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Adaptive polynomial approximation-based virtual coupled cooperative control for high-speed trains

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Virtual coupling is a novel technology that enables trains to run closely together without physical connections through communication and automation systems. The paper addresses an adaptive polynomial approximation algorithm for the cooperative control of high-speed trains (HSTs) under virtual coupling. It aims to solve the cooperative tracking control problem of HST formation operations under various scenarios, including known and unknown parameters. To enable the HST formation system to achieve cooperative operation while ensuring an appropriate spacing distance, the tracking errors of displacement and speed throughout the entire operation converge to zero. The proposed control strategy focuses on adopting polynomial approximation to handle unknown parameters, which are estimated via adaptive laws. Additionally, the unknown parameters of the HSTs are estimated online through adaptive laws. Experimental results verify the effectiveness of this method.

Keywords: high-speed trains (HSTs), cooperative operation, adaptive polynomial approximation, virtual coupling

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1. Introduction

With the urgent demand for efficiency, safety, and comfort, high-speed trains (HSTs) have become the preferred mode of transportation for connecting major cities.[1] However, with the rapid expansion of the high-speed rail network, bottleneck issues in the network have gradually emerged, limiting the further improvement of transportation capacity.[2,3]

Recently, to address the above issue, multi-train cooperative control technology based on virtual coupling has been proposed. Virtual coupling allows for shorter train spacing compared to traditional moving block systems, resulting in higher line capacity through the train-to-train communication system. Scholars have devoted significant efforts to studying the virtual coupling of trains to improve track utilization efficiency and ensure the safety and stability of synchronized train operations.[4] In Ref. [5], the authors solve the cooperative control problem for the virtual coupling of HSTs through the innovative distributed nonlinear state-dependent function (DNSDF). If the system has significant uncertainties, an additional robust design is required to compensate for disturbances. The study in Ref. [6] addresses the issues of safety constraints, input saturation, and dynamic disturbances in the virtual coupling of HSTs through the distributed model predictive control (DMPC) framework. This strategy relies on an accurate dynamic model; if the model parameters are inaccurate, prediction errors will accumulate, necessitating additional robust design. The method proposed in this paper, through online parameter adjustment via adaptive control and a robust design, can flexibly address dynamic uncertainties, external disturbances, and complex constraints in HST cooperative control without relying on off-line training or accurate models.

The train control system is a significant technology for ensuring the safe, efficient, and smooth operation of modern railway transportation systems.[7,8] By utilizing the Global System for Mobile Communications-Railway (GSM-R) network, the system enables frequent two-way information exchange between the radio block center (RBC) and trains, as illustrated in Fig. 1. Based on this communication technology, a moving block system can be implemented to enhance system capacity.[9] In this system, each train can follow the preceding train while maintaining an appropriate spacing distance, thereby optimizing track utilization and improving overall efficiency.[10] Currently, although the operational quality of HSTs still mainly depends on the operator's actions and experience, the development of the automatic train operation (ATO) system suggests that drivers may be completely replaced in the future.[11,12] The ATO system can automatically generate control commands based on the trains operating status. This not only ensures operational safety but also significantly improves transportation efficiency, energy consumption, and passenger comfort.[13] As the speed of HSTs in-

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creases, the complexity of operational control has also grown, involving factors such as energy consumption, track utilization, stopping accuracy, and operational safety.[14] Therefore, strengthening research into the operation control of HSTs and achieving precise speed and position coordination control has become a key factor in ensuring the safe operation of trains.

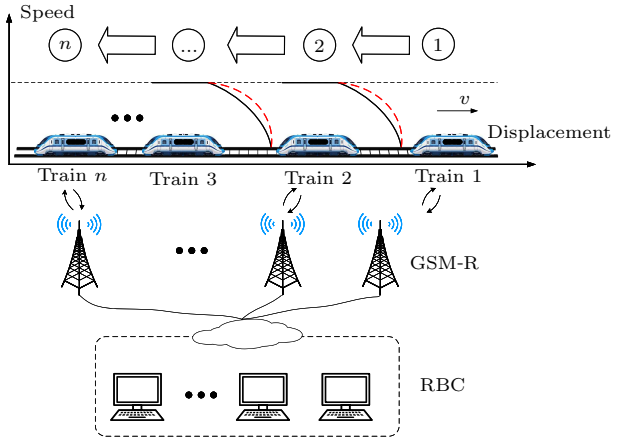


Fig. 1. The cooperative operation framework of virtual coupled HSTs.

