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Adiabatic holonomic quantum computation in decoherence-free subspaces with two-body interaction

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Adiabatic holonomic gates possess the geometric robustness of adiabatic geometric phases, i.e., dependence only on the evolution path of the parameter space but not on the evolution details of the quantum system, which, when coordinated with decoherence-free subspaces, permits additional resilience to the collective dephasing environment. However, the previous scheme [*Phys. Rev. Lett.* **95** 130501 (2005)] of adiabatic holonomic quantum computation in decoherence-free subspaces requires four-body interaction that is challenging in practical implementation. In this work, we put forward a scheme to realize universal adiabatic holonomic quantum computation in decoherence-free subspaces using only realistically available two-body interaction, thereby avoiding the difficulty of implementing four-body interaction. Furthermore, an arbitrary one-qubit gate in our scheme can be realized by a single-shot implementation, which eliminates the need to combine multiple gates for realizing such a gate.

Keywords: adiabatic evolution, holonomic quantum computation, decoherence-free subspaces

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1. Introduction

In 1984, Berry[1] found that a quantum system initially prepared in a nondegenerate eigenstate of the Hamiltonian evolving adiabatically and cyclically along a circuit in the parameter space will acquire a geometric phase in addition to the familiar dynamical phase. In the degenerate case, the geometric phase acquired for the adiabatic and cyclic evolution in the space of degenerate eigenstates should be a matrix-valued operator, termed non-Abelian geometric phases.[2] Different from the dynamical phase that depends on the eigenenergy and evolution timing, the geometric phase is dependent only on the evolution path along a circuit in the parameter space but independent of the evolution details of the quantum system. Due to this geometric robustness, geometric phases are used to realize quantum computation, in which the scenario based on Berry phases is known as adiabatic geometric quantum computation[3] and the scenario based on adiabatic non-Abelian geometric phases is known as adiabatic holonomic quantum computation.[4,5]

The further extension of geometric phases is from adiabatic evolution to nonadiabatic evolution, leading to the proposals of nonadiabatic Abelian geometric phases[6] and nonadiabatic non-Abelian geometric phases.[7] The corresponding quantum computation is referred to as nonadiabatic geometric quantum computation[8,9] and nonadiabatic holonomic quantum computation.[10,11] By avoiding the requirement of adiabatic evolution, nonadiabatic holonomic quantum computation permits high-speed implementation. To date, many protocols of nonadiabatic holonomic quantum computation have been proposed, such as the single-shot scheme,[12,13] the single-loop scheme[14] and the general construction scheme.[15,16] A large number of theoretical realizations were also developed based on various physical systems.[17–29] Besides these theoretical advances, the experimental investigation of nonadiabatic holonomic quantum computation has been highly successful.[30–39] Although nonadiabatic evolution allows for high-speed implementation, the geometric feature between the adiabatic geometric phase and nonadiabatic geometric phase is different, i.e., the adiabatic geometric phase depends on the parameter-space evolution path[1,2] while the nonadiabatic geometric phase depends on the state-space evolution path.[6,7] This results in the adiabatic approach[40,41] of holonomic gates possessing some unique advantages, e.g., the quantum gates are only related to the ratio of parameters and hence robust against fluctuations of their absolute values,[42–45] which is not shared by the nonadiabatic holonomic gates. It is the main reason that adiabatic holonomic quantum computation is still an ongoing research topic.

The aforementioned geometric robustness focuses primarily on the control errors in the quantum computation. Taking the environment noises into account, it is necessary to integrate the decoherence-mitigation methods into geometric quantum computation. For this consideration, adi-

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abatic holonomic quantum computation in decoherence-free subspaces was proposed.[46] It combines the geometric robustness of adiabatic geometric phases and the coherence stabilization of decoherence-free subspaces, thus representing great progress in the community of geometric quantum computation. However, the realization of this scheme requires implementing four-body interaction, which is challenging in an actual physical system with a realistic control. Note that in the nonadiabatic domain, the decoherence-free subspace has been integrated into holonomic quantum computation through the two-body interaction because of the simple three-level configuration.[47,48] Also for this reason, nonadiabatic holonomic quantum computation has been combined with other decoherence-mitigation methods, such as noiseless systems,[49] dynamical decoupling,[50] and quantum error correction.[51] But regarding the adiabatic approach to holonomic quantum computation, the construction of holonomic gates needs quite complicated level configurations and hence the previous methods cannot be directly translated to the realization of universal holonomic quantum computation in decoherence-free subspaces.

