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Citation: Chin. Phys. B, 2025, 34 (**7**): 074210. DOI: 10.1088/1674-1056/adc665
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Pulsed single-longitudinal-mode operation based on modal-gain difference in repetitively passively *Q*-switched lasers

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(Received 23 February 2025; revised manuscript received 23 March 2025; accepted manuscript online 28 March 2025)

The pulsed single-longitudinal-mode (SLM) operation caused by the modal-gain difference in a repetitively passively *Q*-switched (PQS) laser is studied in detail. Firstly, the analytical expressions for the pulse buildup-time difference of repetitively PQS four-level and quasi-three-level lasers have been developed respectively. Then, according to the temporal criterion, the required conditions for repetitively PQS four-level and quasi-three-level lasers to achieve SLM operation are analyzed. The analysis results show that in addition to the short cavity is conducive to obtaining the pulsed SLM laser, the use of a lower pump power (compared to the threshold power) will help to obtain a longer pulse buildup-time difference and thus enabling the SLM operation. Moreover, it is worth noting that for the quasi-three-level lasers, the pulse buildup-time difference also depends on the initial population inversion density. The results also reveal that setting resonator parameters that can obtain large initial population inversion density will be helpful to the SLM operation in both four-level and quasi-three-level regimes. In addition, the use of saturable absorber with a low absorption cross-section ratio between the excited state and ground state also contributes to the realization of the SLM. Finally, the optimization model of passively *Q*-switched single-longitudinal-mode laser is established. In addition to predicting the output performance of the laser, this model can also be used to obtain the optimal resonator parameters and the upper limit of pump power for SLM operation.

Keywords: saturable absorber, single-longitudinal-mode, *Q*-switching, modal-gain difference

PACS: 42.70.Nq, 42.65.Re, 42.60.Gd

DOI: 10.1088/1674-1056/adc665 **CSTR:** 32038.14.CPB.adc665

1. Introduction

Passively *Q*-switched (PQS) lasers with nanosecond pulse duration have been studied extensively in the past decades due to their advantages of simple, compact structure and low energy consumption.[1] Especially with the continuous emergence of new low dimensional materials, PQS and mode-locked lasers using low dimensional materials as saturable absorbers have been widely studied in the past decade and a series of remarkable results have been achieved.[2–7] As for the PQS lasers, the related researches mainly focus on the improvement of energy or peak power,[8,9] the narrowing of pulse width[10] and the extension of laser wavelength to the mid-infrared band.[5–7] Another important advantage of the PQS lasers is the ease of achieving single-longitudinal-mode (single frequency) operation, mainly because the longitudinal modes can fully compete with each other during the long pulse buildup time caused by the passive *Q*-switching mechanism.[11] So far, the single-longitudinal-mode (SLM) operation of Nd-, Er-, Tm-, and Ho-doped PQS solid-state lasers has been widely demonstrated experimentally.[12–22] In order to ensure the SLM operation, the Fabry–Perot etalon[12,13] and volume Bragg grating[13–15] are usually added into these PQS lasers to increase the loss difference between adjacent longitudinal modes, and/or a unidirectional ring cavity,[16–18] a non-planar ring cavity[19] or a twisted-mode cavity[20,21] is used to suppress the spatial hole burning effect.

Compared with the SLM operation by introducing loss difference, the SLM operation can also be realized by using gain difference between adjacent longitudinal modes.[22] In this approach, the PQS laser can achieve SLM operation by only using a passive *Q*-switch through reasonable setting of the resonator parameters. Thus, there are two significant advantages to implementing SLM operation in this way. One is that low threshold and high efficiency SLM lasers can be obtained without the use of mode selection elements. The second is that pulse-operated SLM lasers can be obtained directly, which will greatly reduce the complexity of the laser system compared to the seed injection method.[23,24]

In terms of theoretical research, Sooy first studied the conditions required for PQS lasers to achieve SLM operation.[11] Sooy gives a rough criterion for SLM operation in a *Q*-switched laser, that is, the peak power of the dominant mode should be at least 10 times greater than that of any other mode. According to this criterion, the loss and gain differences required for SLM operation of a given *Q*-switched laser can be estimated respectively. Later, Isyanova *et al.* proposed a more rigorous temporal criterion to ensure SLM operation in a PQS laser, that is, the difference in the buildup time between any two longitudinal modes should be comparable with or greater

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than the laser pulse duration.[25] The analytical expression of pulse buildup-time difference caused by the modal-gain difference was developed in their model and compared with the pulse duration of a PQS Nd:YAG laser which was pumped by a pulsed laser. The results well support their proposed temporal criterion. In another report, Terekhov *et al.* applied this approach to successfully evaluate the SLM operation of a continuously pumped PQS Er:YAG laser.[26] In addition, SLM operation can also be realized in active Q-switched lasers by using pre-lase Q-switching technology.[27,28] The key of this technology is to increase the mode competition time by adjusting the loss of different stages of the resonator, so as to achieve the choice of the central mode.

