

Algorithm for computing time correlation functions in non-stationary complex dynamic systems

Jiu Zhang(张鹫), Lifu Jin(金立孚), Bo Zheng(郑波), Xiongfei Jiang(蒋雄飞), Tingting Chen(陈婷婷), Cong Xu(徐匆), and Yanqing Hu(胡延庆)

Citation: Chin. Phys. B, 2025, 34 (**3**): 038904. DOI: 10.1088/1674-1056/adb8bd
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Node ranking based on graph curvature and PageRank

Hongbo Qu(曲鸿博), Yu-Rong Song(宋玉蓉), Ruqi Li(李汝琦), Min Li(李敏), and Guo-Ping Jiang(蒋国平)
Chin. Phys. B, 2025, 34 (**2**): 028901. DOI: 10.1088/1674-1056/ad9a9b

Accurate prediction of essential proteins using ensemble machine learning

Dezhi Lu(鲁德志), Hao Wu(吴浩), Yutong Hou(侯俞彤), Yuncheng Wu(吴云成), Yuanyuan Liu(刘媛媛), and Jinwu Wang(王金武)
Chin. Phys. B, 2025, 34 (**1**): 018901. DOI: 10.1088/1674-1056/ad8db2

Impact of environmental factors on the coevolution of information-emotions-epidemic dynamics in activity-driven multiplex networks

Liang'an Huo(霍良安), Bingjie Liu(刘炳杰), and Xiaomin Zhao(赵晓敏)
Chin. Phys. B, 2024, 33 (**12**): 128903. DOI: 10.1088/1674-1056/ad7df5

Hyperbolic map unravels eight regions in temperature volatility regionalization of Mainland China

Yuxuan Song(宋雨轩), Changgui Gu(顾长贵), Muhua Zheng(郑木华), Aixia Feng(冯爱霞), Yufei Xi(席雨菲), Haiying Wang(王海英), and Huijie Yang(杨会杰)
Chin. Phys. B, 2024, 33 (**12**): 128902. DOI: 10.1088/1674-1056/ad84c2

Source localization in signed networks with effective distance

Zhi-Wei Ma(马志伟), Lei Sun(孙蕾), Zhi-Guo Ding(丁智国), Yi-Zhen Huang(黄宜真), and Zhao-Long Hu(胡兆龙)
Chin. Phys. B, 2024, 33 (**2**): 028902. DOI: 10.1088/1674-1056/ad1482

SPECIAL TOPIC — Computational programs in complex systems

Algorithm for computing time correlation functions in non-stationary complex dynamic systems

Jiu Zhang(张鹫)[1], Lifu Jin(金立孚)[2], Bo Zheng(郑波)[3,†], Xiongfei Jiang(蒋雄飞)[4], Tingting Chen(陈婷婷)[5], Cong Xu(徐匆)[1,‡], and Yanqing Hu(胡延庆)[1]

[1] *Department of Statistics and Data Science, Southern University of Science and Technology, Shenzhen* 518055, *China*
[2] *School of Economics and Management, Wenzhou University of Technology, Wenzhou* 325035, *China*
[3] *School of Physics and Astronomy, Yunnan University, Kunming* 650091, *China*
[4] *College of Finance and Information, Ningbo University of Finance and Economics, Ningbo* 315175, *China*
[5] *School of Finance, Zhejiang University of Finance and Economics, Hangzhou* 310000, *China*

(Received 27 December 2024; revised manuscript received 17 February 2025; accepted manuscript online 21 February 2025)

For non-stationary complex dynamic systems, a standardized algorithm is developed to compute time correlation functions, addressing the limitations of traditional methods reliant on the stationary assumption. The proposed algorithm integrates two-point and multi-point time correlation functions into a unified framework. Further, it is verified by a practical application in complex financial systems, demonstrating its potential in various complex dynamic systems.

Keywords: complex dynamic systems, non-stationary states, time correlation functions

PACS: 89.75.–k, 89.65.Gh, 05.45.Tp, 05.10.–a

DOI: 10.1088/1674-1056/adb8bd **CSTR:** 32038.14.CPB.adb8bd

1. Introduction

Understanding the dynamics of complex systems remains a fundamental challenge in many scientific disciplines, largely due to the absence of a principled mathematical framework and the intractability of their equations of motion.[1–4] Temporal and spatial correlation functions have proven invaluable for exploring dynamic behaviors in such systems. In stationary or equilibrium states, these functions provide robust characterizations of the underlying dynamics. Nevertheless, for a wide range of dynamic systems, temporal correlations tend to be either negligible or zero if the non-stationary effects are not considered.[5–13]

Complex dynamic systems are often non-stationary, or at least with features of non-stationary states.[14–16] A typical characteristic of non-stationary states is the strong auto-correlation in dynamic variables that evolve significantly over time.[17–22] Such a phenomenon hinders the computational analysis of temporal and spatial correlation functions under the stationary assumption.[12,23–26] Detrended fluctuation analysis allows for the identification of auto-correlations even when non-stationary trends are present,[27–31] and it can be extended to examine the cross-correlations between multiple time series.[32–36] However, developing a systematic approach to compute time correlations in non-stationary complex systems remains a critical open problem.