In this work, we tackle this problem through constructing realistically available two-body interaction to realize a universal set of adiabatic holonomic gates protected by the decoherence-free subspace encoding. Our scheme therefore avoids the difficulties in the implementation of adiabatic holonomic quantum computation in decoherence-free subspaces with the four-body interaction. Furthermore, an arbitrary one-qubit gate in our scheme can be realized by a single-shot implementation, thus eliminating the need to combine multiple gates to implement an arbitrary one-qubit gate. The paper is organized as follows. In Section 2, we illustrate the physical model used in our scheme. In Section 3, we present the realization of an arbitrary one-qubit gate by a single-shot implementation. In Section 4, we demonstrate the two-qubit gate with the two-body interaction. Section 5 is our conclusion.

2. Physical model

Let us first illustrate the physical model used in our implementation of adiabatic holonomic quantum computation in decoherence-free subspaces. Consider a quantum system consisting of N physical qubits and interacting collectively with a dephasing environment. The driving Hamiltonian of the quantum system is given by

$$H_N = \sum_{k<l} \left[J_{kl}^x \left(\sigma_k^x \sigma_l^x + \sigma_k^y \sigma_l^y \right) + J_{kl}^y \left(\sigma_k^x \sigma_l^y - \sigma_k^y \sigma_l^x \right) \right], \quad (1)$$

where $\sigma_{k(l)}^{x(y)}$ represents the Pauli $x(y)$ operator acting on the $k(l)$-th physical qubit. The first term of this Hamiltonian describes the XY coupling between two qubits and the second term describes the Dzialoshinski–Moriya coupling, with the corresponding coupling strengths J_{kl}^x and J_{kl}^y.[52,53] This Hamiltonian is experimentally accessible. For example, it can be realized by using trapped ions with the Sørensen–Mølmer setting.[54,55] Specifically, we apply a blue sideband laser with Rabi frequency $\Omega_1(t)$ and detuning $-(\nu+\delta)$ to the k-th ion, and another blue sideband laser with Rabi frequency $\Omega_2(t)$ and the same detuning to the l-th ion, where ν is the vibrational frequency and δ is an additional detuning. In the rotating frame and with the rotating-wave approximation, the Hamiltonian of this quantum system is given by $H(t) = \mathrm{i}\eta \exp(-\mathrm{i}\delta t)(\Omega_1(t) a^\dagger |1\rangle_k \langle 0| + \Omega_2(t) a^\dagger |1\rangle_l \langle 0|) + \text{H.c.}$ in the Lamb–Dicke regime. Here, a and $a^\dagger$ are the annihilation and creation operators of the vibrational mode, and η denotes the Lamb–Dicke parameter. If the large detuning approximation $\delta \gg \eta\Omega_1(t), \eta\Omega_2(t)$ is satisfied, the Hamiltonian yields an effective one $H(t) = \Omega_{\text{eff}}(t) |01\rangle \langle 10| + \text{H.c.}$ with $\Omega_{\text{eff}}(t) = \eta^2 \Omega_1^*(t) \Omega_2(t)/\delta$. Here, we have ignored the Stark shift terms that can be easily compensated by using additional lasers. The effective Hamiltonian describes the XY interaction if $\Omega_{\text{eff}}(t)$ is a real number, and it describes the Dzialoshinski–Moriya interaction if $\Omega_{\text{eff}}(t)$ is an imaginary number. Therefore, the aforementioned interactions are realistically available in trapped ions.

The environmental noise under consideration is the collective dephasing shared by all physical qubits, with the interaction Hamiltonian between the quantum system and its environment given by

$$H_I = \left(\sum_k \sigma_k^z \right) \otimes B, \quad (2)$$

where the first term represents the system spin, while the second term denotes the corresponding environment. In the following, we shall demonstrate how to use the driving Hamiltonian described in Eq. (1) to realize a universal set of adiabatic holonomic gates in decoherence-free subspaces, including an arbitrary one-qubit gate and a nontrivial two-qubit gate.

