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Dynamical evolution of imaginarity resources in non-Markovian environments

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The pivotal role of complex numbers in quantum mechanics underpins the resource theory of imaginarity. We investigate imaginarity dynamics in a single-qubit open system coupled to a non-Markovian environment. Crucially, cavity field detuning emerges as the dominant regulator, driving continuous conversion between the real and imaginary components of coherence. Nonzero detuning induces characteristic non-periodic oscillations of imaginarity between zero and maximal values, preventing complete decoherence at specific times. We establish that imaginarity resources stem from both intrinsic system evolution and environmental feedback. Significantly, detuning-driven imaginarity generation persists even in Markovian regimes, demonstrating its origin beyond environmental memory effects. These insights offer new perspectives for understanding and harnessing quantum coherence.

Keywords: open systems, non-Markovian effects, imaginarity resources, coherence resources

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1. Introduction

Quantum computing advances have intensified focus on extending coherence time,[1–9] driving studies of system-environment interactions. These induce quantum noise, manifesting as decoherence and dissipation.[10–16] Open quantum systems are classified as Markovian (exhibiting memoryless exponential decay) or non-Markovian, where environmental feedback reclaims lost quantum information, slowing decoherence.

According to the resource theory of coherence, the off-diagonal elements in the quantum state's density matrix quantify quantum coherence.[17–23] In quantum mechanical systems, the imaginary components of these off-diagonal elements give rise to the resource theory of imaginarity.[24–33] Recent experimental results indicate that imaginary components are indispensable in standard quantum mechanics, playing a crucial role in characterizing the dynamical properties of quantum systems.[34,35] This discovery not only provides compelling experimental validation for the resource theory of imaginarity but also deepens our understanding of the foundational elements of quantum theory.

Imaginary components are not only crucial for characterizing the fundamental aspects of the quantum realm but also play a pivotal role in practical applications. For example, Wick rotation, as discussed in Refs. [36–38], is a technique that transforms real-time into its imaginary counterpart, enabling the derivation of Green's functions for quantum many-body systems. In the context of electronic excitation energy transfer systems, including molecular aggregates and biological and artificial light-harvesting systems, imaginarity resources are crucial for determining the actual transport current, as indicated in Ref. [39]. Recent advancements in polarization-encoded photon systems, highlighted in Ref. [30], further underscore the significance of imaginarity resources.

Consequently, studying the evolution of imaginarity resources in open systems may uncover previously unrecognized insights. This paper investigates the dynamics of imaginarity resources within a single-qubit system interacting with a typical open environment. Our findings demonstrate that the imaginarity resources primarily originate from two factors: the intrinsic evolution of the quantum system and feedback from the external environment. A key innovation is the demonstration that cavity field detuning significantly shapes these resources dynamics, influencing decoherence trajectories on the Bloch sphere. Our research introduces a dual-origin framework for imaginarity resources, highlighting their dependence on both intrinsic system evolution and environmental feedback. This approach not only enhances our theoretical understanding of quantum coherence but also offers practical insights for the design of quantum systems with improved resilience to decoherence.

The structure of this paper is organized as follows. Section 2 introduces the foundational concepts of quantum re-

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source theory, with a particular focus on imaginarity and coherence resources. Section 3 examines the decoherence model of a single-qubit system. Section 4 investigates the evolution of imaginarity resources in a single-qubit system interacting with an open environment. Finally, Section 5 summarizes the findings and discusses potential future directions.

2. Preliminaries

We begin by revisiting the fundamental concepts of the resource theory of coherence and the resource theory of imaginarity. A resource theory consists of two core components: free states and free operations. Free states are quantum states that do not contain any resources, while free operations are quantum processes that map free states to free states. In other words, when free operations are applied, quantum states do not generate any corresponding quantum resources.

2.1. Resource theory of coherence

Given a d-dimensional Hilbert space H (assuming d is finite, even though some extensions to infinite-dimensional spaces can be considered), we denote its reference standard orthonormal basis by $\{|i\rangle\}_{i=1}^{d}$. Density matrices that are diagonal in this particular basis are called incoherent, which are the free states in the resource theory of coherence, and can be represented as

$$\mathcal{I}=\left\{\delta=\sum_i p_i|i\rangle\langle i| : p_i\geq 0, \sum_i p_i=1\right\}, \tag{1}$$

where p_i is the probability distribution, and $\mathcal{I}$ denotes the set of free states.

