

Electron-acoustic solitons in multi-species space plasmas: Supersoliton perspectives

Ln Mbuli and Z Mtumela

Citation: Chin. Phys. B, 2025, 34 (**10**): 105204. DOI: 10.1088/1674-1056/add4f4
http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Post-solitons and electron vortices generated by femtosecond intense laser interacting with uniform near-critical-density plasmas

Dong-Ning Yue(岳东宁), Min Chen(陈民), Yao Zhao(赵耀), Pan-Fei Geng(耿盼飞), Xiao-Hui Yuan(远晓辉), Quan-Li Dong(董全力), Zheng-Ming Sheng(盛政明), and Jie Zhang(张杰)
Chin. Phys. B, 2022, 31 (**4**): 045205. DOI: 10.1088/1674-1056/ac46c5

Oblique collisional effects of dust acoustic waves in unmagnetized dusty plasma

M S Alam, M R Talukder
Chin. Phys. B, 2020, 29 (**6**): 065202. DOI: 10.1088/1674-1056/ab829b

Damped electrostatic ion acoustic solitary wave structures in quantum plasmas with Bohm potential and spin effects

S Hussain, H Hasnain, Mahnaz Q. Haseeb
Chin. Phys. B, 2019, 28 (**1**): 015202. DOI: 10.1088/1674-1056/28/1/015202

Nonlinear ion-acoustic solitary waves in an electron-positron-ion plasma with relativistic positron beam

Ridip Sarma, Amar P Misra, Nirab C Adhikary
Chin. Phys. B, 2018, 27 (**10**): 105207. DOI: 10.1088/1674-1056/27/10/105207

Interactions of ion acoustic multi-soliton and rogue wave with Bohm quantum potential in degenerate plasma

M S Alam, M G Hafez, M R Talukder, M Hossain Ali
Chin. Phys. B, 2017, 26 (**9**): 095203. DOI: 10.1088/1674-1056/26/9/095203

Electron–acoustic solitons in multi-species space plasmas: Supersoliton perspectives

Ln Mbuli[1,†] and Z Mtumela[2]

[1] *School of Chemistry and Physics, University of KwaZulu Natal, Durban, South Africa*
[2] *Walter Sisulu University, Department of Chemical and Physical Sciences, Mthatha, South Africa*

(Received 30 January 2025; revised manuscript received 4 April 2025; accepted manuscript online 7 May 2025)

Since the discovery of the electrostatic wave emissions such as broadband electrostatic noise (BEN) and electrostatic hiss in space plasmas, both kinetic and nonlinear fluid studies have been employed to study the properties and characteristics of the solitons. Here, we use the Sagdeev pseudo-potential method to investigate the existence of the high-frequency supersolitons in a four-component unmagnetised plasma model composed of hot, warm, and cool electrons and cool ions species. All species are treated as adiabatic and are considered as stationary in our soliton analysis. Although the model supports both slow and fast electron–acoustic soliton, only the solutions of a negative-polarity supersoliton solution of the fast electron–acoustic type are discussed in this research study. It is shown that high-frequency supersoliton exists in a very narrower region of parameter space. Furthermore, the lower and upper Mach numbers for the supersolitons are computed and discussed. We have constructed the existence domains of the supersolitons, and the maximum potential amplitudes are computed. Positive potential supersolitons are not found in our numerical analysis. The importance and applications of our numerical findings in space-plasma environments are also discussed.

Keywords: solitons, double layers, supersolitons, polarity

PACS: 52.35.Sb, 02.60.–x, 03.50.De, 04.30.Db

DOI: 10.1088/1674-1056/add4f4 **CSTR:** 32038.14.CPB.add4f4

1. Introduction

Electrostatic solitons[1,2] and double layers (DLs),[3–5] have been observed by various space satellites such as fast auroral snapshot (FAST),[4] Vikings,[1,6] S3-3,[7] Freja,[8] and cluster[9] proagating at various earth's magnetosphere regions. These plasma nonlinear waves (solitons and double layers) are widely known as solitary waves.

A number of laboratory experimental[10,11] and theoretical studies have been conducted to understand the physical properties and characteristics of the nonlinear wave fluctuation phenomena[12] associated with the propagation of the solitons and double layers in space plasma environments.[13–21] Multi-species plasma models applicable to space plasmas have been developed to study small and large amplitudes electrostatic solitons.[17]

Recent studies of electrostatic soliton by Dubinov and Kolotkov[22] and Dubinov and Sazonkin[23] in multi-species plasmas have resulted in the discovery of a new kind of nonlinear wave called a supersoliton (low frequency ion-acoustic type). Unlike the traditional (normal) solitons which have a single[15] local minimum in the Sagdeev pseudo-potential[24] profiles (and usually their upper Mach numbers (M-values), are limited by charged species number density becoming complex valued or the formation of a double layer the supersolitons have double (two) local minima on their Sagdeev pseudo-potential profiles, and its normal solution exist beyond the double layers.[22]

For the supersolitons which seem to only arise and propagate in a strongly nonlinear medium (when the dispersion balances the nonlinearity) their electric field profiles have wiggles.[25] Solitons (observed by cluster satellites in space plasmas) exhibiting the same characters and properties of supersolitons have been reported in the literature by Pickett *et al.*[26] and Dubinov and Koltkov.[27] Recently, collisional effect of the supersolitons and normal solitons have been studied by Olivier *et al.*[28] in multi-species plasma model. It was shown in their simulation study that in longer period supersolitons become regular solitons.

Negative potential supersolitons (low frequency) associated with the ion-acoustic solitons were first studied by Dubinov and Kolotkov[22] in a five charged-species plasma model comprising of electrons, positrons as well as positive and negative ion components and negatively charged dust particles.

In the theoretical analysis conducted by Dubinov and Kolotkov,[29] it was shown that ion-acoustic supersolitons could also be found in a four-species plasma model with hot electrons and positrons as well as cold negative and positive ion species. From their numerical analysis of Dubinov and Kolotkov[29] it was concluded that the electrostatic supersolitons could only exist in a plasma model with atleast four-charged species. However, Baluku *et al.*[30] found that the positive potential supersolitons could also be found in a three

†Corresponding author. E-mail: lifa.mbuli@gmail.com

© 2025 Chinese Physical Society and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

charged species plasma models where there are Boltzmann cool electrons and Boltzmann hot electron components as well as cold ions. Verheest *et al.*[31] also studied the negative potential ion-acoustic supersolitons in a three species plasma model composed of cold positive and negative ion components as well as non-thermal electrons. Furthermore, Veerhest *et al.*[32] also studied positive potential ion-acoustic supersolitons in a plasma model with hot and cool electrons (both kappa distributed) and cold positive ion species.

