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Influence of conformity on the evolution of cooperation in games with sampling rules on networks

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We study the influence of conformity on the evolution of cooperative behavior in games under the learning method of sampling on networks. A strategy update rule based on sampling is introduced into the stag hunt game, where agents draw samples from their neighbors and then update their strategies based on conformity or inference according to the situation in the sample. Based on these assumptions, we present the state transition equations in the dynamic evolution of population cooperation, conduct simulation analysis on lattice networks and scale-free networks, and discuss how this mechanism affects the evolution of cooperation and how cooperation evolves under different levels of conformity in the network. Our simulation results show that blindly imitating the strategies of neighbors does not necessarily lead to rapid consensus in the population. Instead, rational inference through samples can better promote the evolution of the same strategy among all agents in the population. Moreover, the simulation results also show that a smaller sample size cannot reflect the true situation of the neighbors, which has a large randomness, and the size of the benefits obtained in cooperation determines the direction of the entire population towards cooperation or defection. This work incorporates the conforming behavior of agents into the game, uses the method of sampling for strategy updates and enriches the theory of evolutionary games with a more realistic significance.

Keywords: cooperative evolution, stag hunt game, conformity, scale-free network

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1. Introduction

Cooperation has always been a goal pursued in human social activities, as cooperation can often create the greatest benefits for society. Exploring the mechanisms of cooperation is of great scientific significance and theoretical value for correctly understanding the conditions and means of cooperation. How cooperation arises and evolves has always been a hot topic of research for biologists and sociologists, and evolutionary game theory, based on the assumption of bounded rationality of agents, has become one of the most effective theoretical methods for solving the problem of cooperation dilemmas.

Coordination games are a typical example in the study of the evolution of cooperation, as the phenomenon of conformity in coordination games is widely present in nature and human society. In coordination games, individuals' decision-making is not only based on maximizing their own interests but is also influenced by the behavior of others and social norms. In reality, individuals often develop conformity in their interactions with social groups, and unconsciously exhibit conformity behavior.

Conformity refers to the tendency of agents to imitate the behavior of others when facing uncertain situations. This behavioral pattern has been deeply studied in fields such as sociology, psychology and behavioral economics. In the financial market, investors' decisions are often influenced by the behavior of other investors, leading to market fluctuations and irrational prosperity. The GameStop incident at the beginning of 2021 is a typical example, where ordinary investors collectively acted through social platforms such as Reddit, causing extreme fluctuations in stock prices. This kind of behavior of blindly imitating others after learning new information is a typical manifestation of conformity. If we cannot correctly understand how conformity affects the mechanism behind evolutionary games, it will be easy for the public to fall into a social dilemma when dealing with such events in the future.

Besides that, in games, individuals often adopt decision-making methods with sampling rules, enabling them to make reasonable choices with limited information when facing complex decisions. A game with sampling rules greatly reduces the complexity of decision-making by allowing individuals to focus on only part of the information rather than all of it. This method is widely used in practical applications. For example, in the economic market, investors often face many choices, but due to the cost and time limitations of information acquisition they usually only conduct in-depth research on a small number of stocks and make investment decisions. Such sampling behavior is ubiquitous in daily life, reflecting the rational decision-making process of individuals under conditions of in-

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complete information.

In this work, we will take the stag hunt game as the background to study the influence of conformity on the evolution of cooperation. Through the establishment and analysis of the stag hunt game model we aim to reveal the incentive or hindrance of conformity behavior among agents under network structure on the evolution of cooperation. In this model, conformity and sampling rules mainly affect the strategy update of agents, and the strategy update rule plays a key role in evolutionary games. By the strategy update rule used in this paper, agents will sample neighbors and can only observe their strategies from the sample. Then agents make inferences from the sample and update their strategies driven by conformity or payoffs.

2. Literature review

Evolutionary game theory considers the behavioral dynamics of a large number of agents, and evolutionary games on networks have always been an important topic of study and exploration by many scholars. In evolutionary games on networks, the network structure of the population and the update rules of agents will affect the evolution of cooperation. The research on network structure in evolutionary games mainly focuses on the impact of different network topological structures on the outcomes of cooperation in specific rules, as well as the correlation between certain properties of the network and the level of cooperation.[1–5] Nowak and May[6] first studied the prisoner's dilemma on a lattice network in 1992, and found that cooperators can form clusters through mutual cooperation to suppress the invasion of defectors. From then on, studies on games on various networks emerged one after another, such as small-world networks, scale-free networks, and so on.[7–16] Most studies have assumed that all individuals in the network only have payoff-driven strategy update rules, while this work considers conformity-driven strategy update rules. By introducing this rule, we can more comprehensively understand the evolutionary game behavior in complex networks. Here we verify the theoretical model through simulation experiments, which can not only test the accuracy and applicability of the theory but also provide specific guidance for practical applications.

In addition to evolutionary games on networks, update learning rules in different environments are also a hot topic of research.[17–22] Acemoglu *et al.*[23] studied the spread of false information in social networks, where the model built random encounters and information exchange between agents. When two agents meet, there is a certain probability of one of three information exchange events occurring to update their beliefs, thus obtaining the stochastic process of belief evolution under this kind of update learning; finally, the literature provides the upper bound or exact result of the distance between the consensus value and the benchmark value of belief to quantify the spread of false information. Mostagir *et al.*[24] discussed when society is susceptible to manipulation, which is essentially a social learning model with network structure. In the model, agents are divided into stubborn agents and DeGroot agents. Finally, the literature describes the social norms and network structures that are easily manipulated, derives the conditions under which social networks are unaffected and cannot be manipulated and develops a new centrality measure in this process. Many works have studied different aspects of how the initial state of the population affects the update learning process, and most of these focus on specific decision rules.[25–27] Kandori *et al.*[28] and Young[29] studied the model of limited populations participating in the social learning process, where agents occasionally make mistakes when updating strategies. They showed that when considering an infinite time range, the path dependence of the social learning process disappears. When the probability of making a mistake is small enough, agents are almost always in a unique "stochastic stable state" that is independent of the initial state. Most studies on game strategy updates use the Fermi rule, that is, randomly selecting a neighbor and imitating it according to a probability function related to the difference in payoffs. The foundation of the model in this paper is similar to the above literature, and the model also adopts a finite population of a large number of agents, so the results of these previous works are of great reference significance for the research described here. However, most previous studies on evolutionary games on networks have assumed that agents can accurately obtain information from all neighbors, but in real life individuals are limited by the cost of obtaining information or the limited channels of obtaining information, and it is difficult to fully understand the situation of all individuals around them, so they can only sample and observe from their neighbors. After this sampling process, if a certain strategy occupies a large proportion of the entire sample, the individual is easily driven by conformity and will unconsciously choose that strategy. Previous studies have not considered this point, and in this paper we further study the influence of conformity on the evolution of cooperation by using this psychological characteristic of social individuals in real life.

