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Citation: Chin. Phys. B, 2025, 34 (**10**): 100304. DOI: 10.1088/1674-1056/add67e
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Entangling operations in a quantum repeater node using synchronized fast adiabatic pulses

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(Received 2 April 2025; revised manuscript received 6 May 2025; accepted manuscript online 9 May 2025)

Solid-state rare-earth ions are promising candidates for implementing repeater nodes for quantum networks. However, the state-of-the-art quantum nodes use only a single qubit per node, which greatly limits the functionality of the node and the scalability of the network. Here, we propose a scheme that utilizes a hybrid system of two ion qubits coupled to a nanophotonic cavity as a quantum node. Simultaneously applying a fast adiabatic pulse to the two ions can lead to an effective interaction between the two ion spin qubits by exchanging virtual photons in the cavity. Using this interaction, a controlled phase gate between the two ion qubits can be realized with a fidelity of 99.6%. Further utilizing this interaction, entangled states within the node can be generated deterministically with high fidelity, and are robust to a variety of noises and fluctuations. These results pave a way for fully functional quantum repeater nodes based on solid-state rare-earth ions.

Keywords: cavity QED, quantum information with hybrid systems, entanglement production

PACS: 03.67.Lx, 42.50.Dv, 42.50.Pq

DOI: 10.1088/1674-1056/add67e **CSTR:** 32038.14.CPB.add67e

1. Introduction

Quantum networks play an important role in both fundamental research in quantum physics and in the application of quantum technologies, ranging from long-distance quantum communication to distributed quantum computing and quantum sensing.[1–4] Quantum networks usually consist of quantum repeater nodes and photonic channels, where quantum nodes have various physical platforms as candidates. In recent years, rare-earth ion doped in solids has emerged as a promising platform that has attracted much attention.[5–17] On the one hand, rare-earth ions have a strong electric dipole moment, which can form an efficient interface with flying photons. On the other hand, the electron and nuclear spins of rare-earth ions have long coherence times and can be used as a long-lived memory for quantum information.

Despite these impressive breakthroughs, current demonstrations use only a single ion qubit per node,[14–16] which constrains the functionality of quantum nodes and thereby greatly impedes the scalability of quantum networks. A fully functional quantum repeater node requires not only transducers and memories to interface with photons, but also processors to manipulate the information encoded in the entangled states. Therefore, designing a multi-ion node capable of preparing and manipulating entangled states is a highly desired, but challenging goal. These challenges include: (i) rare-earth ions have only weak magnetic dipole interactions with each other, making it difficult to achieve strong coupling for entanglement. (ii) Rare-earth ions typically have complex energy level structures and decoherence mechanisms, it remains unclear how to harness this complexity.

Here, we propose a hybrid system consisting of two rare-earth ions in a nanophotonic cavity as a quantum repeater node. We design a fast adiabatic pulse by adjusting the shape and phase of the laser driving pulse. The fast adiabatic pulse is then applied to the two ions simultaneously, causing the spin states of the two ions to interact effectively through a virtual photon process in the cavity. This method is able to maintain the coherence of the hybrid system while effectively manipulating the states of the two ions. Using this synchronized fast adiabatic pulse, we demonstrate that entangling gates can be achieved with a fidelity of up to 99.6% under realistic parameter conditions, which is a significant improvement over the previously reported operation fidelity of the ions.[14,17] In addition, we demonstrate the robustness of this method to various noise and decoherence, as well as high-fidelity on-node entanglement preparation. These results provide a powerful and convenient toolbox for quantum nodes based on rare-earth ions in solids.

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2. Models of quantum repeater nodes

As shown in Fig. 1, we consider a hybrid system that includes two rare-earth ions and a nanophotonic cavity. Such a solid-state hybrid system can act as a fully functional quantum repeater node. Firstly, each ion can act as a transducer to realize an efficient interface with flying photons. Secondly, the spin states of each ion can be used as a memory, capable of storing quantum information for a long time. More importantly, we can design high-fidelity entanglement operations between two ions that can act as a processor for quantum information.