Existence of the ion-acoustic supersolitons was recently reported by Singh and Lakhina[33] in a three-species plasma with Cairns distributed electrons, Boltzmann electrons and cold ion (fluid) species. Recently, Kakad *et al.*[34] reported for the first time on a fluid simulation study of low frequency ion-acoustic supersolitons. The plasma model was composed of cold ions and two kappa-distributed electron species. On the other hand, it has been also found by Varghese and Ghosh[35] in an unmagnetised plasma with hot and cool electrons as well as hot and cool ions that negative polarity supersolitons could also be supported. In addition, based on the concept of low frequency supersoliton waves, Dutta and Sahu[36] found that a three-component unmagnetised plasma with hot and cool electrons (obeying non-thermal Tsallis distribution) and warm ions in a model are found to support negative potential ion-acoustic supersolitons. In their study coexistence of two super-acoustic speed ion-acoustic solitons was also reported.

Verheest and Hellberg[37] in a three component plasma model with negative and positive cold ions and Cairns distributed electrons have recently reported on the existence of ion-acoustic supersolitons and double layers occurring at the acoustic speed. It was reported by Lakhina *et al.*[38] that supersolitons cannot be found in a three species plasma with cool (heavy) and warm (light) ion components and hot (Boltzmann distributed) electron species. In a theoretical investigation conducted by Verheest *et al.*,[39] it was reported recently that the ion-acoustic supersolitons cannot be found in an unmagnetised two component plasma.

Positive polarity dust ion-acoustic supersolitons in a dusty plasma with cold negative dust and positive (adiabatic) dust particles, hot electrons (Boltzmann distributed) and non-thermal ions was first reported by Maharaj *et al.*[15] Hellberg *et al.*[40] studied negative potential dust-acoustic supersolitons in a three-species plasma model with cold negative dust grains, and kappa distributed hot and cool ion components. Negative potential dust-acoustic supersolitons have been studied by Verheest[25] in a multi-species model consisting of a negative dust fluid, and Boltzmann as well as non-thermal Cairns distributed ion components. Since the investigation of supersolitons (no fast electron-acoustic supersolitons) has reported (found) in adiabatic unmagnetised plasma models (with inertial hot, warm, and cool electrons, and background ions). In a magnetised plasmas model such as that considered by Kamalam *et al.*[41] (with adiabatic electron beam and adiabatic electrons, and two hot Boltzmann ions) electron-acoustic supersolitons have been reported. However, Verheest and Hellberg[42] suggested that their conclusion is invalid because in their models their assumption of treating the ions Boltzmann is wrong.

In this paper, we report for the first time nonlinear electron-acoustic wave the existence of high-frequency supersolitons associated with the fast electron-acoustic mode for the multi-electron temperature species plasma model of Mbuli *et al.*[43] As a result in this paper we extend the work of Mbuli *et al.*[43] motivated by the study of Dubinov and kolotkov.[22] We use the Sagdeev pseudopotential method in a parameter space applicable in the plasma sheet boundary layer (PSBL) and search for the solution of the high-frequency fast electron-acoustic supersolitons found in a multi-electron temperature space plasma. Although we found the solution for the supersolitons by varying one plasma parameter at a time and looking at the behavior of the Sagdeev pseudopotential, we do not provide a procedure to find such a solution of the high-frequency supersolitons in multi-species plasma model. However, in this research study we show that high frequency supersolitons (of fast electron-acoustic type) do exist in input parameter regions similar to those of Mbuli *et al.*[44] applicable to the plasma sheet boundary layers (PSBL).[9] Our models as reported in the research work of Olivier, Verheest, and Hereman,[28] the limitation of the Sagdeev pseudo-potential method is that it cannot be used to study the collisional properties of the supersolitons but can be used to study the parameter space regions where they are found. Effects of the plasma parameters on the supersoliton's amplitudes and Mach numbers can also be studied using this approach.

This paper is organized as follows: in Section 2 we present a theoretical unmagnetised plasma model. In Section 3 we discuss numerical results of the fast electron-acoustic supersolitons. Finally, in Section 4 we provide a brief summary of our findings.

2. Theoretical model

In this research study we consider homogeneous and collisionless unmagnetised plasma model containing cool, warm and hot electron species, as well as cool ions where all species are treated as adiabatic. Multi-fluid equations[43,45] are used, namely, the continuity, momentum and pressure equations as follows:

$$\frac{\partial n_j}{\partial t}+\frac{\partial (v_j n_j)}{\partial x}=0,$$
$$\frac{\partial v_j}{\partial t}+v_j\frac{\partial v_j}{\partial x}=-\frac{Z_j}{\mu_i}\frac{\partial \Phi}{\partial x}-\frac{1}{\mu_i n_j}\frac{\partial P_j}{\partial x},$$

$$\frac{\partial P_j}{\partial t}+v_j\frac{\partial P_j}{\partial x}+3P_j\frac{\partial v_j}{\partial x}=0. \tag{1}$$

The above fluid equations are enclosed by the Poisson's equation given by

$$\frac{\partial^2\Phi}{\partial x^2}=n_{\rm we}+n_{\rm he}+n_{\rm ce}-n_{\rm i}, \tag{2}$$

where the plasma parameters Z_j, n_j, v_j, P_j, $\mu_i=m_j/m_{\rm e}$ represent the charge, number density, speed, pressure, and mass ratio of species j ($j=$ ce, we, he, i, representing the cool, warm, and hot electrons, and the cool ions) with $Z_j=-1$ $(+1)$ for electrons (ions). Electrostatic soliton theory for an adiabatic three-electron species model is discussed in Refs. [43,45] where the slow and fast electron-acoustic solitons were analysed. For the purposes of completeness, we briefly repeat the derivation. In general in our mathematical analysis we treat all species as having field-aligned equilibrium drift speed, $v_{{\rm db}jo}$, but we present the numerical results for the high frequency supersolitons in a non-drifting scenario, namely, the time where the nonlinear waves are observed propagating in space plasmas in the absence of the drifting species.