In recent years, with the rapid development of semiconductor lasers, all-solid-state lasers and fiber lasers, PQS lasers pumped by continuous-wave lasers have become the mainstream research direction. Under this background, we developed a model for the pulse buildup-time difference that can be used in the continuous pumping regime. The analytical expressions for the pulse buildup-time difference of four-level and quasi-three-level lasers are given. This paper also discusses more details that affect the pulse buildup-time difference, which in turn affects the SLM operation. Finally, the model presented here can provide a design basis for the SLM operation of PQS lasers. We believe that our work contributes to a comprehensive understanding of how PQS lasers operate in single longitudinal mode.

2. Theoretical model of the pulse buildup-time difference

In this paper, the expression of the pulse buildup-time difference between two adjacent longitudinal modes is derived from the photon rate equation of a PQS laser, which can be written in the form

$$\frac{\mathrm{d}\phi_n}{\mathrm{d}t} = \frac{\phi_n}{t_\mathrm{r}}\left[2\sigma_n N(t)\, l - 2\sigma_n N_{\mathrm{th}(n)} l\right], \tag{1}$$

where ϕ_n and σ_n are the spatially average intracavity photon density and stimulated emission cross section of the n-th mode, respectively, t_r is the round-trip transit time, l is the length of the gain medium, $N(t)$ is the time-dependent average population inversion density in the gain medium which is the reason to push the loop gain above unity, and $N_{\mathrm{th}(n)}$ is the population inversion density of the n-th mode at the laser threshold. Herein, a resonator with no additional mode selection elements is the case we consider, thus the initial round-trip loss $2\sigma_n N_{\mathrm{th}(n)} l$ only depends on unsaturated absorber transmission (T), reflectivity of the output mirror (R), and the remaining dissipative optical loss (L),[29] which can be expressed as

$$2\sigma_n N_{\mathrm{th}(n)} l = \ln\left(\frac{1}{T_0^2}\right) + \ln\left(\frac{1}{R}\right) + L. \tag{2}$$

Figure 1 illustrates the evolution of photon density and population inversion density over time under continuous pumping regime. The pulse buildup time, $t_{sn} - t_n$, describes the pulse evolution from the laser threshold to the saturable absorber (SA) starts to bleach. During this period, the intracavity photon flux is still very small and the SA is not significantly bleached, while the population inversion density continues to increase slightly with the help of the pump and it can be taken to be a linear function over the time, which can be expressed as

$$N(t) = kt - \mu. \tag{3}$$

From Eq. (1), the photon density at time t_{sn} can be expressed as

$$\phi_n(t_{sn}) = \phi_0 \exp\left[\frac{\sigma_n k l\,(t_{sn} - t_n)^2}{t_\mathrm{r}}\right], \tag{4}$$

where ϕ_0 is the initial photon density at the threshold and $\phi_n(t_{sn})$ is equal to the saturated photon density of the SA (ϕ_s in Fig. 1). During the period of pulse establishment, the laser intensity of each mode grows separately. For the m-th mode, it begins to reach the threshold at t_m and its photon density reaches $\phi_n(t_{sn})$ at t_{sm}. Since the laser intensity of the n-th mode first reaches the saturation intensity, if the pulse buildup-time difference, $t_{sm} - t_{sn}$, is greater than the pulse width, the n-th mode consumes a large number of population inversion during this period, which restrains the continuous increase of the m-th mode and obtaining SLM. Thus, according to $\phi_n(t_{sn}) = \phi_m(t_{sm})$, we can get

$$\frac{\sigma_n k l\,(t_{sn} - t_n)^2}{t_\mathrm{r}} = \frac{\sigma_m k l\,(t_{sm} - t_m)^2}{t_\mathrm{r}}. \tag{5}$$

Here the slight difference between σ_n and σ_m is negligible, so $t_{sm} - t_{sn}(\Delta t_\mathrm{s})$ can be expressed as

$$\Delta t_\mathrm{s} = t_m - t_n. \tag{6}$$

Further considering that the round-trip loss remains constant at this stage, we can obtain

$$2\sigma_n N_{\mathrm{th}(n)} l = 2\sigma_m N_{\mathrm{th}(m)} l. \tag{7}$$

By using Eqs. (3), (6), and (7), the pulse buildup-time difference, Δt_s, between two adjacent longitudinal modes (m-th and n-th) is

$$\Delta t_\mathrm{s} = \frac{(\sigma_n - \sigma_m)}{\sigma_m}\frac{N_{\mathrm{th}(n)}}{k}, \tag{8}$$

where $(\sigma_n - \sigma_m)/\sigma_m$ is the modal-gain difference, and k is the population inversion derivative around the threshold. For a four-level laser, $k = (P_\mathrm{p}/P_\mathrm{th} - 1)N_{\mathrm{th}(n)}/\tau$, where P_p and P_th are the pump power and threshold pump power, respectively, and

τ is the excited-state lifetime. Thus, equation (8) can be rewritten as

$$\Delta t_{\mathrm{s}} = \frac{\sigma_n - \sigma_m}{\sigma_m} \frac{\tau}{\left(\dfrac{P_{\mathrm{p}}}{P_{\mathrm{th}}} - 1\right)}. \tag{9}$$