Proportional-integral-derivative (PID) control is one of the most commonly used technologies in train operation control.[15,16] The significant advantages of PID control lie in its simplicity and ease of implementation. Although PID control can provide satisfactory performance in many industrial control scenarios, it still faces some challenges when applied to train tracking control. In Ref. [17], an adaptive controller based on neural networks and PID algorithms (NNPID) is proposed for speed tracking control of HSTs, which addresses dynamic parameter issues and actuator saturation in train operation. In Ref. [18], a fuzzy PID controller based on train speed tracking is studied, with the main goal of solving local oscillations and overshooting in fuzzy PID control under multiple operational constraints, achieving efficient and energy-saving speed tracking control. It is often necessary to conduct multiple field tests and adjust PID parameters to achieve optimal control performance. During train operation, the controller may lead to frequent changes in control commands, which not only affect passenger comfort but may also increase train energy consumption. To improve the performance of PID control, specific constraints are often introduced in practical applications. However, these constraints are typically targeted and difficult to apply directly to other trains or different rail lines, limiting their universality and flexibility.[19] Iterative learning is a learning method based on cyclicality and feedback mechanisms, which continuously improves the control performance of a system by repeatedly executing specific tasks.[20–22] Since iterative learning control methods are suitable for control scenarios with repetition, those are particularly well-suited for precise control of train operations, as train movement is inherently periodic and repetitive.[23] Ji *et al.* propose a speed tracking control strategy that integrates adaptive control with iterative learning control, requiring normal modeling and offering strong adaptability.[24] This provides an innovative solution to address the nonlinear and external disturbance issues encountered in HST operations. Huang *et al.* focus on an adaptive iterative learning control method designed to address the delay and input saturation issues faced by HSTs in speed tracking control, aiming to achieve precise and efficient trajectory tracking control.[25] Trains are subject to various constraint conditions during operation, and model predictive control (MPC) is an optimization-based control strategy that can control complex nonlinear systems while satisfying a set of constraints.[26–29] In Ref. [30], an online trajectory control method is proposed for HSTs based on a T-S fuzzy bilinear model, where the MPC dynamically adjusts the model and controller parameters based on a lazy learning algorithm. Note that in Ref. [30], the additional running resistance of the train is not considered. During actual operation, when trains pass through railway curves, slopes, or tunnels, additional generated resistance can affect the stability of train operation.

This paper addresses the operational consistency of the train cooperation control system and the handling of unknown parameters and unmodeled interference. A polynomial approximation strategy for adaptive cooperative control of HSTs is proposed, with two significant contributions:

1) A cooperative control strategy based on polynomial function approximation is proposed for HSTs with virtual coupling, considering two scenarios: a cooperative control algorithm for HSTs in the scenario of known parameters; and a polynomial approximation algorithm for the adaptive cooperative control of HSTs in the scenario of unknown parameters. The polynomial function can reduce the impact of external disturbances on the system.

2) Under unknown scenarios, the polynomial coefficients can be obtained by continuously adjusting the parameters through adaptive laws. We adopt this method in the polynomial approximation strategy for the adaptive cooperative control of HSTs when applying it to the trains' cooperative operation system.

The remainder of the article is organized as follows. Section 2 introduces the HST cooperative operation model. Section 3 discusses the cooperative operation of train formations when the train parameters are known. Section 4 explores the adaptive coordination controller based on polynomial approximation under the scenario of unknown trains parameters. Section 5 provides some practical examples to validate the above two scenarios. Section 6 concludes the paper.

2. Preliminaries and problem formulation

The cooperative control system of HSTs fundamentally establishes a mechanism for train formation. Comprising mul-

tiple trains, each of them can independently accelerate or decelerate. Precise adjustments by the control system enable the trailing train to maintain an appropriate distance from the front train, thereby allowing it to track the displacement and speed of the train ahead seamlessly throughout the operation of the formation.

Remark 1 This scheme adopts a predecessor–follower virtual coupling cooperative operation mode, achieving dynamic coordination of multiple trains through train-to-train communication. The first train takes the desired curve as the tracking target, and the remaining trains only receive the real-time status information of their preceding trains. The system employs fully distributed architecture without requiring a central control node, enabling global coordination via cascaded command transmission between adjacent trains.

Analyzing the dynamic characteristics of the train operation process, we use the single-point model for modeling the dynamic system of the HSTs because it focuses on describing the overall state of the train. Based on Newton's dynamics laws, the motion processes of HSTs are described by the following dynamic model:

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ M_i\dot{v}_i(t) = u_i(t) - f_i(v_i(t)) - \Delta f_i(v_i(t), p_i(t)), \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, n$, $p_i(t)$, $v_i(t)$ and $\dot{v}_i(t)$ respectively represent the actual displacements, real-time running speeds, and accelerations of train i. M_i indicates the total weight of train i during operation. u_i represents the control inputs (i.e., traction or braking forces) of train i.

In actual operation, the resistances faced by train i can be divided into two main parts: basic running resistances $f_i(v_i(t))$ and additional resistances $\Delta f_i(v_i(t), p_i(t))$. The basic running resistances $f_i(v_i(t))$ are directly related to the train's speed and can be described by the Davis equation in the following form:

$$f_i(v_i(t)) = c_{i,0} + c_{i,1}v_i(t) + c_{i,2}v_i^2(t), \tag{2}$$

where $c_{i,0}$, $c_{i,1}$, and $c_{i,2}$ are basic resistance coefficients that are all positive definite and can be obtained through wind tunnel experiments. The additional resistances $\Delta f_i(v_i(t), p_i(t))$ are encountered by the trains when running on a fixed route and include curve resistance, gradient resistance, tunnel resistance, and other types of resistance caused by unmodeled factors. These additional resistances are often difficult to accurately determine in the actual operation of HSTs, which can adversely affect the stability and efficiency of train operations.

3. Cooperative controller design under known HST parameters

The driving strategy for HST formation is developed within known parameters of the HSTs, which addresses the control challenges of train formations operating in intricate scenarios.