The numerical results in this paper are presented in normalised form, with the proton–electron mass ratio taken as $\mu_{\rm i}=m_{\rm i}/m_{\rm e}=1836$, i.e., for a hydrogen plasma. The number densities are normalised by the total equilibrium plasma density $n_{\rm o}=n_{\rm ce}+n_{\rm he}+n_{\rm we}$ (in a normalized form $n_{\rm co}+n_{\rm ho}+n_{\rm wo}=1$), and thermal temperatures by the hot electron temperature, $T_{\rm he}$. Time is normalised by the inverse of the total electron plasma frequency $\omega_{\rm pe}^{-1}=(4\pi n_{\rm o}e^2/m_{\rm e})^{-1/2}$, speeds by the hot electron thermal speed, $C_{\rm he}=(T_{\rm he}/m_{\rm e})^{1/2}$, and spatial lengths by the electron Debye length $\lambda_{\rm d}=(T_{\rm he}/4\pi n_{\rm o}e^2)^{1/2}$.

The dynamic fluid equations (1) and (2) are transformed to a frame moving with the non-linear wave by considering $\zeta=x-Mt$ where M $(=V/C_{\rm he})$ is the Mach number (normalized) and V is the true speed of the soliton wave. For our analysis we consider the following boundary conditions: $n_j\to n_{jo}$, $v_j\to v_{{\rm db}jo}$, $n_{\rm i}\to 1$, $\Phi\to 0$, and $P_j\to P_{jo}\sim n_{jo}T_j$ as $\zeta\to\pm\infty$ to study the supersoliton solution in the plasma.

The general expression for the perturbed number density describing the physical behavior of the species in the homogeneous or uniform plasma unmagnetized model is given by

$$n_j(\Phi,M)=\frac{n_{jo}}{\sqrt{6\delta_j}}\left(\left((M-v_{{\rm db}jo})^2+3\delta_j-\frac{2Z_j\Phi}{\mu_{\rm i}}\right)\pm\sqrt{\left((M-v_{{\rm db}jo})^2+3\delta_j-\frac{2Z_j\Phi}{\mu_{\rm i}}\right)^2-12(M-v_{{\rm db}jo})^2}\right)^{1/2}, \tag{3}$$

where $\delta_j=P_{jo}/n_{jo}\mu_{\rm i}$. After doing analytical computation and integrating Poisson's equation with respect to Φ, we then obtain the energy integral-like equation given by

$$\frac{1}{2}\left(\frac{{\rm d}\Phi}{{\rm d}\zeta}\right)^2+V(\Phi,M)=0, \tag{4}$$

where the expression for the electrostatic Sagdeev pseudo-potential[24] expression, $V(\Phi,M)$, is given by Ref. [45]

$$\begin{aligned}V(\Phi,M)=&\frac{n_{\rm wo}\left((M-v_{\rm dbwo})+\sqrt{3T_{\rm we}}\right)^3}{6\sqrt{3T_{\rm we}}}\left[1-\left(\sqrt{1-\frac{2\Phi}{\left((M-v_{\rm dbwo})+\sqrt{3T_{\rm we}}\right)^2}}\right)^3\right]\\
&-\frac{n_{\rm wo}\left((M-v_{\rm dbwo})-\sqrt{3T_{\rm we}}\right)^3}{6\sqrt{3T_{\rm we}}}\left[1-\left(\sqrt{1-\frac{2\Phi}{\left((M-v_{\rm dbwo})-\sqrt{3T_{\rm we}}\right)^2}}\right)^3\right]\\
&+\frac{n_{\rm co}\left(M+\sqrt{3T_{\rm ce}}\right)^3}{6\sqrt{3T_{\rm ce}}}\left[1-\left(\sqrt{1-\frac{2\Phi}{\left(M+\sqrt{3T_{\rm ce}}\right)^2}}\right)^3\right]-\frac{n_{\rm co}\left(M-\sqrt{3T_{\rm ce}}\right)^3}{6\sqrt{3T_{\rm ce}}}\left[1-\left(\sqrt{1-\frac{2\Phi}{\left(M-\sqrt{3T_{\rm ce}}\right)^2}}\right)^3\right]\\
&+\frac{n_{\rm ho}\left(M+\sqrt{3}\right)^3}{6\sqrt{3}}\left[1-\left(\sqrt{1-\frac{2\Phi}{\left(M+\sqrt{3}\right)^2}}\right)^3\right]-\frac{n_{\rm ho}\left(M-\sqrt{3}\right)^3}{6\sqrt{3}}\left[1-\left(\sqrt{1-\frac{2\Phi}{\left(M-\sqrt{3}\right)^2}}\right)^3\right]\\
&+\frac{\left(M+\sqrt{3T_{\rm i}/\mu_{\rm i}}\right)^3}{6\sqrt{3}}\left[1-\left(\sqrt{1-\frac{2\Phi}{\left(M+\sqrt{3T_{\rm i}/\mu_{\rm i}}\right)^2}}\right)^3\right]\\
&-\frac{\left(M-\sqrt{3T_{\rm i}/\mu_{\rm i}}\right)^3}{6\sqrt{3}}\left[1-\left(\sqrt{1-\frac{2\Phi}{\left(M-\sqrt{3T_{\rm i}/\mu_{\rm i}}\right)^2}}\right)^3\right].\end{aligned} \tag{5}$$

The second derivative of Sagdeev pseudo-potential expression, $V(\Phi,M)$, i.e., Eq. (5) computed at $\Phi=0$ is given by

$$\left(\frac{{\rm d}^2V(\Phi,M)}{{\rm d}\Phi^2}\right)_{\Phi=0}=\frac{n_{\rm co}}{(M^2-3T_{\rm ce})}+\frac{n_{\rm wo}}{((M-v_{\rm dbwo})^2-3T_{\rm we})}+\frac{n_{\rm ho}}{(M^2-3T_{\rm he})}+\frac{1}{\mu_{\rm i}(M^2-3T_{\rm i}/\mu_{\rm i})}, \tag{6}$$

and the third derivative of $V(\Phi,M)$ at $\Phi=0$ is written as

$$\left(\frac{\mathrm{d}^3V(\Phi,M)}{\mathrm{d}\Phi^3}\right)_{\Phi=0} = -\frac{3n_{\mathrm{co}}(M^2+T_{\mathrm{ce}})}{(M^2-3T_{\mathrm{ce}})^3} - \frac{3n_{\mathrm{wo}}((M-v_{\mathrm{dbwo}})^2+T_{\mathrm{we}})}{((M-v_{\mathrm{dbwo}})^2-3T_{\mathrm{we}})^3} - \frac{3n_{\mathrm{ho}}(M^2+1)}{(M^2-3)^3} + \frac{3(M^2+T_{\mathrm{i}}/\mu_{\mathrm{i}})}{\mu_{\mathrm{i}}^2(M^2-3T_{\mathrm{i}}/\mu_{\mathrm{i}})^3}. \tag{7}$$