Recently, a novel methodology has been proposed for the computation of time correlation functions based on the historical big-data, considering dynamic effects of non-stationary states.[10–12] In particular, the two-point time correlation function has been computed in Ref. [10], and a non-zero correlation is observed in different dynamic systems. This is crucial for both theory and applications, particularly in financial systems, where it relates to the long-debated volatility-feedback effect.[37,38] In stock markets, however, this correlation is typically below 1%–2% and subject to fluctuations, posing challenges for risk management in investment strategies based on such correlations.

In general, multi-point time correlations provide a more comprehensive characterization of dynamic behaviors. Notably, empirical models of complex systems, such as ARCH models, minority games, and agent-based models,[39–44] often incorporate multi-point interactions. Nonetheless, calculating multi-point correlations is significantly more complex, and identifying corresponding physical effects is challenging. Reference [11] introduces an effective method to simplify multi-point time correlations, extending significantly beyond the two-point framework.

Although considerable progress has been achieved in investigating non-stationary time correlation functions, there is currently a lack of standardized algorithms in practical applications, which restrains the widespread dissemination of those theoretical results. In this paper, we formulate a standardized

†Corresponding author. E-mail: zhengbo@zju.edu.cn
‡Corresponding author. E-mail: xuc6@sustech.edu.cn

© 2025 Chinese Physical Society and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved.

algorithm for calculating two-point and multi-point time correlation functions in non-stationary states, providing an efficient framework for investigating complex dynamic systems.

2. Methods

2.1. Two-point time correlation function

Given two dynamic variables in a complex system, $A(t')$ and $B(t')$, where $t' = 1, 2, \ldots, T$. Let $A(t')$ represent the driving force of the dynamic system, and $B(t')$ the corresponding response. To investigate the dynamic behavior of $B(t')$, one can define the two-point time correlation function between $A(t')$ and $B(t')$ as

$$C(t) = \langle A(t')B(t'+t)\rangle - \langle A(t')\rangle\langle B(t'+t)\rangle, \tag{1}$$

where $\langle\cdots\rangle$ denotes the average over time t'. The correlation function $C(t)$ measures the impact of the past dynamic variable $A(t')$ on the future dynamic variable $B(t'+t)$. Notably, $A(t')$ may represent either an internal driving force, such as dynamic fluctuations in various systems,[10] or an external one, such as external information in complex financial systems.[12] Meanwhile, $B(t')$ could stand for various dynamic variables, such as changes in stock prices,[11] atmospheric temperature,[13] and so on.

From the perspective of statistical physics, the computation of $C(t)$ typically assumes that the dynamic system resides in a stationary state. However, complex dynamic systems are usually non-stationary or exhibit features of non-stationary states.[14–16] In such cases, a direct calculation of $C(t)$ can be hindered by non-stationary dynamics, resulting in correlations that fluctuate around zero (see Fig. 1(a)).

A typical example of non-stationary states arises from the strong auto-correlation of dynamic variables, indicating that these variables undergo significant temporal evolution, even when averaged over long time scales. Assuming the auto-correlation time of such variables as $T_{\rm b}$, the system can be approximated as being in a quasi-stationary state within the time window $T_{\rm b}$. As time evolves, however, it switches among different quasi-stationary states. To consider the dynamic effects of such non-stationary states, and following the methodology outlined in Refs. [10–12], we first subtract the background values of $A(t')$ calculated over the time window $T_{\rm b}$ from its instantaneous values within the time window T

$$\tilde{A}(t') = A_T(t') - A_{T_{\rm b}}(t'), \tag{2}$$

where $A_T(t') = \frac{1}{T}\sum_{i=0}^{T-1} A(t'-i)$, and $A_{T_{\rm b}}(t')$ is defined similarly. Using this background-subtracted variable, we redefine Eq. (1) as

$$\tilde{C}(t) = \langle \tilde{A}(t')B(t'+t)\rangle - \langle \tilde{A}(t')\rangle\langle B(t'+t)\rangle. \tag{3}$$

In contrast to $C(t)$ in Eq. (1), the modified correlation function $\tilde{C}(t)$ above is non-zero for a broad class of dynamic systems, as illustrated in Fig. 1(b).

2.2. Multi-point time correlation function

Due to the presence of noise interference and intricate interactions, the two-point time correlation function defined in Eq. (3) may not sufficiently capture the complexities of dynamic systems. To address this limitation, the two-point time correlation is generalized to a multi-point correlation function, allowing for a more comprehensive characterization of the system's dynamics,

$$\tilde{C}_{\rm mp}(t) = \left\langle \tilde{A}(t'-t+1)\cdots\tilde{A}(t')B(t'+1)\right\rangle, \tag{4}$$

which quantifies the relationship between a past configuration of the dynamic variable, expressed as $\tilde{A}(t'-t+1)$, $\tilde{A}(t'-t+2)$,..., $\tilde{A}(t')$, and a future dynamic variable $B(t'+1)$. However, the computation of such multi-point time correlations is significantly more complex than calculating a two-point one. As suggested in Ref. [11], an efficient approach has been developed to simplify the calculation of multi-point time correlations, comprising the following four sequential steps.

