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Coherent feedback ground-state cooling of mechanical resonators assisted by a quantum well

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We theoretically investigate a cooling scheme assisted by a quantum well (QW) and coherent feedback within a hybrid optomechanical system. Although the exciton mode in the QW and the mechanical resonator (MR) are initially uncoupled, their interaction via the microcavity field leads to an indirect exciton-mode–mechanical-mode coupling. The coherent feedback loop is applied by feeding back a fraction of the output field of the cavity through a controllable beam splitter to the cavity's input mirror. It is shown that the cooling capability is enhanced by effectively suppressing the Stokes process through coupling with the QW. Furthermore, the effect of the anti-Stokes process is enhanced through the application of the coherent feedback loop. This particular system configuration enables cooling of the mechanical resonator even in the unresolved sideband regime (USR). This study has some important guiding significance in the field of quantum information processing.

Keywords: ground-state cooling, quantum well, coherent feedback, optomechanical system

PACS: 37.10.De, 37.10.Gh, 42.65.Tg, 42.50.Pq

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1. Introduction

Cavity optomechanics is an interdisciplinary field that explores the intrinsic interaction between electromagnetic radiation and mechanical resonators (MRs).[1–3] Cavity optomechanics holds significant potential for a variety of applications. In the aspect of applications, it can be used for high-sensitivity measurement,[4,5] to realize quantum state transfer between different modes[6] and to achieve quantum information processing[7–9] and quantum communication in optical storage.[10,11] All of the above applications require the MR to be cooled to the ground state to suppress unwanted thermal effects.[12–17] Therefore, improving the cooling process and suppressing the heating process are the main research topics for realizing the quantum effect.

At present, a range of methods for achieving ground-state cooling have been proposed, including sideband cooling[18–24] and feedback cooling.[25–30] Among these methods, the traditional sideband cooling scheme needs to satisfy the condition of the resolved sideband, which is hard to achieve in experiments.[31–34] Therefore, it is essential to develop appropriate cooling methods that can effectively operate in the unresolved sideband regime (USR), i.e., the decay rate of the cavity is larger than the mechanical frequency.

Assisted cooling is one of the commonly used methods. In such methods, cooling is achieved by coupling additional quantum systems to the optomechanical system. This coupling modulates the cavity's optical response, thereby enhancing the cooling effect and inhibiting the heating process.[35–43] Guo *et al.*[36] have successfully improved the cooling process by introducing coupling between two optomechanical cavities. Liu *et al.*[40] have achieved ground-state cooling by establishing coupling between the optomechanical cavity and a two-level atomic ensemble. In this study, we focus on the ground-state cooling of a micromechanical mirror that incorporates a quantum well (QW) with a lower exciton decay rate, situated within an optomechanical cavity. This type of solid-state system possesses distinct advantages in comparison to atomic cavity systems.[44–47] The tunable emission frequency, stable position and the possibility of integrating with cavities and waveguides through advanced semiconductor fabrication methods render it an exceptional tool for delving deeper into optomechanical research.[48–55]

Feedback cooling techniques are employed to realize ground-state cooling. These techniques are mainly categorized into two types: measurement-based feedback (MBF)[56–63] and coherent feedback.[64–67] MBF generates control signals by monitoring the quantum system, but it increases the control complexity of the quantum system due to the back reaction of the measurement process. Additionally, a quantum filter is required to extract important data, thus weakening the coherence of the quantum network.[68–70] In contrast, coherent feedback control avoids the need for measurements and uses an additional quantum system as a controller, which makes implementation simpler.[64,65] Coherent feedback has been shown to

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effectively enhance cooling performance in both the resolved sideband regime[71] and the unresolved sideband.[72]

Motivated by these developments, we present a coherent feedback cooling scheme assisted by a QW. The interaction through the microcavity field between the exciton mode in the QW and the mechanical mode of the system leads to indirect coupling. Compared with the traditional optomechanical scheme,[18–30] this scheme exploits the benefits of the aforementioned methods, improves the cooling capability of the system and approximates the MR closer to its ground state. The incorporation of a QW introduces quantum interference, thereby modulating the cavity profile. This interference is instrumental in the suppression of heating processes and the concurrent enhancement of cooling processes. The introduction of a coherent feedback loop enhances both the anti-Stokes process and the cooling effect, which cannot be achieved by simply adding an extra cavity.

The structure of the paper is as follows. In Section 2, we illustrate the system model, introduce the Hamiltonian equations for the system and derive expressions for the optical force noise spectrum and the final phonon number. In Section 3, the effect of the QW and coherent feedback on the cooling process is investigated through simulation results and discussed in detail. The results and discussions are given in Section 4.