In this paper, to reflect the conformity in coordination games the stag hunt game is taken as the research object. As a typical social dilemma game model, the stag hunt game is often used for analysis of the evolution of cooperation. If most of the population chooses a certain strategy, then the consistency of the population is high. In the evolution of cooperation in the stag hunt game there are often two issues of concern: one is the consistency of the evolutionary outcome of the population and the other is the level of cooperation in the population with high consistency. From the existing literature,[30–32] compared with other games such as the prisoner's dilemma, the stag hunt game is less studied. Most models studying the stag hunt game

focus on strategy update rules and subsequently explore the influence of various factors on the level of cooperation.[33–35] Compared with existing research, this paper incorporates the psychological factors of conformity into the stag hunt game model, considering the diversity of individual strategy update rules and heterogeneous conformity, that is, individuals can update strategies based on their own conformity thresholds. By introducing conformity into the stag hunt game, we can more accurately simulate real individual behavior and analyze the impact of different factors on the evolution of cooperation.

The difference between our model and all the above studies is that our strategy update rule uses the method of sampling inference and considers the influence of conformity. Specifically, in reality it is difficult for agents to know the situation of all neighbors around them, so agents can only randomly understand some of the neighbors' information and then infer and estimate the overall situation of the neighbors before making decisions. Regarding conformity and sampling inference, some past literature has similar ideas but there are fewer papers that conduct in-depth research using this kind of idea. For example, Heller and Mohlin[36] analyzed a social learning process in which agents observe the past actions taken by some other agents in the environment and then make decisions, and studied under what circumstances the initial actions of the population will have a lasting impact. This is essentially an agent sampling other agents in the population and then updating strategies based on the results of the sampling. Some literature[37–39] has also considered the idea of sampling in the game. In fact, the strategy update process can also include the drive of conformity, because conforming behavior is also a common phenomenon in life. Agents in the stag hunt game tend to choose the same strategy, so for them, if a large proportion of a strategy is sampled from some neighbors, there is motivation to choose that strategy as well. However, at present, there is less literature exploring how conformity affects the outcomes of cooperative evolution. Wang and Wang[40] studied how conformity affects the increase in the number of cooperators in complex coordination games, and their results showed that the existence of conformity can enhance network reciprocity in coordination games, thus producing more favorable results in social dilemmas. Pena *et al.*[41] studied the conformity of agents and its influence on the prisoner's dilemma and snowdrift games on scale-free networks. The results showed that when conformity reaches a certain level, the scale-free network can no longer promote the emergence of cooperation.

Unlike previous work,[42,43] in this paper we use sampling and inference in the game on the network and updates strategies according to the sample, and this strategy update process considers the influence of each agent's conformity. We aim to find the key factors for the rapid spread of public opinion or information in society through such settings, and explore the influence of conformity behavior on the evolution of cooperation.

3. Models

3.1. Stag hunt game on the network

Consider a structured finite population with a very large number of agents, denoted as N. The set of agents is $\mathcal{N} = \{1, 2, \ldots, N\}$, and the relationships between agents in the population are described by a network. In the network, each node represents an agent, and each edge represents an undirected connection between two agents, that is, all agents on this network constitute an undirected graph. Without loss of generality, we introduce an $N \times N$ matrix $E = (e_{ij})_{i,j=1,\ldots,N}$ to represent the mutual connection relationship between the structured finite population of size N, where $e_{ij} \in \{0, 1\}$ and $e_{ii} = 0$. For two different agents i and j, $e_{ij} = 0$ if there is no connection between agents i and j, and $e_{ij} = 1$ if there is a connection between agents i and j. If there is a connection between agents i and j, then they are neighbors. The set of neighbors of agent i can be written as $L_i = \{j | e_{ij} = 1, j \in \mathcal{N}\}$, and the degree of node i, k_i, is the number of elements in L_i, that is, $k_i = |L_i|$.

Assume that time is discrete and infinite, and $\mathcal{T} = \{0, 1, \ldots\}$ represents the set of all time steps. At each time step $t \in \mathcal{T}$, all agents play the stag hunt game with each of their neighbors. The stag hunt game is one of the most classic coordination games and social dilemmas, where agents have the motivation to choose the same strategy. In the stag hunt game, each agent has two strategies to choose from: hunting stag (C) or hunting hare (D), meaning that the set of strategies available to each agent at each time step is $\mathcal{S} = \{\mathrm{C}, \mathrm{D}\}$. The payoff matrix of the stag hunt game is shown in Fig. 1.

	C	D
C	R,R	S,T
D	T,S	P,P

Fig. 1. Payoff matrix of the stag hunt game ($R > T > P > S$).

If both sides choose to hunt stag, they will each receive a payoff R; if both choose to hunt hare, they will each receive a payoff P. If the agents adopt different strategies, the hare hunter receives a payoff T and the stag hunter receives a payoff S, where the parameters satisfy $R > T > P > S$. According to the classic model of the stag hunt game, the two Nash equilibria are when both choose to hunt stag or both choose to hunt hare, while the Pareto optimal state is when both choose to hunt stag. Here we study the case when $P = 1$, $T = 2$, $S = 0$, $R > 2$.

At each time step t, agent i will choose a strategy $s_i(t) \in \mathcal{S}$. At time step t, let the set of individuals in agent i's neighborhood who choose the stag hunting strategy be $\Omega_{\mathrm{C}}^{i}(t)$, where

$\Omega_{\rm C}^i(t)=\left\{j|s_j(t)={\rm C},\ \ j\in L_i\right\}$. At time step t, let the number of neighbors of agent i who choose the stag hunting strategy be $n_{\rm C}^i(t)$, where $n_{\rm C}^i(t)=\left|\Omega_{\rm C}^i(t)\right|$. At $t=0$, each agent randomly chooses the stag hunting strategy or the hare hunting strategy with a probability of 0.5, which means that at $t=0$, the probability that the number of neighbors of agent i choosing the stag hunting strategy is $n_{\rm C}^i(0)$ is $\begin{pmatrix} k_i \\ n_{\rm C}^i(0) \end{pmatrix}\frac{1}{2^{k_i}}$.