Step 1: Simplification of $\tilde{A}(t')$ To reduce computational complexity, $\tilde{A}(t')$ is simplified to its sign function $\mathrm{sgn}(\tilde{A}(t'))$, where $\mathrm{sgn}(x)$ indicates the sign of x. For a given t, the possible micro-state configurations of the past states $\{\mathrm{sgn}(\tilde{A}(t'-i))\}$, with $i = t-1,\ldots,0$, grow exponentially as 2^t, making calculations intractable for large t.

Step 2: Micro-state configuration classification To address this, a multi-point time variable is introduced,

$$\tilde{A}(t';t) = \sum_{i=0}^{t-1} \mathrm{sgn}\left(\tilde{A}(t'-i)\right), \tag{5}$$

which maps the micro-state configurations to discrete values $\eta(t) = -t, -t+2, \ldots, t-2, t$. This reduces the number of possible micro-state configurations to $t+1$, significantly simplifying the calculation.

Step 3: Correlation of micro-state configurations For a fixed micro-state configuration $\tilde{A}(t';t) = \eta(t)$, the time correlation function between $\tilde{A}(t';t)$ and $B(t'+1)$ is defined as

$$\begin{aligned}\tilde{C}_{\rm mp}(t,\eta(t)) \;=\; & \langle \mathrm{sgn}\left(\tilde{A}(t';t)|_\eta\right)\cdot B(t'+1)\rangle \\ & -\langle \mathrm{sgn}\left(\tilde{A}(t';t)|_\eta\right)\rangle\cdot\langle B(t'+1)\rangle,\end{aligned} \tag{6}$$

where $\tilde{A}(t';t)|_\eta$ refers to the subset of data with $\tilde{A}(t';t) = \eta(t)$. The occurrence frequency of each micro-state configuration is described by $n_\eta = N_\eta/N$, where N_η is the number of data points with $\tilde{A}(t';t) = \eta(t)$, and N is the total number of data points.

Step 4: Overall multi-point correlation The total multi-point time correlation function is obtained by summing over all $\eta(t)$:

$$\tilde{C}_{\rm mp}(t) \;=\; \langle \mathrm{sgn}(\tilde{A}(t';t))\cdot B(t'+1)\rangle$$

$$-\langle \mathrm{sgn}(\tilde{A}(t';t))\rangle \cdot \langle B(t'+1)\rangle, \quad (7)$$

where $\tilde{C}_{\mathrm{mp}}(t) = \sum_{\eta} \tilde{C}_{\mathrm{mp}}(t, \eta(t))$. This function differs fundamentally from the two-point time correlation function $\tilde{C}(t)$ in Eq. (3) (see Fig. 1(b)). The ratio $\tilde{C}_{\mathrm{mp}}(t)/\tilde{C}(1)$ quantifies the enhancement of the multi-point correlation compared to the two-point case.