In the theory of quantum coherence resources, free operations are referred to as incoherent operations. These operations are completely positive and trace-preserving (CPTP) maps, which can be represented as $\Lambda(\delta)=\sum_i K_i\delta K_i^{\dagger}$, where the Kraus operators satisfy the condition $\sum_i K_i K_i^{\dagger}=\mathbb{I}$, and for any incoherent state and any operator, it satisfies $\frac{K_i\delta K_i^{\dagger}}{\mathrm{Tr}\left(K_i\delta K_i^{\dagger}\right)}\in\mathcal{I}$.

2.2. Resource theory of imaginarity

Similar to the resource theory of coherence, in a d-dimensional Hilbert space, select a set of standard orthogonal basis vectors $\{|m\rangle\}_{m=1}^{d}$. The free states in the resource theory of imaginarity are real states, which can be represented as

$$\mathcal{R}=\left\{|\psi\rangle=\sum_m c_m|m\rangle : c_m\in R, \sum_m |c_m|^2=1\right\}, \tag{2}$$

where c_m is a non-negative real number, and the symbol $R(\mathcal{R})$ represents the set of real numbers (real states). Mixed real states can be written as convex mixtures of real states: $\rho=\sum_m p_m|\psi_m\rangle\langle\psi_m|$. Similarly, we refer to the density matrix corresponding to a real state as the real density matrix: $\langle n|\rho|m\rangle\in R$.

The free operation in the theory of imaginarity resources is a real operation, and the corresponding quantum operation is $\Lambda(\rho)=\sum_m K_m\rho K_m^{\dagger}$, here the corresponding Kraus operators are referred to as real Kraus operators: $\langle n|K_m|m\rangle\in R$. In the theory of imaginarity resources, we transform quantum states through a set of real operations. In this paper, we do not consider probabilistic transformations, so the real Kraus operators satisfy $\sum_m K_m K_m^{\dagger}=\mathbb{I}$.

2.3. Quantification of quantum resources

Generally speaking, in resource theory, the measurement of resources is achieved through a function C defined on the state space. This function needs to satisfy the following conditions:[15]

(C1) Non-negativity: $C(\rho)\geq 0$, and the equality holds only when the quantum state is a free state.

(C2a) Monotonicity: $C(\rho)\geq C(\Lambda[\rho])$, the value of the coherence resource measure does not increase under incoherent completely positive trace-preserving operations.

(C2b) Strong monotonicity: $C(\rho)\geq\sum_i p_i C(\rho_i)$, where $p_i=\mathrm{Tr}\left[K_i\rho K_i^{\dagger}\right], \rho_i=K_i\rho K_i^{\dagger}/p_i$, the measure function exhibits monotonicity under incoherent operations with subselection.

(C3) Convexity: $\sum_i p_i C(\rho_i)\leq C\left(\sum_i p_i\rho_i\right)$, the measure function is a convex function.

The distance measure based on the L_1 matrix norm is a widely used coherence measure, and it is defined as

$$C_{L_1}^{\mathrm{C}}(\rho)=\min_{\delta\in\mathcal{I}}\|\rho-\delta\|_{L_1}. \tag{3}$$

Its analytical expression is $C_{L_1}^{\mathrm{C}}(\rho)=\sum_{i\neq j}|\langle i|\rho|j\rangle|$, which corresponds to the sum of the absolute values of all off-diagonal elements of the density matrix ρ of the quantum state being measured.

Similarly, an imaginarity measure based on the L_1 matrix norm has also been proposed:[33]

$$C_{L_1}^{\mathrm{I}}(\rho)=\sum_{n\neq m}|\langle n|\mathrm{Im}(\rho)|m\rangle|. \tag{4}$$

3. Pure decoherence model

We now consider the dissipative dynamics governed by a single-qubit amplitude damping channel. At zero temperature, the Hamiltonian describing a two-level atom interacting with a high-Q cavity is expressed as[40,41]

$$H=\omega_0\sigma_+\sigma_-+\sum_k\omega_k b_k^{\dagger}b_k+(g_k\sigma_+ b_k+\mathrm{h.c.}), \tag{5}$$

where ω_0 denotes the resonance frequency of the qubit system, and $\sigma_{\pm}$ represents the corresponding raising or lowering operators. The g_k is the qubit coupling constant, and b_k ($b_k^{\dagger}$) is the annihilation operator of the k-th bosonic mode with the reservoir frequency ω_k.