At each time step t, let the payoff obtained by agent i after playing the game with neighbor j be $\pi_{ij}(t)$, and the average payoff of agent i after interacting with all neighbors can be represented as $\sum_{j\in L_i}\pi_{ij}(t)/k_i$. At time step t, let the average payoff of agent i for choosing strategy s be $\Pi_s^i(t)$, that is, the average payoff for agent i choosing the stag hunting strategy is $\Pi_{\rm C}^i(t)$, and the average payoff for choosing the hare hunting strategy is $\Pi_{\rm D}^i(t)$. Then it is easy to obtain the average payoff of agent i for choosing different strategies at time step t, as shown below:

$$\Pi_s^i(t)=\begin{cases}\dfrac{Rn_{\rm C}^i(t)}{k_i}, & s={\rm C},\\ 1+\dfrac{n_{\rm C}^i(t)}{k_i}, & s={\rm D}.\end{cases}\tag{1}$$

From this equation, it can be seen that the average payoff is related to the proportion of neighbors choosing the stag hunting strategy $n_{\rm C}^i(t)/k_i$.

3.2. Sampling and strategy update rule

At each time step, agents draw a certain number of samples from the strategies adopted by all neighbors at the last time step. Specifically, at time step $t+1$, to know the relevant information about the strategies adopted by their neighbors, each agent samples from the strategies adopted by their neighbors at time t. Each agent prepares to draw a sample of size b, but if the number of neighbors of agent i $k_i\le b$ they then draw k_i samples, that is, the size of the sample drawn by agent i is $a_i=\min\{b,k_i\}$. b is exogenous and time-independent. This sampling process represents the fact that, in reality, agents have limited means of obtaining information, or the cost of obtaining information is high, making it difficult to fully understand the situation of all neighbors around them, and they can only resort to sampling from neighbors and then observing the sample. b represents the average number of neighbors that each agent can observe at each time step in the network. Therefore, agents can only randomly understand some of the information of their neighbors, but if the number of neighbors of the agent is less than or equal to b then they can observe all the neighbors. We assume that in a sample of size a_i agent i observes $a_{\rm C}^i(t)$ stag hunting strategies.

After observing the information in the sample, agent i will update their strategies based on conformity or imitation after estimating the payoffs of neighbors. To introduce the consideration of conformity, a conformity threshold Z ($0.5<Z<1$) is set, and when the proportion of a certain strategy in the sample is greater than this threshold the agent will tend to choose this strategy. That is, at time step $t+1$, when $a_{\rm C}^i(t)/a_i>Z$, agent i will choose the stag hunting strategy with a higher probability; when $1-a_{\rm C}^i(t)/a_i>Z$, agent i will choose the hare hunting strategy with a higher probability. Moreover, the proportion of the strategy in the sample that exceeds this threshold is also positively correlated with the probability of the agent choosing this strategy; in other words, the larger the proportion that exceeds the conformity threshold, the more likely the agent is to choose this strategy.

Similar to the Fermi rule expression, agents imitate the strategy with a larger proportion in the sample with a certain probability, so here we assume that when $a_{\rm C}^i(t)/a_i>Z$, regardless of the current strategy s of agent i, the function of the probability $p_{s\to s'}^i$ that agent i will choose strategy s' at time step $t+1$ is

$$\begin{cases}p_{s\to{\rm C}}^i(t+1)=\dfrac{1}{1+\exp\left(-\dfrac{\dfrac{a_{\rm C}^i(t)}{a_i}-Z}{K_1}\right)}, & s\in\mathcal{S}.\\ p_{s\to{\rm D}}^i(t+1)=1-p_{s\to{\rm C}}^i(t+1),\end{cases}\tag{2}$$

Similarly, when $1-a_{\rm C}^i(t)/a_i>Z$, regardless of the current strategy s of agent i, the function of the probability $p_{s\to s'}^i$ that agent i will choose strategy s' at time step $t+1$ is

$$\begin{cases}p_{s\to{\rm C}}^i(t+1)=1-p_{s\to{\rm D}}^i(t+1),\\ p_{s\to{\rm D}}^i(t+1)=\dfrac{1}{1+\exp\left(-\dfrac{1-\frac{a_{\rm C}^i(t)}{a_i}-Z}{K_1}\right)}, & s\in\mathcal{S},\end{cases}\tag{3}$$

where K_1 is the noise intensity, quantifying the uncertainty in the strategy selection process. When $K_1\to 0$, if the proportion of a certain strategy exceeds the conformity threshold, agent i will conform to choosing that strategy; when $K_1\to\infty$, the strategy of agent i will be completely randomly updated.

The conformity threshold reflects the fact that if agents in the network observe a significant proportion of a certain strategy in the group, this conformity phenomenon will exert conformity pressure on the agents, making them actively choose the strategy that most people choose, regardless of other factors. The smaller the conformity threshold Z, the higher the conformity of the population agents.

When $1-Z\le a_{\rm C}^i(t)/a_i\le Z$, agent i will use the maximum likelihood estimation method at time step $t+1$ to infer $\tilde{n}_{\rm C}^i(t)$, the possible number of stag hunting strategies among neighbors at time step t, that is, to solve

$$\tilde{n}_{\rm C}^i(t)\in\underset{n_{\rm C}^i\in\mathbb{N}}{\arg\max}\begin{pmatrix}a_i\\ a_{\rm C}^i(t)\end{pmatrix}\left(\frac{n_{\rm C}^i}{k_i}\right)^{a_{\rm C}^i(t)}\left(1-\frac{n_{\rm C}^i}{k_i}\right)^{a_i-a_{\rm C}^i(t)},\tag{4}$$

where $\mathbb{N}$ is the set of natural numbers. When the solution set of $\tilde{n}^i_{\rm C}(t)$ by the maximum likelihood estimation includes multiple values, agent i will randomly choose one of them with equal probability.