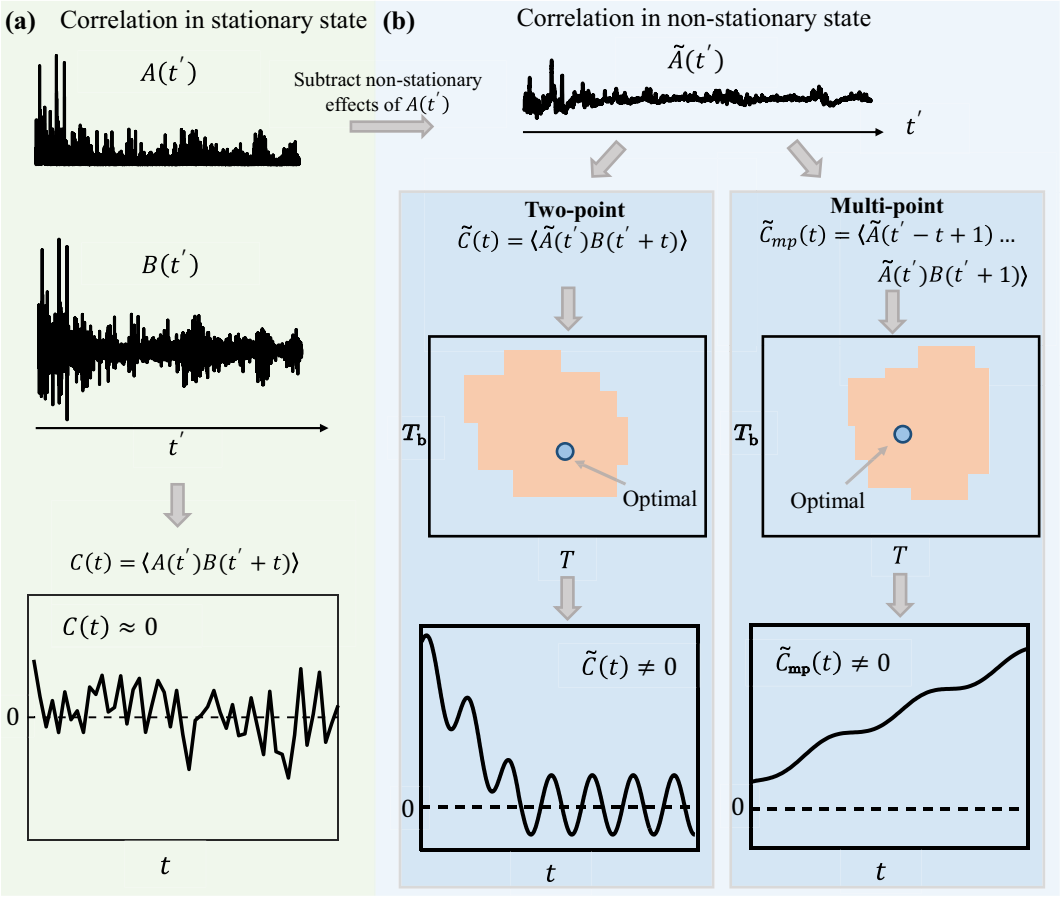


Fig. 1. Diagram of the proposed approach for calculating time correlation functions in (a) stationary and (b) non-stationary states. (a) For two time series, where $A(t')$ exhibits strong auto-correlation and $B(t')$ is uncorrelated with itself, a direct computation of $C(t)$ may be hindered by the non-stationary effects of $A(t')$, causing the correlation to fluctuate around zero. (b) To address this, the non-stationary effect is explicitly taken into account in the computation of the two-point time correlation function $\tilde{C}(t)$ and the multi-point correlation function $\tilde{C}_{\mathrm{mp}}(t)$. First, $\tilde{A}(t')$ is derived by subtracting the background values of $A(t')$ over the time window T_{b} from its instantaneous values over the time window T, as defined in Eq. (2). Second, $\tilde{C}(t)$ and $\tilde{C}_{\mathrm{mp}}(t)$ are calculated across different time window pairs of T and T_{b}. Third, effective and optimal time window pairs are identified based on specific criteria (see Subsection 2.3). Finally, the non-zero correlation functions $\tilde{C}(t)$ and $\tilde{C}_{\mathrm{mp}}(t)$ are obtained at the optimal time window pair.

2.3. The effective pairs of time windows T and T_{b}

Following the methods described in Refs. [10,11], two criteria are proposed to identify the effective time window pairs T and T_{b} for both two-point and multi-point time correlation functions. The schematic representation of these criteria is illustrated in Fig. 2, with the detailed steps outlined below.

(I) Criterion for two-point time correlation functions $\tilde{C}(t)$

(i) For each time window pair T and T_{b}, calculate $\tilde{C}(t)$ and apply a 3-day moving average to smooth the data and reduce fluctuations.

(ii) Identify t_1, the first time where $\tilde{C}(t)$ changes its sign. Define $\tilde{C}(t)$ over the range $1 \le t \le t_1$ as the first sector, and $t_1 \le t \le t_1 + t_2$ as the second sector. In the first sector, $\tilde{C}(t)$ should consistently remain either positive or negative, while in the second sector, it should fluctuate around zero.

(iii) Compute the average absolute values of $\tilde{C}(t)$ in the first and second sectors, denoted as AC_1 and AC_2, respectively. For non-zero $\tilde{C}(t)$, the criteria require that $|\tilde{C}(t)| > AC_2$ for $1 \le t \le t_0$ and $t_0 > 10$, and $|\tilde{C}(t)| < AC_1$ in the second sector for all t. To further quantify the deviation of $\tilde{C}(t)$ from zero, compute the average value of $\tilde{C}(t)$ over the interval $1 \le t \le t_0$, denoted as AC_0.

(iv) The pair of T and T_{b} that maximizes $|AC_0|$ is defined as the optimal time window pair. The sign of AC_0 indicates whether the correlation is positive or negative.

(II) Criterion for multi-point time correlation functions $\tilde{C}_{\mathrm{mp}}(t)$

(i) For each time window pair T and T_{b}, calculate $\tilde{C}_{\mathrm{mp}}(t)$

and apply a 3-day moving average to smooth the data and reduce fluctuations.

(ii) Define $\tilde{C}_{\rm mp}(t)$ over the range $1 \le t \le t_1$ as the first sector, and $t_1 < t \le t_2$ as the second sector. Identify the time window pair T and $T_{\rm b}$ such that $\tilde{C}_{\rm mp}(t)$ does not change its sign for $t \le t_2$.

(iii) Compute the average absolute values of $\tilde{C}_{\rm mp}(t)$ in the first and second sectors, denoted as AC_1 and AC_2, respectively. For the first sector, $|\tilde{C}_{\rm mp}(t)| < AC_2$ must hold for all t, while for the second sector, $|\tilde{C}_{\rm mp}(t)| > AC_1$ must be satisfied. To characterize the deviation from zero, calculate the average value of $\tilde{C}_{\rm mp}(t)$ over $t \le t_2$, denoted as $\overline{C}_{\rm mp}$.

(iv) The pair of T and $T_{\rm b}$ that maximizes $|\overline{C}_{\rm mp}|$ is defined as the optimal time window pair. The sign of $\overline{C}_{\rm mp}$ indicates whether the correlation is positive or negative.

3. Results

3.1. Methodological framework

The primary objective of this paper is to present a standardized algorithm for calculating two-point and multi-point time correlation functions in non-stationary states. The methodological framework is illustrated in Fig. 1. First, the non-stationary effects are considered by subtracting the background values of dynamic variables, as defined in Eq. (2). Next, the two-point time correlation function $\tilde{C}(t)$ and the multi-point one $\tilde{C}_{\rm mp}(t)$ are computed across different pairs of time windows T and $T_{\rm b}$. Subsequently, effective and optimal time window pairs are identified based on specific criteria (details provided in Subsection 2.3 and Fig. 2). Finally, the non-zero correlation functions $\tilde{C}(t)$ and $\tilde{C}_{\rm mp}(t)$ are obtained at the optimal time window pair.