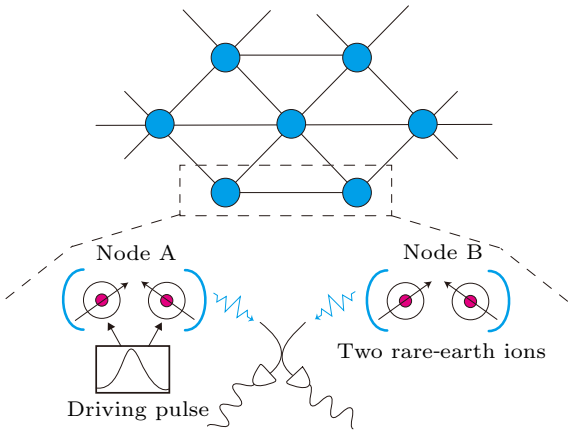


Fig. 1. Schematic of two-ion nodes in a quantum network architecture. Two rare-earth ions are coupled to a nanophotonic cavity, and this hybrid system can act as a fully functional quantum repeater node.

For concreteness, we consider a typical system:[17] two $^{171}Yb^{3+}$ ions doped in a YSO_4 crystal. The Hamiltonian of each ion can be described as

$$\begin{aligned} H_{\rm ion} &= (E_{\rm g}+g\mu_{\rm B}BS_z+D_{\rm e}S_z)\frac{1+\tau_z}{2} \\ &\quad +(g\mu_{\rm B}BS_z+D_{\rm g}S_z)\frac{1-\tau_z}{2}, \end{aligned} \tag{1}$$

where τ_z denotes the orbital state operator with eigenvalue of -1 for the ground state and 1 for the excited state, and S_z denotes the spin state operator with quantum number $m_{\rm s}=1/2$ for the state $|\uparrow\rangle$ and $m_{\rm s}=-1/2$ for the state $|\downarrow\rangle$. The parameter $E_{\rm g}$ denotes the energy gap between the excited state and the ground state, and the term $g\mu_{\rm B}BS_z$ denotes the Zeeman splitting of the spins in an external magnetic field. $D_{\rm g}$ and $D_{\rm e}$ denote zero-field spin splittings of the ground state and the excited state, respectively.

Since rare-earth ions have large electric dipole moments, we can apply a laser driving field to manipulate each ion. Working in a rotating frame with the laser frequency $\omega_{\rm L}$, we can write down the control Hamiltonian

$$H_{\rm L}=g\mu_{\rm B}BS_z+DS_z-\frac{\Delta}{2}S_z+\frac{\delta_{\rm L}}{2}\tau_z+(g_{\rm L}\tau_-+g_{\rm L}^*\tau_+), \tag{2}$$

where we define $D=(D_{\rm g}+D_{\rm e})/2$ and $\Delta=(D_{\rm g}-D_{\rm e})/2$ as the mean and difference between zero-field spin splittings, $\tau_\pm$ represent the transitions between the ground and excited states, $\delta_{\rm L}=E_{\rm g}-\omega_{\rm L}$ is the frequency detuning, and $g_{\rm L}$ is the driving strength of the laser field.

When the whole crystal is placed in a nanophotonic cavity, each ion couples to the cavity field. Therefore, the interaction Hamiltonian can be written as

$$H_{\rm c}=\delta_{\rm c}a^\dagger a+g_{\rm c}(\tau_+a+\tau_-a^\dagger), \tag{3}$$

where $\delta_{\rm c}=\omega_{\rm c}-\omega_{\rm L}$ denotes the frequency detuning between the cavity field and the laser field. $a^\dagger$ and a denote the creation and annihilation operators of the cavity photon mode, and $g_{\rm c}$ is the coupling strength between the ion and the cavity.

Putting things together, the whole Hamiltonian of the hybrid system can be described as

$$H=H_0+V, \tag{4}$$

$$H_0=\delta_{\rm c}a^\dagger a+\sum_{i=1,2}\frac{1+\tau_z^i}{2}(\delta_{{\rm L}i}+\Delta S_z^i), \tag{5}$$

$$V=\sum_{i=1,2}(g_{{\rm L}i}\tau_-^i+g_{{\rm c}i}a^\dagger\tau_-^i+{\rm H.c.}). \tag{6}$$

Here we use the label i to denote the ions.