3. One-qubit gates

For realizing adiabatic holonomic one-qubit gates protected by decoherence-free subspaces, we need to consider a quantum system consisting of four physical qubits. In this case, there exists a four-dimensional subspace, given by

$$\mathcal{S} = \text{Span}\{|1000\rangle, |0100\rangle, |0010\rangle, |0001\rangle\}. \quad (3)$$

Clearly, the quantum states in this subspace are immune to the collective dephasing [induced by the interaction Hamiltonian defined in Eq. (2)] shared by the four physical qubits. Therefore, we can use the basis states in the subspace to encode a logical qubit in order to mitigate decoherence. To this end, we encode the computational logical qubit into a smaller subspace

$$\mathcal{S}^L = \text{Span}\{|0001\rangle, |0010\rangle\}. \quad (4)$$

Here, the last two qubits are used as the data qubits that represent the quantum information while the first two qubits act as auxiliary qubits. As a consequence, the logical-qubit encoding corresponding to the computation subspace reads $|0001\rangle \equiv |00\rangle \otimes |0\rangle_L$ and $|0010\rangle \equiv |00\rangle \otimes |1\rangle_L$. Meanwhile, we regard the remaining basis states in the four-qubit decoherence-free subspace as auxiliary states, denoted as $|a\rangle = |0100\rangle$ and $|e\rangle = |1000\rangle$.

Having accomplished the logical-qubit encoding with the decoherence-free subspace, we now design the adiabatic holonomic operation in this subspace. Let us consider the Hamiltonian in Eq. (1) with $N = 4$. For our purpose, the nonzero parameters of the Hamiltonian are taken as

$$\begin{aligned}
J_{12}^{x} &= -J(t)\cos\frac{\alpha(t)}{2},\\
J_{13}^{x} &= -J(t)\sin\frac{\alpha(t)}{2}\cos\frac{\theta}{2}\cos\beta(t),\\
J_{13}^{y} &= J(t)\sin\frac{\alpha(t)}{2}\cos\frac{\theta}{2}\sin\beta(t),\\
J_{14}^{x} &= -J(t)\sin\frac{\alpha(t)}{2}\sin\frac{\theta}{2}\cos(\beta(t)+\varphi),\\
J_{14}^{y} &= J(t)\sin\frac{\alpha(t)}{2}\sin\frac{\theta}{2}\sin(\beta(t)+\varphi),
\end{aligned} \tag{5}$$

where θ and φ are the time-independent parameters, and $\alpha(t)$ and $\beta(t)$ are the time-dependent parameters, as indicated in the above equation. The remaining parameter $J(t)$ can be either time-independent or time-dependent. In the basis $\{|00\rangle \otimes |0\rangle_L, |00\rangle \otimes |1\rangle_L, |a\rangle, |e\rangle\}$, the above Hamiltonian can be briefly represented as

$$\begin{aligned}
\mathcal{H}_1 = J(t)\Big(&\sin\tfrac{\alpha(t)}{2}\,\mathrm{e}^{\mathrm{i}\beta(t)}\,|e\rangle\langle b|\\
&-\cos\tfrac{\alpha(t)}{2}\,|e\rangle\langle a| + \mathrm{H.c.}\Big),
\end{aligned} \tag{6}$$

in which $|b\rangle$ denotes the state

$$|b\rangle = \sin\frac{\theta}{2}\,\mathrm{e}^{-\mathrm{i}\varphi}\,|00\rangle \otimes |0\rangle_L - \cos\frac{\theta}{2}\,|00\rangle \otimes |1\rangle_L. \tag{7}$$

For this Hamiltonian, there exist two dark states

$$\begin{aligned}
|d\rangle &= \cos\frac{\theta}{2}\,|00\rangle \otimes |0\rangle_L + \sin\frac{\theta}{2}\,\mathrm{e}^{\mathrm{i}\varphi}\,|00\rangle \otimes |1\rangle_L,\\
|D\rangle &= \cos\frac{\alpha(t)}{2}\,|b\rangle + \sin\frac{\alpha(t)}{2}\,\mathrm{e}^{\mathrm{i}\beta(t)}\,|a\rangle.
\end{aligned} \tag{8}$$

