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Periodic modulation of adiabatic dynamics in non-reciprocal Landau–Zener systems

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The control of adiabatic dynamics is essential for quantum manipulation. We investigate the effects of both periodic modulating field and linear sweeping field on adiabatic dynamics based on a non-reciprocal Landau–Zener model with periodic modulation. We obtain adiabatic phase diagrams in the (ω,δ) parameter space, where the adiabatic region is bounded by the modulating frequency ω greater than a critical value ω_c and the non-reciprocal parameter δ less than one. The results show that the adiabaticity of the system is not sensitive to the modulating amplitude. We find that the critical modulating frequency can be expressed as a power function of the modulating period number or the sweeping rate. Our findings suggest that one can change the adiabatic region or improve the adiabaticity by adjusting the parameters of both the modulating and the sweeping fields, which provides an effective means to flexibly control the adiabatic dynamics of non-reciprocal systems.

Keywords: adiabaticity, non-reciprocity, Landau–Zener tunneling, periodic modulation

PACS: 03.65.–w, 03.75.Lm, 74.50.+r

DOI: 10.1088/1674-1056/adab64 **CSTR:** 32038.14.CPB.adab64

1. Introduction

The Landau–Zener (LZ) model is one of the most important models in the two-level systems (TLS), which is universal and has been widely used in various fields of physics for decades.[1–7] It describes quantum tunneling between two levels in a system. Based on this model, a lot of explorations have been conducted, such as composite adiabatic passage techniques,[8] transition time,[9] quantum simulation,[10,11] and adiabatic-impulse approximation,[12] to name a few.

Non-reciprocity can result in many dynamical behaviors and interesting quantum phenomena. In recent years, researchers have begun to focus on non-reciprocal systems[13–17] and have paid much attention to the tunneling dynamics of the systems under periodic modulation. By adding a sine or cosine modulating field to the diagonal term[14,18–20] or non-diagonal term[21,22] of the TLS, different tunneling processes[1,23] and the effects of periodic modulation on the tunneling dynamics have been studied. Of course, both the diagonal and the non-diagonal terms of the Hamiltonian can also be modulated[24] and this dual periodic modulation will greatly enrich the dynamics of the system.

With advances in quantum science and technology, the adiabatic quantum evolution has been widely concerned and applied to many fields.[25–29] According to the adiabatic theorem, by controlling the Hamiltonian of the system, efficient state transition and problem solving can be achieved in quantum computing and other quantum technologies. Recently, adiabatic theory has been extended to the quantum systems with nonlinear interactions[5,30–33] and new properties of adiabatic evolution have been explored in non-Hermitian systems[34–38] including counterdiabatic driving,[39] adiabatic shortcut,[40,41] and Berry phase.[42]

In order to further understand the adiabatic dynamics of the non-reciprocal systems during tunneling process, in this paper we consider a non-reciprocal LZ model with periodic modulation. We explore the influence of both periodic modulating field and linear sweeping field on adiabatic region and adiabaticity of the system. We show the adiabatic phase diagrams in the (ω,δ) parameter space and find the adiabatic region can be determined by the modulating frequency ω greater than a critical value ω_c and the non-reciprocal parameter $\delta<1$. Moreover, we find that the critical modulating frequency can be expressed as a power function of the modulating period number or the sweeping rate and the adiabaticity of the system is not sensitive to the modulating amplitude. These findings allow us to change the adiabatic region or improve the adiabaticity by tuning the parameters of both the modulating and the sweeping fields, which provides an effective way to flexibly control the adiabatic tunneling dynamics of non-reciprocal systems.

The rest of the paper is structured as follows. In Section 2, we introduce a non-reciprocal LZ model with periodic

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modulation. In Section 3, we study the effects of both periodic modulation and linear sweeping on adiabatic tunneling dynamics. Finally, a summary is presented in Section 4.