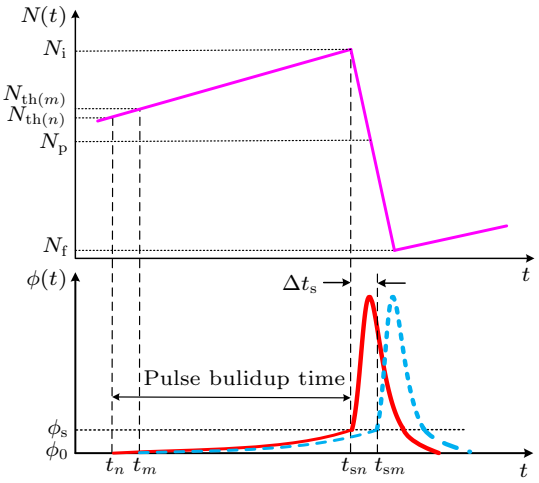


Fig. 1. Illustration of the evolution of photon density and population inversion density over time under continuous pumping regime. N_{i} is the population inversion density at the onset of SA bleaching, N_{p} is the population inversion density at the point of the peak power, N_{f} is the final population inversion density after Q-switch pulse extraction.

For a quasi-three-level laser, $N(t)$ can be written as

$$N(t) = N_2(t) - fN_1(t), \tag{10}$$

where $N_1(t)$ and $N_2(t)$ are the lower and upper laser manifold population densities, respectively, and f is the ratio of Boltzmann factor between the lower and upper laser levels. Meanwhile, the total population density N_{ga} can be expressed as

$$N_{\mathrm{ga}} = N_2(t) + N_1(t). \tag{11}$$

From Eqs. (10) and (11), the corresponding population inversion derivative, k, can be expressed as

$$k = \left(\frac{P_{\mathrm{p}}}{P_{\mathrm{th}}} - 1\right) \frac{fN_{\mathrm{ga}} + N_{\mathrm{th}(n)}}{\tau}. \tag{12}$$

Substituting Eq. (12) into Eq. (8) yields

$$\Delta t_{\mathrm{s}} = \frac{\sigma_n - \sigma_m}{\sigma_m} \frac{\tau}{\left(\dfrac{P_{\mathrm{p}}}{P_{\mathrm{th}}} - 1\right)\left(\dfrac{fN_{\mathrm{ga}}}{N_{\mathrm{th}(n)}} + 1\right)}. \tag{13}$$

For a Lorentzian lineshape gain profile, the modal-gain difference, $(\sigma_n - \sigma_m)/\sigma_m$ can be expressed as $(L_{\mathrm{c}}\Delta\nu)^{-2}$, where L_{c} and $\Delta\nu$ are the resonator length and FWHM (full-width at half-maximum) bandwidth, respectively.[25] Thus, the pulse buildup-time difference of a four-level and a quasi-three-level laser can be expressed as follows:

$$\Delta t_{\mathrm{s}} = \frac{1}{(L_{\mathrm{c}}\Delta\nu)^2} \frac{\tau}{\left(\dfrac{P_{\mathrm{p}}}{P_{\mathrm{th}}} - 1\right)}, \tag{14}$$

$$\Delta t_{\mathrm{s}} = \frac{1}{(L_{\mathrm{c}}\Delta\nu)^2} \frac{\tau}{\left(\dfrac{P_{\mathrm{p}}}{P_{\mathrm{th}}} - 1\right)\left(\dfrac{fN_{\mathrm{ga}}}{N_{\mathrm{th}(n)}} + 1\right)}. \tag{15}$$

It can be seen from the expressions of Eqs. (14) and (15) that the pulse buildup-time difference depends on the gain medium ($\Delta\nu$ and τ), optical length of the resonator (L_{c}) and the power ratio $P_{\mathrm{p}}/P_{\mathrm{th}}$. Because P_{p} is the artificially determined pump power and P_{th} is the threshold pump power which is proportional to the population inversion density at the laser threshold ($N_{\mathrm{th}(n)}$). Therefore, for a given pump power, the power ratio $P_{\mathrm{p}}/P_{\mathrm{th}}$ is inversely proportional to $N_{\mathrm{th}(n)}$ which is determined by Eq. (2). The gain medium owing a higher $\tau/\Delta\nu^2$ has a longer pulse buildup-time difference, which is beneficial to obtain the SLM operation. For a given gain medium, the combination of a short cavity and a small $P_{\mathrm{p}}/P_{\mathrm{th}}$ can help to obtain a longer pulse buildup-time difference. In addition, the pulse buildup-time difference of a quasi-three-level laser also depends on the ratio of Boltzmann factor between the lower and upper laser levels (f), and the ratio of N_{ga} to $N_{\mathrm{th}(n)}$.

3. Analysis of the conditions for realizing single-longitudinal-mode operation

3.1. Single-longitudinal-mode operation in passively Q-switched four-lever lasers

In order to quantitatively analyze the factors affecting the successful operation of SLM with the above analytical expressions, we need to compare the pulse buildup-time difference with the pulse width, and find the conditions that satisfy the pulse buildup-time difference greater than the pulse width. The pulse width of a PQS laser can be stimulated by the rate equation theory. In the case that the ground-state absorption cross-section of the SA is ten times larger than the emission cross-section of the gain medium, which can generally be satisfied, the pulse width can be expressed as[29,30]