When a train formation operates with coordinated movement, the first train, known as the leading train, tracks a desired trajectory. In this work, the desired displacement and desired speed for the leading train tracking are defined as $p_{\mathrm{d}}(t)$ and $v_{\mathrm{d}}(t)$. For the leading train, the displacement error $e_{1,1}(t)$ and speed error $e_{1,2}(t)$ are defined as follows:

$$\begin{cases} e_{1,1}(t) = p_1(t) - p_{\mathrm{d}}(t), \\ e_{1,2}(t) = v_1(t) - v_{\mathrm{d}}(t), \end{cases} \tag{3}$$

where $p_{1,1}(t)$ and $v_{1,1}(t)$ are the displacement and speed of the leading train. Differentiating it and incorporating the train dynamics (2), we can arrange it as follows:

$$\begin{cases} \dot{e}_{1,1}(t) = e_{1,2}(t), \\ \dot{e}_{1,2}(t) = \dfrac{1}{M_1}(u_1(t) - \Phi_1^{\mathrm{T}}\Psi_1(t) \\ \qquad\qquad - \Delta f_1(v_1(t), p_1(t))) - \dot{v}_{\mathrm{d}}(t), \end{cases} \tag{4}$$

where $\Phi_1 \in \Re^3$ and $\Psi_1 \in \Re^3$ collectively describe the basic resistance with the structure as follows:

$$\Phi_1 = \begin{bmatrix} c_{1,0} \\ c_{1,1} \\ c_{1,2} \end{bmatrix}, \quad \Psi_1(t) = \begin{bmatrix} 1 \\ v_1(t) \\ v_1^2(t) \end{bmatrix}.$$

However, the following trains have distinct tracking objectives. Those targets include the status information of the trains in front of them, which collectively ensure the synchronized running of the entire formation. The objective for each following train in the formation is to monitor the status information of the train ahead of them. Next, we define the displacements $e_{i,1}(t)$ and speed errors $e_{i,2}(t)$ for the following train $(i > 1)$

$$\begin{cases} e_{i,1}(t) = p_i(t) - p_{i-1}(t) + l_{\mathrm{s}}, \\ e_{i,2}(t) = v_i(t) - v_{i-1}(t), \end{cases} \tag{5}$$

where $p_{i-1}(t)$ and $v_{i-1}(t)$ indicate the displacements and speeds of the trains preceding train i; and l_{s} is the spacing distance between two adjacent trains.

Similarly, we can obtain the error system

$$\begin{cases} \dot{e}_{i,1}(t) = e_{i,2}(t), \\ \dot{e}_{i,2}(t) = \dfrac{1}{M_i}(u_i(t) - \Phi_i^{\mathrm{T}}\Psi_i(t) \\ \qquad\qquad - \Delta f_i(v_i(t), p_i(t))) - \dot{v}_{i-1}(t), \end{cases} \tag{6}$$

where $\Phi_i \in \Re^3$, $\Psi_i \in \Re^3$, and their structures are

$$\Phi_i = \begin{bmatrix} c_{i,0} \\ c_{i,1} \\ c_{i,2} \end{bmatrix}, \quad \Psi_i(t) = \begin{bmatrix} 1 \\ v_i(t) \\ v_i^2(t) \end{bmatrix}.$$

Assuming that all HST parameters are known, we will develop a control strategy based on polynomial function approximation and apply it to the HST cooperative operation

system. This aims to achieve tracking of displacement and speed, meaning that the tracking errors of displacement and speed throughout the entire operation converge to zero over a short period of time, and the entire system operates coherently.

Theorem 1 In order to ensure that the leading and following trains can still accurately track the desired target curves in complex operating environments, the control law for the leading train is designated as follows:

$$\begin{cases} u_1(t) = M_1(\dot{v}_{\mathrm{d}}(t) - k_{1,2}e_{1,1}(t) - \gamma_{1,2}e_{1,2}(t)) \\ \qquad + \Phi_1^{\mathrm{T}}\Psi_1(t) + \Delta f_1(v_1(t), p_1(t)), \\ u_i(t) = M_i(\dot{v}_{i-1}(t) - k_{i,2}e_{i,1}(t) - \gamma_{i,2}e_{i,2}(t)) \\ \qquad + \Phi_i^{\mathrm{T}}\Psi_i(t) + \Delta f_i(v_i(t), p_i(t)), \quad (i > 1), \end{cases} \tag{7}$$

where $\gamma_{i,2} = k_{i,1} \cdot k_{i,2}$. Otherwise, $k_{i,1}$ and $k_{i,2}$ are control law parameters that are all greater than zero. Then, the tracking errors of displacement and speed converge to zero in a short period of time from any initial conditions, and the train formation system operates cohesively and consistently while ensuring an appropriate spacing distance.