Since from the sample drawn agent i only observes the neighbors' strategies and cannot know their payoffs, it is assumed that at time step $t+1$, agent i approximately believes that for any other agent in the network the proportion of their neighbors holding the stag hunting strategy is the same as agent i, denoted as $\tilde{n}^i_{\rm C}(t)/k_i$. The average payoff is related to the proportion of neighbors choosing the stag hunting strategy, so agent i extends the estimated payoff from the sample to all neighbors. Let the payoff obtained by any neighbor $j \in L_i$ holding strategy $s \in \mathcal{S}$ at time step t, which is perceived by agent i, be $\tilde{\Pi}^i_s(t)$; that is, agent i believes that the payoff of the neighbor choosing the stag hunting strategy is $\tilde{\Pi}^i_{\rm C}(t)$ and the payoff of the neighbor choosing the hare hunting strategy is $\tilde{\Pi}^i_{\rm D}(t)$. From Eq. (1), the payoffs of neighbors choosing different strategies as perceived by agent i are given by

$$\tilde{\Pi}^i_s(t) = \begin{cases} \dfrac{R\tilde{n}^i_{\rm C}(t)}{k_i}, & s = {\rm C}, \\ 1 + \dfrac{\tilde{n}^i_{\rm C}(t)}{k_i}, & s = {\rm D}. \end{cases} \tag{5}$$

Based on such sampling inference, if at time step t, agent i's strategy is $s \in \mathcal{S}$, according to the payoff-driven Fermi rule, agent i will randomly choose a neighbor $j \in L_i$ at time step $t+1$, observe the strategy $s' \in \mathcal{S}$ of agent j at time step t and agent i will imitate the strategy of agent j with a certain probability $P^i_{s\to s'}(t+1)$, where

$$P^i_{s\to s'}(t+1) = \frac{1}{1+\exp\left(-\dfrac{\tilde{\Pi}^i_{s'}(t) - \Pi^i_s(t)}{K_2}\right)}, \tag{6}$$

and K_2 is the noise intensity, quantifying the uncertainty in the strategy selection process. When $K_2 \to 0$, if the payoff of agent j is higher, agent i will imitate the strategy of agent j; when $K_2 \to \infty$, the strategy of agent i will be completely randomly updated. In the simulation part of this paper we will assume that $K_2 = 0.5$ to ensure that agents with higher inferred payoffs have a higher probability of being imitated.

In other words, driven by this rule, by sampling and inferring the strategies and payoffs of neighbors, the probability of an agent changing to the other's strategy at the next moment is reflected by the difference in inferred payoffs, thus making new decisions. Let $p^i_{s\to s'}(t+1)$ represent the probability of agent i choosing strategy s' at time step $t+1$ given that they chose strategy s at time step t. Then, the probability $p^i_{{\rm C}\to{\rm D}}(t+1)$ of agent i choosing the hare hunting strategy at time step $t+1$ if they chose the stag hunting strategy at time step t, and the probability $p^i_{{\rm C}\to{\rm C}}(t+1)$ of agent i choosing the stag hunting strategy at time step $t+1$ if they chose the stag hunting strategy at time step t, are respectively:

$$\begin{cases} p^i_{{\rm C}\to{\rm D}}(t+1) = \left(1 - \frac{n^i_{\rm C}(t)}{k_i}\right) P^i_{{\rm C}\to{\rm D}}(t+1), \\ p^i_{{\rm C}\to{\rm C}}(t+1) = 1 - p^i_{{\rm C}\to{\rm D}}(t+1), \end{cases} \tag{7}$$

and the probability $p^i_{{\rm D}\to{\rm D}}(t+1)$ of agent i choosing the hare hunting strategy at time step $t+1$ if they chose the hare hunting strategy at time step t, and the probability $p^i_{{\rm D}\to{\rm C}}(t+1)$ of agent i choosing the stag hunting strategy at time step $t+1$ if they chose the hare hunting strategy at time step t, are respectively:

$$\begin{cases} p^i_{{\rm D}\to{\rm D}}(t+1) = 1 - p^i_{{\rm D}\to{\rm C}}(t+1), \\ p^i_{{\rm D}\to{\rm C}}(t+1) = \frac{n^i_{\rm C}(t)}{k_i} P^i_{{\rm D}\to{\rm C}}(t+1). \end{cases} \tag{8}$$

In summary, according to the different situations of the observed result $a^i_{\rm C}(t)/a_i$ from the sample at time step t, the functions of the strategy transition probabilities for agent i can be written as

$$p^i_{{\rm C}\to{\rm C}}(t+1) = \begin{cases} \left[1+\exp\left(\dfrac{Za_i - a^i_{\rm C}(t)}{K_1 a_i}\right)\right]^{-1}, & \dfrac{a^i_{\rm C}(t)}{a_i} > Z, \\ \left[1+\exp\left(\dfrac{a_i - a^i_{\rm C}(t) - Za_i}{K_1 a_i}\right)\right]^{-1}, & \dfrac{a^i_{\rm C}(t)}{a_i} < 1-Z, \\ \dfrac{n^i_{\rm C}(t) + k_i \exp(g_i(t))}{k_i + k_i \exp(g_i(t))}, & 1-Z \le \dfrac{a^i_{\rm C}(t)}{a_i} \le Z, \end{cases} \tag{9}$$

$$p^i_{{\rm C}\to{\rm D}}(t+1) = \begin{cases} \left[1+\exp\left(\dfrac{a^i_{\rm C}(t) - Za_i}{K_1 a_i}\right)\right]^{-1}, & \dfrac{a^i_{\rm C}(t)}{a_i} > Z, \\ \left[1+\exp\left(\dfrac{a^i_{\rm C}(t) + Za_i - a_i}{K_1 a_i}\right)\right]^{-1}, & \dfrac{a^i_{\rm C}(t)}{a_i} < 1-Z, \\ \dfrac{k_i - n^i_{\rm C}(t)}{k_i + k_i \exp(g_i(t))}, & 1-Z \le \dfrac{a^i_{\rm C}(t)}{a_i} \le Z, \end{cases} \tag{10}$$

$$p^i_{\mathrm{D}\to\mathrm{C}}(t+1)=\begin{cases}\left[1+\exp\left(\dfrac{Za_i-a^i_{\mathrm{C}}(t)}{K_1a_i}\right)\right]^{-1}, & \dfrac{a^i_{\mathrm{C}}(t)}{a_i}>Z,\\ \left[1+\exp\left(\dfrac{a_i-a^i_{\mathrm{C}}(t)-Za_i}{K_1a_i}\right)\right]^{-1}, & \dfrac{a^i_{\mathrm{C}}(t)}{a_i}<1-Z,\\ \dfrac{n^i_{\mathrm{C}}(t)}{k_i+k_i\exp(h_i(t))}, & 1-Z\le\dfrac{a^i_{\mathrm{C}}(t)}{a_i}\le Z,\end{cases}\tag{11}$$