3. Synchronized fast adiabatic pulses

The hybrid system consists of two ions and a cavity, which contains multiple degrees of freedom. It is therefore subject to various noises and fluctuations. For example, the excited state can suffer from decoherence processes due to charge noise in the environment, and the photon modes can leak due to imperfections in the cavity. Additional noises and fluctuations are introduced when the control pulses are applied. For example, when adjusting the waveform of the laser driving field, it is sensitive to external charge noise, leading to significant errors.

Adiabatic pulses have been developed to compensate for the effects of noises and fluctuations. Adiabatic tuning control pulses, which keep the quantum system in the instantaneous eigenstates, can avoid errors caused by non-adiabatic transitions of quantum states. However, the adiabatic process requires a long evolution time, and the decoherence effects accumulated during this time can introduce significant errors. Therefore, there is a strong interest in developing a shortcut to adiabaticity methods.[18–21] Among them, fast adiabatic pulses can both eliminate the non-adiabatic transitions between quantum states and shorten the manipulation time by designing suitable driving field waveforms. In recent years, fast adiabatic pulses have been demonstrated in various single-qubit systems.[22–25]

However, hybrid systems have complex energy level structures and interactions compared to single-quantum systems. In particular, due to the large differences in the manipulation and decoherence times of each component of the hybrid

system, this poses a completely new challenge in designing the corresponding fast adiabatic pulses.

As shown in Fig. 2, we designed a synchronized fast adiabatic pulse sequence. At the rising stage of the pulse, the laser field drives the ion to the excited state. At the falling stage of the pulse, the laser field drives the ion back to the ground state. Because the laser pulse is detuned from the transition frequency of the ion, the ion does not occupy the excited state and does not emit a real photon into the cavity, but is dispersively coupled to a virtual photon in the cavity field. When this pulse sequence is applied to the two ions at the same time, both ions can interact with a virtual photon in the cavity field. In this way, the excitation of one ion can be transferred to the other ion by exchanging a virtual photon in the cavity.

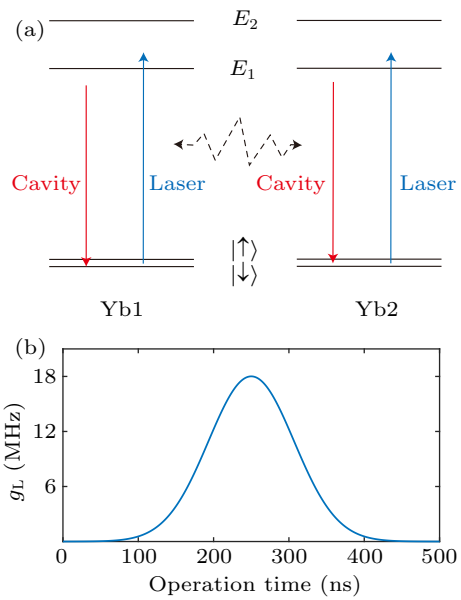


Fig. 2. (a) Schematic of two-ion energy levels with laser pulses. The laser pulses drive the two ions into virtual excited states and back to the ground states synchronizedly, and thereby the two ions are coupled via exchange of virtual photons in the cavity. (b) The laser pulse $g_L(t)$ as a function of time. This pulse is designed as a convolution of a rectangle and a Gaussian shape.

On the other hand, because the driving pulse is slowly varying, the adiabatic theorem ensures that the system stays in the instantaneous eigenstate of the time dependent Hamiltonian. Such a fast adiabatic pulse allows efficient gate operations between ions and avoids errors due to non-adiabatic transitions outside the computational subspace.

To better understand this synchronized fast adiabatic pulse, we utilize the Schrieffer–Wolff transformation to describe the hybrid system.[26,27] First, we write the total Hamiltonian as the sum of the free part H_0 and perturbed part V as in Eq. (4). Then we apply a unitary transformation $U = \mathrm{e}^S$, which gives

$$H_{\rm eff} = \mathrm{e}^S H \mathrm{e}^{-S} \simeq H + [S,H] + \frac{1}{2}[S,[S,H]] + \frac{1}{6}[S,[S,[S,H]]] + \frac{1}{24}[S,[S,[S,[S,H]]]]. \quad (7)$$

Notice that the coupling between each ion and the cavity field under the fast adiabatic pulse is a second order virtual photon process. So the interaction between the two ions is a fourth order virtual photon process. Here we calculate the Schrieffer–Wolff transformation to the fourth order of perturbation theory.