In the limit of small potential amplitude soliton, with appropriate expansions, equation (5) reduces to

$$V(\Phi,M) \approx C_2\Phi^2 + C_3\Phi^3, \tag{8}$$

with the solution of Eq. (4) given by

$$\Phi = -\left(\frac{C_2}{C_3}\right)\mathrm{sech}^2\left(\sqrt{\left(-\frac{C_2}{4}\right)}\zeta^2\right), \tag{9}$$

where

$$C_2 = \frac{1}{2}\left(\frac{\mathrm{d}^2V(\Phi,M)}{\mathrm{d}\Phi^2}\right)\Big|_{\Phi=0},$$
$$C_3 = \frac{1}{6}\left(\frac{\mathrm{d}^3V(\Phi,M)}{\mathrm{d}\Phi^3}\right)\Big|_{\Phi=0}.$$

It should be noted that equation (6) has the numerical solution or roots of the phase speeds of the three different linear wave modes, *viz*, ion-acoustic, fast, and slow electron-acoustic waves.[43] From Eqs. (8) and (9), the polarity of the small amplitude super-acoustic solitons in general is determined by the sign of the third derivative, Eq. (7), of the Sagdeev pseudo-potential equation at $\Phi=0$.[46]

The lower limit or critical Mach number M_{crit} for a soliton to occur are evaluated numerically by setting $C_2=0$[46] and solving for various M-values.[15,45] In this paper existence domains[47] are defined as a regions in parameter space bounded by the lower and upper Mach number values for which fast electron-acoustic supersoliton and soliton solutions is valid.

Solitons exist in an unmagnetised plasma model when the following conditions are satisfied, *viz*, (I) $V(\Phi,M)=0$, $\mathrm{d}V(\Phi,M)/\mathrm{d}\Phi=0$, and $\mathrm{d}^2V(\Phi,M)/\mathrm{d}\Phi^2<0$ at $\Phi=0$; $V(\Phi,M)=0$, and $\mathrm{d}V(\Phi,M)/\mathrm{d}\Phi>$ $(<)$ 0 for $\Phi_{\mathrm{o}}>$ $(<)$ 0 at some $\Phi=\Phi_{\mathrm{o}}$; and $V(\Phi,M)<0$ for Φ ranging between 0 and Φ_{o}. We also note the condition that $0<\Phi<\Phi_{\mathrm{o}}$ $(\Phi_{\mathrm{o}}<\Phi<0)$, where Φ_{o} is the amplitude of the soliton potential must be satisfied for positive [compressive] (negative [rarefactive]) solitons to exist. For double layer solutions[48] to exist another condition must be satisfied, i.e., $\mathrm{d}V(\Phi,M)/\mathrm{d}\Phi|_{\Phi=\Phi_{\mathrm{o}}}=0$. In the next section we discuss the numerical result of supersolitons found in our model and point out the plasma regions in the earth's magnetosphere were the numerical or theoretical findings are applicable.

3. Numerical results and discussion

We search for the solution of arbitrary amplitude electron-acoustic supersolitons in multi-electron species unmagnetised plasma model comprising of the hot, warm and cool electrons and cool ions. All the species are treated as stationary and adiabatic. In our search for supersolitons (using arbitrary amplitude method) in a plasma parameter space we fix all other parameters and vary one parameter systematically until we obtain the numerical solution that exhibit the characteristics of the supersolitons discussed in different articles including the research study of Baluku *et al.*,[30] Dubinov and Kolotkov,[22] and Maharaj and Bharuthram.[15] This is similar to the parametric research method of searching for the coexistence of the electron-acoustic solitons. The difficult part of finding the high-frequency supersoliton and coexistence in adiabatic models is to find a set of plasma parameter on input parameter space applicable in space plasmas and laboratory plasmas that admit the occurrence of such solutions.

For the numerical results presented in this paper, we consider the following typical standard input plasma parameters[44] applicable in various terrestrial planetary magnetospheric regions such as the plasma sheet boundary layer (PSBL)[9] and dayside auroral zone,[49] *viz* $T_{\mathrm{we}}=0.14$, $T_{\mathrm{ce}}=0.04$, $T_{\mathrm{i}}=5.71$, $n_{\mathrm{wo}}=0.03$, and $n_{\mathrm{co}}=0.64$ supporting the solution of the fast electron-acoustic supersolitons.

In our numerical investigation we find that the numerical solution, namely, critical and upper Mach number for the high-frequency supersolitons exist between n_{co} (0.298–0.3012). The lower or critical Mach number limit (M_{crit}) for normal fast electron-acoustic solitons in subregion S of Fig. 1(a) is represented by the dotted line, while the upper Mach number limit (M_{max}) is represented by the dashed-dot line.

However, lower and upper values for the range of Mach numbers for the fast electron-acoustic supersolitons in both regions I and II of subregion SUP of Fig. 1(a) are represented by a dashed-dot curve and a solid curve, respectively.

The supersoliton existence domain is divided into regions I ($0.298 \lesssim n_{\mathrm{co}} < 0.2998$) and II ($0.2998 < n_{\mathrm{co}} \lesssim 0.3012$) of Figs. 1(a) and 1(b). In region I of the figure we find as the Mach number increases there is a direct transition from the solitons to supersolitons, whereas in region II (Figs. 1(a) and 1(b)) the solutions of the negative potential supersolitons (of fast electron-acoustic type) exist beyond the negative potential fast electron-acoustic double layer solutions.

In this study we strictly focus our attention on the properties and characteristics of the fast electron-acoustic supersolitons found in subregion SUP of Fig. 1(a). As a result we do not show any Sagdeev pseudo-potential profiles of the normal

fast electron-acoustic solitons found in region S of Fig. 1 since their profiles are shown and discussed in the numerical work of Ref. [43].