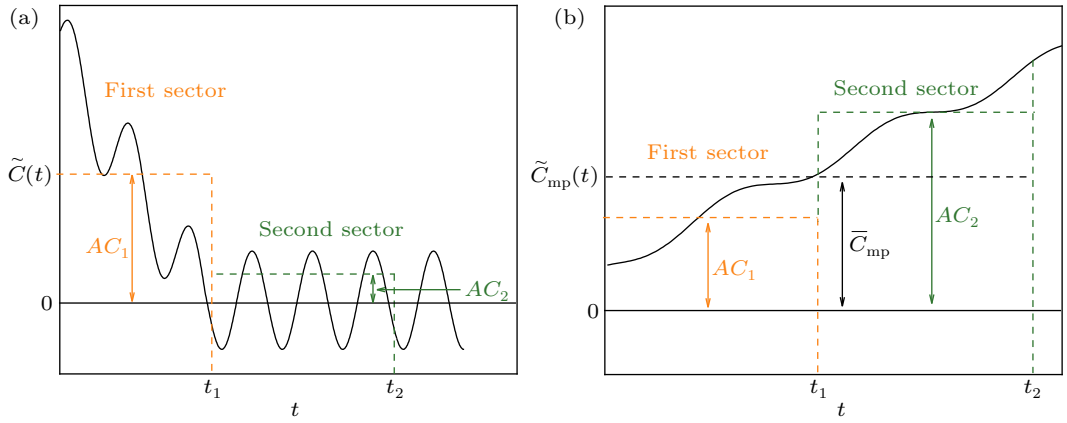


Fig. 2. A schematic representation of the criteria for identifying the non-zero (a) two-point time correlation function $\tilde{C}(t)$ and (b) multi-point time correlation function $\tilde{C}_{\rm mp}(t)$. The solid curves depict that $\tilde{C}(t)$ and $\tilde{C}_{\rm mp}(t)$ are divided into two sectors: the first sector spans $1 \le t \le t_1$, while the second sector spans $t_1 < t \le t_2$. The average absolute values in these sectors are denoted as AC_1 and AC_2, respectively. Additionally, in (b), $\overline{C}_{\rm mp}$ denotes the average value of $\tilde{C}_{\rm mp}(t)$ over the range $t \le t_2$.

Algorithm 1: Pseudo code to compute the two-point time correlation function in non-stationary states.

Inputs:

- $A(t')$ and $B(t')$: Normalized time series data.
- $T_{\rm para}$: Time windows parameters $[T^{\rm start}, T^{\rm step}, T^{\rm stop}, T_{\rm b}^{\rm start}, T_{\rm b}^{\rm step}, T_{\rm b}^{\rm stop}]$.
- t: Length of the two-point time correlation function.

Outputs:

- $\tilde{C}(t)$: Two-point time correlation function at the optimal time windows pair of T and $T_{\rm b}$.
- Effective and optimal time window pairs for the two-point time correlation function.

Steps:

1. For each time window pair $(T, T_{\rm b})$ in $T_{\rm para}$:
   (i) Compute the background-removed time series:
   $$\tilde{A}(t') = \text{removeBG_func}(A(t'), T, T_{\rm b}).$$
   (ii) Calculate the two-point correlation:
   $$\tilde{C}(t) = \text{correlationTP_func}(\tilde{A}(t'), B(t'), t_{\rm c}).$$
2. Identify effective time window pairs based on the criterion:
   $$[T^{\rm eff}, T_{\rm b}^{\rm eff}, T^{\rm opt}, T_{\rm b}^{\rm opt}] = \text{window_selection_TP_func}(\tilde{C}(t)).$$
3. Select the optimal $(T^{\rm opt}, T_{\rm b}^{\rm opt})$ pair based on the maximum absolute value of AC_0.
4. Using the optimal $(T^{\rm opt}, T_{\rm b}^{\rm opt})$, calculate the two-point correlation:
   $$\tilde{C}(t) = \text{correlationTP_func}(\text{removeBG_func}(A(t'), T^{\rm opt}, T_{\rm b}^{\rm opt}), B(t'), t).$$

Functions:

- removeBG_func: Remove background values to compute a new time series.
- correlationTP_func: Compute two-point correlation function.
- window_selection_TP_func: Select effective and optimal time windows for two-point correlation.

Algorithm 2: Pseudo code to compute the multi-point time correlation function in non-stationary states.

Inputs:

- $A(t')$ and $B(t')$: Normalized time series data.
- T_{para}: Time windows parameters $[T^{\text{start}}, T^{\text{step}}, T^{\text{stop}}, T_{\text{b}}^{\text{start}}, T_{\text{b}}^{\text{step}}, T_{\text{b}}^{\text{stop}}]$.
- t: Length of the multi-point time correlation function.