They span a time-dependent space of degenerate eigenstates with a common zero-energy eigenvalue. As seen below, the existence of the space spanned by two zero-energy eigenstates is the key point for the construction of adiabatic holonomic gates. For this, we need a driving Hamiltonian corresponding to the space containing at least four dimensions, as indicated in Eq. (6). The decoherence-free subspace of the four-qubit physical system depicted by Eq. (3) can provide precisely such a space. The driving Hamiltonian determined by the parameters in Eq. (5) then mediates the couplings of two basis states $|00\rangle \otimes |0\rangle_L$ and $|00\rangle \otimes |1\rangle_L$ along with an auxiliary state $|a\rangle$ to another auxiliary state $|e\rangle$. Among the couplings, the XY interaction generates a Pauli X-like operation acting on $|01\rangle$ and $|10\rangle$ and the Dzialoshinski–Moriya interaction produces a Pauli Y-like operation. It follows that the XY interaction and the Dzialoshinski–Moriya interaction applied to the first and third physical qubits facilitate the transition $|00\rangle \otimes |1\rangle_L \leftrightarrow |e\rangle$ and the similar interactions applied to the first and fourth qubits facilitate the transition $|00\rangle \otimes |0\rangle_L \leftrightarrow |e\rangle$. The remaining XY interaction applied to the first and second physical qubits facilitates the transition between two auxiliary states $|a\rangle$ and $|e\rangle$. As such, we construct the Hamiltonian in Eq. (6), thereby producing the degenerate subspace containing two zero-energy eigenstates in Eq. (8). This is the main consideration that we choose the decoherence-free subspace defined in Eq. (3) and the parameters in Eq. (5) to generate the desired holonomic gates.

To realize adiabatic holonomic gates, we let the parameters $\alpha(t)$ and $\beta(t)$ transport slowly along a circuit in the parameter space with the starting and ending points $\alpha(0) = \alpha(\tau) = 0$. In this case, the quantum system prepared in the initial eigenstate (i.e., the state $|b\rangle$) corresponding to the dark state $|D\rangle$ will undergo an adiabatic and cyclic evolution, consequently acquiring a geometric phase after the time evolution. The geometric phase is given by $\gamma_{\mathcal{C}} = -\oint_{\mathcal{C}}(1-\cos\alpha)\,\mathrm{d}\beta/2$. Meanwhile, the quantum system prepared in the other dark state $|d\rangle$ remains unchanged because all the parameters in this state are time independent. As a result, the holonomic matrix confined to the subspace $\mathcal{S}^L$ reads

$$U = |d\rangle\langle d| + \mathrm{e}^{\mathrm{i}\gamma_{\mathcal{C}}}\,|b\rangle\langle b|. \tag{9}$$

Acting on the logical-qubit space spanned by $\{|0\rangle_L, |1\rangle_L\}$, this operator is equivalent to $U_L = \exp(-\mathrm{i}\gamma_{\mathcal{C}}\boldsymbol{n}\cdot\boldsymbol{\sigma}/2)$, up to an unimportant global phase factor $\exp(\mathrm{i}\gamma/2)$. Here, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the standard Pauli operator with respect to the logical qubit, $\boldsymbol{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ is an arbitrary unit vector determining the orientation of a rotation axis, and γ represents the rotation angle. Therefore, the realized operator is an adiabatic holonomic gate around an arbitrary axis with an arbitrary angle. Additionally, it is realized by a single-shot implementation, hence eliminating the need for the combination of multiple one-qubit gates to realize an arbitrary one-qubit gate. It is worth emphasizing that the Dzialoshinski–Moriya interaction in Eq. (1) plays an indispensable role in the realization of geometric gates because it provides an additional degree of freedom, thereby creating the geometric phase. This can be seen from the fact that if the parameter $\beta(t)$ is absent, the Berry connection $\langle D(t)|\dot{D}(t)\rangle$ is equal to zero and hence the geometric phase $\gamma_{\mathcal{C}}$ is zero. Additionally, the parameter φ induced by the Dzialoshinski–Moriya interaction is also important, because if there is no φ, the rotation axis along an

arbitrary orientation yields an axis in the x-z plane. This is the main reason that our scheme needs the Dzialoshinski–Moriya coupling.