$$W = \frac{t_{\mathrm{r}}}{2\sigma l N_{\mathrm{i}}} \frac{\dfrac{N_{\mathrm{i}}}{N_{\mathrm{p}}} \ln\left(\dfrac{N_{\mathrm{i}}}{N_{\mathrm{f}}}\right)}{\dfrac{N_{\mathrm{i}}}{N_{\mathrm{p}}} - \ln\left(\dfrac{N_{\mathrm{i}}}{N_{\mathrm{p}}}\right) - 1}, \tag{16}$$

where W is the pulse width, $t_{\mathrm{r}} = 2L_{\mathrm{c}}/c$ is the round-trip transit time in the resonator of optical length L_{c} and c is the speed of light, σ is the laser stimulated emission cross-section, l is the length of the gain medium, N_{p} is the population inversion density at the point of the maximum power, N_{i} and N_{f} are the initial and final population inversion densities, respectively (see Fig. 1). N_{i}, N_{p} and N_{f} are related by the transcendental equation

$$N_{\mathrm{i}} - N_{\mathrm{f}} = N_{\mathrm{p}} \ln\left(\frac{N_{\mathrm{i}}}{N_{\mathrm{f}}}\right), \tag{17}$$

where $N_{\rm i}$ can be obtained by expression (2) because of $N_i \approx N_{{\rm th}(n)}$, and $N_{\rm p}$ can be expressed as[31]

$$N_{\rm p} = \frac{\delta \ln\left(\frac{1}{T_0^2}\right) + \ln\left(\frac{1}{R}\right) + L}{2\sigma l}, \tag{18}$$

where $\delta = \sigma_{\rm es}/\sigma_{\rm gs}$, $\sigma_{\rm gs}$ and $\sigma_{\rm es}$ are the absorption cross-sections for the SA ground and excited states, respectively. By using Eqs. (2), (16), (17), and (18), the pulse width can be calculated. For a four-lever laser, the pulse buildup-time difference can be obtained with Eq. (14). To illustrate the utility of the present model, the Nd:YAG is served as the gain medium in a four-level laser. The value for the $\Delta\nu$ chosen from Ref. [32] is estimated to be 9 cm^{-1} and the excited state fluorescence lifetime τ is equal to 230 μs. The remaining dissipative optical loss L is set to 0.05.

Figure 2 shows the pulse buildup-time difference and pulse width as a function of R for several values of T_0, $L_{\rm c}$ (in unit cm), and δ. Here, the upper limit of $P_{\rm p}/P_{\rm th}$ is used to evaluate the difficulty of achieving SLM operation. The higher the upper limit of $P_{\rm p}/P_{\rm th}$, the easier it is for the laser to achieve the SLM operation. It should be noted that the same dashed line in Fig. 2 represents the same power ratio $P_{\rm p}/P_{\rm th}$, but each point on the dashed line does not represent the same pump power or threshold pump power. This is because the threshold pump power changes with the change of R and T_0. As for the same solid line, each point also corresponds to a different threshold pump power. In general, the smaller the T_0 and R, the smaller the pulse width can be obtained, that is, the more beneficial the laser to achieve the SLM operation. In the meantime, the influence of T_0 on the pulse width is much greater than that of R, so the influence of T_0 on the SLM operation is also greater. Another very critical factor is the resonator length. As the resonator length decreases, the upper limit of $P_{\rm p}/P_{\rm th}$ increases significantly. When $L_{\rm c} = 5$ cm and $\delta = 0.05$, the upper limits of $P_{\rm p}/P_{\rm th}$ for $T_0 = 0.99$, $T_0 = 0.98$, and $T_0 = 0.95$ are about 2.4, 3.6, and 6.4, respectively, as shown in Fig. 2(a). When $L_{\rm c}$ is increased to 15 cm, the corresponding values drop to 1.05, 1.09, and 1.2, respectively, as shown in Fig. 2(c), which does mean that the realization of the SLM operation becomes more difficult. By comparing Figs. 2(a), 2(b), and 2(c) with Figs. 2(d), 2(e), and 2(f), it can be seen that δ also has some impact on the realization of the SLM operation by affecting the size of the pulse width. When δ increases from 0.05 to 0.5, the corresponding $P_{\rm p}/P_{\rm th}$ decreases, and the reduction amplitude of short cavity is slightly larger.