2. Model

We consider a non-reciprocal LZ model with periodic modulation, its Hamiltonian reads

$$\boldsymbol{H}(t)=\begin{pmatrix}\gamma/2 & \nu/2\\ (\nu/2)[1-(\delta+A\sin(\omega t))] & -\gamma/2\end{pmatrix}, \tag{1}$$

where δ represents the non-reciprocal parameter, which leads to the non-Hermiticity of the system. We apply a monochromatic field with amplitude A and frequency ω to periodically modulate the non-reciprocal parameter. ν denotes the coupling between two levels. For convenience, we set $\nu=1$ in the following calculations. γ is the level bias. The LZ tunneling occurs when γ changes linearly with time, i.e., $\gamma=\alpha t$, where α is the sweeping rate.

The dynamics of the system is governed by the dimensionless Schrödinger equation ($\hbar=1$)

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}a\\ b\end{pmatrix}=\boldsymbol{H}(t)\begin{pmatrix}a\\ b\end{pmatrix}, \tag{2}$$

where a and b are the probability amplitudes.

In this paper, we assume that the system is initially prepared in the lower or upper level, i.e.,

$$[a(t\to-\infty),\, b(t\to-\infty)]^{\mathrm{T}}=(1,0)^{\mathrm{T}}\quad\text{or}$$
$$[a(t\to-\infty),\, b(t\to-\infty)]^{\mathrm{T}}=(0,1)^{\mathrm{T}}.$$

Due to the non-Hermiticity of the Hamiltonian (1), the time evolution is no longer unitary and the total population $N=n_a+n_b$ (with $n_a=|a|^2$ and $n_b=|b|^2$) is not a conserved quantity. Accordingly, the tunneling probability from the lower level to the upper level or *vice versa* can be defined as

$$P_a\equiv\frac{n_a(t\to+\infty)}{N(t\to+\infty)}\quad\text{or}\quad P_b\equiv\frac{n_b(t\to+\infty)}{N(t\to+\infty)}. \tag{3}$$

In the absence of non-reciprocity and periodic modulation, i.e., $\delta=A=0$, the Hamiltonian (1) simplifies to the standard LZ model and the tunneling probability is given by the famous formula[43]

$$P_{\mathrm{LZ}}=\exp\left(-\frac{\pi\nu^2}{2\alpha}\right). \tag{4}$$

Equation (4) implies that a complete tunneling (i.e., $P_{\mathrm{LZ}}=1$) will occur in the sudden limit (i.e., $\alpha\to\infty$) and that no tunneling (i.e., $P_{\mathrm{LZ}}=0$) will occur in the adiabatic limit (i.e., $\alpha\to0$). We expect this result to be substantially modified by non-reciprocity and periodic modulation.

3. Results and analysis

3.1. Energy-level structure and tunneling dynamics

We can obtain the instantaneous eigenenergies of the Hamiltonian (1) as follows:

$$\varepsilon_{\pm}(t)=\pm\frac{1}{2}\sqrt{(\alpha t)^2+\nu^2[1-(\delta+A\sin\omega t)]}, \tag{5}$$

which determine the adiabatic levels of the system. In the following, we discuss the influence of the non-reciprocal parameter and periodic modulation on the energy-level structure of the system.

The non-Hermiticity of the system will cause the energy eigenvalues to be complex. For convenience, in Fig. 1 we plot the real [i.e., $\varepsilon_{\mathrm{r}}\equiv\mathrm{Re}(\varepsilon_{\pm})$, see Fig. 1(a)] and imaginary [i.e., $\varepsilon_{\mathrm{i}}\equiv\mathrm{Im}(\varepsilon_{\pm})$, see Fig. 1(b)] parts of the energy eigenvalues for different non-reciprocal parameters, respectively. Here we fix the parameters of the periodic modulating and linear sweeping fields, i.e., $A=0.4$, $\omega=20$, and $\alpha=1$. We see that when $\delta<1$ (e.g., $\delta=0$ or $\delta=0.5$) the real parts of adiabatic levels show an avoided crossing structure and the imaginary parts are always zero. In this case, the effect of periodic modulation is only to make the smooth energy levels fluctuate somewhat. However, when $\delta\geq1$ (e.g., $\delta=1$ or $\delta=3$), the exceptional points (where the upper and lower levels begin to cross) appear and the adiabatic levels form a number of ring structures. This multi-loop structure is different from the energy-level structure without periodic modulation, which implies that the periodic modulation can effectively change the topology of energy-level structure. Furthermore, we find that the number of rings formed by the imaginary part is one more than that of rings formed by the real part. For example, the numbers are 4 and 3 for $\delta=1$, respectively. The reason is that the regions where the imaginary part is zero exactly correspond to the regions where the real part is nondegenerate, and *vice versa*.