Before ending this section, let us briefly outline the realization of adiabatic holonomic one-qubit gates in decoherence-free subspaces based on the previous scheme[46] to highlight the advantages of our approach. In the previous scheme, the first holonomic gate is the rotation gate around the z axis. It is implemented by letting $\theta = 0$, implying that the nonzero parameters in Eq. (5) are taken as $J_{12}^{x} = -J(t)\cos(\alpha(t)/2), J_{13}^{x} = -J(t)\sin(\alpha(t)/2)\cos\beta(t)$ and $J_{13}^{y} = J(t)\sin(\alpha(t)/2)\sin\beta(t)$. In this case, the first dark state of the Hamiltonian is reduced to $|d\rangle = |00\rangle \otimes |0\rangle_L$ and the second dark state is reduced to $|D\rangle = -\cos(\alpha(t)/2)]\,|00\rangle \otimes |1\rangle_L + \sin(\alpha(t)/2)\exp(\mathrm{i}\beta(t))\,|a\rangle$. If the parameters $\alpha(t)$ and $\beta(t)$ transport slowly along a circuit in the parameter space with $\alpha(0) = \alpha(\tau) = 0$, the dark state $|D\rangle$ starting from $|00\rangle \otimes |1\rangle_L$ will acquire a geometric phase γ_C while the dark state $|d\rangle = |00\rangle \otimes |0\rangle_L$ remains unchanged. As a consequence, the quantum gate confined to the computational subspace is reduced to $U = |0\rangle\langle 0| + \exp(\mathrm{i}\gamma_C)\,|1\rangle\langle 1|$, which is nothing but a rotation gate around the z axis with the angle γ up to an unimportant global phase. The second one-qubit gate is the rotation gate around the y axis. It is implemented by letting $\varphi = \beta = 0$ but requiring $\alpha(t)$ and $\theta(t)$ to be time-dependent parameters. As such, the dark states are reduced to $|d\rangle = \cos(\theta(t)/2)\,|00\rangle \otimes |0\rangle_L + \sin(\theta(t)/2)\,|00\rangle \otimes |1\rangle_L$ and $|D\rangle = \cos(\alpha(t)/2)\sin(\theta(t)/2)\,|00\rangle \otimes |0\rangle_L - \cos(\alpha(t)/2)\cos(\theta(t)/2)\,|00\rangle \otimes |1\rangle_L + \sin(\alpha(t)/2)\,|a\rangle$. If we require the parameters $\alpha(t)$ and $\theta(t)$ to transport slowly along a circuit in the parameter space with $\alpha(0) = \alpha(\tau) = 0$, the quantum system will acquire a non-Abelian geometric phase. Acting on the computational subspace, it is just a holonomic gate around the y axis. For realizing an arbitrary one-qubit gate, we need to combine multiple implemented holonomic gates around the z and y axes. Compared with this kind of realization, our present scheme allows an arbitrary one-qubit gate to be realized by a single-shot implementation and hence eliminates the need to combine multiple gates to implement an arbitrary one-qubit gate. This clearly simplifies the process for the implementation of a general one-qubit gate.

4. The nontrivial two-qubit gate

To realize universal quantum computation, we also need a nontrivial adiabatic holonomic two-qubit gate protected by decoherence-free subspaces in addition to the arbitrary one-qubit gate described above. To this end, let us consider a quantum system consisting of six physical qubits. When two physical qubits reside in the $|1\rangle$ state while the remaining four reside in the $|0\rangle$ state, there exists a fifteen-dimensional decoherence-free subspace, given by

$$\begin{aligned}\mathcal{S} = \mathrm{Span}\{&|000101\rangle, |000110\rangle, |001001\rangle, |001010\rangle, |010010\rangle,\\ &|000011\rangle, |010001\rangle, |011000\rangle, |010100\rangle, |101000\rangle,\\ &|100100\rangle, |100001\rangle, |001100\rangle, |100010\rangle, |110000\rangle\}. \quad (10)\end{aligned}$$