Assume the qubit initial state as $|\varphi_q(0)\rangle = C_0(0)|0\rangle_q + C_1(0)|1\rangle_q$. In this case, we can use a reduced density matrix to describe the dynamic behavior of qubits

$$\rho(t) = \begin{pmatrix} \rho_{11}(0)|q(t)|^2 & \rho_{10}(0)q(t) \\ \rho_{01}(0)q^*(t) & \rho_{00}(0)+\rho_{11}(0)(1-|q(t)|^2) \end{pmatrix}, \quad (6)$$

where $\rho_{ij}(0) = C_i(0)C_j^*(0)$, the $q(t)$ is determined by $q(t) = -\int_0^t \mathrm{d}\tau f(t-\tau)q(\tau)$, here the correlation function $f(t-\tau)$ is related to the spectral density $J(\omega)$ of the reservoir: $f(t-\tau) = \int \mathrm{d}\omega J(\omega)\mathrm{e}^{\mathrm{i}(\omega_0-\omega)(t-\tau)}$, this function characterizes the temporal correlation of the interaction between the qubit and the environment. The spectral density of the electromagnetic field in a high-Q cavity, influenced by the detuning Δ of ω_0, and the cavity's central frequency, arises from the interplay between the ambient spectrum and the system coupling.

The spectral density $J(\omega)$ can be characterized by Lorentzian functions, Voigt functions, and other distributions. In this context, the Lorentzian distribution is expressed as follows:[40,41]

$$J(\omega) = \frac{1}{2\pi}\frac{\kappa_0\lambda^2}{(\omega_0-\Delta-\omega)^2+\lambda^2}, \quad (7)$$

where κ_0 is related to the coupling strength of the atom and cavity field environment, and λ represents the width of the cavity spectrum. The correlation time of noise caused by the cavity field $\tau_\mathrm{B} = \lambda^{-1}$, and the relaxation time of the system $\tau_\mathrm{R} = \kappa_0$. In a Markovian system, there is $\tau_\mathrm{R} > 2\tau_\mathrm{B}$, in a non-Markovian system, $\tau_\mathrm{R} < 2\tau_\mathrm{B}$.

In the Laplace transform, we obtain[42,43]

$$q(t) = \mathrm{e}^{-\frac{1}{2}(\lambda-\mathrm{i}\Delta)t}\left[\cosh\left(\frac{\Omega t}{2}\right)+\frac{\lambda-\mathrm{i}\Delta}{\Omega}\sinh\left(\frac{\Omega t}{2}\right)\right], \quad (8)$$

where $\Omega = \sqrt{(\lambda-\mathrm{i}\Delta)^2-2\kappa_0\lambda}$. The $q(t)$ describes the oscillations of quasi-coherent energy exchange between qubits and the environment, which are the process that destroys quantum coherence.

The master equation governing the single-qubit system reveals the fundamental mechanism of imaginarity generation. Starting from the time-dependent master equation[12]

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\frac{\mathrm{i}}{2}S(t)[\sigma_+\sigma_-,\rho(t)]+\gamma(t) \times\left(\sigma_-\rho\sigma_+-\frac{1}{2}\{\sigma_+\sigma_-,\rho\}\right), \quad (9)$$

where $S(t) = -2\mathrm{Im}\left[\frac{\dot{q}(t)}{q(t)}\right]$ represents the Lamb shift, this term corresponds to the imaginary component of the complex ratio $\frac{\dot{q}(t)}{q(t)}$, reflecting the phase accumulation induced by environmental coupling. And $\gamma(t) = 2\mathrm{Re}\left[\frac{\dot{q}(t)}{q(t)}\right]$ encodes the instantaneous decay rate, governing dissipative dynamics through the real part of the same ratio.