Proof First, the following Lyapunov function is constructed:

$$V_i(t) = \frac{1}{2}e_{i,1}^2(t) + \frac{1}{2k_{i,2}}e_{i,2}^2(t), \tag{8}$$

verifying the stability of the train formation. Carrying out the derivation of Eq. (8), we can then obtain

$$\dot{V}_i(t) = \begin{cases} e_{1,1}(t)e_{1,2}(t) + \dfrac{1}{k_{1,2}}e_{1,2}(t)\Big(\dfrac{1}{M_1}(u_1(t) - \Phi_1^{\mathrm{T}}\Psi_1(t) \\ -\Delta f_i(v_i(t), p_i(t))) - \dot{v}_{\mathrm{d}}(t)\Big), \\ e_{i,1}(t)e_{i,2}(t) + \dfrac{1}{k_{i,2}}e_{i,2}(t)\Big(\dfrac{1}{M_i}(u_i(t) - \Phi_i^{\mathrm{T}}\Psi_i(t) \\ -\Delta f_i(v_i(t), p_i(t))) - \dot{v}_{i-1}(t)\Big), \quad (i > 1). \end{cases} \tag{9}$$

For the entire HST cooperative operation system, it is necessary to consider the sequential stability of the train formation

$$V_{\mathrm{Coop}}(t) = \sum_{i=1}^{n} V_i(t), \tag{10}$$

and by taking the derivative of V_{Coop} along time, we obtain

$$\dot{V}_{\mathrm{Coop}}(t) = \sum_{i=1}^{n} \dot{V}_i(t) \le -\sum_{i=1}^{n} k_{i,1}e_{i,2}^2(t) \le 0. \tag{11}$$

Based on the above, it can be inferred that V_{Coop} is a non-decreasing function. According to the Lyapunov stability theory, it can be known that the system is stable, meaning that the train formation system can operate in a coordinated and consistent manner. The proof is completed.

Remark 2 We know that it is unrealistic to obtain train data accurately during operation on a real railway because of various environmental factors. Therefore, we will design an adaptive controller in the following section to make up for the deficiency in the above controller.

4. Cooperative controller design under unknown HST parameters

In this section, by utilizing an adaptive control method in conjunction with polynomial function approximation methods we design the driving strategy for HST formation, which solves the conundrum of not being able to get the actual parameters.

During the operation of an HST on a real route, it is always difficult to obtain the additional resistances. This work deals with that issue by applying an polynomial function approximation. The additional resistances can be described as follows:

$$\Delta f_i(v_i(t), p_i(t)) = P_{i,m} + \kappa_i(z(t)), \tag{12}$$

where $\kappa_i(z(t))$ is the reconstruction error of the polynomial approximation and has a unknown upper bound, namely, $\kappa_i(z(t))$ satisfies $|\kappa_i(z(t))| \le \bar{\kappa}_i(z(t)) < \infty$. $P_i = \sum_{k=0}^{m} a_{ki}z_i^k(t)$ is the least-squares polynomial and $z(t) \in \Re$ is the independent variable. Therefore, applying the polynomial approximation to additional resistances, the dynamic equations of train i turn into

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ M_i\dot{v}_i(t) = u_i(t) - f_i(v_i(t) - \sum_{k=0}^{m} a_{ki}z_i^k(t) - \kappa_i(z(t))). \end{cases} \tag{13}$$

Remark 3 In the approximation theory, we should note that a polynomial can approximate any continuous function. Therefore, it is rational to apply polynomial approximation to handle the adverse effects of the additional resistance.

Taking Eq. (13) into Eq. (4), we can obtain

$$\begin{cases} \dot{e}_{1,1}(t) = e_{1,2}(t), \\ \dot{e}_{1,2}(t) = \dfrac{1}{M_1}(u_1(t) - \Phi_1^{\mathrm{T}}\Psi_1(t) \\ \qquad - \Theta_1^{\mathrm{T}}\mathcal{H}_1(t) - \kappa_1(z(t))) - \dot{v}_{\mathrm{d}}(t), \end{cases} \tag{14}$$

where the additional resistances are represented by $\Theta_1 \in \Re^{m+1}$ and $\mathcal{H}_1 \in \Re^{m+1}$, having the following form:

$$\Theta_1 = \begin{bmatrix} a_{1,0} \\ a_{1,1} \\ a_{1,2} \\ \vdots \\ a_{1,m} \end{bmatrix}, \quad \Psi_1(t) = \begin{bmatrix} z_1^0(t) \\ z_1(t) \\ z_1^2(t) \\ \vdots \\ z_1^m(t) \end{bmatrix}.$$

Similarly, we can obtain the error system

$$\begin{cases} \dot{e}_{i,1}(t) = e_{i,2}(t), \\ \dot{e}_{i,2}(t) = \dfrac{1}{M_i}(u_i(t) - \Phi_i^{\mathrm{T}}\Psi_i(t) \\ \qquad - \Theta_i^{\mathrm{T}}\mathcal{H}_i(t) - \kappa_i(z(t))) - \dot{v}_{i-1}(t), \end{cases} \tag{15}$$

where $\Theta_i \in \Re^{m+1}$ and $\mathcal{H}_i \in \Re^{m+1}$,

$$\Theta_i = \begin{bmatrix} a_{i,0} \\ a_{i,1} \\ a_{i,2} \\ \vdots \\ a_{i,m} \end{bmatrix}, \quad \mathcal{H}_i(t) = \begin{bmatrix} z_i^0(t) \\ z_i(t) \\ z_i^2(t) \\ \vdots \\ z_i^m(t) \end{bmatrix}.$$

Compared to system (6), this system contains unknown parameters; therefore, the previously designed controller is no longer applicable to this system. Regarding the aforementioned unknown parameters, we define the form of the estimation error, $\tilde{\bar{\kappa}}_i = \bar{\kappa}_i - \hat{\bar{\kappa}}_i$, $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$ and $\tilde{\Phi}_i = \Phi_i - \hat{\Phi}_i$. We can design the adaptive law as

$$\dot{\hat{\bar{\kappa}}}_i(t) = -\frac{e_{i,2}(t)}{M_i \cdot k_{i,2}} \cdot k_{\kappa i} \cdot \mathrm{sgn}(e_{i,2}(t)), \tag{16}$$

$$\dot{\hat{\Theta}}_i(t) = -\frac{e_{i,2}(t)}{M_i \cdot k_{i,2}} \cdot \Gamma_{i,1} \mathcal{H}_i(t), \tag{17}$$

$$\dot{\hat{\Phi}}_i(t) = -\frac{e_{i,2}(t)}{M_i \cdot k_{i,2}} \cdot \Gamma_{i,2} \Psi_i(t), \tag{18}$$

where $\Gamma_{i,1}$ and $\Gamma_{i,2}$ are control law parameters with all values being positive constants.

