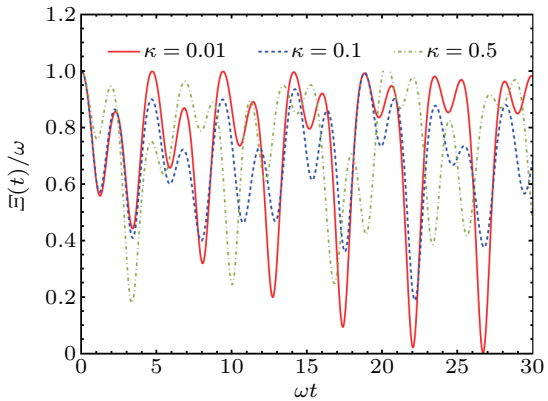


Fig. 4. Behavior of energy fluctuations $\Xi(t)$ over time. The red, blue, and green lines respectively correspond to three cavity–cavity coupling constants. The average photon number is $N = 1$ and $g = 0.5$.

4. Effects of cavity–cavity interaction on the entanglement dynamics

In Section 3, we investigated the impact of cavity–cavity interaction on the performance of QBs. Our findings revealed that when the cavity–cavity interaction is in the ultra-weak coupling regime, the quantum battery exhibits its optimal performance. Under such conditions, the battery can store the maximum amount of energy while achieving a relatively ideal

charging power. However, as the coupling coefficient increases, the battery power gradually diminishes. We conjecture that this phenomenon is closely associated with the entanglement between atoms. In this section, we aim to explore this relationship and the underlying mechanisms.

A common parameter for the entanglement between atoms is the von Neumann entropy. To obtain the reduced state ρ representing the density matrix of the two atoms, we trace out the degrees of freedom of the two cavities from the time-evolved state $|\psi(t)\rangle$. It can be written as[38]

$$\rho = |\psi(t)\rangle\langle\psi(t)|. \tag{12}$$

Substituting the expression of $|\psi(t)\rangle$ into it, we obtain

$$\rho = \begin{pmatrix} r_{11} & 0 & 0 & r_{14} \\ 0 & r_{22} & 0 & 0 \\ 0 & 0 & r_{33} & 0 \\ r_{41} & 0 & 0 & r_{44} \end{pmatrix}, \tag{13}$$

where r_{ij} are non-trivial functions of the system parameters ω, g, ν, κ, derived in Ref. [30].

Given this form of the two-atom reduced density matrix ρ, we now have all the necessary tools to investigate how the cavity–cavity interaction term affects the entanglement dynamics. To this end, we utilize the von Neumann entropy, a well-established measure, defined as follows:[39,40]

$$S(t) = -\mathrm{Tr}[\rho \log \rho]. \tag{14}$$

From Fig. 5, we observe that since the initial state of the system is an entangled state, the two atoms exhibit a certain degree of entanglement at the initial stage. When $\kappa = 0.01$, i.e., the intensity of the cavity–cavity interaction is much smaller than the atom-cavity interaction coefficient g, the cavity–cavity interaction has negligible effect on the entanglement weakening. When κ increases to 0.1, the entanglement is slightly reduced, but not significantly. This indicates that enhancing the interaction between the cavities restricts entanglement loss caused by environmental disturbances from the cavities. When the cavity–cavity interaction strength is comparable to the atom-cavity interaction, entanglement loss over time largely disappears, and a high entanglement state is maintained between the two atoms.

From this, we infer that when there is an interaction between the cavities, entanglement between the atom–atom and cavity–cavity subsystems undergoes back-and-forth quantum transfer. When κ is very low, entanglement between the two atoms is completely transferred to the cavities through the atom-cavity channel, resulting in sudden death of atom-atom entanglement. Therefore, during charging, the QB can reach a high charging power, and the battery performance remains relatively ideal. In contrast, when the cavity–cavity interaction reaches a certain strength, quantum correlations between the two cavities inhibit complete transfer of entanglement, maintaining high atom-atom entanglement throughout. This partially reduces the charging power and hinders the full charging efficiency of the QB.

