

Phase sensitivity of a lossy truncated SU(1,1) interferometer with double-port homodyne detection

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Phase sensitivity of a lossy truncated SU(1,1) interferometer with double-port homodyne detection

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We theoretically investigate the phase sensitivity of a truncated SU(1,1) interferometer fed with a two-mode coherent state and employing double-port homodyne detection. On the one hand, we analytically demonstrate that the two-mode coherent state provides better phase sensitivity than the single-mode coherent state. In addition, we show that the double-port homodyne detection is a quasi-optimal measurement. For a bright coherent-state input, the sensitivity of this scheme saturates the phase-sensitivity bound determined by the quantum Fisher information. On the other hand, we quantitatively illustrate the advantage of double-port homodyne detection over the single-port scheme under ideal conditions and in the presence of photon loss, respectively. Furthermore, our analysis indicates that the scheme we propose is robust against photon loss.

Keywords: quantum-enhanced interferometer, parameter estimation, homodyne detection

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1. Introduction

The important aim of quantum metrology is to achieve highly precise estimation of an unknown parameter by employing quantum resources.[1–6] Both the Mach–Zehnder interferometer and the SU(1,1) interferometer are widely used as important setups in quantum metrology. For the Mach–Zehnder interferometer, one mainly utilizes various nonclassical input states and diverse detection methods to improve the sensitivity of phase estimation.[7–14] It has been shown that the phase sensitivity of the Mach–Zehnder interferometer with nonclassical states can beat the quantum standard limit (SQL), $\Delta\phi = 1/\sqrt{\bar{N}}$, where $\bar{N}$ is the mean photon number inside the interferometer. On the other hand, in an SU(1,1) interferometer,[15,16] the beam splitters in the Mach–Zehnder interferometer are replaced by nonlinear beam splitters, such as optical parametric amplifiers (OPA) or four-wave mixers. In such a nonlinear interferometer, entanglement is generated between the two arms. It has been shown that it can surpass the SQL with two coherent states as inputs via intensity detection.[17] Consequently, phase estimation within an SU(1,1) interferometer has been extensively explored in both theoretical[18–23] and experimental studies.[24–26]

However, the SU(1,1) interferometer cannot make full use of the photons carrying phase information because many photons are absorbed by the second nonlinear beam splitter.[27] As a consequence, only part of the photons carrying phase information contribute to the measurement, which limits the phase sensitivity. To improve phase sensitivity, several modified SU(1,1) interferometers have been proposed, such as nonlinear–linear hybrid interferometers[28,29] and truncated SU(1,1) interferometers.[30,31] As a modified version of the SU(1,1) interferometer, the truncated SU(1,1) removes the second OPA, which can be considered as part of the detection stage. According to the quantum Fisher information (QFI), this does not alter the fundamental phase sensitivity of the device because the second OPA does not add information about the estimated phase. Anderson *et al.*[31] have considered a single-mode coherent state as the input of the truncated SU(1,1) interferometer and studied the phase sensitivity with double-port homodyne detection. It is known that the input state of an interferometer plays a crucial role in phase estimation. For an SU(1,1) interferometer with a two-mode coherent state $|\alpha,\beta\rangle$ as the input, it has been demonstrated that when the mean photon numbers in the two input ports reach a certain proportion, the optimal phase sensitivity can be achieved.[32] In addition, for a coherent state and a squeezed state as the input, the mean photon numbers in the two input ports also need to reach a certain proportion to approach the optimal sensitivity in an SU(1,1) interferometer or a Mach–Zehnder interferometer.[32,33] Consequently, different from the work in Ref. [31], here we consider a two-mode coherent state as the input and implement double-port homodyne detection at the output ports. Similar to the analyses in Refs. [32,33], we examine the proportion of the mean photon numbers in the two input ports required to approach the optimal sensitivity. Moreover, in both ideal and lossy scenarios, we investigate the advantages of utilizing a two-mode coherent state as the input for enhancing phase-measurement precision.

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This paper is organized as follows. In Section 2, we briefly review the model of the truncated SU(1,1) interferometer. Then, by utilizing double-port homodyne measurement and the quantum Fisher information, we analyze the sensitivity of the estimated phase in an ideal scenario. In Section 3, considering that photon loss is inevitable in practical applications, we investigate the effects of photon loss on phase sensitivity. Finally, we summarize our results in Section 4.