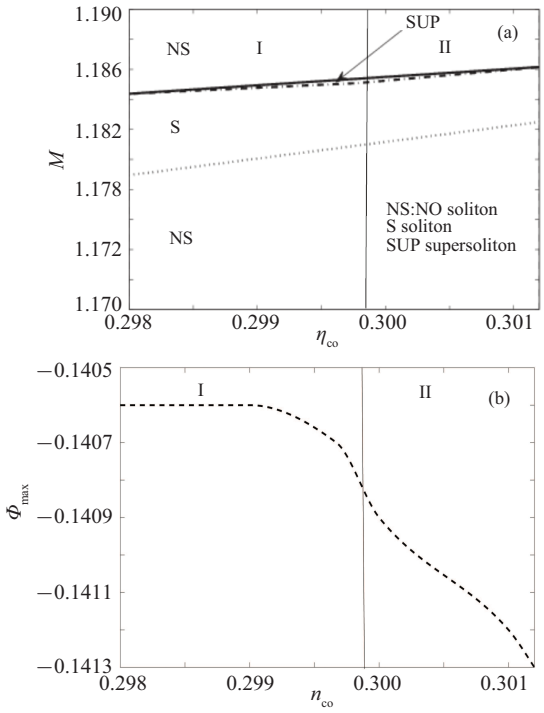


Fig. 1. Existence domains of the fast electron-acoustic solitons and supersolitons for varying cool electron number density, n_{co}. In the lower panel of the figure we show the behaviour of the maximum potential amplitude $|\Phi_o|$ of the fast electron-acoustic supersolitons found in subregion SUP for $M = M_{max}$. The fixed plasma parameters are: $\mu_i = 1836$, $T_{we} = 0.142857$, $T_{ce} = 0.0428571$, $T_i = 5.71429$, $n_{wo} = 0.12$ and $v_{dbjo} = 0$ for all species j.

Sagdeev pseudo-potential profiles of the fast electron-acoustic supersolitons exist in subregion SUP and for normal solitons exist in the existence region S of Fig. 1(a). From our numerical analysis of the result obtained, we note that the physical reason for the occurrence of the upper Mach number limitation in the existence domain of the electrostatic supersolitons in subregion SUP is determined by the warm electron density

$$n_{we}(\Phi, M) \sim \frac{n_{wo}}{2\sqrt{3T_{we}}} \times \left[\left(M + \sqrt{3T_{we}}\right) \sqrt{1 - \frac{2\Phi}{\left(M + \sqrt{3T_{we}}\right)^2}} - \left(M - \sqrt{3T_{we}}\right) \sqrt{1 - \frac{2\Phi}{\left(M - \sqrt{3T_{we}}\right)^2}} \right]. \quad (10)$$

As discussed by Mbuli *et al.*[43] again here we find in our numerical analysis that expression of n_{we} becomes unreal when $\Phi < \Phi_{min,warm} (= -(1/2)\left(M - \sqrt{3T_{we}}\right)^2)$ so that it defines the upper Mach number limit, M_{max} for the solitary waves, (i.e., supersolitons and solitons) studied in this paper. We find that outside of the regions I ($0.298 \lesssim n_{co} < 0.2998$) and II ($0.2998 < n_{co} \lesssim 0.3012$) of the existence subregion SUP of Fig. 1(a) bounded by $0.298 \lesssim n_{co} \lesssim 0.3012$ only regular fast electron-acoustic solitons solution whose characteristic were studied in Ref. [43] are supported. For a value of M fixed at M_{max}, the behavior of the potential amplitude $\Phi_{max}(\cdots)$ of the corresponding supersoliton is shown in Fig. 1(b). As illustrated in the Fig. 1(b), it is noted that the potential amplitude of the negative polarity solitons increases with cool electron density, n_{co}. The general well known procedure for computing the upper (M_{max}) and lower (M_{crit}) Mach numbers for existence domain is discussed by Maharaj and Bharuthram,[15] and Verheest *et al.*,[32] and in our previous numerical study of Mbuli *et al.*[45]

For a fixed value of cool electron number density, $n_{co} = 0.299$ (obtained from region I of Figs. 1(a) and 1(b)), in Fig. 2(a) we show a direct transition (occurring at $M = 1.184561$) from normal solitons ($M = 1.1839$) to supersolitons ($M = 1.1847291$) without the occurrence or formation of a fast electron-acoustic double layer (DL). This transitions from solitons to supersolitons (or supersolitary waves) could be the results of the overtaking (or collisional) properties of the supersolitons as mentioned by Olivier, Verheest, and Hereman.[28] These collisional properties of the supersolitons are beyond the scope of the paper as they can be studied using small smplitude theory.[28] In Fig. 2(b) we illustrate the behavior of the amplitude in region I (of Fig. 1) for normal solitons for M in the range $1.180056 < M < 1.184561$. The physical behavior of the potential amplitude region II (of Fig. 1) for supersolitons for M-values between $1.184561 < M < 1.18493$ is illustrated. It is seen that the (negative) amplitude increases with M.

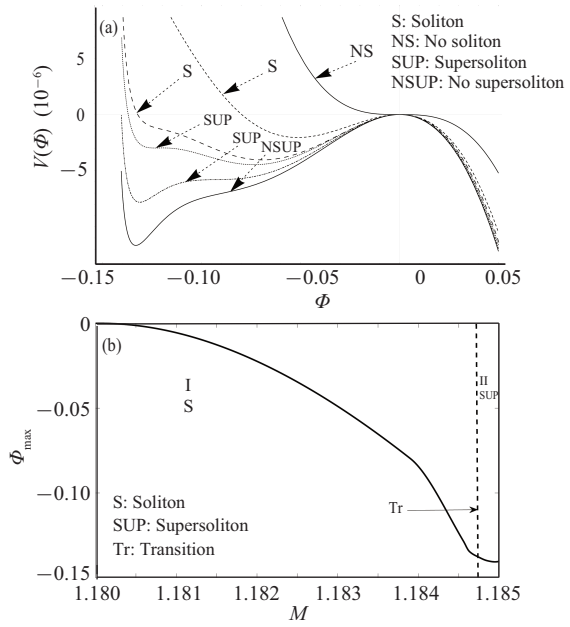


Fig. 2. The Sagdeev pseudo-potential[24] profile of the fast electron-acoustic soliton waves as function of the electrostatic potential (upper panel) for different values of Mach number, $M = 1.180056$, $M = 1.1839$, $M = 1.184561$, $M = 1.1847291$, $M = 1.18492999$, and $M = 1.18505$. The lower panel of this figure illustrate the behavior of the potential amplitude $|\Phi_o|$ *versus* the Mach number M. The input plasma parameters are: $\mu_i = 1836$, $T_{we} = 0.142857$, $T_{ce} = 0.0428571$, $T_i = 5.71429$, $n_{co} = 0.299$, $n_{wo} = 0.12$, and $v_{dbjo} = 0$ for all species j.