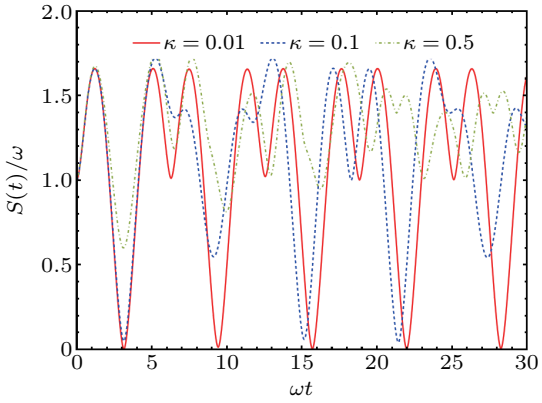


Fig. 5. Von Neumann entropy of the two-atom reduced density matrix ρ over time. The three colors in the figure correspond to three different coupling coefficients, with $g = 0.5$.

5. Conclusion

In this work, we consider a double JC model with cavity–cavity interaction, consisting of two spatially separated atoms, each in a lossless cavity. We study the performance and entanglement dynamics of the QB formed from this system, and analyze the influence of cavity–cavity interaction on QB performance, including energy storage, average charging power, and von Neumann entropy. The results show that the charging performance of the QB with cavity–cavity interaction is significantly improved compared with the single-cavity case. Different cavity–cavity coupling strengths affect the battery performance to varying degrees, which we attribute to the entanglement dynamics between atoms. Analysis of the von Neumann entropy indicates that energy storage and power are consistent with entanglement behavior, suggesting that entanglement is a primary factor limiting charging efficiency. When the coupling is very weak, the atom-atom entanglement is low, allowing full energy storage and high charging power. However, when the cavity–cavity interaction is comparable to the atom-cavity interaction, the charging performance is negatively affected. Future work will explore this further.

Experimentally, current platforms such as superconducting circuits and optical cavity systems can meet some requirements for realizing cavity–cavity interaction QBs, but performance is still affected by experimental limitations, mainly cavity loss and decoherence. Cavity loss can attenuate battery energy and reduce charging power, while decoherence can diminish quantum properties, affecting energy storage and extraction. Mitigation methods include quantum error correction and low-temperature technology, which can substantially improve experimental performance. Our work provides a foundation for implementing cavity–cavity interaction QBs and

will guide experimental research in this direction.

Moreover, the dual-cavity system has a more complex composition and interactions compared with the single-cavity system. Studying QBs of dual-cavity systems with different compositions is practically significant for improving various QB performances. Not only atomic systems, but trapped ions may also be applied. Research exists on QBs based on mechanical oscillator-driven trapped ion chains,[41] and whether trapped ions can be applied to cavity–cavity systems is a direction of future research.

Acknowledgments

Project supported by the Zhejiang Provincial Natural Science Foundation of China (Grant No. LD25A050001), the National Natural Science Foundation of China (Grant Nos. 61975184, 12175199, and 12075209), the Quantum Science and Technology-National Science and Technology Major Project (Grant No. 2023ZD0300904), and the Science Foundation of Zhejiang Sci-Tech University (Grant Nos. 19062151-Y and 18062145-Y).