2. Model and phase sensitivity of the truncated SU(1,1) interferometer

2.1. Model

In the truncated SU(1,1) interferometer, the second OPA in the conventional SU(1,1) interferometer is removed, as shown in Fig. 1. The two-mode coherent state evolves into a two-mode squeezed coherent state via the OPA. The resulting state then undergoes a phase shift ϕ on one arm of the interferometer. To model photon loss, we insert fictitious beam splitters with reflectivities L_1 and L_2 (photon-loss rates) into the two arms of the interferometer. Finally, double-port homodyne detection is performed, in which the photons at each output port of the interferometer are simultaneously detected.

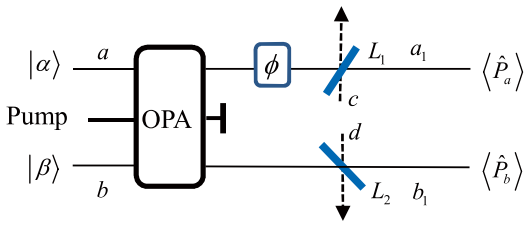


Fig. 1. Schematic of the double-port homodyne detection in a truncated SU(1,1) interferometer with two-mode coherent states. Photon loss processes in the interferometer are modeled by adding fictitious beam splitters.

In general, the action of the OPA is described by the two-mode squeezing operator $\hat{S}_2(g) = \exp\left(g\mathrm{e}^{\mathrm{i}\theta}\hat{a}^\dagger\hat{b}^\dagger - g\mathrm{e}^{-\mathrm{i}\theta}\hat{a}\hat{b}\right)$, where g is the parametric gain and θ is the phase. The operators $\hat{a}$ ($\hat{a}^\dagger$) and $\hat{b}$ ($\hat{b}^\dagger$) denote the annihilation (creation) operators associated with modes a and b of the interferometer, respectively. Here we consider an unknown phase shift applied to the upper arm of the interferometer, which is described by the unitary operator $\hat{U}_a(\phi) = \exp\left[\mathrm{i}\phi\hat{a}^\dagger\hat{a}\right]$. Let us first consider a lossless truncated SU(1,1) interferometer, i.e., $L_1 = L_2 = 0$. Consequently, for an arbitrary input state $\hat{\rho}_{\mathrm{in}}$ propagating through the lossless interferometer, its output state is

$$\hat{\rho}_{\mathrm{out}} = \hat{U}_a(\phi)\hat{S}_2(g)\hat{\rho}_{\mathrm{in}}\hat{S}_2^\dagger(g)\hat{U}_a^\dagger(\phi). \tag{1}$$

In the Heisenberg picture, the relations between the output and input modes of the truncated SU(1,1) interferometer are

$$\begin{aligned}\hat{a}_1 &= \hat{S}_2^\dagger(g)\hat{U}_a^\dagger(\phi)\hat{a}\hat{U}_a(\phi)\hat{S}_2(g)\\ &= \hat{a}\mathrm{e}^{\mathrm{i}\phi}\cosh g + \mathrm{e}^{\mathrm{i}\theta}\hat{b}^\dagger\sinh g,\\ \hat{b}_1 &= \hat{S}_2^\dagger(g)\hat{U}_a^\dagger(\phi)\hat{b}\hat{U}_a(\phi)\hat{S}_2(g)\\ &= \mathrm{e}^{\mathrm{i}\theta}\hat{a}^\dagger\sinh g + \hat{b}\cosh g.\end{aligned} \tag{2}$$

For precision phase estimation, the mean photon number inside the interferometer is an important physical quantity. For a two-mode coherent state $|\alpha,\beta\rangle$ with amplitudes $\alpha = |\alpha|\,\mathrm{e}^{\mathrm{i}\psi}$ and $\beta = |\beta|\,\mathrm{e}^{\mathrm{i}\omega}$, the mean photon number becomes

$$\begin{aligned}\bar{N} &= \langle\alpha,\beta|\hat{S}_2^\dagger(g)\left(\hat{a}^\dagger\hat{a}+\hat{b}^\dagger\hat{b}\right)\hat{S}_2(g)|\alpha,\beta\rangle\\ &= \left(|\alpha|^2+|\beta|^2\right)\cosh 2g + 2\sinh^2 g\\ &\quad +2|\alpha\beta|\cos(\psi+\omega-\theta)\sinh 2g.\end{aligned} \tag{3}$$