In order to eliminate the cavity field term, we choose the transformation matrix S to satisfy $[S,H_0] = -V$, and we can obtain the effective Hamiltonian in the spin subspace of the two ions as

$$H_{\rm eff} = \begin{pmatrix} \varepsilon_{+\frac{1}{2},+\frac{1}{2}} & 0 & 0 & 0 \\ 0 & \varepsilon_{+\frac{1}{2},-\frac{1}{2}} & 0 & 0 \\ 0 & 0 & \varepsilon_{-\frac{1}{2},+\frac{1}{2}} & 0 \\ 0 & 0 & 0 & \varepsilon_{-\frac{1}{2},-\frac{1}{2}} \end{pmatrix}. \quad (8)$$

This is a diagonal matrix whose diagonal terms are

$$\varepsilon_{m_{s1},m_{s2}} = -\frac{2|g_{L1}g_{L2}|g_{c1}g_{c2}\cos(\phi_1-\phi_2)}{(\delta_{L1}+\Delta m_{s1})(\delta_{L2}+\Delta m_{s2})\delta_c}, \quad (9)$$

where ϕ is the phase of the laser pulse. Here we define the two-qubit computational subspace in the logic bases of two-ion spins: $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$.

This effective Hamiltonian evolves over time t, generating a quantum gate $U_{2q} = \exp(-\mathrm{i}H_{\rm eff}t)$. As a single-qubit operation, it realizes a controlled-phase gate operation between two ion spins $U_{\rm cp} = \mathrm{diag}(\mathrm{e}^{\mathrm{i}\gamma},1,1,1)$ with phase γ. In particular, this gate operation is realized by applying a fast adiabatic pulse and using the virtual photon process. So this scheme is immune to some noises and fluctuations in the hybrid system while accomplishing gate operations.

4. Master equation of the hybrid system

To evaluate the performance of this synchronized fast adiabatic operation, we use the master equation to model the dynamics of the hybrid system

$$\dot{\rho} = -\mathrm{i}[H,\rho] + \kappa\mathcal{D}[a]\rho + \sum_{i=1,2}\left(\gamma_1\mathcal{D}[\sigma_i^-]\rho + \gamma_\phi\mathcal{D}[\sigma_i^z]\rho\right), \quad (10)$$

where ρ is the density matrix of the hybrid system, H is the Hamiltonian of the hybrid system, and the Lindblad operator $\mathcal{D}[O]\rho = O\rho O^\dagger - (O^\dagger O\rho + \rho O^\dagger O)/2$ describes the environmentally induced decoherence processes: the relaxation process $\gamma_1 = 1/T_1$ and the dephasing process $\gamma_\phi = 1/T_\phi$ of the ion spins, as well as the leakage process κ of the cavity. On the other hand, we design the control pulse $g_L(t)$ as a convolution of a rectangular and a Gaussian shape. As shown in Fig. 2(b), such a control pulse can be characterized by the following parameters: the frequency detuning δ_L of the laser field, the maximum amplitude $g_{L,\max}$ of the pulse, and the full widths at half-maximum $t_{\rm FWHM,r}$, $t_{\rm FWHM,g}$ of the rectangle and the Gaussian shapes.

In order to obtain a scheme that is instructive for experiments, we use experimentally feasible control parameters and noise parameters in our simulations:[14,28–30] the spin relaxation time $T_1 = 10$ ms, the spin dephasing time $T_\phi = 0.8$ ms and $\kappa = 20$ MHz; $g_c = 100$ MHz, $g_{L,max} = 18$ MHz, $t_{FWHM,r} = 20$ ns and $t_{FWHM,g} = 133$ ns.