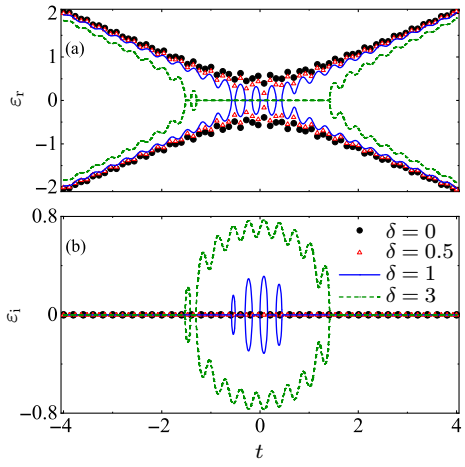


Fig. 1. Adiabatic levels for different non-reciprocal parameters. We plot the real (a) and imaginary (b) parts of the energy eigenvalues, respectively. We set $\alpha=1$, $A=0.4$, and $\omega=20$.

To explore the effects of periodic modulation on the tunneling dynamics, we evolve the system according to Eq. (2) for a sufficiently long time (e.g., t from $-1000T$ to $1000T$ with $T = 2\pi/\omega$ being the modulating period) and calculate the final tunneling probability from Eq. (3). The results (see scatters) for different non-reciprocal parameters and the sweeping rates with $A = 0.5$ and $\omega = 10$ are shown in Fig. 2. For comparisons, we also give the corresponding results without periodic modulation (see solid lines).[5]

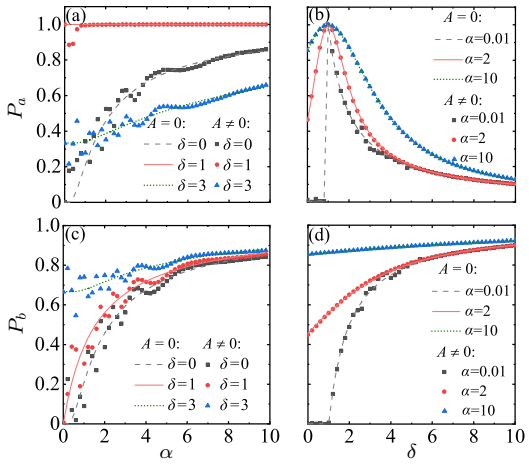


Fig. 2. Tunneling probability of the non-reciprocal LZ model with periodic modulation (scatters). Panels (a) and (c) show the tunneling probability as a function of the sweeping rate for different non-reciprocal parameters. Panels (b) and (d) show the tunneling probability as a function of non-reciprocal parameter for different sweeping rates. The lines give the results without periodic modulation for comparison. We set $A = 0.5$ and $\omega = 10$.

Compared with the case without periodic modulation, we find that (i) in the adiabatic limit, even for $\delta = 0$, the tunneling probability with periodic modulation still does not tend to zero [see solid squares in Figs. 2(a) and 2(c)], which indicates the adiabaticity is broken. (ii) For any δ, the results with periodic modulation deviate greatly from those without periodic modulation in a wide range of α. (iii) The periodic modulation does not change the critical point of the non-reciprocal TLS [see solid squares in Figs. 2(b) and 2(d)], i.e., $\delta = 1$. In the next section, we will focus on the case where the sweeping rate is very small to study the corrections of both the adiabatic region and the adiabaticity of the system by periodic modulation.