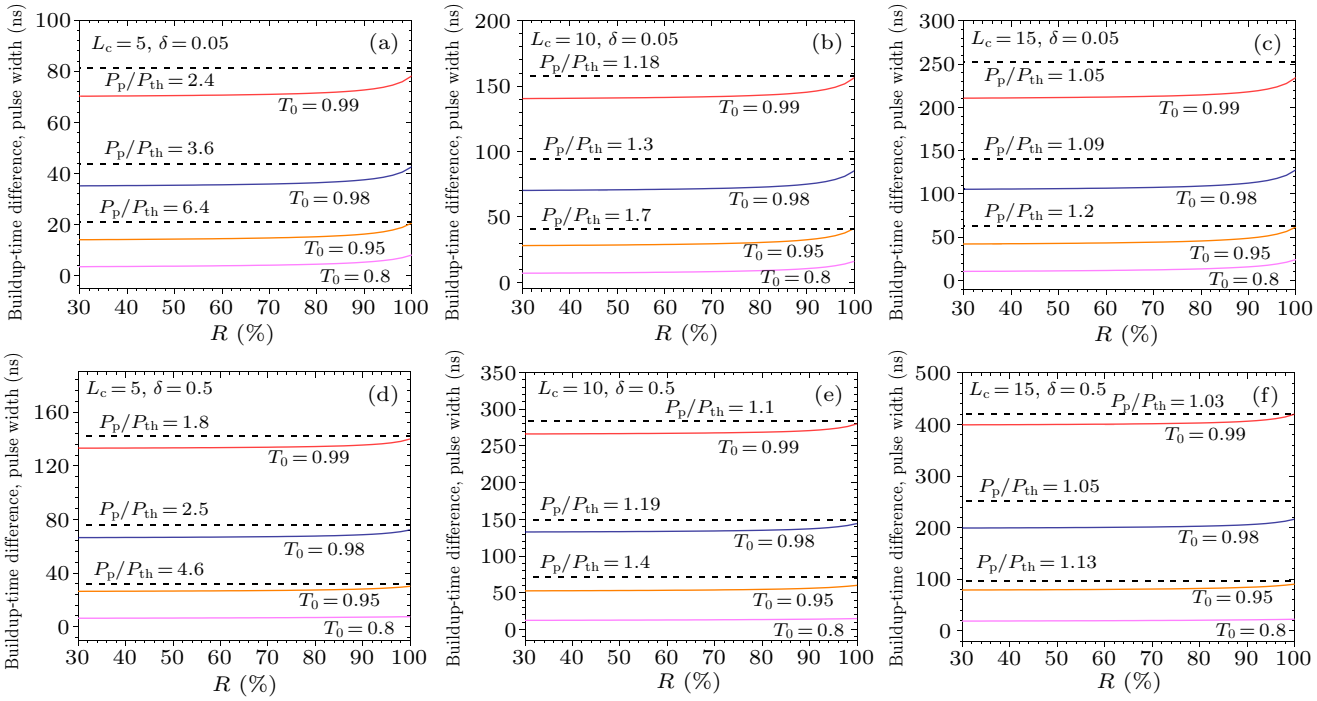


Fig. 2. The pulse buildup-time difference (dashed lines) and pulse width (solid lines) obtained in a PQS four-level laser.

3.2. Single-longitudinal-mode operation in passively *Q*-switched quasi-three-lever lasers

The pulse buildup-time difference of a quasi-three-level laser can be obtained by using Eqs. (2) and (15). Here, a Tm:YAG with a 3-at.% thulium doping concentration acts as the gain medium. The value for the $\Delta\nu$ chosen from Ref. [33] is estimated to be 74 cm^{-1}. The effective emission cross-section is 2.2×10^{-21} cm^2, and the excited state fluorescence lifetime is 10.5 ms, both of which are taken from Ref. [34]. The ratio of Boltzmann factor between the lower and upper laser levels is 0.037.[35] The remaining dissipative optical loss L is also set to 0.05.

The pulse buildup-time difference of the quasi-three-level laser and four-level laser is very different, that is, for a given gain medium and resonator length, the pulse buildup-time difference of the four-level laser only depends on P_p/P_{th}, while the pulse buildup-time difference of the quasi-three-level laser is related to P_p/P_{th}, T_0, R, and the length (l) of the gain medium. Figure 3 shows the pulse buildup-time difference of a PQS quasi-three-level laser obtained with $L_c = 5$ cm and $\delta = 0.05$. In general, the pulse buildup-time difference decreases with the increase of R, T_0, l (in unit cm), and P_p/P_{th}, respectively.

As shown in Fig. 4, the ratio of pulse buildup-time difference to pulse width decreases rapidly with the increase of R, which means that a smaller P_p/P_{th} is required to achieve SLM in the case of high reflectivity. For quasi-three-level lasers, it is also worth noting that the gain medium of short length is conducive to obtaining SLM. Figures 4(a), 4(b), and 4(c) also show that the upper limit of P_p/P_{th} decreases with the increase of T_0 and δ, respectively. It can be seen from Figs. 4(d), 4(e), and 4(f) that the upper limit of P_p/P_{th} rapidly decreases when the resonator length increases from 5 cm to 10 cm.

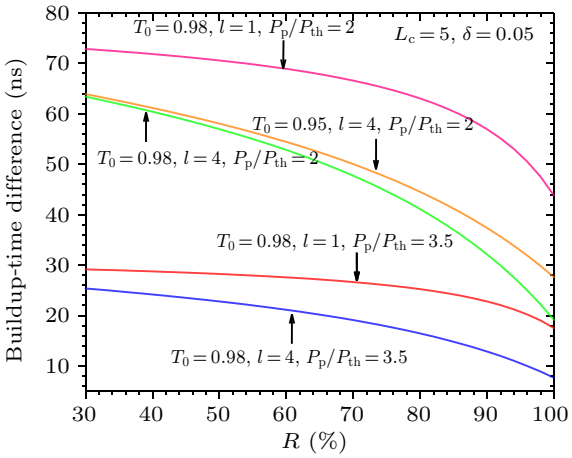


Fig. 3. The pulse buildup-time difference of a PQS quasi-three-level laser obtained with $L_c = 5$ cm and $\delta = 0.05$.