5. Performance of synchronized fast adiabatic pulses

First, we demonstrate that this synchronized fast adiabatic operation enables controlled-phase gates. As shown in Fig. 2, the pulse sequence consists of two stages. Initially, both ions are in the ground state. At the rising stage of the pulse, the laser field slowly increases, driving both ions to the virtual excited state. At the falling stage of the pulse, the laser field slowly decreases, driving both ions back to the ground state. Since the two ions are driven simultaneously, they interact effectively by exchanging virtual photons. In this way, the final state is a coherent superposition of the two-ion spin states.

In Fig. 3(a), we simulate the phase of the non-diagonal element ρ_{ij} of the density matrix in the two-ion spin subspace using the master equation. As the driving pulse is applied, ρ_{ij} accumulates a phase dependent on the difference in eigenvalues of the evolving Hamiltonian H_{eff}. For example, ρ_{14} and ρ_{41} acquire the largest phase accumulation, while the phases of ρ_{23} and ρ_{32} do not change due to the fact $\varepsilon_{+\frac{1}{2},-\frac{1}{2}} = \varepsilon_{-\frac{1}{2},+\frac{1}{2}}$.

Second, to evaluate the performance of the synchronized fast adiabatic pulse, we calculate the average fidelity of the gate operation. The gate fidelity is determined by the overlap between the target ideal case and the noisy case:[31,32]

$$F_g(\varepsilon, U) = \frac{1}{5} + \frac{1}{80}\sum_{a,b} \mathrm{Tr}[U\sigma_a^1\sigma_b^2 U^\dagger \varepsilon(\sigma_a^1\sigma_b^2)]. \quad (11)$$

Here ε is the quantum evolution process, U is the target entangling gate, and σ_a^1 and σ_b^2 with $a, b = I, x, y, z$ are the Pauli operators for two spin qubits. In the first step, we start from a product state of the two-ion spins. Then we select the corresponding control pulse parameters and noise parameters and use the master equation numerically to obtain the density matrix of the two-spin final state for the noisy case. In the second step, we start from the same initial state as the first step. Then we select the same control pulse parameters while setting the noise parameters to zero. Thus we use the master equation to obtain the density matrix of the two-spin final state for the noiseless case, i.e., the ideal case. In the third step, by using the definition of the gate operation fidelity in Eq. (11), the overlap of the density matrices of the noisy case and the ideal case can be calculated, and the value of the corresponding gate fidelity can be obtained.

Figure 3(b) shows the dynamics of the controlled-phase gate operation fidelity with pulse duration. At the end of the pulse, the fidelity reaches a high value of 99.6%. We would like to emphasize that this value meets the fault-tolerance threshold condition for distributed quantum computing.[4]

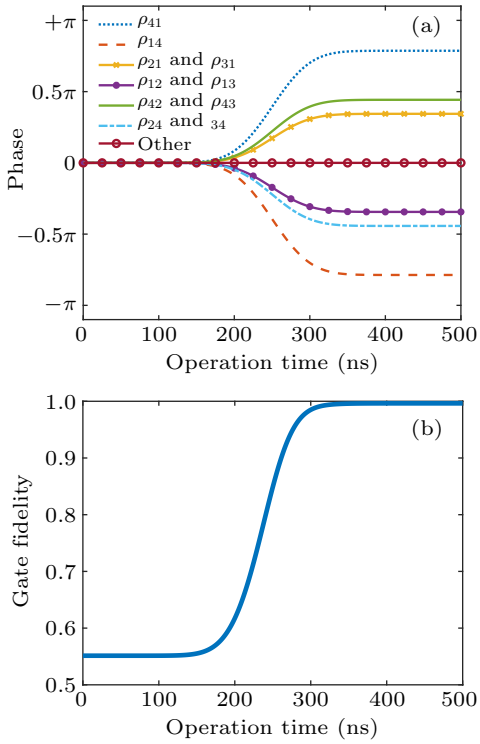


Fig. 3. (a) The evolution of phase during a controlled-phase gate operation. (b) The gate fidelity F_g as a function of time. Here we use the parameters as given in the text.