Theorem 2 Due to the nonlinearity of HST operation and the unavailability of exact parameters, we adopt an adaptive control strategy. Let us design the following control input:

$$\begin{cases} u_1(t) = M_1(\dot{v}_\mathrm{d}(t) - k_{1,2}e_{1,1}(t) - \gamma_{1,2}e_{1,2}(t)) + \hat{\Theta}_1^\mathrm{T}\mathcal{H}_1(t) \\ \qquad + \hat{\Phi}_1^\mathrm{T}\Psi_1(t) + \mathrm{sgn}(e_{1,2}(t)) \cdot \hat{\bar{\kappa}}_1(z(t)), \\ u_i(t) = M_i(\dot{v}_{i-1}(t) - k_{i,2}e_{i,1}(t) - \gamma_{i,2}e_{i,2}(t)) + \hat{\Theta}_i^\mathrm{T}\mathcal{H}_i(t) \\ \qquad + \hat{\Phi}_i^\mathrm{T}\Psi_i(t) + \mathrm{sgn}(e_{i,2}(t)) \cdot \hat{\bar{\kappa}}_i(z(t)), \quad (i > 1), \end{cases} \tag{19}$$

where $\gamma_{1,2} = k_{1,1} \cdot k_{1,2}$, $\gamma_{i,2} = k_{i,1} \cdot k_{i,2}$. Otherwise, $k_{1,1}$, $k_{1,2}$, $k_{i,1}$, $k_{i,2}$ are all greater than zero. Then, the tracking errors of displacement and speed converge to zero within a short period of time from any initial conditions. Consequently, the virtual coupling train formation system operates cohesively and consistently while maintaining an appropriate spacing distance.

Proof In order to validate the stability of the proposed control strategy, this part presents the stability analysis. First, the following Lyapunov function is constructed:

$$\begin{aligned} V_1(t) \ = \ & \frac{1}{2}e_{1,1}^2(t) + \frac{1}{2k_{1,2}}e_{1,2}^2(t) + \frac{1}{2k_{\kappa 1}}\tilde{\bar{\kappa}}_1^2(z) \\ & + \frac{1}{2}\tilde{\Theta}_1^\mathrm{T}\Gamma_{1,1}^{-1}\tilde{\Theta}_1 + \frac{1}{2}\tilde{\Phi}_1^\mathrm{T}\Gamma_{1,2}^{-1}\tilde{\Phi}_1, \end{aligned} \tag{20}$$

verifying the stability of the lead train. Carrying out the derivation of Eq. (20), we can then obtain

$$\begin{aligned} \dot{V}_1(t) \ = \ & e_{1,1}(t)e_{1,2}(t) + \frac{1}{k_{1,2}}e_{1,2}(t)\Big(\frac{1}{M_1}(u_1(t) - \Phi_1^\mathrm{T}\Psi_1(t) \\ & - \theta_1^\mathrm{T}\mathcal{H}_1(t) - \kappa_1(z(t))) - \dot{v}_\mathrm{d}(t)\Big) \\ & + \frac{1}{k_{\kappa 1}}\dot{\tilde{\bar{\kappa}}}_1(z)\tilde{\bar{\kappa}}_1(z) + \dot{\tilde{\Theta}}_1^\mathrm{T}\Gamma_{1,1}^{-1}\tilde{\Theta}_1 + \dot{\tilde{\Phi}}_1^\mathrm{T}\Gamma_{1,2}^{-1}\tilde{\Phi}_1. \end{aligned} \tag{21}$$

Incorporating Eqs. (16)–(19) into Eq. (21), we can obtain

$$\begin{aligned} \dot{V}_1(t) \ = \ & -\frac{\gamma_{1,2}}{k_{1,2}}e_{1,2}^2(t) + \frac{e_{1,2}(t)}{k_{1,2} \cdot M_1} \cdot \hat{\bar{\kappa}}_1(z) \cdot \mathrm{sgn}(e_{1,2}(t)) \\ & - \frac{1}{k_{\kappa 1}}\dot{\hat{\bar{\kappa}}}_1(z)\tilde{\bar{\kappa}}_1(z(t)) - \frac{e_{1,2}(t)}{k_{1,2} \cdot M_1}\kappa_1(z(t)) \\ & - \dot{\hat{\Theta}}_1^\mathrm{T}\Gamma_{1,1}^{-1}\tilde{\Theta}_1 - \frac{e_{1,2}(t)}{k_{1,2} \cdot M_1}\tilde{\Theta}_1^\mathrm{T}\mathcal{H}_1(t) \\ & - \tilde{\Phi}_1^\mathrm{T}\Gamma_{1,2}^{-1}\tilde{\Phi}_1 - \frac{e_{1,2}(t)}{k_{1,2} \cdot M_1}\tilde{\Phi}_1^\mathrm{T}\Psi_1(t). \end{aligned} \tag{22}$$