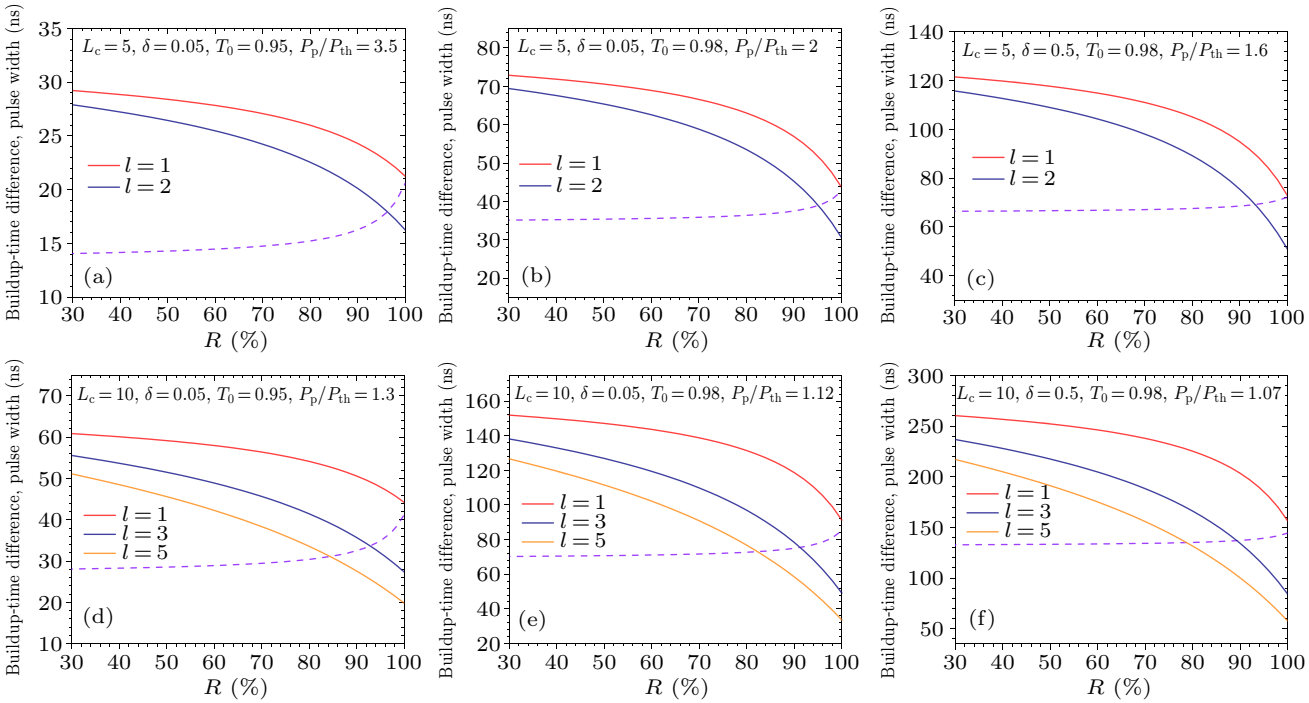


Fig. 4. The pulse buildup-time difference (solid lines) and pulse width (dashed lines) obtained in a PQS quasi-three-level laser.

4. Optimization of passively Q-switched single-longitudinal-mode lasers

Herein, we study the optimization of PQS lasers from the perspective of efficiency or energy maximization. According to the work of Zhang,[31] the optimal solution to the rate equation can be expressed as a function of a variable b, which can be expressed as

$$b = \delta + \frac{1-\delta}{2\sigma N_i l} L. \tag{19}$$

By using the expressions of Eqs. (23), (24), and (35) to Eq. (38) in Ref. [31], the analytical expressions of the key parameters of the energy-maximized PQS laser can be derived, as shown below:

$$\left(\frac{N_i}{N_f}\right)_{opt} = \frac{1}{b}, \tag{20}$$

$$\left(\frac{N_i}{N_p}\right)_{opt} = \frac{\ln(b)}{b-1}, \tag{21}$$

$$R_{opt} = \exp\left(\frac{b\ln(b) - b + 1}{\ln(b)} \frac{L}{b-\delta}\right), \tag{22}$$

$$T_{0(opt)} = \exp\left(\frac{b - \ln(b) - 1}{2\ln(b)} \frac{L}{b-\delta}\right), \tag{23}$$

and the optimal output energy can be written as

$$E_{\rm opt} = \frac{h\nu A}{2\sigma\gamma} \ln\left(\frac{1}{R}\right)_{\rm opt} \ln\left(\frac{N_{\rm i}}{N_{\rm f}}\right)_{\rm opt}, \tag{24}$$

where $h\nu$ is the photon energy, A is the active area of the laser beam in the gain medium, and γ is the inversion reduction factor.

Once the dissipative resonator loss (L), the saturable absorber (δ is fixed), and the gain medium ($\sigma N_{\rm i} l$ is fixed) are given, the optimal R and T_0 can be calculated from formulas (19)–(23). In Fig. 5, we plot the optimal reflectivity of the output mirror, $R_{\rm opt}$, and the optimal unsaturated absorber transmission, $T_{0({\rm opt})}$, as a function of $2\sigma N_{\rm i} l$ and six values of δ in the case of $L = 0.05$. In fact, $2\sigma N_{\rm i} l$ represents the round-trip small-signal gain. Both parameters decrease monotonically with $2\sigma N_{\rm i} l$, and we note that, as δ increases, the $R_{\rm opt}$ decreases while the $T_{0({\rm opt})}$ increases.