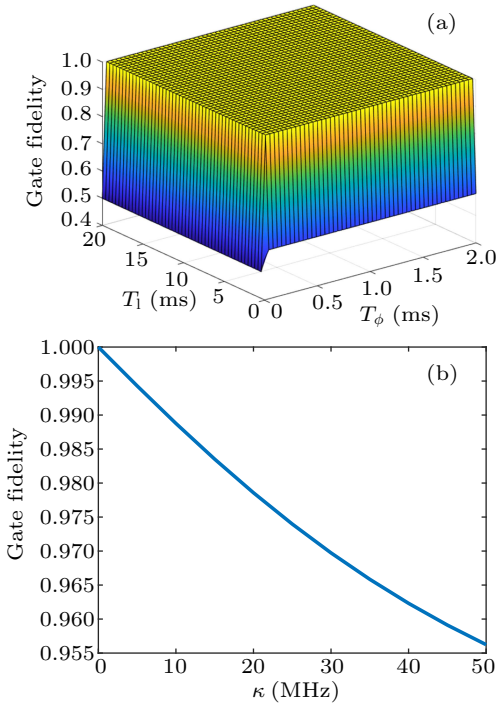


Fig. 4. (a) Gate fidelity F_g as a function of relaxation time T_1 and dephasing time T_ϕ at $\kappa = 20$ MHz. (b) Gate fidelity F_g as a function of κ at $T_1 = 10$ ms and $T_\phi = 0.8$ ms.

Further, we investigate the robustness of synchronized fast adiabatic operation to various noises and fluctuations. In

the hybrid system, the fidelity of gate operations is affected by the decoherence process of ions and cavities. In Fig. 4(a), we show the gate operation fidelity versus spin relaxation time and dephasing time. We find that the fidelity is more sensitive to the spin relaxation time than to the spin dephasing time. This result implies that, since our scheme is based on adiabatic operation, the system is always in the instantaneous eigenstate of the time-dependent Hamiltonian, and thus is somewhat immune to phase fluctuations. On the other hand, since this pulse operation is still somewhat non-adiabatic, it can lead to unwanted transitions, and this error is sensitive to the relaxation process between energy eigenstates.

Figure 4(b) shows the relationship between the gate operation fidelity and the cavity leakage rate. For example, when κ is increased by a factor of 5, the fidelity decreases only slightly from 99.6% to 97.5%. Because this scheme is based on a virtual photon process and does not actually excite the cavity field, it is robust to leakage due to cavity imperfections.

6. Entanglement preparation on a quantum repeater node

As a full-functional quantum repeater node, it is not only capable of storing quantum information, but also capable of processing entangled state information. Here we demonstrate the use of synchronized fast adiabatic operation to achieve arbitrary entangled state preparation on the repeater node. As shown in Fig. 5(a), the scheme in generating entangled states on the node consists of three steps. In the first step, we prepare the initial state of two ion qubits with four product states. In the second step, we apply a controlled-phase gate and three single-qubit rotation gates. In the third step, we perform quantum state tomography on the generated states. In this way, we can generate the following entangled states:

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}[|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle], \tag{12}$$

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle]. \tag{13}$$

To characterize entanglement at the repeater node, we model the dynamics of entanglement generation using the master equation and reconstruct the density matrix of two-spin qubits. Figure 5(b) shows the real part of the density matrix for the final state. We calculate the fidelity of the entangled state $F_e = \mathrm{Tr}(\sqrt{\sqrt{\rho_i}\rho_r\sqrt{\rho_i}})$ with the realistic density matrix ρ_r compared to the target ideal density matrix ρ_i.[33,34] The fidelity of these four entangled states is about 95%, which has reached the fault-tolerance threshold condition for a variety of quantum network architectures.[35,36] Thus our entangling gate scheme provides a simple but powerful elementary component for realizing a fully-functional quantum repeater node. In realizing the controlled-phase (CP) gate, we need to apply only one operation pulse sequence. While in preparing the Bell states, we need to apply three single-qubit gates and one two-qubit CP gate, i.e., we need more operation sequences and longer operation time. In this process, the external noise accumulates more errors, so it results in the fidelity of the prepared Bell state being lower than that of the gate operation.