By the same token, the Lyapunov functions for constructing the following trains are

$$\begin{aligned} V_i(t) \ = \ & \frac{1}{2}e_{i,1}^2(t) + \frac{1}{2k_{i,2}}e_{i,2}^2(t) + \frac{1}{2k_{\kappa i}}\tilde{\bar{\kappa}}_i^2(z(t)) \\ & + \frac{1}{2}\tilde{\Theta}_i^\mathrm{T}\Gamma_{i,1}^{-1}\tilde{\Theta}_i + \frac{1}{2}\tilde{\Phi}_i^\mathrm{T}\Gamma_{i,2}^{-1}\tilde{\Phi}_i. \end{aligned} \tag{23}$$

Then, we have

$$\begin{aligned} \dot{V}_i(t) \ = \ & e_{i,1}(t)e_{i,2}(t) + \frac{1}{k_{i,2}}e_{i,2}(t)\Big(\frac{1}{M_i}(u_i(t) - \Phi_i^\mathrm{T}\Psi_i(t) \\ & - \Theta_i^\mathrm{T}\mathcal{H}_i(t) - \kappa_i(z)) - \dot{v}_{i-1}(t)\Big) \\ & + \frac{1}{k_{\kappa i}}\dot{\tilde{\bar{\kappa}}}_i(z)\tilde{\bar{\kappa}}_i(z) + \dot{\tilde{\Theta}}_i^\mathrm{T}\Gamma_{i,1}^{-1}\tilde{\Theta}_i + \dot{\tilde{\Phi}}_i^\mathrm{T}\Gamma_{i,2}^{-1}\tilde{\Phi}_i. \end{aligned} \tag{24}$$

Considering Eqs. (16)–(19), Eq. (24) is transformed into

$$\begin{aligned} \dot{V}_i(t) \ = \ & -\frac{\gamma_{i,2}}{k_{i,2}}e_{i,2}^2(t) + \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i} \cdot \hat{\bar{\kappa}}_i(z) \cdot \mathrm{sgn}(e_{i,2}(t)) \\ & - \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i} \cdot \kappa_i(z) - \frac{1}{k_{\kappa i}}\dot{\hat{\bar{\kappa}}}_i(z)\tilde{\bar{\kappa}}_i(z) \\ & - \dot{\hat{\Theta}}_i^\mathrm{T}\Gamma_{i,1}^{-1}\tilde{\Theta}_i - \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i}\tilde{\Theta}_i^\mathrm{T}\mathcal{H}_i(t) \\ & - \tilde{\Phi}_i^\mathrm{T}\Gamma_{i,2}^{-1}\tilde{\Phi}_i - \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i}\tilde{\Phi}_i^\mathrm{T}\Psi_i(t). \end{aligned} \tag{25}$$

For the entire HST cooperative operation system, it is necessary to consider the sequential stability of train formations:

$$V_\mathrm{Coop}(t) = \sum_{i=1}^{n} V_i(t), \tag{26}$$

and taking the derivative of $V_\mathrm{Coop}(t)$ along time, we obtain

$$\begin{aligned} & \dot{V}_\mathrm{Coop}(t) = \sum_{i=1}^{n} \dot{V}_i(t) \\ & = \sum_{i=1}^{n}\Big(-\frac{\gamma_{i,2}}{k_{i,2}}e_{i,2}^2(t) + \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i} \cdot \hat{\bar{\kappa}}_i(z) \cdot \mathrm{sgn}(e_{i,2}(t)) \\ & \quad - \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i} \cdot \kappa_1(z) + \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i} \cdot \tilde{\bar{\kappa}}_i(z) \cdot \mathrm{sgn}(e_{i,2}(t))\Big) \\ & \le \sum_{i=1}^{n}\Big(-\frac{\gamma_{i,2}}{k_{i,2}}e_{i,2}^2(t) + \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i} \cdot \hat{\bar{\kappa}}_i(z) \cdot \mathrm{sgn}(e_{i,2}(t) \\ & \quad - \frac{e_{i,2}(t)}{k_{i,2} \cdot M_i} \cdot \bar{\kappa}_1(z) \cdot \mathrm{sgn}(e_{i,2}(t)) \end{aligned}$$

$$+\frac{e_{i,2}(t)}{k_{i,2}\cdot M_i}\cdot\tilde{\tilde{\kappa}}_i(z)\cdot\mathrm{sgn}(e_{i,2}(t)))\Big)$$
$$\leq -\sum_{i=1}^{n}k_{i,1}e_{i,2}^2(t). \tag{27}$$

By Lyapunov's theorem, we can draw the same conclusion, namely that $e_i(t)$ and $\dot{e}_i(t)$ will converge to zero, meanwhile the HSTs operate uniformly in formation. The proof is completed.