To unravel the mechanism of real-imaginary coherence conversion, we decompose the density matrix into its Bloch components $\rho(t) = \frac{1}{2}(I + r_x(t)\sigma_x + r_y(t)\sigma_y + r_z(t)\sigma_z)$, where $r_x(t) = 2\mathrm{Re}[\rho_{01}(0)q(t)]$, $r_y(t) = 2\mathrm{Im}[\rho_{01}(0)q(t)]$, and $r_z(t) = \langle\sigma_z(t)\rangle$. Substituting this into the master equation (Eq. (9)), the dynamics of $r_x(t)$ and $r_y(t)$ are governed by

$$\begin{cases} \dot{r_x}(t) = -S(t)r_y(t)-\gamma(t)r_x(t), \\ \dot{r_y}(t) = S(t)r_x(t)-\gamma(t)r_y(t), \end{cases} \quad (10)$$

which reveals two distinct contributions: the coherent rotation induced by the Lamb shift $S(t)$, and the dissipative damping driven by $\gamma(t)$.

The Lamb shift $S(t)$ is fundamentally tied to the detuning Δ. For large detuning ($\Delta \gg \lambda, \kappa_0$), $S(t) \approx \Delta$, leading to persistent oscillations between real part of the coherence (r_x) and imaginary part of the coherence (r_y) with frequency Δ. This is evident from the simplified equations

$$\dot{r_x}(t) \approx -\Delta r_y(t), \quad \dot{r_y}(t) \approx \Delta r_x(t), \quad (11)$$

which describe a rotation in the x–y plane. In contrast, when $\Delta = 0$, the rotational term vanishes ($S(t) \approx 0$), and the coherence decays monotonically:

$$\dot{r_x}(t) = -\gamma(t)r_x(t), \quad \dot{r_y}(t) = -\gamma(t)r_y(t). \quad (12)$$

Here, real and imaginary parts dissipate independently without conversion, highlighting the critical role of detuning in enabling coherence mixing.

To rigorously quantify this effect, we define the coherence conversion rate $\mathcal{K}(t)$ as the time derivative of the phase angle $\theta(t) = \arctan\left(\frac{r_y(t)}{r_x(t)}\right)$

$$\mathcal{K}(t) = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{r_x\dot{r_y}-r_y\dot{r_x}}{r_x^2+r_y^2}. \quad (13)$$

Substituting the equations of motion, this reduces to $\mathcal{K}(t) = S(t) \approx \Delta$, proving that the conversion rate is directly proportional to the detuning. When $\Delta = 0$, $\mathcal{K}(t) = 0$, and no interconversion occurs; nonzero Δ dynamically redistributes coherence between real and imaginary parts.

Physically, detuning introduces an effective magnetic field along the z-axis (via $S(t)$), causing precession in the x–y plane. This mechanism suppresses independent decay of coherence and enables quantum state revival in non-Markovian regimes, where $\gamma(t)$ temporarily becomes negative. The coherence dynamics are thus universally governed by

$$r_x(t)+\mathrm{i}r_y(t) = [r_x(0)+\mathrm{i}r_y(0)]\,\mathrm{e}^{-\Gamma(t)}\,\mathrm{e}^{-\mathrm{i}\Delta t}, \quad (14)$$

where $\Gamma(t) = \int_0^t \gamma(\tau)\mathrm{d}\tau$. This expression encapsulates both the exponential decay of coherence magnitude and the detuning-driven phase accumulation.

Detuning Δ serves as a dual control parameter: it suppresses monotonic decoherence by coupling real and and imaginary parts of the coherence through $S(t)$, while simultaneously enhancing phase-space mixing via a rotation frequency $\mathcal{K}(t) \propto \Delta$. These effects are universal across Markovian and non-Markovian systems, as demonstrated by the analytic solution $q(t)$ and the master equation. The ability to tune Δ provides a powerful tool for coherence management in quantum technologies, enabling precise control over quantum state evolution and resource generation.

4. Results and discussion

Our primary objective is to investigate the evolution of imaginarity resources in open systems. To achieve this, we selected four different initial quantum states $|\psi_0\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $|\psi_1\rangle = (|0\rangle + e^{i\frac{\pi}{4}}|1\rangle)/\sqrt{2}$, $|\psi_2\rangle = (|0\rangle + e^{i\frac{\pi}{2}}|1\rangle)/\sqrt{2}$, and $|\psi_3\rangle = |0\rangle$, based on their coherence and imaginarity. We simulated their evolution in both Markovian and non-Markovian environments, and observed some intriguing phenomena. In this study, we have set the coupling strength κ_0 to a constant value of 1.