$$p^i_{\mathrm{D}\to\mathrm{D}}(t+1)=\begin{cases}\left[1+\exp\left(\dfrac{a^i_{\mathrm{C}}(t)-Za_i}{K_1a_i}\right)\right]^{-1}, & \dfrac{a^i_{\mathrm{C}}(t)}{a_i}>Z,\\ \left[1+\exp\left(\dfrac{a^i_{\mathrm{C}}(t)+Za_i-a_i}{K_1a_i}\right)\right]^{-1}, & \dfrac{a^i_{\mathrm{C}}(t)}{a_i}<1-Z,\\ \dfrac{k_i-n^i_{\mathrm{C}}(t)+k_i\exp(h_i(t))}{k_i+k_i\exp(h_i(t))}, & 1-Z\le\dfrac{a^i_{\mathrm{C}}(t)}{a_i}\le Z,\end{cases}\tag{12}$$

where $g_i(t)=\frac{Rn^i_{\mathrm{C}}(t)-\tilde{n}^i_{\mathrm{C}}(t)-k_i}{K_2k_i}$ and $h_i(t)=\frac{k_i+n^i_{\mathrm{C}}(t)-R\tilde{n}^i_{\mathrm{C}}(t)}{K_2k_i}$.

On a scale-free network, under the same sample size, nodes with higher degrees are less likely to infer the true situation of neighbors, thus making decisions more randomly. On the other hand, nodes with higher degrees are more likely to be chosen as samples by other agents, so the strategies of such agents can influence a larger number of other agents.

3.3. Evolutionary dynamics of population cooperation

All agents update their strategies according to the above strategy update rules. At time step t, let the set of all agents in the population who choose the stag hunting strategy be A_t, and the ratio $|A_t|/N$ of the number of agents choosing the stag hunting strategy to the total population size, denoted as α_t, is called the cooperation level of this population (also known as the state of this population). Since the population size is finite, any $\alpha_t N$ should be a non-negative integer because it represents the number of agents in the population choosing cooperation at this time, that is, α_t should satisfy $0\le\alpha_t\le 1$ and $\alpha_t N\in\mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers. Therefore, the feasible state set of the population is $\mathcal{W}=\left\{0,\frac{1}{N},\frac{2}{N},\ldots,\frac{N-1}{N},1\right\}$, and all situations discussed in the following should satisfy $\alpha_t\in\mathcal{W}$. After each strategy update, a new cooperation level value is obtained, so a corresponding population state can be obtained at each time step on the time step sequence, which also constitutes a sequence $\{\alpha_t\}, t\in\mathcal{T}$.

Since the cooperation level reflects the proportion of cooperation in the population, it is a characteristic quantity in the network, representing the degree of willingness of the public in society to participate in cooperation, which helps to explore the factors affecting cooperation, so the cooperation level will be the key object of study here. From the above strategy update rules, it can be seen that the choices of agents are random and not deterministic strategy updates, so the initial state of the population cooperation level and the strategy update rules determine the new state of the population cooperation level. Since the population size is finite and the choices of each agent are random, this evolutionary model is not deterministic but stochastic.

Let the set of all subsets of a set A with x elements be denoted as $(A)_x$, that is, $(A)_x=\{B|B\subseteq A,|B|=x\}$. At time step t, let the set of all agents in the population who choose the stag hunting strategy be A_t, that is, $A_t=\left\{j|s_j(t)=C,j\in\mathcal{N}\right\}$, then the set of all agents who choose the hare hunting strategy is $\mathcal{N}\backslash A_t$. From this, the probability $p_x(t+1)$ that exactly x agents who chose the stag hunting strategy at time step t continue to choose the stag hunting strategy at time step $t+1$ is

$$p_x(t+1)=\sum_{M\in(A_t)_x}\left[\prod_{i\in M}p^i_{\mathrm{C}\to\mathrm{C}}(t+1)\prod_{j\in A_t\backslash M}p^j_{\mathrm{C}\to\mathrm{D}}(t+1)\right].\tag{13}$$

Similarly, the probability $q_y(t+1)$ that exactly y agents who chose the hare hunting strategy at time step t choose the stag hunting strategy at time step $t+1$ is

$$q_y(t+1)=\sum_{M\in(\mathcal{N}\backslash A_t)_y}\left[\prod_{i\in M}p^i_{\mathrm{D}\to\mathrm{C}}(t+1)\prod_{j\in\mathcal{N}\backslash A_t\backslash M}p^j_{\mathrm{D}\to\mathrm{D}}(t+1)\right].\tag{14}$$

With the above two formulae, the recursive formula of this stochastic process can be further derived. For any $w,w'\in\mathcal{W}$, assume that $p_{w\to w'}(t+1)$ represents the transition probability of the population state w at time step t changing to the state w' at time step $t+1$, then the state transition probability equation of α_t is

$$\begin{aligned}&p_{w\to w'}(t+1)\\&=P(\alpha_{t+1}=w'|\alpha_t=w)\\&=\sum_{m=\max\{0,(w+w'-1)N\}}^{\min\{wN,w'N\}}p_m(t+1)q_{w'N-m}(t+1),\end{aligned}\tag{15}$$

where $\mathbb{N}$ is the set of natural numbers, and the initial condition of the population state is

$$P(\alpha_0=w_0)=\binom{N}{w_0N}\frac{1}{2^N},\ w_0\in\mathcal{W}.\tag{16}$$

In the stochastic process, the stationary distribution refers to the distribution that remains unchanged under the action of the transition matrix, and the limit distribution refers to the limit of the state probability distribution when the time step tends to infinity for any initial state distribution. According to the state transition probability equation for α_t, it can be seen that no matter what state it starts from, the probability of a certain state transitioning to any state in the feasible state set at each time step is positive, so this stochastic process has ergodicity. A property of stochastic processes is that an ergodic stochastic process must be stationary, and the limit distribution of this ergodic stochastic process is the stationary distribution. From this, a conclusion can be drawn about the existence of the limit distribution of the population state under this dynamic system; that is, for any network and initial state, under this type of strategy update rule with sampling inference, when t tends to positive infinity there is a limit distribution on $\mathcal{W}$, the feasible state set of the population. So when t tends to positive infinity, for any $w, w' \in \mathcal{W}$, the limit of $p_{w\to w'}(t)$ exists, which is also the limit of $P(\alpha_t = w')$, and is independent of the initial state α_0. Let the limit distribution vector of the feasible state be $\pi = (\pi_0, \pi_1, \ldots, \pi_N)$, then

$$\lim_{t\to+\infty} p_{w\to w'}(t) = \lim_{t\to+\infty} P(\alpha_t = w') = \pi_{w'N}. \tag{17}$$

According to this limit distribution, the possible limit states of the system can be determined.