When the total input photon number $\bar{N}_{\mathrm{in}} = |\alpha|^2+|\beta|^2$ is fixed, the conditions $|\alpha| = |\beta|$ and $\psi+\omega-\theta = 0$ maximize the mean photon number. Therefore, we set $\psi = \omega = \theta = 0$ in the following. From Eq. (3), the SQL becomes

$$\Delta\phi_{\mathrm{SQL}} = \frac{1}{\left(|\alpha|^2+|\beta|^2\right)\cosh 2g + 2\sinh^2 g + 2|\alpha\beta|\sinh 2g}. \tag{4}$$

2.2. Phase sensitivity

In quantum estimation theory, the phase sensitivity of a measurement scheme is given by

$$\Delta\phi = \frac{\Delta\hat{M}}{\left|\partial\langle\hat{M}\rangle/\partial\phi\right|}, \tag{5}$$

where $\hat{M}$ is an observable and $\Delta\hat{M} = \sqrt{\langle\hat{M}^2\rangle - \langle\hat{M}\rangle^2}$ is its variance. Phase sensitivity can be improved by reducing $\Delta\hat{M}$, by increasing the slope $\left|\partial\langle\hat{M}\rangle/\partial\phi\right|$, or by optimizing both.

Homodyne detection is a primary tool in experiments. Inspired by Ref. [31], we define the double-port homodyne observable as

$$\hat{M} = \hat{P}_a + \lambda\hat{P}_b = \frac{\mathrm{i}\hat{a}^\dagger - \mathrm{i}\hat{a}}{\sqrt{2}} + \frac{\lambda}{\sqrt{2}}\left(\mathrm{i}\hat{b}^\dagger - \mathrm{i}\hat{b}\right), \tag{6}$$

where $\hat{P}_a$ ($\hat{P}_b$) is the phase quadrature. To improve sensitivity, we introduce a classical gain factor λ multiplying the homodyne signal of mode b before summation.

For the two-mode coherent input, using Eq. (2) we obtain

$$\langle\hat{M}\rangle = \sqrt{2}\sin\phi\left(|\alpha|\cosh g + |\beta|\sinh g\right), \tag{7}$$

where we have again set $\psi = \omega = \theta = 0$. When $|\alpha| = |\beta| = 0$, the signal vanishes, meaning the measurement yields no phase information. The second moment is

$$\begin{aligned}\langle\hat{M}^2\rangle &= 2\sin^2\phi\left(|\alpha|\cosh g + |\beta|\sinh g\right)^2\\ &\quad -\lambda\cos\phi\,\sinh 2g + \left(1+\lambda^2\right)\left(\sinh^2 g + \frac{1}{2}\right).\end{aligned} \tag{8}$$

Thus, the phase sensitivity is

$$\Delta\phi = \frac{\sqrt{-\lambda\cos\phi\sinh 2g + \left(1+\lambda^2\right)\left(\sinh^2 g + 1/2\right)}}{\left|\sqrt{2}\cos\phi\left(|\alpha|\cosh g + |\beta|\sinh g\right)\right|}. \tag{9}$$

Optimizing the gain factor yields

$$\lambda = \tanh 2g. \tag{10}$$

In the lossless case, the gain factor depends solely on the OPA gain. From Eq. (10), λ increases with g and remains below 1 for all finite g, approaching 1 when $g > 1$. Therefore, the gain factor improves sensitivity mainly for small g, while for $g > 1$ its effect becomes negligible. According to Eq. (9), the optimal phase is $\phi = 0$ for double-port homodyne detection. Moreover, when the input photon number $N_{\rm in} = |\alpha|^2 + |\beta|^2$ is constrained, the optimal condition for sensitivity is $|\alpha| = |\beta| \coth g$. Thus, compared with the truncated SU(1,1) interferometer using a single-mode coherent input,[31] the two-mode coherent state further improves phase sensitivity.