In Fig. 1(a), we observe that in a non-Markovian environment with zero detuning, the L_1 norm of the imaginary part of the initial states with imaginary resources exhibits an overall periodic oscillatory decay, eventually approaching zero. This indicates the complete loss of imaginary resources. Throughout this process, the magnitude does not exceed the initial value. In contrast, for initial states with zero imaginary resources, the L_1 norm remains zero. In Fig. 1(b), when the detuning is non-zero, the L_1 norm of the imaginary part exhibits a non-periodic oscillatory decay. It is worth noting that, during this process, the norm may temporarily exceed its initial value. This oscillatory decay reflects the reverse flow of imaginary information from the environment back to the system, thereby extending the duration for which the system retains its imaginary resources.

In Fig. 1(c), within a Markovian environment where the detuning is zero, the L_1 norm of imaginarity for initial states with zero imaginarity resources remains at zero. For initial states possessing imaginarity resources, the L_1 norm rapidly decreases to zero. This decline reflects the reduction of imaginarity resources in the system, attributed to the weak coupling environment — characterized by a correlation time longer than the cavity field correlation time and $q(t)$ exhibiting a Markovian exponential decay — leading to the loss of quantum imaginarity resources. In Fig. 1(d), in a Markovian environment where the detuning is non-zero, only initial states with coherence resources exhibit oscillatory behavior during the evolution process. This counterintuitive phenomenon warrants attention because, typically, there is no memory effect in a weakly coupled environment, implying that imaginarity information does not flow back from the environment to the system.

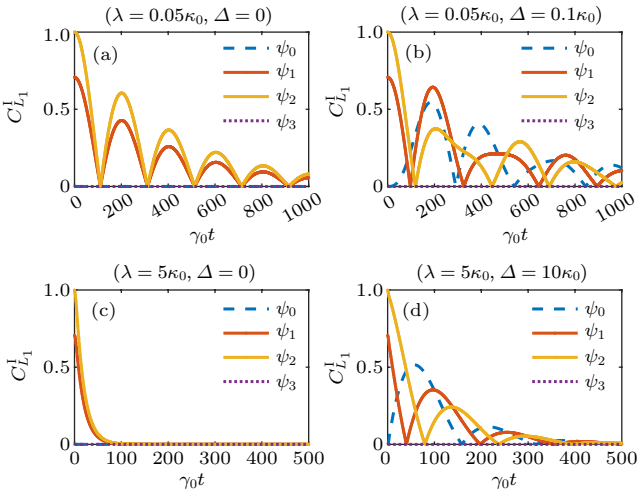


Fig. 1. The L_1 norm of imaginarity as a function of dimensionless quantities for initial states $|\psi_{0,1,2,3}\rangle$ under different environmental conditions: (a) non-Markovian ($\lambda = 0.05\kappa_0$, $\Delta = 0$); (b) non-Markovian ($\lambda = 0.05\kappa_0$, $\Delta = 0.1\kappa_0$); (c) Markovian ($\lambda = 5\kappa_0$, $\Delta = 0$); (d) Markovian ($\lambda = 5\kappa_0$, $\Delta = 10\kappa_0$).

This verifies our previous proof that detuning will promote the mutual transformation between the coherent real part and the imaginary part.

To validate this hypothesis, we selected the initial state $|\psi_0\rangle$. It is the maximum coherent state, which has no imaginarity resources, making it an ideal candidate for the experiment.

In Fig. 2(a), we observe that under a non-Markovian environment with zero detuning, the L_1 norm of coherence exhibits periodic oscillatory decay, consistently reaching zero before rebounding at specific moments in time. This indicates that the system's coherence completely vanishes at certain instances, suggesting the occurrence of complete decoherence periods. In contrast, Fig. 2(b) illustrates the behavior in a non-Markovian environment with non-zero detuning. Here, the decay rate of the L_1 norm of coherence is significantly reduced and does not reach zero, demonstrating enhanced resistance to decoherence compared to the environment depicted in Fig. 2(a). This phenomenon can be attributed to the fact that imaginarity resources serve as a lower bound for coherence resources.[32] However, it should be noted that even with non-zero detuning, the oscillations observed in a non-Markovian environment do not directly imply feedback from the environment to the system.