3.2. The effect of periodic modulation on adiabaticity

To visualize the effects of periodic modulation on the adiabaticity of non-reciprocal TLS, we plot the final tunneling probability (P_a or P_b) in the (ω, δ) parameter plane to form an adiabatic phase diagram. For brevity, we introduce a modulating period number K representing the evolution of the system from $t = -KT$ to $t = KT$. For a given T, the initial time of system evolution is completely determined by K, and the larger the K, the closer the initial state is to the actual eigenstate, which is more favorable for adiabatic evolution. The adiabatic phase diagrams for different modulating period numbers ($K = 10$, 100, and 1000) with $\alpha = 0.01$ and $A = 0.5$ are shown in Fig. 3.

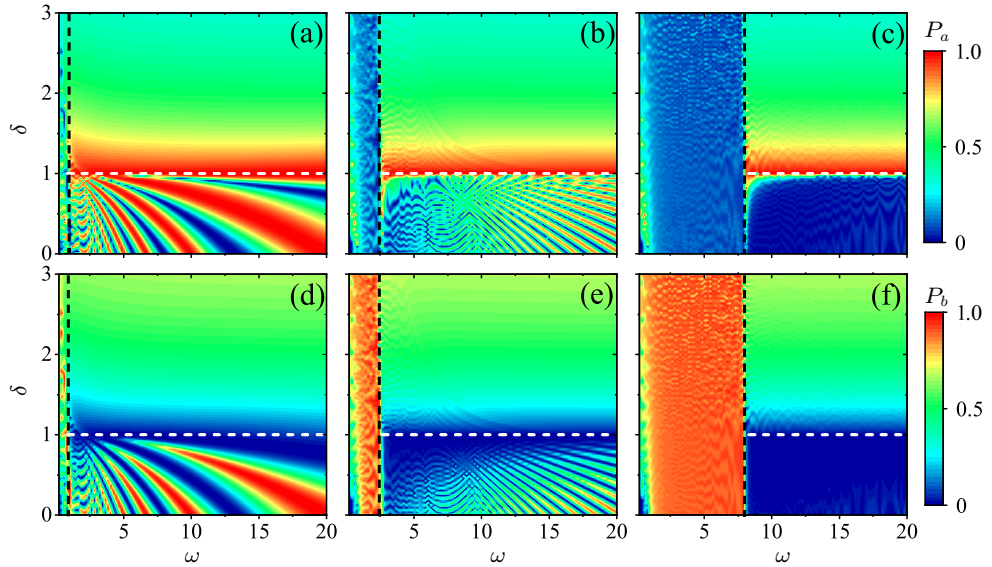


Fig. 3. Adiabatic phase diagrams of P_a (upper panels) and P_b (lower panels) in the (ω, δ) parameter space for different modulating period numbers (for each row, from left to right, $K = 10$, 100, and 1000). The horizontal lines mark the critical non-reciprocal parameter $\delta = 1$ and the vertical lines denote the critical modulating frequency ω_c. We set $\alpha = 0.01$ and $A = 0.5$.

We see that there is a critical modulating frequency (denoted by ω_c, see vertical lines) in the phase diagram and the distributions of P_a or P_b on both sides of ω_c are significantly different. When $\omega > \omega_c$ (see the right half of vertical line), we can divide the phase diagram into two regions (right upper and right lower regions) by using $\delta = 1$. We find that all right upper regions (i.e., $\omega > \omega_c$ and $\delta > 1$) are non-adiabatic, where $P_a > 0$ and $P_b > 0$. However, the adiabaticity of the right lower region (i.e., $\omega > \omega_c$ and $\delta < 1$) depends on the value of K. When K is small (e.g., $K = 10$ or 100), the interference fringes[44] can be seen in this region. When K is large enough (e.g., $K = 1000$), the tunneling probability in this region ap-

proaches zero and it becomes adiabatic. Interestingly, the adiabaticity of the left half region (i.e., $\omega < \omega_c$) depends on the choice of initial state. If the initial state is in the lower level, the left half region is quasi-adiabatic, where P_a is very small (see upper panels). If the initial state is in the upper level, the left half region is non-adiabatic, where P_b approaches 1 (see lower panels).