For comparison, the phase sensitivity of a conventional SU(1,1) interferometer using single-port homodyne detection ($\hat{P}_a$) on the arm where the phase shift occurs is

$$\Delta\phi\big|_{\rm SU(1,1)} = \frac{\sqrt{2\,(1-\cos\phi)\sinh^2 g\cosh^2 g + 1/2}}{\left|\sqrt{2}\cos\phi\,\cosh g\,(|\alpha|\cosh g + |\beta|\sinh g)\right|}. \tag{11}$$

Again, the optimal ratio is $|\alpha| = |\beta| \coth g$. Equation (11) differs from Eq. (16) in Ref. [32] because the phase is distributed on a different arm when detection is performed on the same arm. A comparison shows that single-port detection on the phase-shift arm leads to better sensitivity, providing a useful perspective for practical homodyne implementations.

In Fig. 2, we compare the sensitivities of double-port and single-port homodyne detection. The double-port scheme surpasses the SQL around $\phi = 0$ and achieves better sensitivity than the single-port method at the optimal working point. This improvement arises because the single-port measurement uses only one quadrature and therefore loses part of the phase information. Moreover, based on classical Fisher information, Zhong *et al.*[34] also showed that the double-port scheme is superior in interferometric phase estimation.

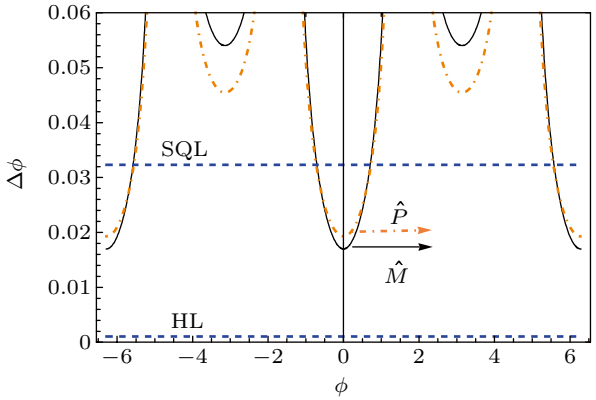


Fig. 2. For $|\alpha| = |\beta| = 12$ and $g = 0.6$, the phase sensitivity as a function of the estimated phase ϕ. Sensitivities are shown for two configurations: double-port homodyne detection ($\hat{M}$, black line) and single-port homodyne detection ($\hat{P}$, orange dot-dashed line). The upper blue dashed line denotes the SQL, while the lower blue dashed line represents the HL.

The ultimate limit of the phase sensitivity is given by the quantum Cramér–Rao bound (QCRB), defined by $\Delta\phi_{\rm QCRB} = 1/\sqrt{F}$. When the estimated phase shift ϕ appears in the upper arm, as shown in Fig. 1, the QFI is given by[35]

$$\begin{aligned} F \;=\; & \left(|\alpha|^2 + |\beta|^2\right)(\cosh 4g + 1) + 2\left(|\alpha|^2 - |\beta|^2\right)\cosh 2g \\ & + 2|\alpha||\beta|\sinh 4g + \sinh^2 2g. \end{aligned} \tag{12}$$

Similarly, under the constraint on the input photon number, the maximum QFI is achieved when $|\alpha| = |\beta| \coth g$. By performing numerical analysis, we compare Eq. (9) with Eq. (12) at the optimal working point $\phi = 0$. The phase sensitivity of this scheme can saturate the QCRB in the limit of $|\alpha| \gg 1$ (or equivalently $|\beta|^2 \gg 1$), as shown in Fig. 3. Thus, the double-port homodyne detection constitutes a quasi-optimal measurement for the two-mode coherent input in this bright-input limit.

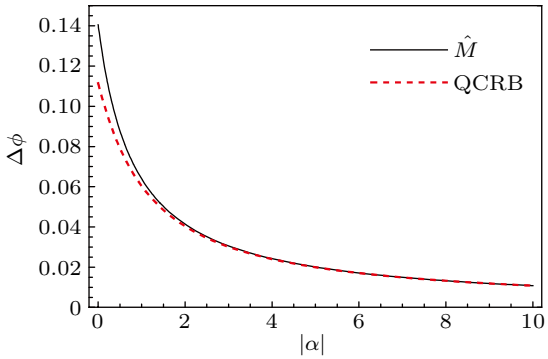


Fig. 3. For $|\beta| = 1$ and $g = 1.2$, the phase sensitivity as a function of $|\alpha|$ at the optimal working point $\phi = 0$. The black line denotes the phase sensitivity of the double-port homodyne detection, while the lower red dashed line represents the QCRB.