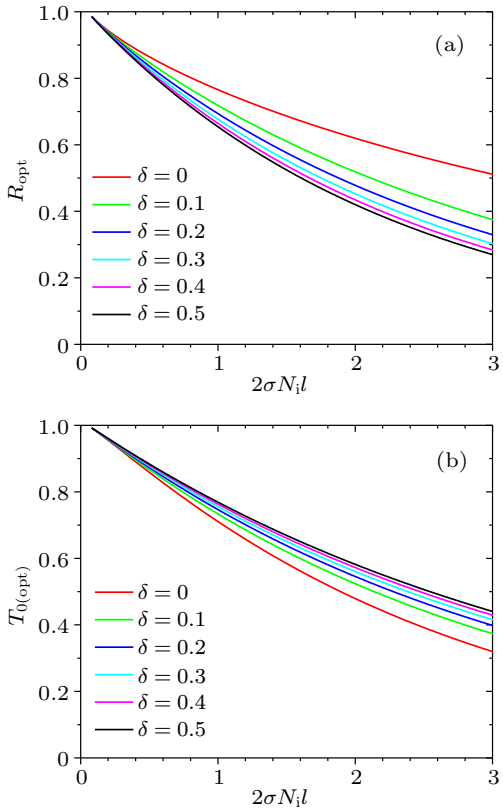


Fig. 5. Dependence of the optimal reflectivity of the output mirror ($R_{\rm opt}$) and the optimal unsaturated absorber transmission ($T_{0({\rm opt})}$) on $2\sigma N_{\rm i} l$ and δ in the case of $L = 0.05$.

Substituting Eqs. (19), (20), and (21) into Eq. (16) yields the optimal pulse width, $W_{\rm opt}$, and substituting Eqs. (19), (20), and (22) into Eq. (24) yields the optimal output energy, $E_{\rm opt}$, both of which are functions of $2\sigma N_{\rm i} l$, respectively. It should be noted that the optimal pulse width here refers to the pulse width when the maximum energy is obtained. Although this pulse width is not the minimum pulse width, it is relatively close to the minimum pulse width. According to Degnan's findings, significant sacrifices in energy and efficiency must generally be made to achieve relatively minor improvements in pulse width.[29] So for most cases of interest, efficiency (or energy) optimization is the most appropriate strategy. As shown in Fig. 6, the optimal pulse width decreases as the $2\sigma N_{\rm i} l$ increases, especially in the lower value region (Fig. 6(a)), $2\sigma N_{\rm i} l$ has a greater effect on the pulse width. On the other hand, the optimal pulse width increases with the increase of δ. Figure 7 shows the dependence of the optimal output energy on $2\sigma N_{\rm i} l$ and δ. The optimal output energy increases linearly with the increase of $2\sigma N_{\rm i} l$ and decreases with the increase of δ. To conclude, a PQS laser using an SA which has no excited state absorption ($\delta = 0$) is more suitable for obtaining high energy SLM laser. In the actual design of the SLM laser with passive Q-switching regime, the round-trip small-signal gain ($2\sigma N_{\rm i} l$) can first be obtained according to the required pulse energy, and then the

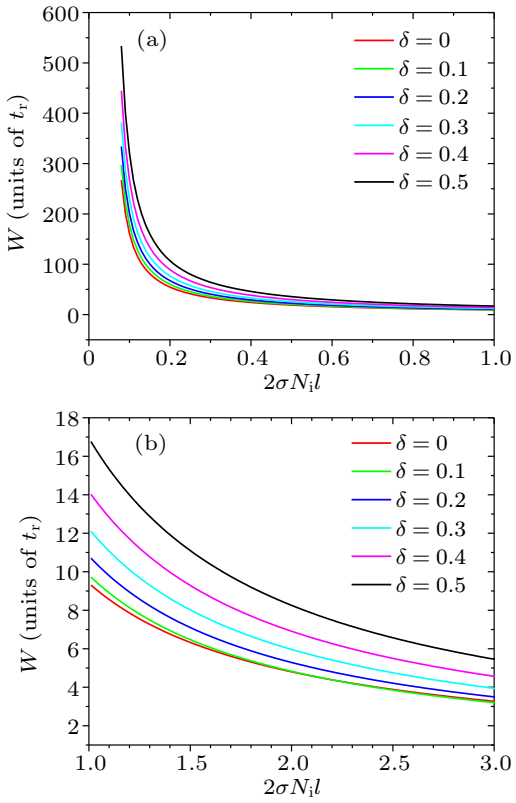


Fig. 6. Dependence of the optimal pulse width, W, on $2\sigma N_{\rm i} l$ and δ.