In order to demonstrate the influence of modulating amplitude A on the adiabaticity, we present the adiabatic phase diagrams under strong periodic modulation ($A = 3$) in Fig. 4 for comparison with the results under weak periodic modulation [$A = 0.5$, see Figs. 3(c) and 3(f)]. We show that for the same sweeping rate and modulating period number (e.g., $\alpha = 0.01$ and $K = 1000$), increasing the modulating amplitude does not change the value of ω_c and the region division of the phase diagram. However, the adiabaticity of all right lower regions is weakened. The adiabaticity of the left half region is significantly improved for the initial state in the lower level [see Fig. 4(a)], while the non-adiabaticity is significantly enhanced for the initial state in the upper level [see Fig. 4(b)].

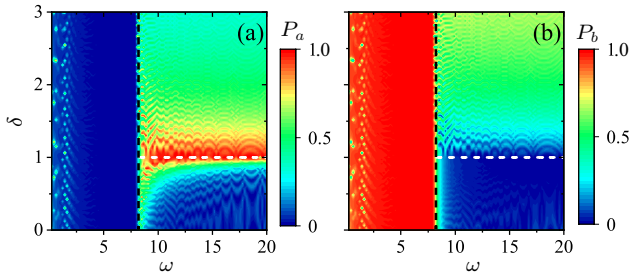


Fig. 4. Adiabatic phase diagrams of (a) P_a and (b) P_b in the (ω, δ) parameter space with $K = 1000$. The horizontal lines mark $\delta = 1$ and the vertical lines denote ω_c. We set $\alpha = 0.01$ and $A = 3$.

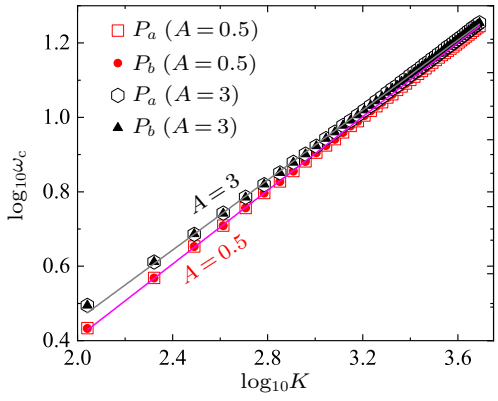


Fig. 5. The critical modulating frequency ω_c as a function of K for $A = 0.5$ (hollow squares and solid circles) and $A = 3$ (hollow hexagons and solid triangles). The solid lines show the linear fits with slopes respectively being 0.5 and 0.47 for $A = 0.5$ and 3. We set $\alpha = 0.01$.

The critical modulating frequency is pivotal in delineating the boundaries of adiabatic regions; therefore, it is essential to establish both qualitative and quantitative relationships between this frequency and other relevant parameters. To this end, we fix $\alpha = 0.01$ and calculate ω_c for different K when $A = 0.5$ and $A = 3$, respectively, and the results are shown in Fig. 5. Obviously, the values of ω_c obtained by P_a and P_b are completely consistent. As a result, we can write ω_c as a function of K in the following form:

$$\omega_c = \lambda K^{\beta}, \tag{6}$$

where λ and β are the two fitting constants associated with A. When $A = 0.5$, $\lambda \approx 0.26$ and $\beta \approx 0.5$. When $A = 3$, $\lambda \approx 0.32$ and $\beta \approx 0.47$. We see that increasing A does not result in a significant correction to the power exponent β, indicating that the boundaries of the adiabatic region are not sensitive to the modulating amplitude (see Fig. 6).

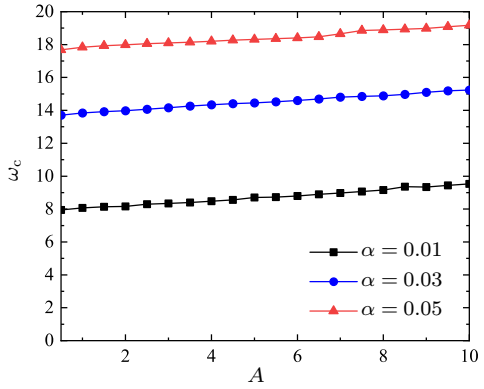


Fig. 6. The critical modulating frequency ω_c as a function of A for $\alpha = 0.01$, 0.03, and 0.05. We set $K = 1000$.