3. Analysis of robustness regarding photon loss

In practical applications, photon loss is inevitable and affects the attainable phase sensitivity. In the following, we investigate the effects of photon loss on the phase sensitivity. As shown in Fig. 1, the process of photon loss can be modeled by inserting a fictitious beam splitter with reflectivity L (photon-loss rate), where a signal input beam and a vacuum state are injected into the two input ports of the fictitious beam splitter.[32,34,36,37] Passing through the fictitious beam splitters, the mode transformation of the field operators is given by $\hat{a}' = \sqrt{1-L_1}\hat{a} + \sqrt{L_1}\hat{c}$ and $\hat{b}' = \sqrt{1-L_2}\hat{b} + \sqrt{L_2}\hat{d}$. Therefore, for a lossy scenario, the operator relationship between the input and output modes of the interferometer reads

$$\begin{aligned} \hat{a}_1 &= \sqrt{1-L_1}\hat{a}\,{\rm e}^{{\rm i}\phi}\cosh g + \sqrt{1-L_1}\hat{b}^\dagger\,{\rm e}^{{\rm i}\phi}\sinh g + \sqrt{L_1}\hat{c}, \\ \hat{b}_1 &= \sqrt{1-L_2}\hat{b}\cosh g + \sqrt{1-L_2}\hat{a}^\dagger\sinh g + \sqrt{L_2}\hat{d}. \end{aligned} \tag{13}$$

Then, for the two-mode coherent state as the input, we obtain

$$\langle \hat{M}_L \rangle = \sqrt{2(1-L_1)}\,\sin\phi\,(|\alpha|\cosh g + |\beta|\sinh g). \tag{14}$$

Further, we calculate

$$\langle \hat{M}_L^2 \rangle = 2(1-L_1)(\alpha\cosh g+\beta\sinh g)^2\sin^2\phi+(1-L_1)\sinh^2 g+\frac{1}{2}+\lambda^2(1-L_2)\sinh^2 g-\lambda\sqrt{(1-L_1)(1-L_2)}\cos\phi\,\sinh 2g+\frac{\lambda^2}{2}. \quad (15)$$

Combining Eqs. (14) and (15), we obtain the phase sensitivity

$$\Delta\phi\big|_L = \frac{\sqrt{-\lambda\sqrt{(1-L_1)(1-L_2)}\cos\phi\,\sinh 2g+(1+\lambda^2-L_1-\lambda^2 L_2)\sinh^2 g+(1+\lambda^2)/2}}{\left|\sqrt{2(1-L_1)}\cos\phi\,(|\alpha|\cosh g+|\beta|\sinh g)\right|}. \quad (16)$$

To obtain the optimal phase sensitivity, the classical gain factor λ is found to be

$$\lambda = \frac{\sqrt{(1-L_1)(1-L_2)}\,\sinh 2g}{\cosh 2g-2L_2\sinh^2 g}. \quad (17)$$

Equation (17) indicates that, in the presence of loss, the factor λ depends not only on the parametric gain g of the OPA but also on the photon-loss rates.

As previously pointed out, the phase sensitivity in the ideal scenario can beat the SQL when the estimated phase approaches zero. In the following, we investigate the impact of the photon-loss rates in both arms of the interferometer on the phase sensitivity. In particular, we analyze the maximal tolerable loss before the phase sensitivity drops below the SQL. For the lossy truncated SU(1,1) interferometer with double-port homodyne detection, the contour lines of the phase sensitivity as a function of the photon-loss rates L_1 and L_2 at the optimal working point are shown in Fig. 4. The phase sensitivities in the lower-left region and within the SQL boundary in Fig. 4 can surpass the SQL. For a given input photon number $|\alpha|^2+|\beta|^2$, both the phase sensitivity and the maximal tolerable loss can be enhanced by increasing the proportion of photons sent into the upper arm of the interferometer, up to the point $|\alpha|/|\beta| = \coth g$. When $|\alpha| > |\beta|\coth g$, although the maximal tolerable loss can still be increased by raising $|\alpha|^2$, the phase sensitivity becomes worse, as shown in Fig. 4(c). In addition, Fig. 4 shows that photon loss L_1 in the upper arm, where the estimated phase is encoded, has a stronger impact on phase sensitivity. On the other hand, for a fixed input-photon proportion $|\alpha|^2/|\beta|^2$, combining Eqs. (4) and (16), our numerical results indicate that the maximal tolerable loss does not depend on the total input photon number.