Here we will use computer simulation methods to study the evolutionary results of the stag hunt game on different networks, analyze the impact of different parameters such as sample size and conformity threshold on group cooperation behavior and the evolutionary equilibrium of the population system, and then draw conclusions and management insights.

4. Simulation results on different networks

4.1. Stag hunt game on a lattice network

Consider that the relationships between agents in the population are described by a lattice network with periodic boundary conditions. The lattice network is one of the simplest networks — a regular arrangement of squares on a two-dimensional plane with periodic boundaries. It is a classic network used to study the evolution of cooperation. On a lattice network, each agent plays the stag hunt game with four neighbors to obtain payoffs and then obtain average payoffs. Each agent has four neighbors, that is, $k_i = 4$, where $i = 1, \ldots, N$. Since all agents on the lattice network have fewer neighbors, the value of K_1 can be set slightly smaller to ensure that agents have a higher probability of imitating the more common strategy when driven by conformity. In this section of the lattice network, the relevant parameters are set as follows: $N = 4900$ and $K_1 = 0.05$.

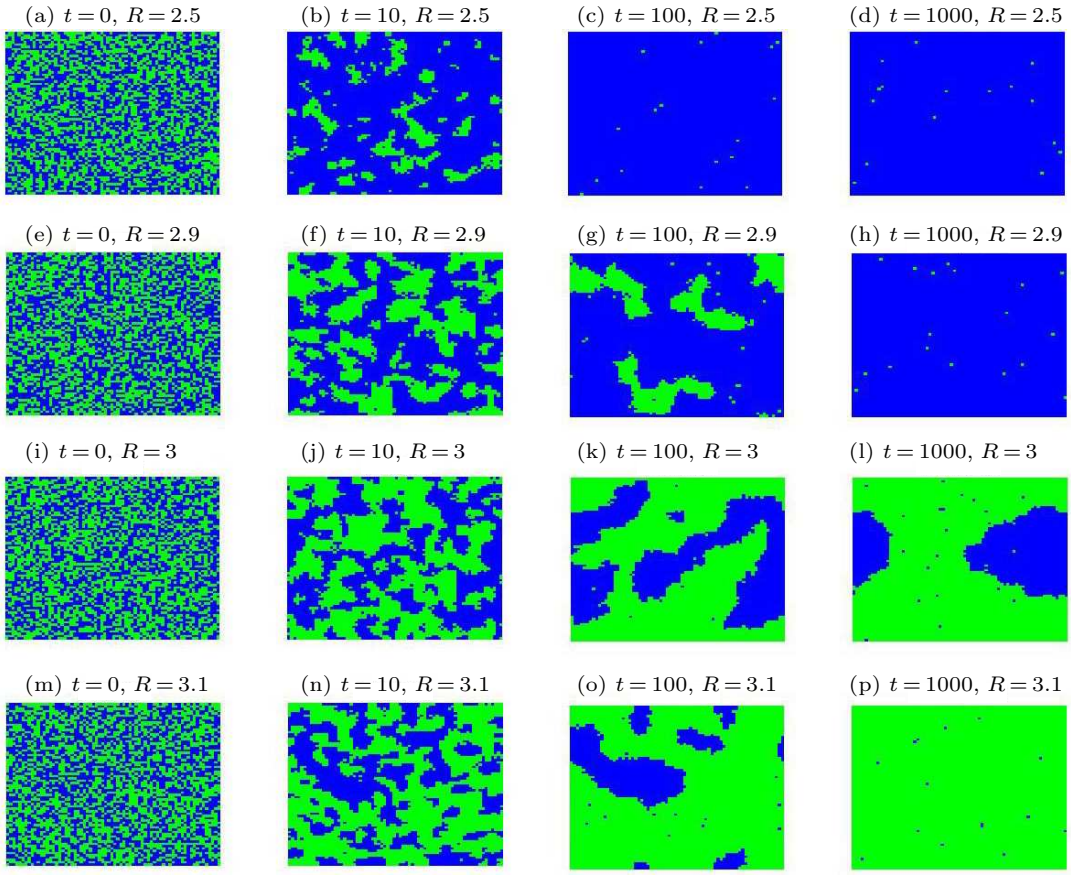


Fig. 2. The distribution of strategies on the lattice network at different time steps t and R values ($b = 3$, $Z = 0.7$; green and blue represent cooperators and defectors, respectively).