3.3. The effect of linear sweeping on adiabatic region

Now we discuss the effect of the sweeping rate α on the adiabatic region. In Fig. 7, we show the adiabatic phase diagrams in the (ω, δ) parameter space for different sweeping rates (α = 0.01, 0.03, and 0.05) with $K = 1000$ and $A = 0.5$. We see that using ω_c and $\delta = 1$ as the boundaries can still clearly divide the whole phase diagram into three regions, which is similar to the results in Fig. 3. It should be emphasized that for the three values of α we choose, in Fig. 7 all right lower regions ($\omega > \omega_c$ and $\delta < 1$) of the phase diagrams are adiabatic and the effect of α on the adiabaticity can be negligible. However, we find that the adiabatic region (marked in dark blue) shrinks significantly with a small increase in α (from 0.01 to 0.05) and the left half region ($\omega < \omega_c$) expands rapidly accompanied by the appearance of interference fringes. In addition, when the initial state is in the lower level, we see that the left half region rapidly changes from a quasi-adiabatic region [where the average tunneling probability between 2% and 10%, see Fig. 7(a)] to a non-adiabatic region [see Fig. 7(c)].

Considering the importance of the critical modulating frequency in controlling adiabatic tunneling dynamics, we establish a quantitative relationship between ω_c and the sweeping rate. For simplicity, we fix $K = 1000$ and calculate ω_c for different α when $A = 0.5$ and $A = 3$, respectively. The results for

both P_a and P_b are illustrated in Fig. 8. We find that ω_c can be expressed as a power function with α as the base, i.e.,

$$\omega_c = \zeta \alpha^{\eta}, \tag{7}$$

where ζ and η are two fitting constants associated with A. When $A = 0.5$, $\zeta \approx 77.6$, and $\eta \approx 0.49$. When $A = 3$, $\zeta \approx 75.9$, and $\eta \approx 0.47$. Although the change of A does not cause significant corrections to the power exponent η, it still offers potential for fine-tuning of adiabatic regions and adiabaticity.