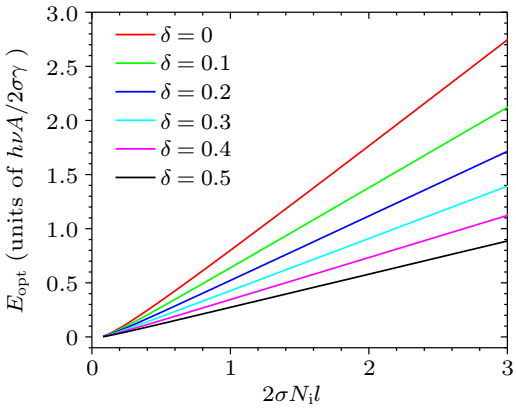


Fig. 7. Dependence of the optimal output energy, $E_{\rm opt}$, on $2\sigma N_{\rm i} l$ and δ.

optimal design values of the output mirror reflectivity (R) and unsaturated absorber transmission (T_0) can be obtained.

According to the pulse width obtained in Fig. 6 and Eqs. (14) and (15), the upper limits of P_p/P_{th} to ensure the SLM operation of four-level and quasi-three-level PQS lasers can be estimated respectively. Figure 8 shows the dependence of the upper limits of P_p/P_{th} on $2\sigma N_i l$ and δ in a PQS Nd:YAG laser when $L_c = 10$ cm. The upper limit of P_p/P_{th} increases monotonically with $2\sigma N_i l$ and decreases with the increase of δ. Figure 9 depicts the change of the upper limit of P_p/P_{th} for a Tm:YAG laser. The dependence between the upper limit of P_p/P_{th} and $2\sigma N_i l$ and δ in Fig. 9(a) is the same as that in Fig. 8. In addition, the upper limit of P_p/P_{th} decreases as the length of the gain medium (l) increases, as shown in Fig. 9(b).

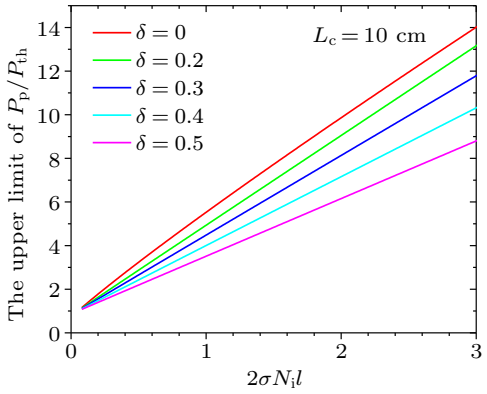


Fig. 8. Dependence of the upper limit of P_p/P_{th} on $2\sigma N_i l$ and δ in a PQS four-level laser.

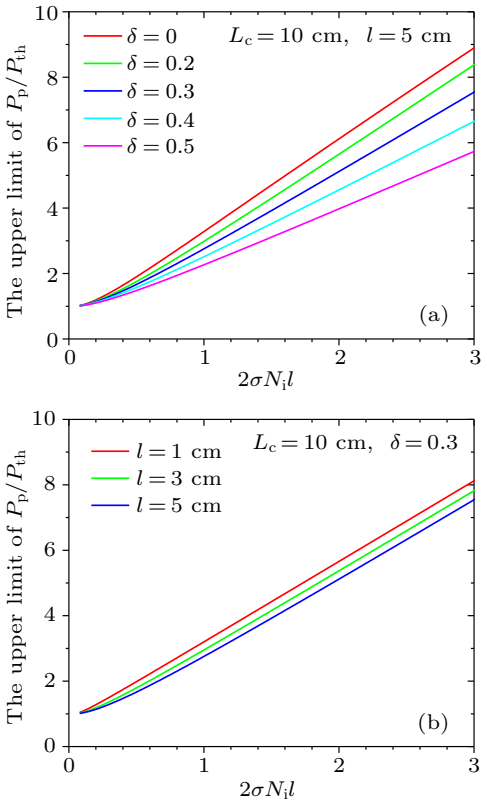


Fig. 9. Dependence of the upper limit of P_p/P_{th} on $2\sigma N_i l$ and δ in a PQS quasi-three-level laser.

5. Conclusion

In summary, the factors affecting the pulsed SLM operation by using the modal-gain difference of repetitively PQS lasers are analyzed in detail from the perspective of temporal criterion. The analytical expressions for the pulse buildup-time difference of PQS four-level and quasi-three-level lasers are given. For a given gain medium and resonator length, the pulse buildup-time difference of a PQS four-level laser only depends on the ratio of pump power to threshold pump power (P_p/P_{th}), while the pulse buildup-time difference of a PQS quasi-three-level laser also depends on unsaturated absorber transmission (T_0), reflectivity of the output mirror (R), and the length of the gain medium (l). Shortening the resonator length and reducing the value of P_p/P_{th} is an effective means to realize SLM operation. In addition, the choice of a small R and an SA with small T_0 and δ (the absorption cross-section ratio between the excited and ground states) is also conducive to the realization of SLM. It is also worth noting that the short-length gain medium is beneficial to the SLM operation of the PQS quasi-three-level laser. Finally, the optimization of passively Q-switched SLM laser is studied. Both of the $T_{0(opt)}$ and R_{opt} decrease monotonically with the increase of the round-trip small-signal gain and as δ increases, the R_{opt} decreases while the $T_{0(opt)}$ increases. On the contrary, the upper limit of P_p/P_{th} to ensure the SLM operation increases with the increase of the round-trip small-signal gain. The approach presented in this paper can be used to evaluate and optimize passively Q-switched SLM lasers.

Acknowledgement

Project supported by National Natural Science Foundation of China (Grant No. 62205102).

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