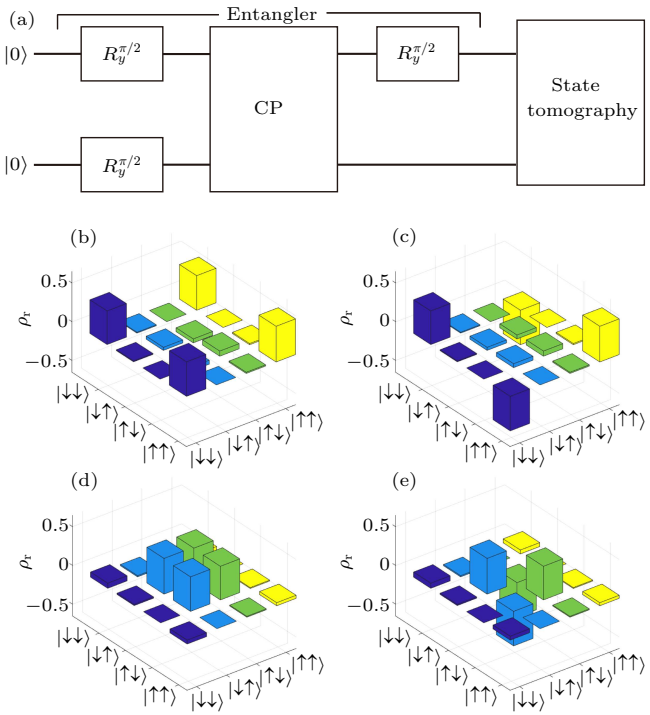


Fig. 5. (a) Gate sequences for generating intra-node entanglement between two ion spin qubits. (b)–(e) Quantum state tomography of the final density matrix ρ_r for different entangled states.

7. Conclusion

To summarize, we propose a scheme to manipulate a quantum repeater node composed of two ion qubits and a cavity. By applying a synchronized fast adiabatic pulse sequence, this hybrid system can operate in the dispersive regime and use the exchange of virtual photon process to realize the interaction between two ion qubits. We find that this scheme can realize control-phase gates between two qubits with 99.6% fidelity under current experimental parameter conditions. Further, this scheme can also generate local entangled states at quantum repeater nodes. These on-node entangled states are an important quantum resource that can improve the flexibility of quantum network architectures and reduce the demanding requirements of quantum error-correction codes and fault-tolerant quantum computation. We also investigate the robustness of this operation to various noises and fluctuations in the hybrid system. In this paper, we use a simple pulse waveform design. By designing more complex pulse waveforms,[19,25] the fidelity of the gate operation and entangled states can be further improved. This scheme provides a promising approach for distributed quantum information processing in network architec-

tures and can be generalized to various other solid-state spin systems.[37–42]

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant Nos. 12304401 and 12350006), the Innovation Program for Quantum Science and Technology (Grant No. 2021ZD0301200), and USTC Research Funds of the Double First-Class Initiative (Grant No. YD2030002026).