The quantum states in this subspace are immune to the collective dephasing shared by all the six physical qubits. The computational logical qubit is encoded into a smaller subspace (spanned by the first four basis states), given by

$$\mathcal{S}_L = \mathrm{Span}\{|000101\rangle, |000110\rangle, |001001\rangle, |001010\rangle\}. \quad (11)$$

Here, the last four qubits act as the data qubits while the first two qubits play the role of auxiliary qubits. It is compatible with the one-logical-qubit encoding and the specific two-logical-qubit encoding is $|000101\rangle \equiv |00\rangle \otimes |00\rangle_L$, $|000110\rangle \equiv |00\rangle \otimes |01\rangle_L$, $|001001\rangle \equiv |00\rangle \otimes |10\rangle_L$, and $|001010\rangle \equiv |00\rangle \otimes |11\rangle_L$.

To realize a nontrivial two-qubit gate, we need to construct an equivalent three-level quantum system (see the first line in Eq. (14) below) to produce the dark state $|d\rangle$ in Eq. (15). In this way, a cyclic change of the parameters $\omega(t)$ and $\mu(t)$ attaches a π phase to the state $|00\rangle \otimes |11\rangle_L$. For this, we resort to the basis states $|e\rangle \equiv |010010\rangle$ and $|a\rangle \equiv |000011\rangle$ (i.e., the fifth and sixth basis states in the decoherence-free subspace $\mathcal{S}$), and then mediate the couplings $|e\rangle \leftrightarrow |00\rangle \otimes |11\rangle_L$ and $|e\rangle \leftrightarrow |a\rangle$ through facilitating the interactions between the second and third physical qubits and between the second and sixth physical qubits. This can be done by considering the Hamiltonian described in Eq. (1) with $N = 6$ and then setting the nonzero parameters of this Hamiltonian as

$$\begin{aligned}J_{23}^{x} &= \lambda(t)\sin\frac{\omega(t)}{2}\cos\mu(t),\\ J_{23}^{y} &= \lambda(t)\sin\frac{\omega(t)}{2}\sin\mu(t),\\ J_{26}^{x} &= -\lambda(t)\cos\frac{\omega(t)}{2}, \quad (12)\end{aligned}$$

where $\omega(t)$ and $\mu(t)$ are time-dependent parameters while $\lambda(t)$ can be either time dependent or time independent, as indicated in the above equation. It is worth noting that the above driving Hamiltonian not only facilitates the desired couplings $|e\rangle \leftrightarrow |00\rangle \otimes |11\rangle_L$ and $|e\rangle \leftrightarrow |a\rangle$ but also unavoidably mediates the transitions $|00\rangle \otimes |10\rangle_L \leftrightarrow |a_1\rangle$, $|00\rangle \otimes |10\rangle_L \leftrightarrow |a_2\rangle$, and $|00\rangle \otimes |00\rangle_L \leftrightarrow |a_3\rangle$ with $|a_1\rangle \equiv |010001\rangle$, $|a_2\rangle \equiv |011000\rangle$, and $|a_3\rangle \equiv |010100\rangle$ (i.e., the seventh to ninth basis states in the decoherence-free subspace $\mathcal{S}$). This in turn implies that the other six basis states are decoupled from the quantum system. Therefore, the closed decoherence-free subspace that really works is given by

$$\begin{aligned}\tilde{\mathcal{S}} = \mathrm{Span}\{&|000101\rangle, |000110\rangle, |001001\rangle, |001010\rangle,\\ &|e\rangle, |a\rangle, |a_1\rangle, |a_2\rangle, |a_3\rangle\}. \quad (13)\end{aligned}$$