However, for a slightly large value of n_{co} ($= 0.30$) (in region II of Fig. 1) we show in Fig. 3 the transition from normal solitons (e.g., $M = 1.185281$) to supersolitons (e.g., $M = 1.185381$) is preceded by the occurrence of a double layer at $M = 1.185232$. The lower panel in Fig. 3 reveals $|\Phi_o|$ increasing with M while moving from normal soliton to a double layer to supersoliton.

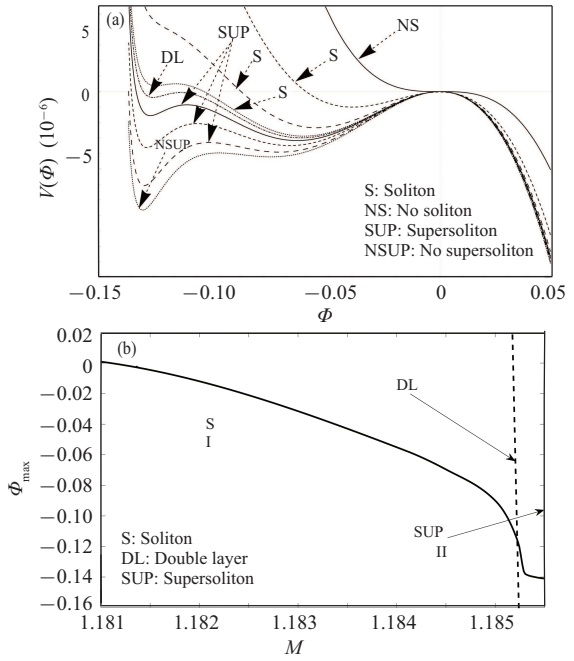


Fig. 3. The Sagdeev pseudo-potential profile of the fast electron-acoustic solitons as function of the electrostatic potential (upper panel) for the different values of the Mach number M, $M = 1.181161$, $M = 1.184181$, $M = 1.1850008$, $M = 1.185232$, $M = 1.185281$, $M = 1.185381$, $M = 1.185485$, and $M = 1.185581$, with the lower panel showing the variation of the potential amplitude $|\Phi_o|$ with M. The fixed plasma parameters are: $\mu_i = 1836$, $T_{we} = 0.142857$, $T_{ce} = 0.0428571$, $T_i = 5.71429$, $n_{co} = 0.3$, $n_{wo} = 0.12$, and $v_{dbjo} = 0$ for all species j.

The potential and electric field profiles (obtained numerically from the energy integral) of the soliton, double layer, and supersoliton potential corresponding to Fig. 3 are shown in Figs. 4(a) and 4(b).

The electrostatic potential, and electric field profiles (computed from our four-component unmagnetised plasma model) for the high frequency electron-acoustic supersolitons are consistent with those reported earlier in the literature by Doninov and Kolotkov,[50] Verheest *et al.*,[51] Lakhina *et al.*,[38] Verheest and Hellberg,[37] and Maharaj *et al.*[15] However, most of their study were focusing on the low frequency or speed modes. As pointed out by Maharaj and Bharuthram,[15] Verheest *et al.*,[32] and Kakad *et al.*,[34] the distinction between a supersoliton ($M \sim 1.185281$) and a regular soliton ($M = 1.184181$; $M = 1.1850008$) in Fig. 4(b) is more visible on the electric field profiles (lower panel) rather than the potential profiles. The wiggles on the electric field (E) profiles (Fig. 4(b)) of the supersolitons correspond to the two sub-wells shown in the corresponding electrostatic Sagdeev pseudo-potential (Fig. 3).

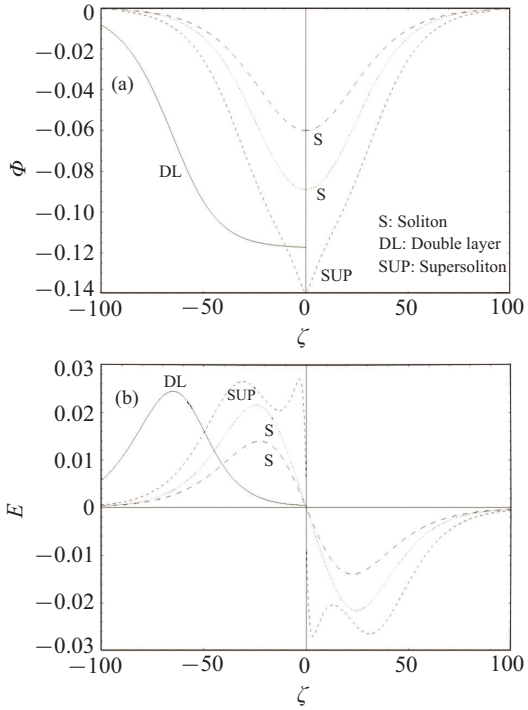


Fig. 4. The upper panel represents the potential, Φ, profile of the fast electron-acoustic soliton, whereas the lower panel represents the corresponding electric field E, for $M = 1.184181$ (soliton), $M = 1.1850008$ (soliton), $M = 1.185232$ (double layer), and $M = 1.185281$ (supersoliton). The input plasma parameters are $\mu_i = 1836$, $T_{we} = 0.142857$, $T_{ce} = 0.0428571$, $T_i = 5.71429$, $n_{co} = 0.3$, $n_{wo} = 0.12$, and $v_{dbjo} = 0$ for all species j.

The electric field profile presented in Fig. 5 was measured by the cluster setallites[26] on the right side of the graph, which have wiggles (or humps) that are almost similar to those presented in Fig. 4(b).

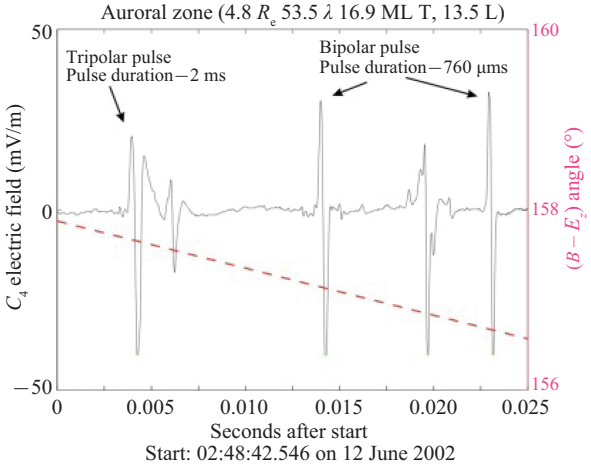


Fig. 5. The behavior of the electric field profile of the "supersolitons-like" on the right part of the graph taken from the research paper of Pickett *et al.*,[26] is similar to the electric profiles of the curve labelled SUP in Fig. 4(b).