Outputs:

- $\tilde{C}_{\text{mp}}(t)$: Multi-point time correlation function at the optimal time window pair.
- $\tilde{C}_{\text{mp}}(t, \eta(t))$: Micro-state multi-point time correlation function at the optimal time window pair.
- Effective and optimal time window pairs for the multi-point time correlation function.

Steps:

1. For each time window pair (T, T_{b}) in T_{para}:
   (i) Compute the background-removed time series:
   $$\tilde{A}(t') = \text{removeBG_func}(A(t'), T, T_{\text{b}}).$$
   (ii) Calculate the two-point correlation:
   $$\tilde{C}_{\text{mp}}(t) = \text{correlationMP_func}(\tilde{A}(t'), B(t'), t).$$
2. Identify effective time window pairs based on the criteria:
   $$[T^{\text{eff}}, T_{\text{b}}^{\text{eff}}, T^{\text{opt}}, T_{\text{b}}^{\text{opt}}] = \text{window_selection_MP_func}(\tilde{C}_{\text{mp}}(t)).$$
3. Select the optimal $(T^{\text{opt}}, T_{\text{b}}^{\text{opt}})$ pair based on the maximum absolute value of $\overline{C}_{\text{mp}}$.
4. Using the optimal $(T^{\text{opt}}, T_{\text{b}}^{\text{opt}})$ to calculate the micro-state multi-point correlation $\tilde{C}_{\text{mp}}(t, \eta(t))$ for fixed $\eta(t)$:
   $$\tilde{C}_{\text{mp}}(t, \eta(t)) = \text{correlationMPmicro_func}(\text{removeBG_func}(A(t'), T^{\text{opt}}, T_{\text{b}}^{\text{opt}}), B(t'), t).$$
5. Using the optimal $(T^{\text{opt}}, T_{\text{b}}^{\text{opt}})$ to calculate the multi-point correlation:
   $$\tilde{C}_{\text{mp}}(t) = \text{correlationMP_func}(\text{removeBG_func}(A(t'), T^{\text{opt}}, T_{\text{b}}^{\text{opt}}), B(t'), t).$$

Functions:

- removeBG_func: Remove background values to compute a new time series.
- correlationMPmicro_func: Compute multi-point correlation function for micro-state configurations.
- correlationMP_func: Compute multi-point correlation function.
- window_selection_MP_func: Select effective and optimal time windows for multi-point correlation.

To further elucidate the specific calculations involved in Figs. 1 and 2, we provide the above pseudo-codes to explicitly present the computational procedures. These algorithms describe the detailed steps to compute the two-point and multi-point time correlation functions in non-stationary states, including the identification of effective and optimal time window pairs.

Algorithm 1 outlines the steps for calculating the two-point time correlation function $\tilde{C}(t)$, while Algorithm 2 focuses on the computation of the multi-point correlation function $\tilde{C}_{\text{mp}}(t)$. Each algorithm provides a clear set of inputs, outputs, and computational steps to ensure accuracy and reproducibility of results. The criteria for effective time window selection are explicitly incorporated into both algorithms.

3.2. Practical application

A large class of dynamic systems may usually be non-stationary, such as social, biological and atmospheric systems. To demonstrate the effectiveness of our proposed algorithm, we apply it to a practical case in complex financial systems, where non-stationary effects are prevalent. Specifically, we compute the time correlation functions between the stock price volatility $A(t')$ and return $B(t')$ using real-world financial data from Hang Seng Index (HSI) and S&P 500 Index (S&P500).[46] The time period of data covers from 2010 to 2024 for HSI and from 2013 to 2024 for S&P 500. It is well-known that the stock price volatility $A(t')$ exhibits strong auto-correlation, whereas the stock price return $B(t')$ is uncorrelated with itself.[47] Thus, when calculating Eq. (1), it is essential to take into account the non-stationary effects of $A(t')$. The results are summarized in Fig. 3.

Let the time series of the stock price be denoted as $Y(t')$. The stock price return $y(t')$ is defined as the change in $Y(t')$ over a unit time step: $y(t') = \ln Y(t') - \ln Y(t'-1)$. To facilitate the comparison of different time series, we introduce the normalized return: $B(t') = \frac{y(t') - \langle y(t') \rangle}{\sigma}$, where $\langle \cdots \rangle$ represents the time average over t', and σ is the standard deviation of $y(t')$. The stock price volatility is then defined as the absolute value of the normalized return: $A(t') = |B(t')|$.

In practical computations, the time window T is swept from 5 to 44 days, reflecting the instant time scale of the traders in stock markets.[48] T_{b} is swept from 45 to 250 days, representing the correlating time scale of the stock price volatility.[47] There are a total of 1680 pairs of time windows T and T_{b}. A pair of T and T_{b} for which the time correlation function is non-zero is defined as an effective pair, while the effective pair exhibiting the largest correlation is referred to as the optimal pair. As shown in Figs. 3(a) and 3(b), the effective time window pairs form a specific region for both two-point and multi-point correlation functions. For the two-point and multi-point correlation functions, there are 219 and 159 effective time window pairs, respectively. In our algorithm, we primarily focus on the representative results associated with the optimal effective pair.