2. System model and Hamiltonian

As shown in Fig. 1, the system we consider here comprises a single-sided Fabry–Pérot cavity with a vibrating end mirror, an asymmetric beam splitter, a laser pump, and a single QW, positioned midway within the optical cavity defined by two mirrors. The cavity is driven by the transmitted portion of the input laser, which enters through an asymmetric beam splitter. The exciton mode within the QW and the mechanical mode of the system are indirectly coupled through their interaction with the microcavity field. The output field, after being totally reflected by mirror M, sends a part back into the cavity via a beam splitter, thus establishing a coherent feedback loop. The beam splitter can be characterized using transmission coefficient, t, and reflection coefficient, r, with the conditions $t, r > 0$ and $t^2 + r^2 = 1$ being met. The strength of the feedback loop can be modulated by altering the reflection coefficient r of the beam splitter. Due to the low density of excitons, exciton–exciton scattering can be ignored, which allows us to linearize the interaction between the exciton mode and the cavity optical mode for simplicity.[45]

The total system can be described by the Hamiltonian ($\hbar = 1$)[44,72]

$$\begin{aligned} H = {} & \omega_a a^\dagger a + \omega_d d^\dagger d + \omega_{\rm m} b^\dagger b - g_{\rm om} a^\dagger a (b^\dagger + b) \\ & + g(a^\dagger d + a d^\dagger) + {\rm i}t(\Omega a^\dagger {\rm e}^{-{\rm i}\omega_{\rm L} t} - \Omega^* a {\rm e}^{{\rm i}\omega_{\rm L} t}), \end{aligned} \tag{1}$$

where the first three terms describe the free Hamiltonian of the system. Here a, b and d are the annihilation operators of the cavity optical mode, the MR and the QW exciton mode, with ω_a, $\omega_{\rm m}$, and ω_d being their frequencies, respectively. Given that we are studying with quasi-resonant coherent excitation, we can disregard the higher-lying exciton states within the QW. The fourth term accounts for the optomechanical interaction, where $g_{\rm om}$ denotes the single-photon optomechanical coupling strength. The coupling between the cavity mode and the exciton mode is described by the fifth term with g being the interaction strength. The last term represents the Hamiltonian of laser driving, where $\Omega(\omega_{\rm L})$ denotes the driving strength (frequency) of the optical fundamental mode.

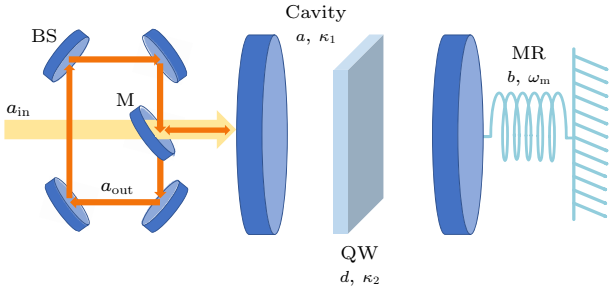


Fig. 1. A schematic description of an optomechanical system with a quantum well in the cavity. The coherent feedback loop is established using a controllable beam splitter (BS).

The Hamiltonian of the system, in the frame rotating at the input laser frequency $\omega_{\rm L}$, reads

$$\begin{aligned} H' = {} & \Delta_a^0 a^\dagger a + \Delta_d d^\dagger d + \omega_{\rm m} b^\dagger b - g_{\rm om} a^\dagger a (b^\dagger + b) \\ & + g(a^\dagger d + a d^\dagger) + {\rm i}t(\Omega a^\dagger - \Omega^* a), \end{aligned} \tag{2}$$

where $\Delta_a^0 = \omega_a - \omega_{\rm L}$ and $\Delta_d = \omega_d - \omega_{\rm L}$ describe the detuning of the cavity mode and the exciton mode, respectively.

By taking into account the feedback loop and the appropriate damping and noise operators, we obtain the quantum Langevin equations of the system, which are expressed as

$$\begin{aligned} \dot{a} = {} & -{\rm i}[\Delta_a^0 - g_{\rm om}(b^\dagger + b)]a - {\rm i}gd + t\Omega - \kappa_1 a \\ & + \sqrt{2\kappa_1}(t a_{\rm in} + r a_{\rm out}), \\ \dot{d} = {} & -{\rm i}\Delta_d d - {\rm i}ga - \kappa_2 d + \sqrt{2\kappa_2} d_{\rm in}, \\ \dot{b} = {} & -{\rm i}\omega_{\rm m} b + {\rm i}g_{\rm om} a^\dagger a - \gamma_{\rm m} b + \sqrt{2\gamma_{\rm m}} b_{\rm in}, \end{aligned} \tag{3}$$

where $a_{\rm out}$ represents the output field of the optomechanical cavity, $t a_{\rm in}$ denotes the transmitted component of the input noise of the cavity and $r a_{\rm out}$ signifies the portion of the output field that is reflected by the beam splitter. In addition, κ_1, κ_2 and $\gamma_{\rm m}$ are the dissipation rates of the optical mode, the exciton mode and the MR, respectively. $a_{\rm in}$, $d_{\rm in}$ and $b_{\rm in}$ are the noise operators associated with the dissipations, which satisfy