For the Sagdeev pseudo-potential $V(\Phi, M)$, profiles shown in Fig. 3, following Refs. [15,22,52] in Fig. 6 we also illustrate the behavior of the solitons, double layers, and supersolitons using phase portrait.

The phase portrait plots obtained from our numerical analysis are presented in Fig. 6; a single loop (represented by

a solid line) represents existence of the soliton solution in our three electron plasma model. On the other hand, the loop represented by the dashed lines (enclosing small values of potential) that cuts a Φ axis and form another loop at larger values of Φ represents the double layer solution in a model. The outer double loops represent the solutions of the supersolitons in our unmagnetised plasma model.

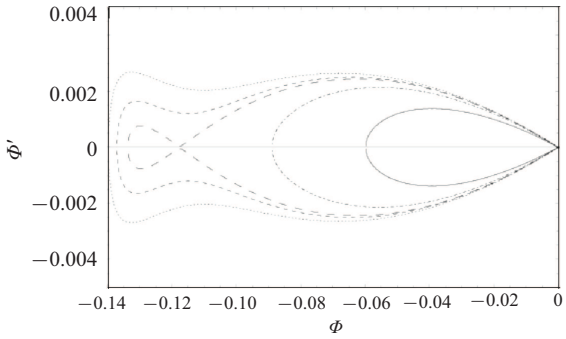


Fig. 6. The phase portraits plot of the fast electron-acoustic soliton, double layer and supersoliton, Φ' *versus* Φ, for $M = 1.184181$ (soliton), $M = 1.1850008$ (soliton), $M = 1.185232$ (double layer), $M = 1.185281$ (supersoliton), and $M = 1.185381$ (supersoliton) corresponding to Fig. 3. Other input plasma parameters are $\mu_i = 1836$, $T_{we} = 0.142857$, $T_{ce} = 0.0428571$, $T_i = 5.71429$, $n_{co} = 0.3$, $n_{wo} = 0.12$, and $v_{dbjo} = 0$ for all species j.

Our results for the electrostatic fast electron-acoustic supersolitons (displayed in regions I ($0.298 \lesssim n_{co} \lesssim 0.2998$) and II ($0.2998 \lesssim n_{co} \lesssim 0.3012$), subregion SUP of Figs. 1(a) and (b)) presented here suggest that the high frequency fast electron-acoustic supersolitons (best described by the wiggles on electric fields profiles) could be observed in the homogeneous space (where inhomogeneity scale length are negligible[53]) and laboratory plasma environments when the above plasma parameter conditions are satisfied.

Lastly, we would like to mention that our numerical analysis is applicable for one dimensional supersoliton propagations such that the solution for the unmagnetised plasma model with adiabatic ($\gamma = 3$) species be valid. The effect of the Boltzmann effect of hot electron as in the research work of Mace *et al.*[54] is neglected in this paper. In this paper we reported the existence of the solution of the negative polarity supersolitons in multi-electron species plasmas. As a reminder to a reader is that Mbuli *et al.*[43] reported polarity switches of the fast electron-acoustic soliton in the same adiabatic unmagnetised plasma model considered in this paper. In this paper we strictly focused on the parameter range where negative potential supersolitons exist (e.g., in Fig. 2). We do not rule-out the possibility that positive potential supersolitons can be found in this model. However, in the investigation conducted in this paper, positive potential double layers beyond the positive potential supersolitons were not found. Our arbitrary amplitudes model can only be used to study the arbitrary amplitude supersoliton, existence region in parameter space, the effect of plasma parameters on the potential amplitudes and Mach number. Thus, collisional properties of the supersolitons cannot be studied.

4. Conclusion

Using the Sagdeev pseudo-potential technique,[24] we have studied fast electron-acoustic solitons in a multi-electron species plasma model composed of adiabatic, cool, warm and hot electrons and cool ions. As such, we report for the first time the existence of numerical solution of the high frequency fast electron-acoustic supersolitons in adiabatic plasma model.

For increasing Mach numbers with other plasma parameters fixed, we find that the transition from regular solitons to a supersoliton is possible. However, at a different (slightly larger) values of cool electron density, $0.2998 \lesssim n_{co}$, the soliton and supersoliton are separated by the formation of a negative potential fast electron-acoustic double layer. The broadband electrostatic wave emissions with the electric field structures similar to that of the supersolitons have reported to have been observed by satellites in the research work of Lakhina *et al.*,[55] and solitons study of Pickett *et al.*[26] This research results found in this study can be applied to explain the occurrences of these nonlinear waves structures in various space plasma regions, such as plasma sheet boundary layers, magnetopause, and dayside auroral zone as reported in the study of Pickett *et al.*[26]

We did not find positive potential fast electron-acoustic supersolitons in our three-electron unmagnetised plasma model. The results presented here can be relevant to the understanding of nonlinear fluctuations observed by various satellites in different regions of the earth's magnetosphere such as PSBL regions. This numerical research study can be extended further to include other plasma effects such as the field-aligned warm electron beam and anti-field-aligned cool electron beam on the electron-acoustic supersolitons searching for the positive polarity. Another research approach that can be considered in the future would be to investigate the effects of warm and hot electron inertialess (Boltzmann) on the polarity switches of supersolitons[47,54,56] From the research studies of Mbuli, Bharuthram, and Maharaj,[57] and Verheest and Hellberg,[56] it is expected that warm electron beam and hot electron-inertialess might have effects on the polarity of the fast electron-acoustic supersolitary waves.

Acknowledgment

We would like to thank the support of the University of KwaZulu Natal and the Walter Sisulu University.