In Figs. 2(c) and 2(d), within a Markovian environment where the detuning equals zero, the L_1 norm of the coherence curve rapidly decays to zero. When the detuning is non-zero, the L_1 norm of the imaginarity curve increases from its initial value of zero but does not exceed the value of the L_1 norm of coherence. Although it is commonly understood that in a Markovian environment, the environment does not feedback information to the system, the increase in imaginarity

observed is not due to environmental feedback but rather an intrinsic characteristic of the interaction between the system and the environment under non-resonant conditions. This interaction leads to the generation of imaginarity resources, but it does not imply that the environment is feeding 'imaginarity information' back to the system.

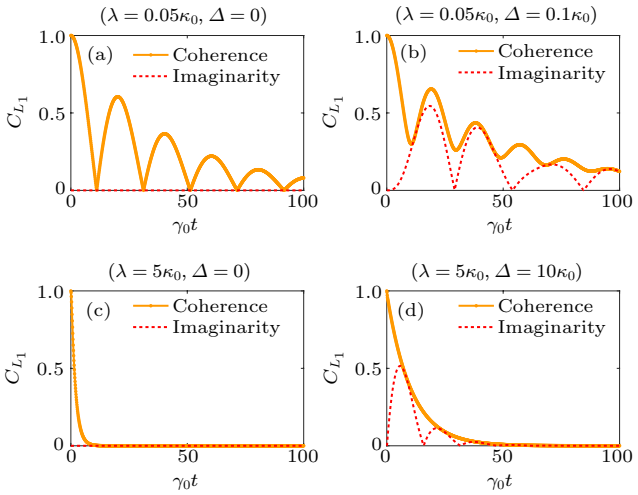


Fig. 2. The L_1 norm of imaginarity and coherence as a function of $\gamma_0 t$ for the initial state $|\psi_0\rangle$ under different environmental conditions: (a) non-Markovian ($\lambda = 0.05\kappa_0$, $\Delta = 0$); (b) non-Markovian ($\lambda = 0.05\kappa_0$, $\Delta = 0.1\kappa_0$); (c) Markovian ($\lambda = 5\kappa_0$, $\Delta = 0$); (d) Markovian ($\lambda = 5\kappa_0$, $\Delta = 10\kappa_0$).

Similarly, we selected the initial state $|\psi_2\rangle$ as the maximum imaginary state and observed its evolution.

In Fig. 3, the L_1 norm of coherence evolves the same as in Fig. 2, but the L_1 norm of imaginarity shows different features. At zero detuning, the L_1 norm of the imaginarity overlaps with that of coherence, indicating the coherence is purely imaginary with no real part. At non-zero detuning, the L_1 norm of the imaginarity decays like in Fig. 2. This again confirms the previous findings, showing that detuning significantly impacts the mutual transformation between imaginarity and coherence resources, and highlighting its key role in quantum system evolution.

To provide a more intuitive understanding of the "trajectories" of decoherence, this study employs the Bloch sphere to illustrate the decoherence process of the initial state under different environmental conditions. This approach aims to achieve a clear visualization of the decoherence dynamics.

In Fig. 4, blue circles mark the initial state position on the Bloch sphere, black stars denote the final state position, and red curves trace the real-time evolution paths composed of dense red dots (representing intermediate states), illustrating the decoherence process of the initial state $|\psi_0\rangle$ under various environmental conditions. When the decoherence in the environment is zero, the evolution of the quantum state is primarily confined to the X–Z plane. However, when decoherence is nonzero, the interaction between the environment and the quantum system causes the state to evolve not only within the X–Z plane but also along the Y-axis. This interaction facilitates the mutual transformation between the real and imaginary parts of the coherence resources.

The imaginarity resource generation capacity of the quantum state $\rho(t)$ in an open system can be measured by the following functional:

$$N(\rho) := \sup_{\rho(t)} \int_{I(t)>0} I(t)\,\mathrm{d}t, \tag{15}$$

where sup represents all binary states, $I(t)$ denotes the rate of change of the L_1 norm of imaginarity curve.

As shown in Fig. 5, in the non-Markovian environment, the accumulated imaginarity resources exhibit monotonic enhancement with increasing detuning Δ. This detuning-induced amplification effect remains operative even under Markovian conditions. The persistent scaling behavior fundamentally demonstrates detuning's governing role in manipulating quantum resource longevity.