References

[1] Kimble H J 2008 *Nature* **453** 1023
[2] Sangouard N, Simon C, de Riedmatten H and Gisin N 2011 *Rev. Mod. Phys.* **83** 33
[3] Northup T E and Blatt R 2014 *Nat. Photon.* **8** 356
[4] Wehner S, Elkouss D and Hanson R 2018 *Science* **362** eaam9288
[5] Lauritzen B, Minar, J, de Riedmatten H, Afzelius M, Sangouard N, Simon C and Gisin N 2010 *Phys. Rev. Lett.* **104** 080502
[6] Kolesov R, Xia K, Reuter R, *et al*. 2012 *Nat. Commun.* **3** 1029
[7] Utikal T, Eichhammer E, Petersen L, *et al*. 2014 *Nat. Commun.* **5** 3627
[8] Pierre Jobez1, Cyril Laplane1, Nuala Timoney, *et al*. 2015 *Phys. Rev. Lett.* **114** 230502
[9] Zhong M, Hedges M P, Ahlefeldt R L, Bartholomew J G, Beavan S E, Wittig S M, Longdell J J and Sellars M J 2015 *Nature* **517** 177
[10] Gundogan M, Ledingham P M, Kutluer K, Mazzera M and de Riedmatten H 2015 *Phys. Rev. Lett.* **114** 230501
[11] Zhou Z Q, Hua Y L, Liu X, Chen G, Xu J S, Han Y J, Li C F and Guo G C 2015 *Phys. Rev. Lett.* **115** 070502
[12] Zhong T, Kindem J M, Bartholomew J G, *et al*. 2018 *Phys. Rev. Lett.* **121** 183603
[13] Dibos A M, Raha M, Phenicie C M and Thompson J D 2018 *Phys. Rev. Lett.* **120** 243601
[14] Kindem J M, Ruskuc A, Bartholomew J G, *et al*. 2020 *Nature* **580** 201
[15] Lago-Rivera D, Grandi S, Rakonjac J V, Seri A and de Riedmatten H 2021 *Nature* **594** 37
[16] Liu X, Hu J, Li Z F, Li X, Li P Y, Liang P J, Zhou Z Q, Li C F and Guo G C 2021 *Nature* **594** 41
[17] Ruskuc A, Wu C J, Green E, *et al* 2025 *Nature* **639** 54
[18] Guery-Odelin D, Ruschhaupt A, Kiely A, *et al*. 2019 *Rev. Mod. Phys.* **91** 045001
[19] Chen X, Lizuain1 I, Ruschhaupt A, *et al*. 2010 *Phys. Rev. Lett.* **105** 123003
[20] Bason M G, Viteau M, Malossi N, *et al*. 2012 *Nat. Phys.* **8** 147
[21] del Campo A 2013 *Phys. Rev. Lett.* **111** 100502
[22] An S, Lv D, Campo A, *et al*. 2016 *Nat. Commun.* **7** 12999
[23] Wang T, Zhang Z X, Xiang L, *et al*. 2018 *New J. Phys.* **20** 065003
[24] Li Y C, Chen Xi, Muga J G, *et al*. 2018 *New J. Phys.* **20** 113029
[25] Zhou B B, Baksic A, Ribeiro H, *et al*. 2017 *Nat. Phys.* **13** 330
[26] Winkler R, Papadakis S, De Poortere E and Shayegan M 2003 *Spin-orbit coupling in two-dimensional electron and hole systems* Vol. 41 (Springer)
[27] Bravyi S, DiVincenzo D P and Loss D 2011 *Ann. Phys.* **326** 2793
[28] Bottger T, Sun Y, Thiel C and Cone R 2006 *Phys. Rev. B* **74** 075107
[29] Bottger T, Thiel C, Cone R and Sun Y 2009 *Phys. Rev. B* **79** 115104
[30] Ulanowski A, Merkel B and Reiserer A 2022 *Sci. Adv.* **8** eabo4538
[31] Nielsen M A 2002 *Phys. Lett. A* **303** 249
[32] Gilchrist A, Langford N K and Nielsen M A 2005 *Phys. Rev. A* **71** 062310
[33] Simmons S, Brown R M, Riemann H, *et al*. 2011 *Nature* **470** 69
[34] Steffen M, Ansmann M, Bialczak R C, *et al*. 2006 *Science* **313** 1423
[35] Nickerson N H, Li Y and Benjamin S C 2013 *Nat. Commun.* **4** 1756
[36] Nickerson N H, Fitzsimons J F and Benjamin S C 2014 *Phys. Rev. X* **4** 041041
[37] Liu C, Zhu T X, Su M X, Ma Y Z, Zhou Z Q, Li C F and Guo G C 2020 *Phys. Rev. Lett.* **125** 260504
[38] Zhu T X, Liu C, Jin M, Su M X, Liu Y P, Li W J, Ye Y, Zhou Z C, Li C F and Guo G C 2022 *Phys. Rev. Lett.* **128** 180501
[39] Liu D C. Li P Y, Zhu T X, Zheng L, Huang J Y, Zhou Z Q, Li C F and Guo G C 2023 *Phys. Rev. Lett.* **129** 120501
[40] Zhu X Y, Zhu L T, Guo G C, Tu T and Li C F 2025 *Phys. Rev. A* **111** 012621
[41] Guo A L, Tu T, Zhu L T and Li C F 2021 *Chin. Phys. Lett.* **38** 094203
[42] Zhu X Y, Zhu L T, Tu T and Li C F 2024 *Chin. Phys. B* **33** 020315