References

[1] Dubouloz N, Pottelette R, Malingre M, Holmgren G and Lindqvist P A 1991 *Journal of Geophysical Research: Space Physics* **96** 3565
[2] Bounds S R, Pfaff R F, Knowlton S F, Mozer F S, Temerin M A and Kletzing C A 1999 *Journal of Geophysical Research: Space Physics* **104** 28709
[3] Osmane A and Pulkkinen T I 2014 *Journal of Geophysical Research: Space Physics* **119** 8243
[4] Ergun R, Andersson L, Tao J, *et al.* 2009 *Phys. Rev. Lett.* **102** 155002
[5] Ergun R, Andersson L, Carlson C, Newman D and Goldman M 2003 *Nonlinear Processes in Geophysics* **10** 45
[6] Boström R, Gustafsson G, Holback B, Holmgren G, Koskinen H and Kintner P 1988 *Phys. Rev. Lett.* **61** 82
[7] Temerin M, Cerny K, Lotko W and Mozer F 1982 *Phys. Rev. Lett.* **48** 1175
[8] Dovner P, Eriksson A, Bostrm R and Holback B 1994 *Geophys. Res. Lett.* **21** 1827
[9] Teste A and Parks G K 2009 *Phys. Rev. Lett.* **102** 075003
[10] Ikezi H, Kiwamoto Y and Lonngren K 1972 *Phys. Lett. A* **42** 29
[11] Ikezi H 1973 *The Physics of Fluids* **16** 1668
[12] Gurnett D A and Frank L A 1977 *Journal of Geophys. Research* **82** 1031
[13] Asano N, Taniuti T and Yajima N 1969 *Journal of Mathematical Physics* **10** 2020
[14] Ghosh S, Ghosh K and Sekar Iyengar A 1996 *Phys. Plasmas* **3** 3939
[15] Maharaj S, Bharuthram R, Singh S and Lakhina G 2013 *Phys. Plasmas* **20** 8
[16] Maharaj S, Bharuthram R, Singh S V and Lakhina G 2012 *Phys. Plasmas* **19** 7
[17] Bharuthram R, Singh S, Maharaj S, Moolla S, Lazarus I, Reddy R and Lakhina G 2014 *Journal of Plasma Physics* **80** 825
[18] Verheest F, Hellberg M A and Kourakis I 2008 *Phys. Plasmas* **15** 11
[19] Lakhina G, Singh S, Kakad A, Goldstein M, Vinas A and Pickett J 2009 *Journal of GeoPhys. Research: Space Physics* **114** No. A9
[20] Lakhina G, Singh S and Kakad A 2014 *Phys. Plasmas* **21** 6
[21] Verheest F and Hellberg M A 2010 *Journal of Plasma physics* **76** 277
[22] Dubinov A E and Kolotkov D Y 2012 *IEEE Transactions on Plasma Science* **40** 1429
[23] Dubinov A, Kolotkov D Y and Sazonkin M 2012 *Plasma Physics Reports* **38** 833
[24] Sagdeev R 1966 *Rev. Plasma Physics* **4** 23
[25] Verheest F 2014 *Journal of Plasma Physics* **80** 787
[26] Pickett J, Chen L J, Kahler S, Santolik O, Gurnett D, Tsurutani B and Balogh A 2015 *Annales Geophysicae* **22** 2515
[27] Dubinov A E and Kolotkov D Y 2018 *Rev. Mod. Plasma Phys.* **2** 2
[28] Olivier C, Verheest F and Hereman W 2018 *Phys. Plasmas* **25** 3
[29] Dubinov A and Kolotkov D Y 2012 *Plasma Physics Reports* **38** 909
[30] Baluku T K, Hellberg M and Verheest F 2010 *Europhys. Lett.* **91** 15001
[31] Verheest F, Hellberg M A and Kourakis I 2013 *Phys. Plasmas* **20** 1
[32] Verheest F, Hellberg M and Kourakis I 2013 *Phys. Plasmas* **20** 8
[33] Singh S and Lakhina G 2015 *Communications in Nonlinear Science and Numerical Simulation* **23** 274
[34] Kakad A, Lotekar A and Kakad B 2016 *Phys. Plasmas* **23** 110702
[35] Varghese S S and Ghosh S 2016 *Phys. Plasmas* **23** 8
[36] Dutta D and Sahu B 2016 *Phys. Plasmas* **23** 062313
[37] Verheest F and Hellberg M A 2015 *Phys. Plasmas* **22** 1
[38] Lakhina G S, Singh S V and Kakad A P 2014 *Phys. Plasmas* **21** 062311
[39] Verheest F, Lakhina G S and Hellberg M A 2014 *Phys. Plasmas* **21** 062303
[40] Hellberg M A, Baluku T K, Verheest F and Kourakis I 2013 *Journal of Plasma Physics* **79** 1039
[41] Kamalam T, Steffy S and Ghosh S 2018 *Journal of Plasma Physics* **84** 905840406
[42] Verheest F and Hellberg M A 2019 *Journal of Plasma Physics* **85** 905850111
[43] Mbuli L, Maharaj S, Bharuthram R, Singh S and Lakhina G 2016 *Phys. Plasmas* **23** 6
[44] Mbuli L, Maharaj S and Bharuthram R 2014 *Phys. Plasmas* **21** 5
[45] Mbuli L, Maharaj S, Bharuthram R, Singh S and Lakhina G 2015 *Phys. Plasmas* **22** 6
[46] Maharaj S, Bharuthram R, Singh S and Lakhina G 2012 *Phys. Plasmas* **19** 122301
[47] Cattaert T, Verheest F and Hellberg M A 2005 *Phys. Plasmas* **12** No. 4
[48] Bharuthram R and Shukla P 1986 *The Physics of fluids* **29** 3214
[49] Mbuli L, Maharaj S and Bharuthram R 2013 *Phys. Plasmas* **20** 12
[50] Dubinov A and Kolotkov D Y 2012 *High Energy Chemistry* **46** 349
[51] Verheest F, Hellberg M A and Kourakis I 2013 *Phys. Rev. E* **87** 043107
[52] Dubinov A and Sazonkin M 2013 *Physics of Wave Phenomena* **21** 118
[53] Kompaneets R, Ivlev A V, Nosenko V and Morfill G E 2014 *Phys. Rev. E* **89** 043108
[54] Mace R L, Baboolal S, Bharuthram R and Hellberg M A 1991 *Journal of Plasma Physics* **45** 323
[55] Lakhina G S, Singh S, Rubia R and Devanandhan S 2021 *Plasma* **4** 681
[56] Verheest F and Hellberg M A 2015 *Phys. Plasmas* **22** 7
[57] Mbuli L, Bharuthram R and Maharaj S 2018 *Phys. Plasmas* **25** 1