$$\begin{aligned} & \langle a_{\rm in}(t) a_{\rm in}^\dagger(t') \rangle = \langle d_{\rm in}(t) d_{\rm in}^\dagger(t') \rangle = \delta(t - t'), \\ & \langle a_{\rm in}^\dagger(t) a_{\rm in}(t') \rangle = \langle d_{\rm in}^\dagger(t) d_{\rm in}(t') \rangle = 0, \\ & \langle b_{\rm in}(t) b_{\rm in}^\dagger(t') \rangle = (n_{\rm th} + 1)\delta(t - t'), \\ & \langle b_{\rm in}^\dagger(t) b_{\rm in}(t) \rangle = n_{\rm th} \delta(t - t'), \end{aligned} \tag{4}$$

and $n_{\rm th}=1/(\mathrm{e}^{\hbar\omega_{\rm b}/k_{\rm B}T}-1)$ is the mean thermal phonon number, where T represents the environmental temperature and $k_{\rm B}$ is the Boltzmann constant. In addition, the relationship involving $a_{\rm out}$, the cavity mode and the input noise is given by

$$a_{\rm out}=\sqrt{2\kappa_1}a-ta_{\rm in}, \tag{5}$$

which is different from the standard input–output relation due to the presence of the beam splitter. By substituting Eq. (6) into Eq. (5), the Langevin equations become

$$\begin{aligned}\dot{a}=&-\mathrm{i}[\Delta_a^0-g_{\rm om}(b^\dagger+b)]a-\mathrm{i}gd+t\Omega-\kappa_{\rm eff}a\\&+\sqrt{2\kappa_1}t(1-r)a_{\rm in},\\ \dot{d}=&-\mathrm{i}\Delta_d d-\mathrm{i}ga-\kappa_2 d+\sqrt{2\kappa_2}d_{\rm in},\\ \dot{b}=&-\mathrm{i}\omega_{\rm m}+\mathrm{i}g_{\rm om}a^\dagger a-\gamma_{\rm m}b+\sqrt{2\gamma_{\rm m}}b_{\rm in},\end{aligned} \tag{6}$$

where $\kappa_{\rm eff}=\kappa_1(1-2r)$ is the effective decay rate of the cavity.

In the case of the optomechanical cavity under strong drive, Eq. (3) can be linearized by expressing the operators as the sum of their steady-state values and small fluctuations, i.e., $a=a_{\rm s}+\delta a$, $d=d_{\rm s}+\delta d$, and $b=b_{\rm s}+\delta b$. The steady-state solutions of Eq. (6) are obtained as

$$\begin{aligned}a_{\rm s}&=\frac{t\Omega}{\mathrm{i}\Delta_a'+\kappa_{\rm eff}+\frac{g^2}{\mathrm{i}\Delta_d+\kappa_2}},\\ d_{\rm s}&=\frac{-ga_{\rm s}}{\mathrm{i}\Delta_d+\kappa_2},\quad b_{\rm s}=\frac{\mathrm{i}g_{\rm om}|a_{\rm s}|^2}{\mathrm{i}\omega_{\rm m}+\gamma_{\rm m}},\end{aligned} \tag{7}$$

where $\Delta_a=\Delta_a^0-g_{\rm om}(b_{\rm s}+b_{\rm s}^*)$ is the frequency detuning of the cavity field caused by the input laser. Under the condition of $a_{\rm s}\gg 1$, the linearized system Hamiltonian can be inferred as

$$\begin{aligned}H_{\rm L}=&\Delta_a a^\dagger a+\Delta_d d^\dagger d+\omega_{\rm m}b^\dagger b+g(a^\dagger d+ad^\dagger)\\&-G(b^\dagger+b)(a^\dagger+a),\end{aligned} \tag{8}$$

where $G=g_{\rm om}a_{\rm s}$ is the enhanced optomechanical coupling strength for the linearized system. For simplicity, we can omit the fluctuation operator notation "δ" for all terms. Consequently, the following equations of motion for the linearized system are derived:

$$\begin{aligned}\dot{a}=&-\mathrm{i}\Delta_a a+\mathrm{i}G(b^\dagger+b)-\mathrm{i}gd+t\Omega-\kappa_{\rm eff}a\\&+\sqrt{2\kappa_1}t(1-r)a_{\rm in},\\ \dot{d}=&-\mathrm{i}\Delta_d d-\mathrm{i}ga-\kappa_2 d+\sqrt{2\kappa_2}d_{\rm in},\\ \dot{b}=&-\mathrm{i}\omega_{\rm m}b+\mathrm{i}G(a^\dagger+a)-\gamma_{\rm m}b+\sqrt{2\gamma_{\rm m}}b_{\rm in}.\end{aligned} \tag{9}$$

Among these, since the decay rate is always positive, it is clear that $r\le 0.5$.