References

[1] Zhang Q, Xu F H, Li L, Liu N L and Pan J W 2019 *Quantum Sci. Technol.* **4** 040503
[2] Raymer M G and Monroe C 2019 *Quantum Sci. Technol.* **4** 020504
[3] Carrega M, Sassetti M and Weiss U 2019 *Phys. Rev. A* **99** 062111
[4] Levy A and Kosloff R 2012 *Phys. Rev. Lett.* **108** 070604
[5] Vinjanampathy S and Anders J 2016 *Contemp. Phys.* **57** 545
[6] Alicki R and Fannes M 2013 *Phys. Rev. E* **87** 042123
[7] Hovhannisyan K V, Llobet M P, Huber M and Acín A 2013 *Phys. Rev. Lett.* **111** 240401
[8] Campaioli F, Pollock F A, Binder F C, Céleri L and Vinjanampathy S 2017 *Phys. Rev. Lett.* **118** 150601
[9] Binder F C, Vinjanampathy S, Modi K and Goold J 2015 *New J. Phys.* **17** 075015
[10] Yang X, Yang Y H, Alimuddin M, Salvia R, Fei S M, Zhao L M and Luo M X 2023 *Phys. Rev. Lett.* **131** 030402
[11] Šafránek D, Rosa D and Binder F C 2023 *Phys. Rev. Lett.* **130** 210401
[12] Chen P, Yin T S, Jiang Z Q and Jin G R 2022 *Phys. Rev. E* **106** 054119
[13] Seah S, Llobet M P, Haack G and Brunner N 2021 *Phys. Rev. Lett.* **127** 100601
[14] Liu J X, Shi H L, Shi Y H, Wang X H and Yang W L 2021 *Phys. Rev. B* **104** 245418
[15] Andolina G M, Farina D, Mari A, Pellegrini V, Giovannetti V and Polini M 2018 *Phys. Rev. B* **98** 205423
[16] Santos T F F, Almeida Y V D and Santos M F 2023 *Phys. Rev. A* **107** 032203
[17] Hao X, Yan K, Tan J and Wu Q Y 2022 *Phys. Rev. A* **107** 012207
[18] Abah O, Chiara G D, Paternostro M and Puebla R 2022 *Phys. Rev. Research* **4** L022017
[19] Arjmandi M B, Shokri A, Faizi E and Mohammadi H 2022 *Phys. Rev. A* **106** 062609
[20] Joshi J and Mahesh T S 2022 *Phys. Rev. A* **106** 042601
[21] Zheng R H, Ning W, Yang Z B, Xia Y and Zheng S B 2022 *New J. Phys.* **24** 063031
[22] Wenniger I M B, Thomas S E, Maffei M, Wein S C, Pont M, Harouri A, Lemaître A, Sagnes I, Somaschi N, Auffèves A and Senellart P 2023 *Phys. Rev. Lett.* **131** 260401
[23] Zhu G, Chen Y, Hasegawa Y and Xue P 2023 *Phys. Rev. Lett.* **131** 240401
[24] Dou F Q, Lu Y Q, Wang Y J and Sun J A 2022 *Phys. Rev. B* **105** 115405
[25] Jiang Y X, Chen T H, Xiao C, Pan K Y, Jin G R, Yu Y B and Chen A X 2022 *Entropy* **24** 1821
[26] Zhang D Y, Ma S Q, Jiang Y X, Yu Y B, Jin G R and Chen A X 2024 *Phys. Rev. A* **110** 032211
[27] Liu F, Yang H Y, Wang S L, Wang J Z, Zhang K and Wang X H 2025 *Chin. Phys. B* **34** 020306
[28] Yu W L, Zhang Y, Li H, Wei G F, Han L P, Tian F and Zou J 2023 *Chin. Phys. B* **32** 010302
[29] Yang Z Q, Zhou L K, Zhou Z Y, Jin G R, Cheng L and Wang X G 2023 *Chin. Phys. B* **32** 110301
[30] Pandit M, Das S, Roy S S, Dhar H S and Sen U 2018 *J. Phys. B: At. Mol. Opt. Phys.* **51** 045501
[31] Zheng H and Takada Y 2011 *Phys. Rev. A* **84** 043819
[32] Schiró M, Bordyuh M, Öztop B and Türeci H E 2012 *Phys. Rev. Lett.* **109** 053601
[33] Pandit M, Das S, Roy S S, Dhar H S and Sen U 2016 arXiv:1612.01165
[34] Ferraro D, Campisi M, Andolina G M, Pellegrini V and Polini M 2018 *Phys. Rev. Lett.* **120** 117702
[35] Crescente A, Carrega M, Sassetti M and Ferraro D 2020 *Phys. Rev. B* **102** 245407
[36] Crescente A, Carrega M, Sassetti M and Ferraro D 2020 *New J. Phys.* **22** 063057
[37] Fink J M, Bianchetti R, Baur M, Göppl M, Steffen L, Filipp S, Leek P J, Blais A and Wallraff A 2009 *Phys. Rev. Lett.* **103** 083601
[38] Hill S and Wootters W K 1997 *Phys. Rev. Lett.* **78** 5022
[39] Boes P, Eisert J, Gallego R, Müller M P and Wilming H 2019 *Phys. Rev. Lett.* **122** 210402
[40] Wang L, Liu S Q, Wu F L, Fan H and Liu S Y 2023 *Phys. Rev. A* **108** 062402
[41] Wen J, Wen Z and Li G Q 2025 arXiv:2502.08065