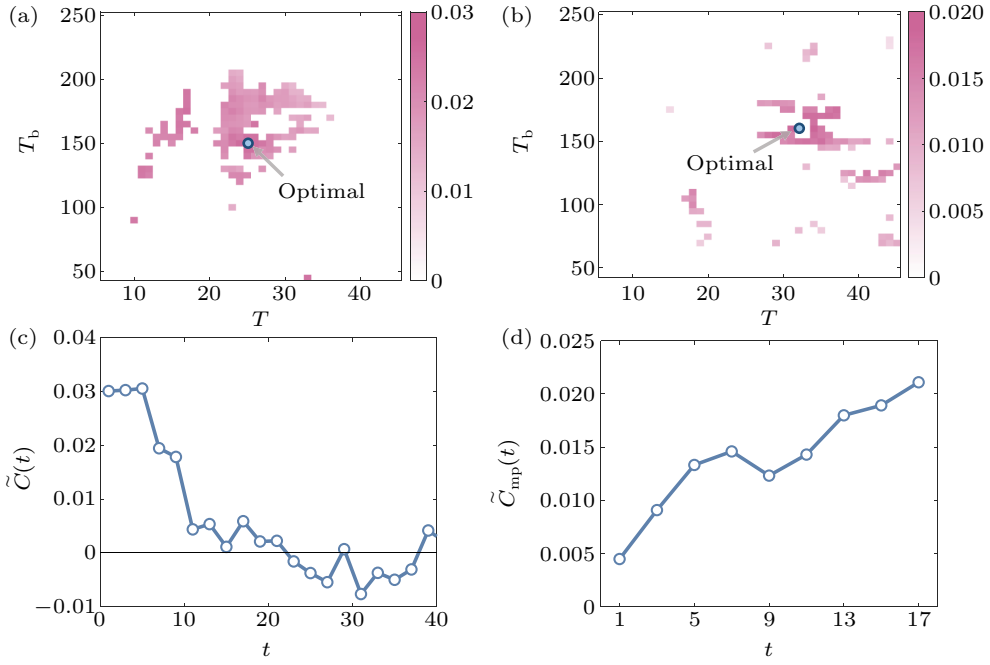


Fig. 3. Practical examples of the two-point $\tilde{C}(t)$ and multi-point $\tilde{C}_{mp}(t)$ time correlation functions in complex financial systems. Here we apply HSI and S&P500 to calculate $\tilde{C}(t)$ and $\tilde{C}_{mp}(t)$, respectively. Panels (a) and (b) illustrate the effective time window pairs for two-point and multi-point time correlation functions, respectively. The color from white (minimum) to pink (maximum) indicates the strength of the correlation. The optimal time window pair is highlighted with a blue circle and labeled as "Optimal". Panels (c) and (d) show $\tilde{C}(t)$ and $\tilde{C}_{mp}(t)$ computed based on the optimal time window pair, respectively.

The results of the time correlation functions with the optimal time window pair of T and T_b are presented in Figs. 3(c) and 3(d). As expected, the two-point time correlation function $\tilde{C}(t)$ and the multi-point time correlation function $\tilde{C}_{mp}(t)$ are observed to be non-zero. For an initial time t, $\tilde{C}(t)$ exhibits a positive correlation with values exceeding 0.02, persisting with a relaxation time of up to 10 days. The multi-point correlation function $\tilde{C}_{mp}(t)$ generally increases with time t, despite minor fluctuations. The ratio $\tilde{C}_{mp}(17)/\tilde{C}_{mp}(1)$ quantifies the enhancement of the multi-point time correlation and is found to be significantly greater than 4.

In practical applications, one may exploit the time correlation to develop profitable investment strategies and managing risks in financial dynamic systems.[11,12,45] Beyond finance systems, it also finds utility in predicting climate change[13] and understanding other intricate dynamic phenomena.[10] Therefore, our standardized algorithm provides a systematic framework for investigating dynamic behaviors across diverse fields.

4. Conclusion

In this paper, a standardized algorithm is established for computing time correlation functions in non-stationary states. By considering the non-stationary effects, the algorithm integrates both two-point and multi-point correlation functions into a unified framework. Applications to complex financial systems illustrate its robustness and generalizability, and researchers can leverage this methodology to investigate other complex dynamic systems in non-stationary states.

Program availability

The source code is openly available in the Science Data Bank at https://www.doi.org/10.57760/sciencedb.21039.

Acknowledgments

Project supported by the Postdoctoral Fellowship Program of China Postdoctoral Science Foundation (Grant No. GZC20231050), the National Natural Science Foundation of China (Grant Nos. 12175193 and 11905183), and the 13th Five-year plan for Education Science Funding of Guangdong Province (Grant No. 2021GXJK349).