In the frequency domain, the quantum Langevin equations are presented as

$$\begin{aligned}-\mathrm{i}\omega a(\omega)=&-\mathrm{i}\Delta_a a(\omega)-\mathrm{i}gd(\omega)-\kappa_{\rm eff}a(\omega)\\&+\sqrt{2\kappa_1}t(1-r)a_{\rm in}(\omega),\\ -\mathrm{i}\omega d(\omega)=&-\mathrm{i}\Delta_d d(\omega)-\mathrm{i}ga(\omega)-\kappa_2 d(\omega)+\sqrt{2\kappa_2}a_{\rm in}(\omega),\end{aligned} \tag{10}$$

$a(\omega)$ is calculated as

$$a(\omega)=\frac{1}{A(\omega)}\left[\frac{-\mathrm{i}g\sqrt{2\kappa_2}}{\kappa_2-\mathrm{i}(\omega-\Delta_d)}d_{\rm in}(\omega)+\sqrt{2\kappa_1}t(1-r)a_{\rm in}(\omega)\right]. \tag{11}$$

To investigate the cooling characteristics, an analysis of the fluctuation spectrum of the optical force is essential. The last term in Eq. (8) describes the linear optomechanical coupling interaction and the effective optical force applied on the movable mirror. In the weak coupling regime, we can get the optical force on the MR $F=a^\dagger+a$. Therefore, the optical force spectrum $S_{\rm FF}(\omega)$ can be obtained as

$$S_{\rm FF}(\omega)=\frac{2}{A(\omega)A^*(\omega)}\left[\frac{g^2\kappa_2}{\kappa_2^2+(\omega-\Delta_d)^2}+\kappa_1 t^2(1-r)^2\right], \tag{12}$$

where

$$A(\omega)=\kappa_{\rm eff}-\mathrm{i}(\omega-\Delta_a)+\frac{g^2}{\kappa_2-\mathrm{i}(\omega-\Delta_d)}. \tag{13}$$

The cooling rate is an important index for further ground-state cooling research on the MR. According to Fermi's golden rule,[73,74] the cooling and heating rates are given by $A_-=S_{\rm FF}(\omega_{\rm m})x_{\rm ZPF}^2$ and $A_+=S_{\rm FF}(-\omega_{\rm m})x_{\rm ZPF}^2$, corresponding to the ability to absorb and emit a phonon by the cavity, respectively. The zero-point fluctuation $x_{\rm ZPF}$ is defined as $x_{\rm ZPF}=\sqrt{\hbar/(2m_{\rm eff}\omega_{\rm m})}$ where $m_{\rm eff}$ denotes the effective mass of the mechanical mode. These rates subsequently dictate the net cooling rate $\Gamma=g^2(A_--A_+)$ and the quantum cooling limit $n_{\rm c}=A_+/(A_--A_+)$.

The final phonon number can be written as

$$n_{\rm f}=\frac{\gamma_{\rm m}n_{\rm th}+\Gamma n_{\rm c}}{\gamma_{\rm m}+\Gamma}, \tag{14}$$

and this value is mainly associated with $S_{\rm FF}(\pm\omega_{\rm m})$, obviously. To cool the MR close to its ground state, it is necessary to control the fluctuation spectrum $S_{\rm FF}(\omega)$ of the optical force, i.e., enhance the positive-frequency component $S_{\rm FF}(+\omega_{\rm m})$ and inhibit the negative-frequency component $S_{\rm FF}(-\omega_{\rm m})$.

3. Results and discussion

In order to achieve the optimal cooling effect, we next investigate the dependence of the critical indicators of ground-state cooling on various parameters, e.g., the coupling coefficient between the QW and the optical cavity, and the dissipation of the optical cavity. It is worthwhile to mention that in the following simulations we have used experimentally accessible parameters inspired from some experimental works.[44,47]

3.1. Dependence of the ground-state cooling on the quantum well

Here, we study the dependence of ground-state cooling on the QW. First, we investigate the dependence of the QW and the ground-state cooling of MRs without feedback ($r = 0$) and plot the optical fluctuation spectrum as a function of frequency ω for different values of the optical coupling coefficient g in Fig. 2. In the absence of coupling between the QW and the optical cavity, the optical force fluctuation spectrum $S_{\rm FF}(\omega)$ exhibits a Lorentzian profile with a single peak at $\omega = \Delta_a$. After establishing the coupling ($g > 0$), this peak bifurcates into two narrower peaks, with a distinct dip emerging between them, which can be pictured as an outcome of quantum interference. This suggests that the presence of the QW induces a modulation of the cavity profile, which results in asymmetry between the cooling and heating processes. The physical reason for these phenomena is similar to that of the two-photon resonance. This resonance gives rise to a quantum destructive interference effect, thereby effectively suppressing the heating process of the MR. Therefore, the presence of the QW can be harnessed to suppress the heating effect while enhancing the cooling effect.