5. Simulation

In this section, there are four identical trains serving as experimental subjects, each with mass of 5×10^5 kg. The reason for selecting four trains is to illustrate the coordinated operation between the leading train and the following trains, the tail train and the preceding trains, as well as the trains in the middle of the formation. Considering the effects of basic resistance and additional resistances during HST operation, where the inertial resistance coefficient, rolling resistance coefficient, and aerodynamic resistance coefficient are 2.25 kN, 1.19×10^3 kN·s/m and 3.2×10^4 kN·s/m^2, respectively, simulation experiments are conducted to verify the effectiveness of the proposed control method. Table 1 provides the characteristic parameters of the trains and the track parameters.

Table 1. Simulation parameters.

Name	Numerical values
Initial position of HST 1	0 m
Initial position of HST 2	−1010 m
Initial position of HST 3	−2020 m
Initial position of HST 4	−3030 m
Train space distance	1000 m
Curve central angle	120°
Railway line gradient	0→0.015
Tunnel length	1500 m

The objective of this paper is to design an appropriate control strategy to ensure that the train formation operates collaboratively while maintaining a safe distance. Additionally, the actual position and speed of the leading train should accurately track the desired displacement and speed.

Case 1 Cooperative control algorithm for HSTs within known parameters.

Using the cooperative control strategy (7) from Theorem 1, the simulation results mainly include the following: Under the condition of known parameters for the HST formation, figure 2 shows the displacement tracking errors and speed tracking errors of each train in the formation, demonstrating the effectiveness of the control scheme proposed in Theorem 1. Figure 3 illustrates the cooperative operation of the entire formation, from which it can be observed that the train formation operates collaboratively while maintaining the required spacing distance.

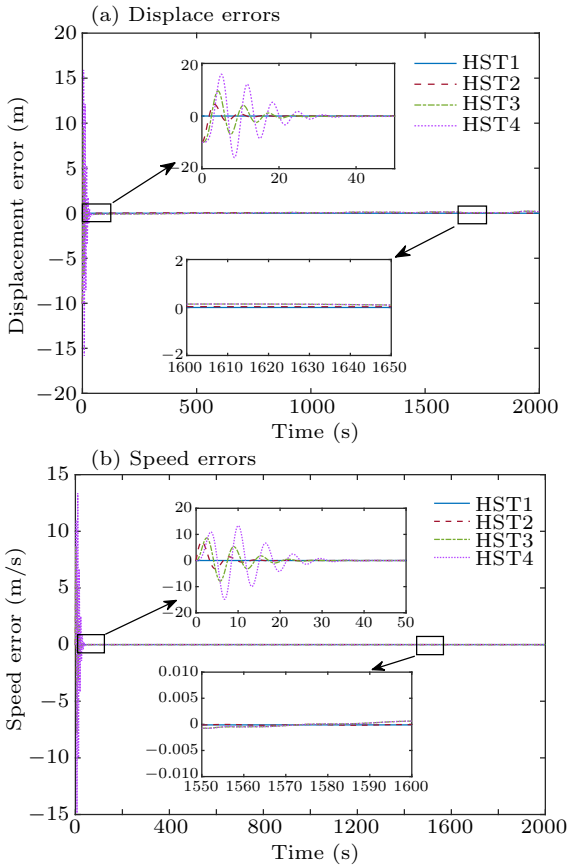


Fig. 2. (a) Displacement tracking errors and (b) speed tracking errors distributions of the trains under the condition that the parameters are known in the formation. Each color represents a train.

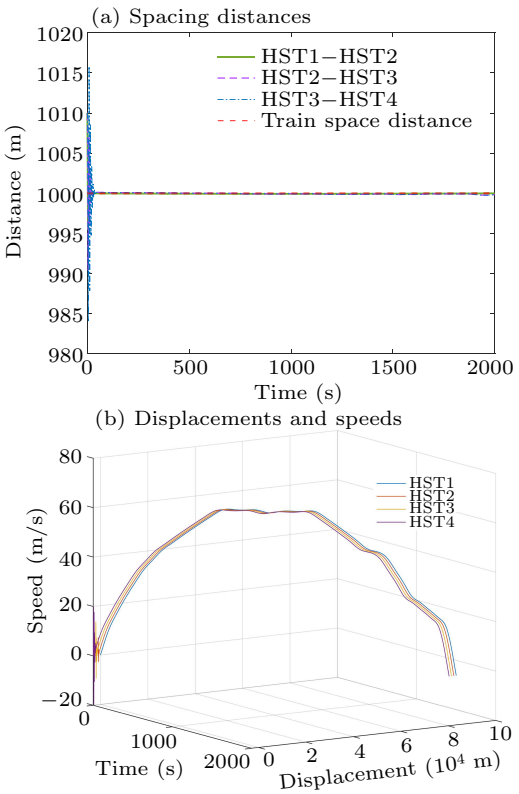


Fig. 3. (a)–(b) The coupling and cooperative operation processes of the trains under the condition that the parameters are known in the formation.

Case 2 Polynomial approximation algorithm for adaptive cooperative control of HSTs with unknown parameters.

The adaptive cooperative control strategy, applied to the scenario of unknown parameters and based on the polynomial function approximation method from Theorem 2, ensures effective system performance. The simulation results are as follows. Figure 4 demonstrates the effectiveness of the control strategy in handling unknown factors. Figure 5 shows that under the influence of this control strategy, HSTs can still operate collaboratively and stably.