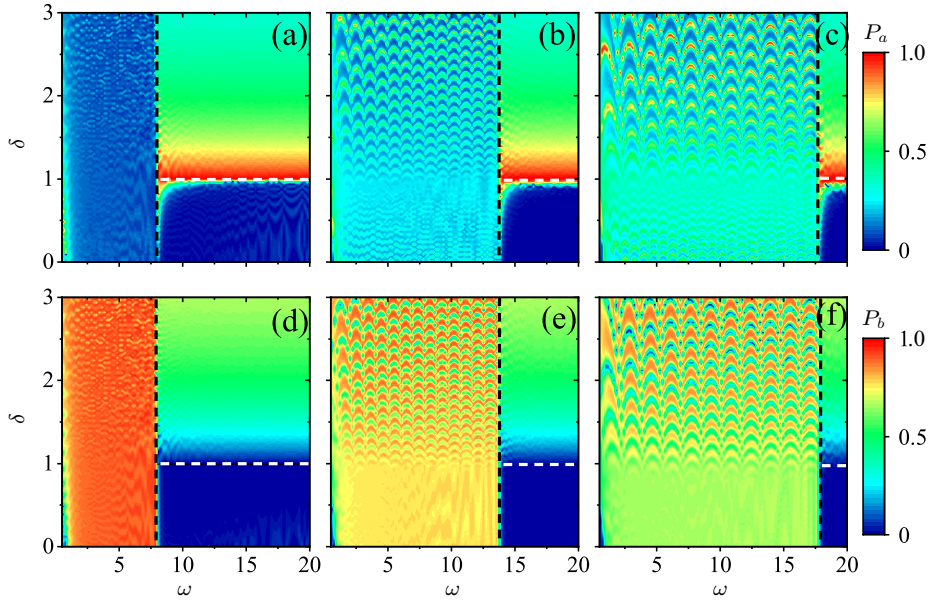


Fig. 7. Adiabatic phase diagrams of P_a (upper panels) and P_b (lower panels) in the (ω, δ) parameter space for different sweeping rates (for each row, from left to right, $\alpha = 0.01$, 0.03, and 0.05). The horizontal lines mark $\delta = 1$ and the vertical lines denote ω_c. We set $K = 1000$ and $A = 0.5$.

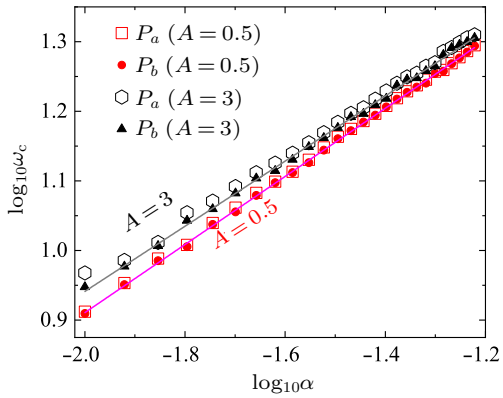


Fig. 8. The critical modulating frequency ω_c as a function of α for $A = 0.5$ (hollow squares and solid circles) and $A = 3$ (hollow hexagons and solid triangles). The solid lines show the linear fits with slopes respectively being 0.49 and 0.47 for $A = 0.5$ and 3. We set $K = 1000$.

4. Conclusion

In summary, we have studied the effects of both periodic modulating field and linear sweeping field on adiabatic tunneling dynamics by using a non-reciprocal Landau–Zener model subject to periodic modulation. We have obtained the adiabatic phase diagrams in the (ω, δ) parameter plane, where the adiabatic region is determined by $\omega > \omega_c$ and $\delta < 1$. We have further discussed the influence of initial state selection, modulating-field parameters, and sweeping-field parameter on the adiabaticity and adiabatic region of the system. It is found that the critical modulating frequency ω_c can be expressed as a power function of the modulating period number K or the sweeping rate α. Moreover, it is shown that the adiabaticity is not sensitive to the modulating amplitude A and the adiabatic region can be flexibly scaled by fine-tuning the sweeping rate α. Our findings imply that one can change the adiabatic region or improve the adiabaticity by adjusting the parameters of both the modulating and the sweeping fields, which provides a potential way to effectively manipulate the adiabatic tunneling dynamics of non-reciprocal systems and to explore the adiabatic tunneling dynamics of more complex non-reciprocal systems such as nonlinear non-reciprocal systems.

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References

[1] Ashhab S, Johansson J R, Zagoskin A M and Nori F 2007 *Phys. Rev. A* **75** 063414
[2] Shevchenko S N, Ashhab S and Nori F 2010 *Phys. Rep.* **492** 1
[3] Shytov A V, Ivanov D A and Feigel'man M V 2003 *Eur. Phys. J. B* **36** 263
[4] Wu B and Niu Q 2000 *Phys. Rev. A* **61** 023402
[5] Wang W Y, Sun B and Liu J 2022 *Phys. Rev. A* **106** 063708
[6] Dai X, Trappen R, Chen H, *et al*. 2022 arXiv: 2207.02017v1 [quant-ph]
[7] Liu J, Fu L B, Ou B Y, Chen S G, Choi D I, Wu B and Niu Q 2002 *Phys. Rev. A* **66** 023404
[8] Hu J Y, Mao T F, Dou F Q and Zhao Q 2013 *Acta Phys. Sin.* **62** 170303 (in Chinese)