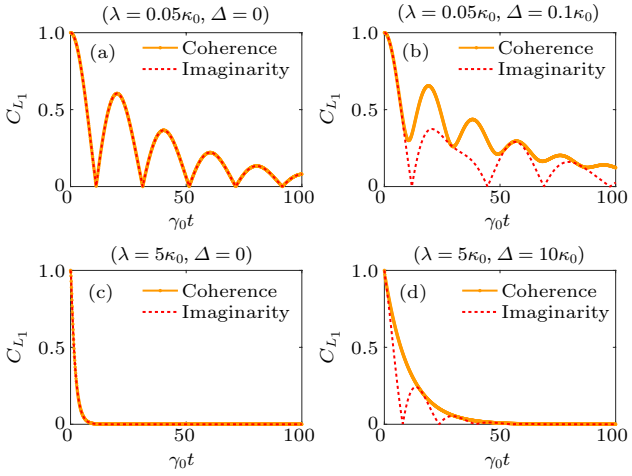


Fig. 3. The L_1 norm of imaginarity and coherence as a function of $\gamma_0 t$ for the initial state $|\psi_2\rangle$ under different environmental conditions: (a) non-Markovian ($\lambda = 0.05\kappa_0$, $\Delta = 0$); (b) non-Markovian ($\lambda = 0.05\kappa_0$, $\Delta = 0.1\kappa_0$); (c) Markovian ($\lambda = 5\kappa_0$, $\Delta = 0$); (d) Markovian ($\lambda = 5\kappa_0$, $\Delta = 10\kappa_0$).

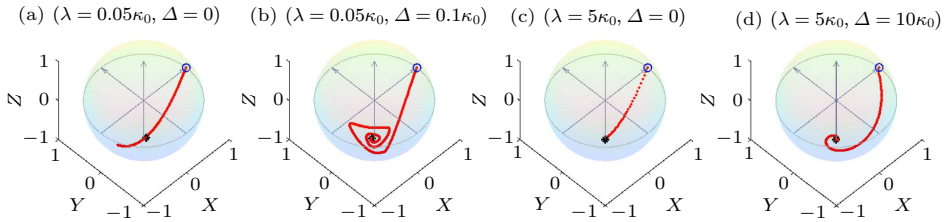


Fig. 4. Illustration of the decoherence process of the initial state $|\psi_0\rangle$ in different environments: (a) non-Markovian regime ($\lambda = 0.05\kappa_0$, $\Delta = 0$); (b) non-Markovian regime ($\lambda = 0.05\kappa_0$, $\Delta = 0.1\kappa_0$); (c) Markovian regime ($\lambda = 5\kappa_0$, $\Delta = 0$); (d) Markovian regime ($\lambda = 5\kappa_0$, $\Delta = 10\kappa_0$).

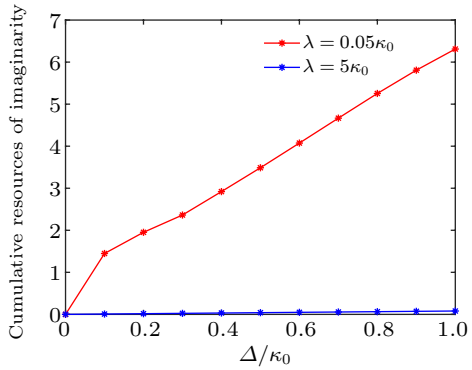


Fig. 5. The cumulative idleness of the initial state $|\psi_0\rangle$ in both the non-Markov environment ($\lambda = 0.05\kappa_0$) and the Markov environment ($\lambda = 5\kappa_0$).

5. Conclusions

Coherence is widely recognized as the most fundamental form of quantum correlation, with imaginarity resources arising from the imaginary component of coherence. By focusing on the imaginary part, researchers can uncover unique potential applications and gain a deeper insight into the nature of coherence.

Our investigation into the dynamics of imaginarity resources in a single-qubit system interacting with an open environment has yielded several important insights. We have demonstrated that these resources are primarily influenced by the intrinsic evolution of the quantum system and feedback from the external environment. The cavity field detuning was found to play a pivotal role in shaping the dynamic behavior of these resources, particularly in non-Markovian environments where memory effects are significant.

These findings are crucial for advancing quantum technologies that rely on sustained coherence and for deepening our understanding of the operational significance of imaginarity in quantum systems. Future research should focus on further elucidating the practical utility of imaginarity in quantum processes,[44] as well as its role in facilitating electronic excitation energy transport.[39]

Acknowledgements

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