In Fig. 5, we show the dependence of the maximal tolerable loss on the gain parameter of the OPA. The maximal tolerable loss L_2 in the lower arm decreases as the gain parameter increases, whereas the maximal tolerable loss L_1 in the upper arm increases slightly with increasing gain. Furthermore, for fixed photon-loss rates, the phase sensitivity improves as the gain parameter increases, as shown in Fig. 5.

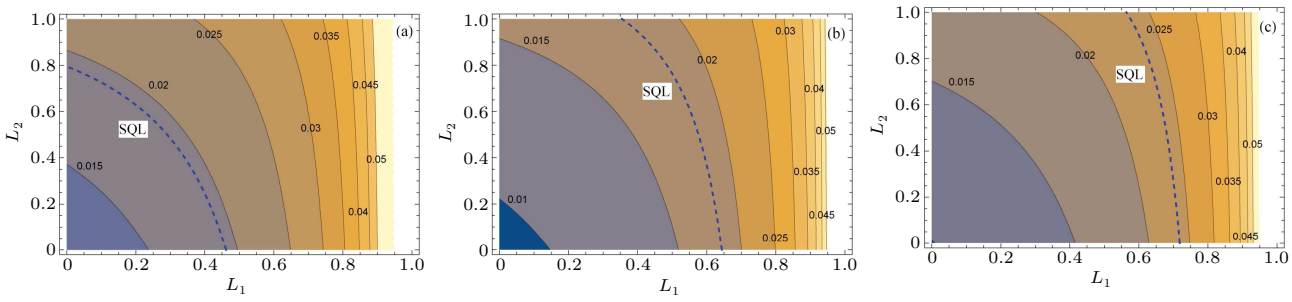


Fig. 4. For a given input photon number $|\alpha|^2+|\beta|^2 = 1000$ and $g = 0.6$, the contour lines of the phase sensitivity as a function of the photon lossy rates L_1 and L_2 at the optimal working point for different ratios of $|\alpha|/|\beta|$. (a) $|\alpha|/|\beta| = 1/3$; (b) $|\alpha|/|\beta| = \coth g$; (c) $|\alpha|^2 = 1000$, $|\beta|^2 = 0$. The phase sensitivities in the lower-left region within the blue dashed SQL line can beat the SQL.

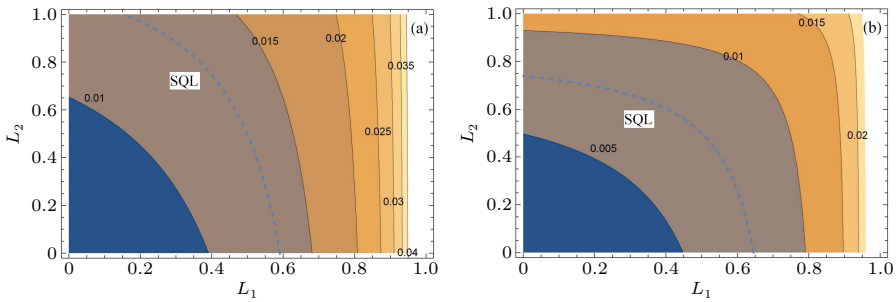


Fig. 5. For $|\alpha| = |\beta| = 30$, the contour lines of the phase sensitivity as a function of the photon lossy rates L_1 and L_2 at the optimal working point for different parametric gains of the OPA. (a) $g = 0.6$; (b) $g = 1.2$. The phase sensitivities in the lower-left region within the blue dashed SQL line can beat the SQL.

Since the lengths of the two arms are close in a practical scenario, for simplicity we impose that the two arms have identical photon loss rates, i.e., $L_1 = L_2 = L$. Then the optimal phase sensitivity reads

$$\Delta\phi|_L = \frac{\sqrt{-\lambda\,(1-L)\cos\phi\sinh 2g + (1+\lambda^2)\,(1-L)\sinh^2 g + (1+\lambda^2)/2}}{\left|\sqrt{2\,(1-L)}\cos\phi\,(|\alpha|\cosh g + |\beta|\sinh g)\right|}, \tag{18}$$