[9] Liu X Z, Tian D P and Chong B 2016 *Mod. Phys. Lett. B* **30** 1650194
[10] Stehli A, Brehm J D, Wolz T, Schneider A, Rotzinger H, Weides M and Ustinov A V 2023 *npj Quantum Inform.* **9** 61
[11] Xu X Y, Han Y J, Sun K, Xu J S, Tang J S, Li C F and Guo G C 2014 *Phys. Rev. Lett.* **112** 035701
[12] Tong X Q, Gao X L and Kou S P 2023 *Phys. Rev. B* **107** 104306
[13] Liu K Y, Zhang H, Cheng H Y, Dai T, Zhao Y J and Su J 2024 *Results Phys.* **64** 107941
[14] Dong J, Li X L, Dou F Q and Wang W Y 2023 *Phys. Rev. A* **108** 063506
[15] Ghaemi-Dizicheh H 2023 *Phys. Rev. B* **107** 125155
[16] Huang X Y, Lu C C, Liang C, Tao H G and Liu Y C 2021 *Light Sci. Appl.* **10** 30
[17] Hamann A R, Müller C, Jerger M, Zanner M, Combes J, Pletyukhov M, Weides M, Stace T M and Arkady F 2024 *Phys. Rev. Lett.* **121** 123601
[18] Li S C, Fu L B and Liu J 2018 *Phys. Rev. A* **98** 013601
[19] Wang X, Liu H D and Fu L B 2023 *New J. Phys.* **25** 043032
[20] Wang G F, Fu L B and Liu J 2006 *Phys. Rev. A* **73** 013619
[21] Cáceres J J, Domínguez D and Sánchez M J 2023 *Phys. Rev. A* **108** 052619
[22] Wubs M, Saito K, Kohler S, Kayanuma Y and Hänggi P 2005 *New J. Phys.* **7** 218
[23] Sarreshtedari F and Hosseini M 2017 *Phys. Rev. A* **95** 033834
[24] Tchapda A B, Tchapda M B, Kammogne A D and Fai L C 2017 arXiv: 1708.04184v2 [quant-ph]
[25] Sun J and Lu S F 2019 *Quantum Inf. Process.* **18** 211
[26] Zhang Q 2016 *Phys. Rev. A* **93** 012116
[27] Zhuang M, Huang J H, Ke Y G and Lee C H 2020 *Ann. Phys.* **532** 1900471
[28] Wei Z H and Ying M S 2006 *Phys. Lett. A* **356** 312
[29] Wang Z Y and Plenio M B 2016 *Phys. Rev. A* **93** 052107
[30] Liu J, Wu B and Niu Q 2003 *Phys. Rev. Lett.* **90** 170404
[31] Sun J and Lu S F 2016 *Commun. Theor. Phys.* **66** 207
[32] Zhou X F, Zhang Y S, Zhou Z W and Guo G C 2010 *Phys. Rev. A* **81** 043614
[33] Fermanian-Kammerer C and Joye A 2020 *Nonlinearity* **33** 4715
[34] Zhang Q and Wu B 2019 *Phys. Rev. A* **99** 032121
[35] Wu Z Y, Yu T and Zhou H W 1994 *Phys. Lett. A* **186** 59
[36] Cheniti S, Koussa W, Koussa A and Maamache M 2020 *J. Phys. A: Math. Theor.* **53** 405302
[37] Li H, Shen H Z, Wu S L and Yi X X 2017 *Opt. Express* **25** 30135
[38] Luan T Z, Sun J Y and Shen H Z 2023 *Europhys. Lett.* **142** 58001
[39] Song Y H, Wang X, Liu H D and Yi X X 2023 *J. Phys. A: Math. Theor.* **56** 015304
[40] Yang J and Zhang Y 2020 *Int. J. Theor. Phys.* **59** 3593
[41] Dong X P, Feng Z B, Lu X J, Li M and Zhao Z Y 2023 *Chin. Phys. B* **32** 034201
[42] Singhal Y, Martello E, Agrawal S, Ozawa T, Price H and Gadway B 2023 *Phys. Rev. Research* **5** L032026
[43] Zener G 1932 *Proc. R. Soc. Lond. A* **137** 696
[44] Li S C and Fu L B 2020 *Phys. Rev. A* **102** 033313