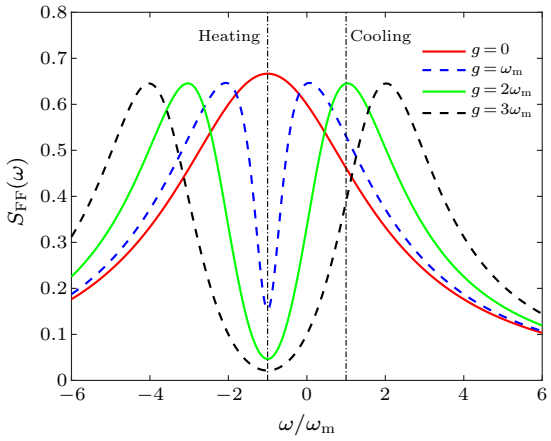


Fig. 2. The noise spectrum $S_{\rm FF}(\omega)$ (in arb. units) versus the normalized frequency $\omega/\omega_{\rm m}$ for different optical coupling strengths. Other parameters are $\Delta_a = \Delta_d = -\omega_{\rm m}$, $\kappa_1 = 3\omega_{\rm m}$ and $\kappa_2 = 0.1\omega_{\rm m}$.

Furthermore, it is found from Fig. 2 that the positions of the peaks of $S_{\rm FF}(\omega)$ depend significantly on the optical coupling coefficient g. In fact, the positions of the two peaks can be determined by diagonalizing the optical Hamiltonian. To optimize the cooling transition rate, the fluctuation spectrum value of cooling $S_{\rm FF}(+\omega_{\rm m})$ should be maximized, i.e., fix the center of the right-hand peak around $\omega = +\omega_{\rm m}$. Thus, we can obtain the optimal optical coupling coefficient as[36]

$$g = \sqrt{2\omega_{\rm m}(\omega_{\rm m} - \Delta_a)}. \tag{15}$$

Assuming the coherent feedback does not work in the system (i.e., $r = 0$), we plot the changing properties of spectrum $S_{\rm FF}(\omega)$ as a function of $\omega/\omega_{\rm m}$ in four different decay rates κ_2 of the QW under the optimal conditions that $\Delta_d = -\omega_{\rm m}$ and the optical coupling g satisfies Eq. (15). To achieve ground-state cooling, the amplitude of the dip in the fluctuation spectrum must approach zero even in the optimal optical coupling coefficient, while the value at the right peak should be maximized. It can be observed from Fig. 3 that the amplitude of the right peak of the optical fluctuation spectrum $S_{\rm FF}(\omega)$ gradually increases with the decrease of κ_2, while the amplitude of the left peak gets closer and closer to zero. The feasibility of this cooling approach is facilitated by the interaction between the high-Q QW and the mechanical mode mediated by the cavity field. This implies that the heating and cooling rates of an MR can be controlled by choosing the appropriate QW decay rate, κ_2, and a smaller decay rate is preferable for the cooling of the MR in this system. In particular, achieving a smaller decay rate within the feedback system is crucial as it enables the protocol's validity to be extended across a broader parameter range.

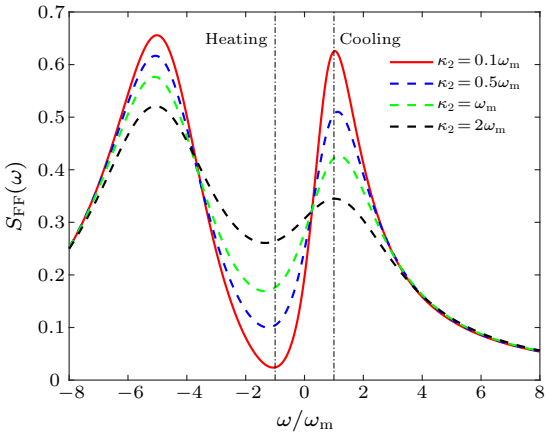


Fig. 3. The noise spectrum $S_{\rm FF}(\omega)$ (in arb. units) versus the normalized frequency $\omega/\omega_{\rm m}$ for different decay rates of the exciton mode and no feedback ($r = 0$). The other parameters are taken as $\Delta_a = -3\omega_{\rm m}$, $\Delta_d = -\omega_{\rm m}$, $\kappa_1 = 3\omega_{\rm m}$ and $g = 2\sqrt{2}\omega_{\rm m}$.