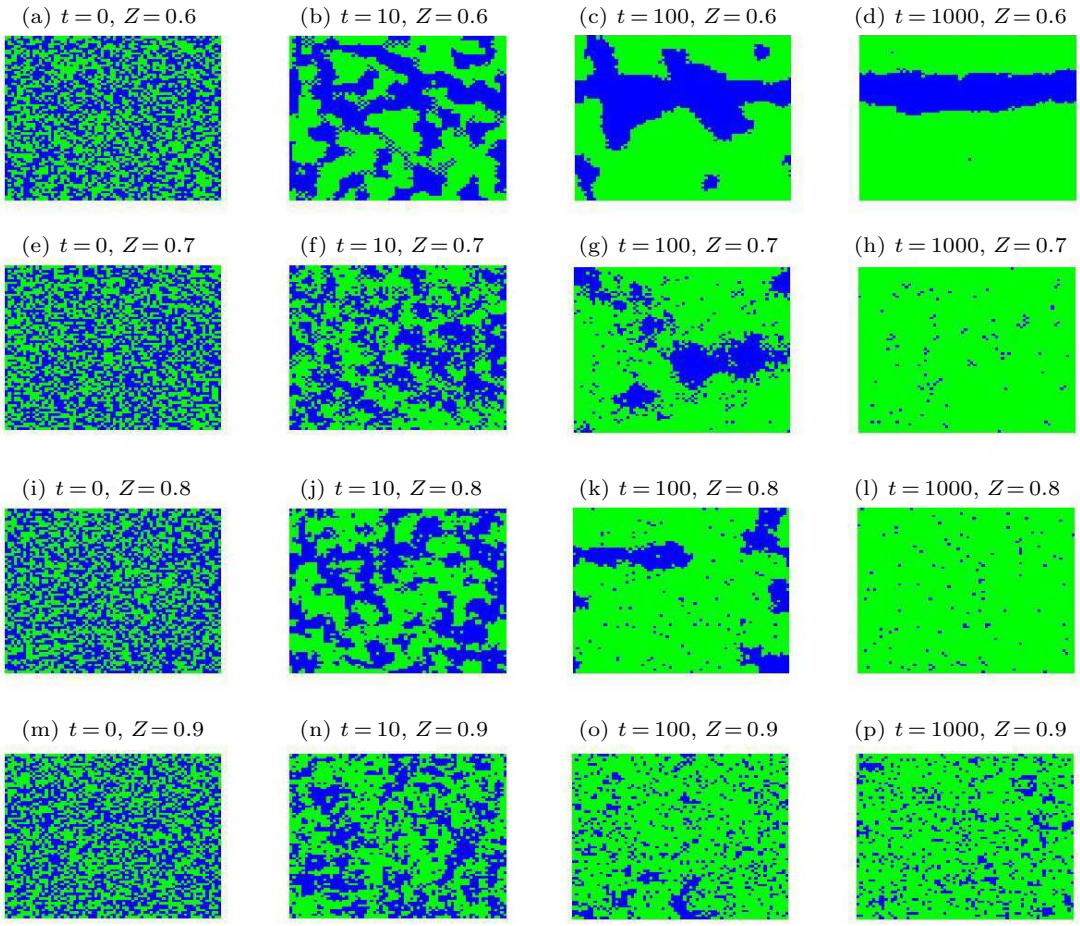


Fig. 3. The distribution of strategies on the lattice network at different time steps t and conformity thresholds Z ($b=4$, $R=3.1$; green and blue represent cooperators and defectors, respectively).

To compare how different cooperation payoffs affect the cooperation level, we first studied the evolution of strategies in the population under different R, as shown in Fig. 2. Under the four different conditions, strategies are distributed in clusters on the network. When R is less than 3, the evolutionary stable state is basically only defectors, and the clusters of cooperators disappear faster at $R=2.5$ than at $R=2.9$; when R is greater than 3, the evolutionary stable state is basically only cooperators; when $R=3$, at $t=1000$, there are still a large number of cooperators and defectors in the population, with cooperators accounting for the majority of the population and defectors forming a larger cluster in the population. From Fig. 2, it can be concluded that higher cooperation payoffs can lead to higher levels of cooperation, and the size of cooperation payoffs can affect the speed at which the population evolves to the evolutionary stable state.

On a lattice network, clusters of cooperators or defectors naturally emerge as individuals adopt the same strategy that maximizes their payoffs, and there is no motivation to change the strategy. From a micro mechanism perspective, the formation of strategy clusters is closely related to the mutual influence between individuals. In the group of defectors, individuals choose defection strategies because they gain higher payoffs through interactions with other defectors, forming a self reinforcing mechanism. Similarly, the formation of a cluster of cooperators stems from the positive feedback effect among cooperators, where each individual's payoff is maximized when cooperators choose cooperation strategies, leaving no motivation to change the strategy. This clustering effect can be explained by the social identity theory in social psychology: individuals tend to interact with similar others, thereby enhancing the cohesion of the group. Through collective action, cooperators can resist the invasion of defection strategies to a certain extent, forming a "protective barrier" within the group.

Next, we compared the evolution of strategies in the population under different conformity thresholds Z to explore how conformity affects the cooperation level, as shown in Fig. 3. When $Z=0.6$, both cooperators and defectors in the population formed clusters at the beginning, and finally cooperators occupied most of the population, with defectors forming a larger cluster in the population. This is because, at this time, agents have a high level of conformity and are very inclined to choose the strategy that is more common among neighbors, so the ability to gather for each strategy is very strong. From Fig. 3, it can be seen that as Z increases step by step, the conformity of agents decreases, the ability to gather for the same strategy in the population during the evolution process weakens and the clusters formed become smaller. This result means

that on the lattice network the amount of conformity can determine the ability of the same strategy agents in the population to gather and the size of the cluster, thereby affecting the ability to resist the invasion of other strategies. When $Z = 0.7$ or 0.8, defectors in the evolutionary stable state also gradually disappear, and the population no longer has a larger cluster of defectors, with the cooperation level close to 1. However, when $Z = 0.9$, defectors are scattered among cooperators in the evolutionary stable state, and the cooperation level is not as high as when $Z = 0.7$ or 0.8. This is because, at this time, agents have a low level of conformity and do not rely on conformity for strategy updates but update strategies through inference, so even for populations where cooperators are the majority, agents do not blindly follow a trend, and the cooperation level at this time is not as high as when the conformity threshold is lower.