with $\lambda = (1-L)\sinh 2g/\left(\cosh 2g - 2L\sinh^2 g\right)$. Naturally, when $L = 0$, i.e., in the ideal scenario, Eq. (18) reduces to Eq. (10) as expected. In the limit of $L = 1$, where the inputs are completely lost, we have $\langle \hat{M}_L^2\rangle - \langle \hat{M}_L\rangle^2 = 1/2$, which matches the variance of a vacuum state that carries no information on the estimated phase. Similarly, for the conventional SU(1,1) interferometer, we can obtain the phase sensitivity with the single-port homodyne detection under symmetric photon losses ($L_1 = L_2 = L$), i.e.,

$$\Delta\phi|_{\mathrm{SU}(1,1),L} = \frac{\sqrt{2\,(1-L)\,(1-\cos\phi)\cosh^2 g\sinh^2 g + L\sinh^2 g + 1/2}}{\sqrt{2\,(1-L)}\,|\cos\phi\cosh g\,(|\alpha|\cosh g + |\beta|\sinh g)|}. \tag{19}$$

Particularly, in the case of $L = 50\%$, according to Eqs. (18) and (19) we have

$$\begin{aligned}\Delta\phi|_{L=0.5} &= \Delta\phi|_{\mathrm{SU}(1,1),L=0.5} \\ &= \frac{\sqrt{2\,(1-\cos\phi)\cosh^2 g\sinh^2 g + \sinh^2 g + 1}}{\sqrt{2}\,|\cos\phi\cosh g\,(|\alpha|\cosh g + |\beta|\sinh g)|}.\end{aligned} \tag{20}$$

Equation (20) indicates that the phase sensitivity of the double-port homodyne detection is equal to that of the single-port one in the case of $L = 50\%$.

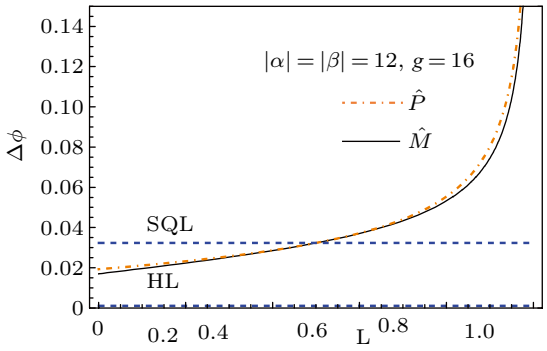


Fig. 6. For $|\alpha| = |\beta| = 12$ and $g = 0.6$, the phase sensitivity as a function of the photon lossy rate at the optimal working point $\phi = 0$. Sensitivity is shown for two configurations: double-port homodyne detection ($\hat{M}$, black line) and single-port homodyne detection ($\hat{P}$, orange dot-dashed line). The upper blue dashed line denotes the SQL, while the lower blue dashed line represents the HL.

In the symmetric photon loss scenario, the phase sensitivity can still approach the SQL as the loss approaches 50%, as shown in Fig. 6. In addition, the double-port homodyne detection provides better phase sensitivity than the single-port homodyne detection except at $L = 50\%$. In the case of $L = 50\%$, the phase sensitivities obtained by the double-port and single-port homodyne detections are identical, corresponding to Eq. (20). It should be pointed out that when $L > 50\%$, the phase sensitivities of both the double-port and single-port homodyne detection schemes do not surpass the SQL.

4. Conclusion

In summary, we have investigated the phase estimation performance of the truncated SU(1,1) interferometer with a two-mode coherent state $|\alpha,\beta\rangle$ in both ideal and lossy scenarios. We focus on the double-port homodyne detection scheme, in which the photons at each output port of the interferometer are simultaneously detected. Our results show that, compared with injecting a single-mode coherent state and a vacuum state, using a two-mode coherent state yields better phase sensitivity under the constraint of a fixed total mean input photon number. At the optimal working point $\phi = 0$, the phase sensitivity of our scheme can saturate the QCRB when the coherent amplitude satisfies $|\alpha| \gg 1$ (or $|\beta|^2 \gg 1$). In addition, for a given total input photon number, the condition $|\alpha| = |\beta|\coth g$ is optimal for achieving both improved phase sensitivity and a tighter QCRB.

On the other hand, in the presence of photon loss, our results indicate that homodyne detection remains robust. The double-port homodyne detection consistently provides better phase sensitivity than the single-port scheme in both ideal and lossy scenarios. Moreover, based on currently available experimental techniques, our scheme is feasible for achieving phase sensitivities that saturate the QCRB with a bright-seeded SU(1,1) interferometer.

Acknowledgments

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