In this basis, the Hamiltonian can be briefly recast as

$$\begin{aligned}\mathcal{H}_2(t) =\ & \lambda(t)\Big(\sin\frac{\omega(t)}{2}\,\mathrm{e}^{\mathrm{i}\mu(t)}\,|e\rangle\langle 00|\otimes\langle 11|_L \\ & -\cos\frac{\omega(t)}{2}\,|e\rangle\langle a|+\text{H.c.}\Big) \\ & +\lambda(t)\Big(\sin\frac{\omega(t)}{2}\,\mathrm{e}^{-\mathrm{i}\mu(t)}\,|00\rangle\otimes|10\rangle_L\langle a_1| \\ & -\cos\frac{\omega(t)}{2}\,|00\rangle\otimes|10\rangle_L\langle a_2|+\text{H.c.}\Big) \\ & -\lambda(t)\cos\frac{\omega(t)}{2}(|00\rangle\otimes|00\rangle_L\langle a_3|+\text{H.c.}),\end{aligned} \tag{14}$$

where H.c. represents the corresponding Hermitian conjugate term. For this Hamiltonian, there exist two dark states

$$\begin{aligned}|d\rangle &= \cos\frac{\omega(t)}{2}\,|00\rangle\otimes|11\rangle_L+\sin\frac{\omega(t)}{2}\,\mathrm{e}^{\mathrm{i}\mu(t)}\,|a\rangle\,, \\ |\bar{d}\rangle &= \cos\frac{\omega(t)}{2}\,|a_1\rangle+\sin\frac{\omega(t)}{2}\,\mathrm{e}^{-\mathrm{i}\mu(t)}\,|a_2\rangle\,.\end{aligned} \tag{15}$$

Clearly, for the quantum system initially residing in the computational subspace $\mathcal{S}_L$ and then undergoing an adiabatic evolution, the dark state $|d\rangle$ governs the time evolution of the quantum system while the dark state $|\bar{d}\rangle$ is decoupled from this system. As a consequence, when the parameters $\omega(t)$ and $\mu(t)$ transport slowly along a circuit in the parameter space with the starting and ending points $\omega(0)=\omega(\tau)=0$, the basis state $|00\rangle\otimes|11\rangle_L$ acquires an adiabatic geometric phase $\gamma_{\mathcal{C}}=-\oint_{\mathcal{C}}(1-\cos\omega)\,\mathrm{d}\mu/2$. For the other basis states in the computational subspace, $|00\rangle\otimes|00\rangle_L$ and $|00\rangle\otimes|10\rangle_L$ cannot acquire purely geometric phases while $|00\rangle\otimes|01\rangle_L$ is decoupled from the quantum system and hence acquires no phase. To eliminate the non-geometric parts in the adiabatic evolution, we additionally require $\lambda(t)$ to be an asymmetric function and $\omega(t)$ to be a symmetric function in the sense of evolution time. As such, the quantum system initially residing in the basis states $|00\rangle\otimes|00\rangle_L$ and $|00\rangle\otimes|10\rangle_L$ will undergo the spin echo evolution, leading to the phases acquired by $|00\rangle\otimes|00\rangle_L$ and $|00\rangle\otimes|10\rangle_L$ being compensated with each other during the time evolution. This technique has been regularly used to remove the dynamical phases in adiabatic geometric manipulation.[56–58] After operating as above, the unitary operator acting on the subspace $\mathcal{S}^L$ yields a holonomic matrix. The resulting quantum gate with respect to the two logical qubits is nothing but a controlled-phase gate. This completes our demonstration of the nontrivial adiabatic holonomic two-qubit gate in decoherence-free subspaces.

5. Conclusion

In conclusion, we have put forward a scheme for the realization of adiabatic holonomic quantum computation in decoherence-free subspaces. Our scheme is based on realistically available two-body interactions for coupling qubits. It is worth emphasizing that the previous scheme relied on four-body interactions to realize adiabatic holonomic quantum computation in decoherence-free subspaces, which is challenging in practical implementation. Our scheme thus avoids the difficulties of implementing four-body interactions in a realistic physical system. This makes adiabatic holonomic quantum computation in decoherence-free subspaces more accessible in practice. Furthermore, an arbitrary one-qubit gate in our scheme can be realized by a single-shot implementation, which eliminates the need to combine multiple gates in order to realize an arbitrary one-qubit operation. This additionally enhances the practicality of our scheme. Further investigation may lie in how to speed up such an adiabatic holonomic scheme while maintaining its geometric robustness determined by the evolution path in parameter space rather than on the evolution details of the quantum system.

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