As illustrated in Fig. 4, we further investigate the final phonon number $n_{\rm f}$ and the quantum cooling limit $n_{\rm c}$ as functions of g in the case of optimal conditions and reflection coefficient $r = 0$, respectively. The quantum cooling limit $n_{\rm c}$ of the incoherent feedback optomechanical system declines within a range $0 < g/\omega_{\rm m} < 100$ and gradually tends to zero. Instead, the behavior of the final phonon number $n_{\rm f}$ changes. The minimum of $n_{\rm f}$ decreases with g and achieves the minimum value in the vicinity of $g = 9\omega_{\rm m}$, then it experiences a slow increase. It is noted that under specific conditions, such as $g = 16\omega_{\rm m}$, the value of $n_{\rm f}$ can be less than 1. This proves that even in USR, efficient cooling of the MR to near its ground state can be achieved by incorporating a high-quality QW. As depicted in Fig. 4, the optimal values for the frequency detuning of the cavity and the optical coupling strength determined to be $\Delta_{a,\rm opt} = -39.5\omega_{\rm m}$ and $g_{\rm opt} = 9\omega_{\rm m}$, while the final phonon number is $n_{\rm f} \approx 0.01$.

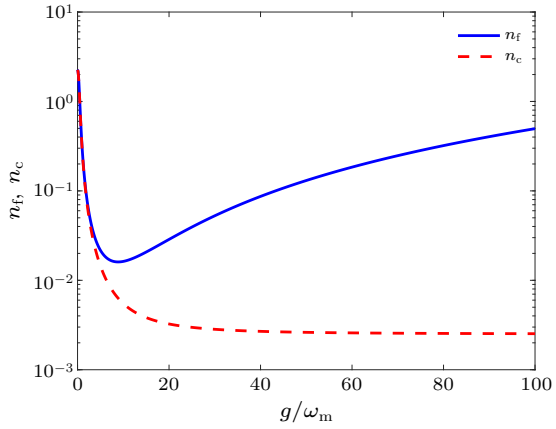


Fig. 4. The mean phonon number n_f and the quantum cooling limit n_c as functions of the dimensionless optimal optical coupling coefficient g without feedback ($r = 0$). The other parameters are taken as $\Delta_d = -\omega_m$, $\kappa_1 = 3\omega_m$, $\kappa_2 = 0.1\omega_m$, $n_{th} = 312$ and $\gamma_m = 1/8 \times 10^4$.

3.2. The impact of coherent feedback on the ground-state cooling

In the above discussions, we note that the incorporation of a QW contributes to modulating the cooling effect of the optomechanical system. In this section, we investigate the impact of coherent feedback on the cooling process of the system that incorporates a QW. To assess the final cooling of the MR, we utilize a set of experimentally viable parameters.[36,72]

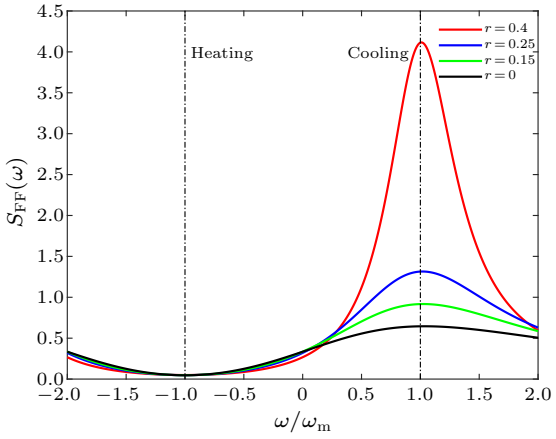


Fig. 5. The noise spectrum $S_{FF}(\omega)$ (in arb. units) versus the normalized frequency ω/ω_m for different values of r. The other parameters are taken as $\Delta_a = \Delta_d = -\omega_m$, $\kappa_1 = 3\omega_m$, $\kappa_2 = 0.1\omega_m$ and $g = 2\omega_m$.

According to Fig. 5, we show the changing properties of spectrum $S_{FF}(\omega)$ under various values of r in the case of optimal conditions. It is observed that an increase in r leads to a rise in the right peak amplitude, whereas the amplitude of the left dip remains relatively stable. In addition, the positions of the peak and the dip remain unaffected since they are determined by the optical Hamiltonian. This is due to the fact that coherent feedback maintains the internal dynamics of the system. As previously discussed, optimal cooling is achieved by maintaining small values of $S_{FF}(-\omega_m)$ and large values of $S_{FF}(+\omega_m)$. The results indicate that the difference between $S_{FF}(-\omega_m)$ and $S_{FF}(+\omega_m)$ could be greatly enhanced by adding a coherent feedback loop to the QW-assisted optomechanical system.