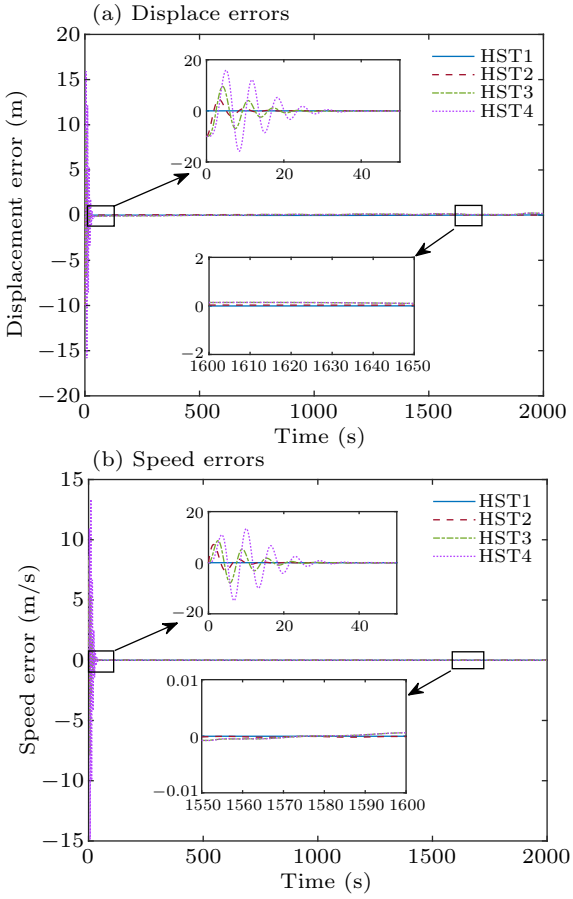


Fig. 4. (a) Displacement tracking error and (b) speed tracking error distributions of the trains under the condition that the parameters are unknown in the formation. Each color represents a train.

Regarding the experimental results in different scenarios, we compared the speed errors in the scenarios where the parameters are known, the parameters are unknown without polynomial approximation, and the parameters that are unknown with polynomial approximation in Fig. 6. Taking the first train as an example, it can be seen that in the scenario where the parameters are known, the curve of the velocity error has smaller fluctuations. However, during the operation of trains, it is difficult to obtain the relevant parameters. Therefore, we compared the speed errors in scenarios where the parameters are unknown. It can be observed that after adopting polynomial approximation, the fluctuation of the speed error curve significantly decreases.

From the experimental results, it can be seen that the proposed control strategy is effective to ensure the stability of the resulting HST formation cooperative closed-loop system as well as the tracking performance of displacement and speed.

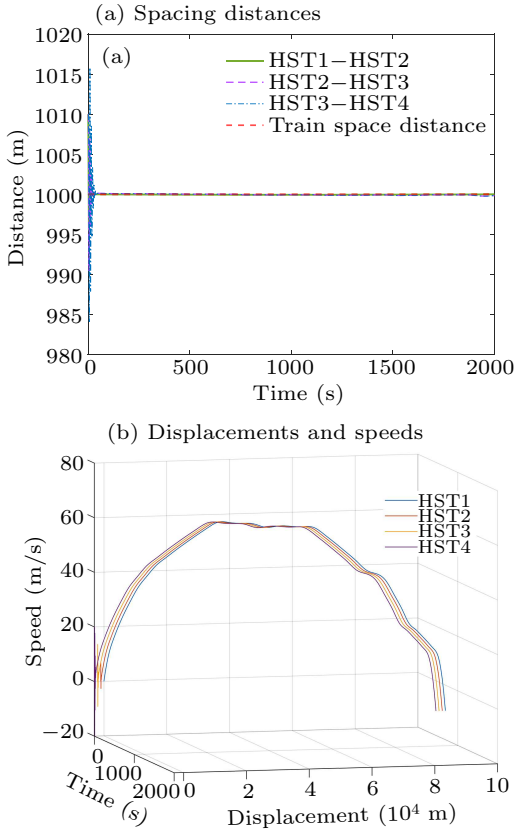


Fig. 5. (a)–(b) The coupling and cooperative operation processes of the trains under the condition that the parameters are unknown in the formation.

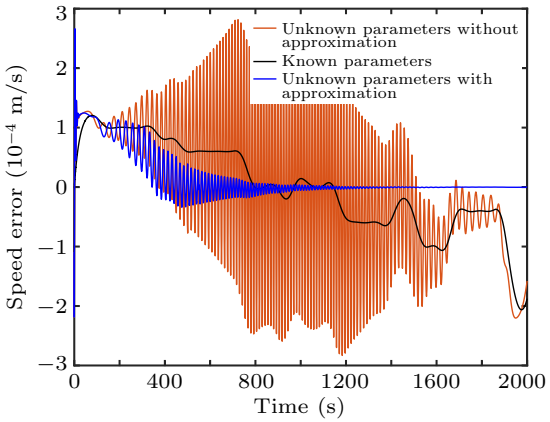


Fig. 6. Comparison of different scenarios.

6. Conclusion

In this paper, a polynomial function approximation cooperative control algorithm for HST formation addresses the issue of HST formation cooperative control in scenarios with unknown parameters under virtual coupling environments. By introducing the polynomial function approximation method and an adaptive control strategy, the proposed approach enables online estimation of unknown parameters and effectively handles external disturbances, and ensures the safety and sta-

bility of HST formation under complex operating conditions. Theoretical speed analysis and simulation results demonstrate that this method significantly meets the consistency requirements for coordinated operation.

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