References

[1] Kwapień J and Drożdż S 2012 *Phys. Rep.* **515** 115
[2] Young J G, St O G, Laurence E, Murphy C, Hébert D L and Desrosiers P 2019 *Phys. Rev. X* **9** 041056
[3] Papaefthymiou E S, Iordanou C and Papadopoulos F 2024 *Phys. Rev. Lett.* **132** 257401
[4] Wang J, Zhang Y J, Xu C, Li J, Sun J, Xie J, Feng L, Zhou T and Hu Y 2024 *Nat. Commun.* **15** 2849
[5] Plerou V, Gopikrishnan P, Rosenow B, Amaral L A N and Stanley H E 1999 *Phys. Rev. Lett.* **83** 1471
[6] Bouchaud J P, Matacz A and Potters M 2001 *Phys. Rev. Lett.* **87** 228701
[7] Wang Y J, Cheng H, Edwards R L, He Y Q, Kong X G, An Z S, Wu J Y, Kelly M J, Dykoski C A and Li X D 2005 *Science* **308** 854
[8] Shen J and Zheng B 2009 *Europhys. Lett.* **86** 48005
[9] Jiang X F and Zheng B 2012 *Europhys. Lett.* **97** 48006
[10] Chen T T, Zheng B, Li Y and Jiang X F 2018 *New J. Phys.* **20** 073005
[11] Zhang J, Jin L F, Zheng B, Li Y and Jiang X F 2022 *Physica A* **589** 126615

[12] Zhang J, Zheng B, Jin L F, Li Y and Jiang X F 2024 *Chin. J. Phys.* **88** 756
[13] Zhang X, Huang T, Wang C P and Zeng C H 2023 *Physica A* **609** 128316
[14] Ivanov P C, Rosenblum M G, Peng C K, Mietus J, Havlin S, Stanley H E and Goldberger A L 1996 *Nature* **383** 323
[15] Amaral L A N, Goldberger A L, Ivanov P C and Stanley H E 1998 *Phys. Rev. Lett.* **81** 2388
[16] Mallika M C, Prabhaa S S, Asokan K, Kumar K S A, Ramamohan T R and Kumar K S 2021 *Phys. Rev. E* **104** 054217
[17] Podobnik B and Stanley H E 2008 *Phys. Rev. Lett.* **100** 084102
[18] Bassler K E, McCauley J L and Gunaratne G H 2007 *Proc. Natl Acad. Sci. USA* **104** 17287
[19] Qiu T, Zheng B and Chen G 2010 *New J. Phys.* **12** 043057
[20] Pan C P, Zheng B, Wu Y Z, Wang Y and Tang X W 2004 *Phys. Lett. A* **329** 130
[21] Cherstvy A G, Vinod D, Aghion E, Chechkin A V and Metzler R 2017 *New J. Phys.* **19** 063045
[22] Breakspear M 2017 *Nat. Neurosci.* **20** 340
[23] Laloux L, Cizeau P, Bouchaud J P and Potters M 1999 *Phys. Rev. Lett.* **83** 1467
[24] Plerou V, Gopikrishnan P, Rosenow B, Amaral L A N, Guhr T and Stanley H E 2002 *Phys. Rev. E* **65** 066126
[25] Qiu T, Zheng B, Ren F and Trimper S 2006 *Phys. Rev. E* **73** 065103
[26] Nagy M, Akos Z, Biro D and Vicsek T 2010 *Nature* **464** 890
[27] Peng C K, Buldyrev S V, Havlin S, Simons M, Stanley H E and Goldberger A L 1994 *Phys. Rev. E* **49** 1685
[28] Buldyrev S V, Goldberger A L, Havlin S, Mantegna R N, Matsa M E, Peng C K, Simons M and Stanley H E 1995 *Phys. Rev. E* **51** 5084
[29] Hu K, Ivanov P C, Chen Z, Carpena P and Stanley H E 2001 *Phys. Rev. E* **64** 011114
[30] Chen Z, Ivanov P C, Hu K and Stanley H E 2002 *Phys. Rev. E* **65** 041107
[31] Ma Q D Y, Bartsch R P, Bernaola-Galván P, Yoneyama M and Ivanov P C 2010 *Phys. Rev. E* **81** 031101
[32] Zhou W X 2008 *Phys. Rev. E* **77** 066211
[33] Podobnik B, Jiang Z Q, Zhou W X and Stanley H E 2011 *Phys. Rev. E* **84** 066118
[34] Zebende G F 2011 *Physica A* **390** 614
[35] Oświeçimka P, Drożdż S, Forczek M, Jadach S and Kwapień J 2014 *Phys. Rev. E* **89** 023305
[36] Kwapień J, Oświeçimka P and Drożdż S 2015 *Phys. Rev. E* **92** 052815
[37] French K R, Schwert G W and Stambaugh R F 1987 *J. Financ. Econ.* **19** 3
[38] Bekaert G and Wu G J 2000 *Rev. Financ. Stud.* **13** 1
[39] Engle R F 1982 *Econometrica* **50** 987
[40] Challet D and Zhang Y C 1997 *Physica A* **246** 407
[41] Fehr E and Fischbacher U 2003 *Nature* **425** 785
[42] Camerer C F and Fehr E 2006 *Science* **311** 47
[43] Chakraborti A, Toke I M, Patriarca M and Abergel F 2011 *Quant. Financ.* **11** 1013
[44] Chen J J, Tan L and Zheng B 2015 *Sci. Rep.* **5** 8399
[45] Li Y, Zheng B, Chen T T and Jiang X F 2017 *Plos One* **12** 12
[46] The website of the dataset is https://cn.investing.com
[47] Liu Y, Gopikrishnan P, Cizeau, Meyer, Peng and Stanley H E 1999 *Phys. Rev. E* **60** 1390
[48] Menkhoff L 2010 *J. Bank. Fin.* **34** 2573