Figure 6 displays the variation of the net cooling rate with respect to r. It is observed that, as r approaches its maximum value, Γ increases significantly because $S_{FF}(+\omega_m) - S_{FF}(-\omega_m)$ decreases and $S_{FF}(+\omega_m)$ increases simultaneously. It is noteworthy that for various values of the effective detuning of the optomechanical cavity mode Δ_a and the corresponding optical coupling strength g calculated using Eq. (15), the incorporation of a coherent feedback loop can further improve the cooling performance. Consequently, the proposed method is capable of reaching significantly lower energy states of the MR.

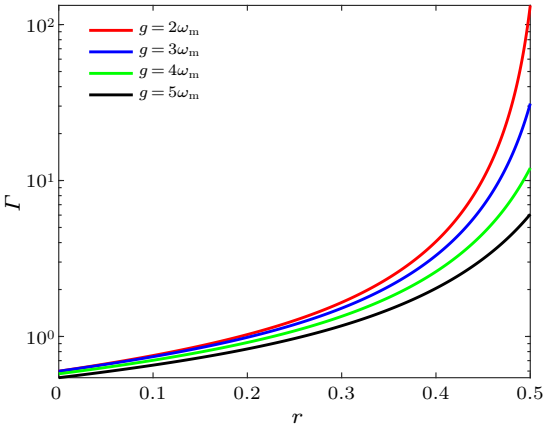


Fig. 6. The net cooling rate $\Gamma = A_- - A_+$ versus r with $g = 2\omega_m$ (the red solid curve), $g = 3\omega_m$ (the blue solid curve) and $g = 4\omega_m$ (the green solid curve) $g = 5\omega_m$ (the black solid curve). The other parameters are taken as $\Delta_d = -\omega_m$, $\kappa_1 = 3\omega_m$ and $\kappa_2 = 0.1\omega_m$.

Figure 7 illustrates the net cooling rate Γ of the system with the feedback coefficient r for the optimal parameters g_{opt} and $\Delta_{a,opt}$. It is observed that Γ increases exponentially as r approaches its maximum value. It can be explained that the incorporation of coherent feedback within the system introduces supplementary degrees of freedom that facilitate the cooling of the mechanical mode to levels surpassing those attainable without feedback. Therefore, our QW-assisted cooling scheme can enhance the cooling performance by incorporating coherent feedback technology.

The relationship between the final phonon number n_f and r for different values of g is depicted in Fig. 8. For each value of g, Δ_a is calculated using Eq. (15) to ensure that the optimal cooling condition is met. As anticipated, augmenting the feedback intensity, i.e., increasing the value of r, results in a reduction of n_f. As previously discussed, this enhancement widens the difference between $S_{FF}(-\omega_m)$ and $S_{FF}(+\omega_m)$, which subsequently improves the net cooling rate Γ while the cooling limit n_c is decreased. This phenomenon occurs due to the reduced dissipation of the optical cavity and the increased quality factor of the QW, which makes ground-state cooling more readily achievable. Here, the exciton mode of the QW and the

mechanical mode establish an indirect coupling due to their interaction with the cavity optical field. Moreover, it should be pointed out that the value of the final mean phonon number is $n_{\rm f} \approx 2.02$ without coherent feedback and the QW, i.e., $g = 0$ and $r = 0$. When $g = 0.4\omega_{\rm m}$, the final phonon number in the absence of the feedback loop is $n_{\rm f} = 1.26$. Both suggest that the MR does not reach the ground state. However, upon increasing r to 0.5, there is a significant reduction in $n_{\rm f}$ and the MR can be cooled more efficiently to the ground state.

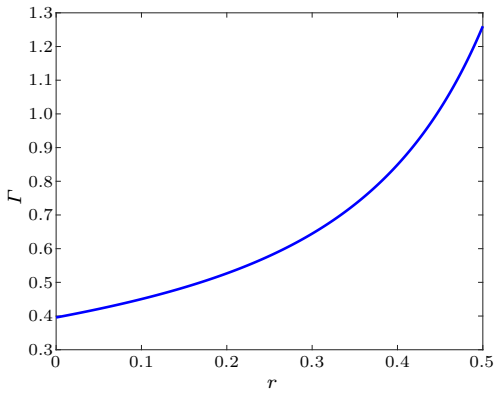


Fig. 7. The net cooling rate $\Gamma = A_- - A_+$ as functions of r with $g_{\rm opt} = 9\omega_{\rm m}$ and $\Delta_{a,{\rm opt}} = -39.5\omega_{\rm m}$. The values of the other parameters are set as $\Delta_d = -\omega_{\rm m}$, $\kappa_1 = 3\omega_{\rm m}$, $\kappa_2 = 0.1\omega_{\rm m}$, $n_{\rm th} = 312$ and $\gamma_{\rm m} = 1/8 \times 10^4$.

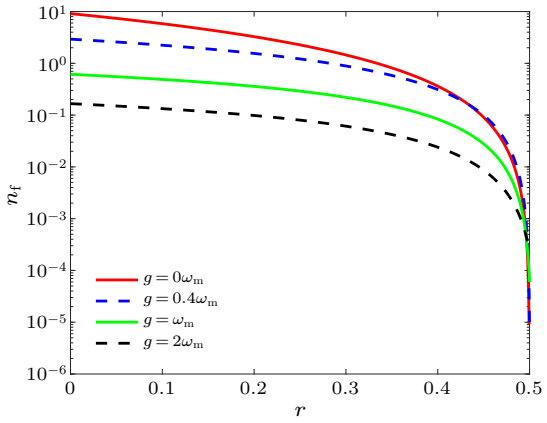


Fig. 8. The final mean phonon number $n_{\rm f}$ as a function of r with $g = 0\omega_{\rm m}$ (the red solid curve), $g = 0.4\omega_{\rm m}$ (the blue dotted curve), $g = \omega_{\rm m}$ (the green solid curve) and $g = 2\omega_{\rm m}$ (the black dotted curve). Here, $\Delta_d = -\omega_{\rm m}$, $\kappa_1 = 6\omega_{\rm m}$, $\kappa_2 = 0.1\omega_{\rm m}$ and $g = 2\omega_{\rm m}$.

As mentioned above, the incorporation of a coherent feedback loop into the QW-assisted optomechanical system markedly amplifies the anti-Stokes processes, resulting in a larger amplitude of the fluctuation spectrum at $\omega = +\omega_{\rm m}$. To further explore the impact of coherent feedback on the final phonon number and the cooling limit, the functional relationships of $n_{\rm f}$ and $n_{\rm c}$ with respect to the feedback coefficient r is depicted in Fig. 9. It can be distinctively seen that the values of both $n_{\rm f}$ and $n_{\rm c}$ change slightly as the feedback coefficient r varies from 0 to 0.5. The overall trend for both values is a decrease, indicating improved cooling efficiency for the MR. Specifically, the outcome without feedback is $n_{\rm f} \approx 0.022$ with $n_{\rm c} \approx 0.003$ for the given the specified parameters. In contrast, employing coherent feedback with $r = 0.5$ and the same parameters, the MR can be cooled to near the quantum limit, with final phonon numbers $n_{\rm f} \approx 0.013$ and $n_{\rm c} \approx 0.002$. This phenomenon can be explained by the physical mechanisms mentioned above, where the enhancement of the feedback coefficient results in an effective decay rate of the cavity that is lower than the effective decay rate without feedback. The presence of high-Q elements within the cavity modifies the noise spectrum, leading to asymmetric cooling and heating rates. Therefore, the coherent feedback approach offers an alternative way for quantum manipulation of the MR.

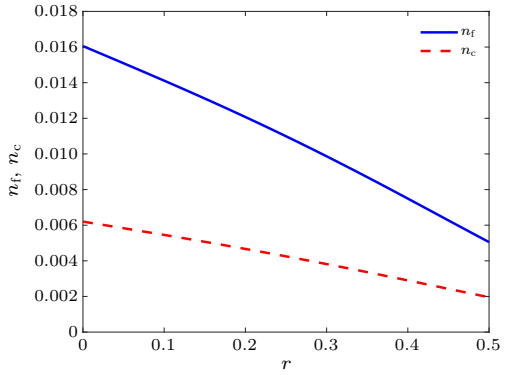


Fig. 9. The mean phonon number $n_{\rm f}$ and the quantum cooling limit $n_{\rm c}$ as functions of r with $g_{\rm opt} = 9\omega_{\rm m}$ and $\Delta_{a,{\rm opt}} = -39.5\omega_{\rm m}$. The blue solid curve shows $n_{\rm f}$. The red dotted curve denotes $n_{\rm c}$. The other parameters are taken as $\Delta_d = -\omega_{\rm m}$, $\kappa_1 = 3\omega_{\rm m}$, $\kappa_2 = 0.1\omega_{\rm m}$, $n_{\rm th} = 312$ and $\gamma_{\rm m} = 1/8 \times 10^4$.

4. Conclusion

In this study, we have proposed a coherent feedback scheme assisted by a QW to achieve the ground-state cooling of an MR. The coherent feedback loop is implemented by reflecting a portion of the optomechanical cavity's output field back to the fixed mirror using an adjustable beam splitter. The incorporation of the QW exciton mode introduces quantum interference, modulating the cavity response and effectively suppressing Stokes processes. Additionally, the effect of the anti-Stokes processes is enhanced by applying the coherent feedback loop. It is also observed that the cooling performance is enhanced with increased values of r and the optimal outcomes are achieved with maximum coherent feedback strength. Consequently, the proposed cooling scheme has the potential to enable ground-state cooling of an MR across a broader parameter range. The present results may provide a guideline for theoretical research on the cooling of MRs in cavity optomechanical systems.

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