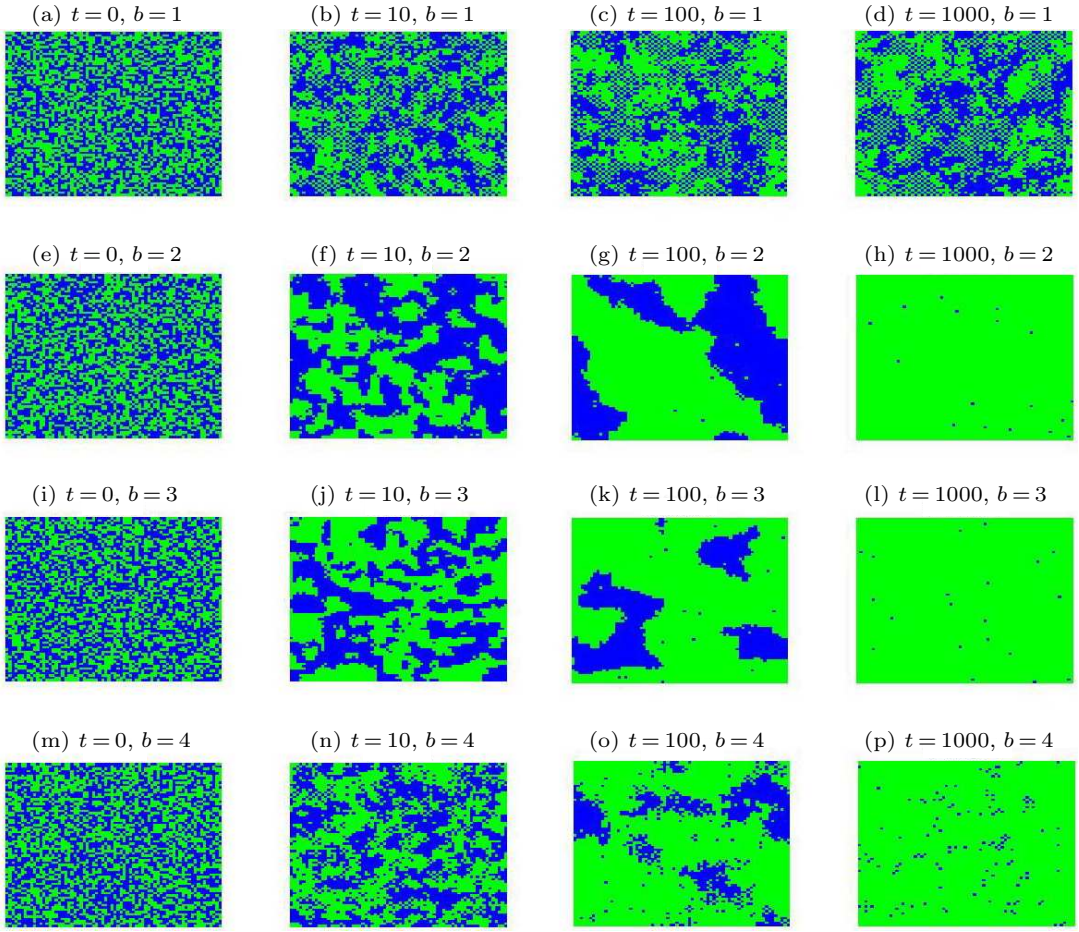


Fig. 4. The distribution of strategies on the lattice network at different time steps t and sample sizes b ($R = 3.1$, $Z = 0.7$; green and blue represent cooperators and defectors, respectively).

The above phenomenon can be explained by social impact theory: when individuals are influenced by the majority group, their decisions tend to follow the group's behavioral norms, and low conformity reduces the constraint of group norms on individual behavior. This result reveals the dual effect of conformity: conformity that is either too high or too low may hinder the unity of population strategies. When conformity is low, both cooperation and defection strategies are difficult to successfully promote and occupy the population. Taking human society as an example, if individuals in society tend to think rationally rather than blindly follow the crowd, although the spread of negative public opinion and rumors will be suppressed, positive public propaganda and policy promotion may also face greater resistance. This indicates that a moderate balance of conformity is of great significance for promoting consistency and efficiency in social group behavior.

Finally, we compared the evolution of strategies in the population under different sample sizes b to further explore how sample size affects the cooperation level, as shown in Fig. 4. When $b = 1$, agents will be completely driven by conformity to choose the strategy obtained by sampling. Since the sample size is very small, the sampling results have a large randomness and agents cannot accurately judge the real situation at this time, so the cooperation level is close to 0.5 with a large number of cooperators and defectors coexisting in the population, and only a small number of clusters in the population. When b is greater than 1, the cooperation level in the evolutionary stable state is close to 1, with a small number of defectors existing. When $b = 3$ is compared with $b = 2$, defec-

tors in the population disappear faster, the clusters of defectors also become multiple clusters under the invasion of cooperative strategies, the clusters become smaller and they gradually disappear faster under the encirclement of cooperative strategies. When $b = 4$, the clusters of defectors become smaller, the distribution of defectors in the network becomes more dispersed and the ability of defectors to resist cooperative strategies weakens obviously.

With a small sample size, agents cannot accurately assess neighbors' states, leading to low consistency in the population's behavior. When the sample size is larger, the sample size will affect the ability of clusters in the population to resist invasion. From a social psychology perspective, this process can be explained as follows: a larger sample of information reduces uncertainty in individual decision-making, thereby enhancing the consistency of collective behavior. This phenomenon indicates that the adequacy of information acquisition is an important driving force for the evolution of population strategies. In society, if individuals have limited understanding of their surrounding environment, the uncertainty of decision-making will intensify, making it difficult to form social consensus, thus reducing group coordination. When individuals are able to obtain sufficient authentic information, group behavior tends to be more consistent, achieving a high degree of social coordination. Therefore, in practical society, information transparency and accurate dissemination are crucial.

4.2. Stag hunt game on a scale-free network

Consider that the relationships between agents in the population are described by the Barabási–Albert model of a scale-free network. On a scale-free network, each node represents an agent, and each edge represents a connection between two agents. In this section, a scale-free network of size N is generated by a preferential attachment mechanism, starting from a fully connected network with m_0 $(m_0 < N)$ nodes, and each time a new node is added it is randomly connected to m $(m \leq m_0)$ different existing nodes with a certain probability until the number of nodes in the network is N. The probability that the newly added node is connected to the existing node i is $\Phi_i = k_i / \sum_j k_j$. In the scale-free network in this section, the relevant parameters are set as follows: $N = 4900$, $m_0 = 20$, $m = 10$, $K_1 = 0.5$.

Scale-free networks have heterogeneity, with a few nodes having many connections while most nodes have very few connections. In the simulation analysis below, nodes with $k \geq 50$ are referred to as high-degree nodes and nodes with $k < 50$ are referred to as low-degree nodes.

We first study the impact of R on the evolution of the strategy's stationary distribution in the game under the given conformity threshold Z and sample size b. R is the payoff obtained when both in the game cooperate, and the size of R represents the level of encouragement for both agents to choose cooperation in this game. Let $P(\mathrm{C})$ represent the proportion of agents choosing the cooperation strategy in the population. To test the robustness of the results, simulations were conducted for $Z = 0.85$, $b = 10$ and $Z = 0.75$, $b = 100$ when R takes different values, and the relationship between $P(\mathrm{C})$ and time step t is shown in Fig. 5.

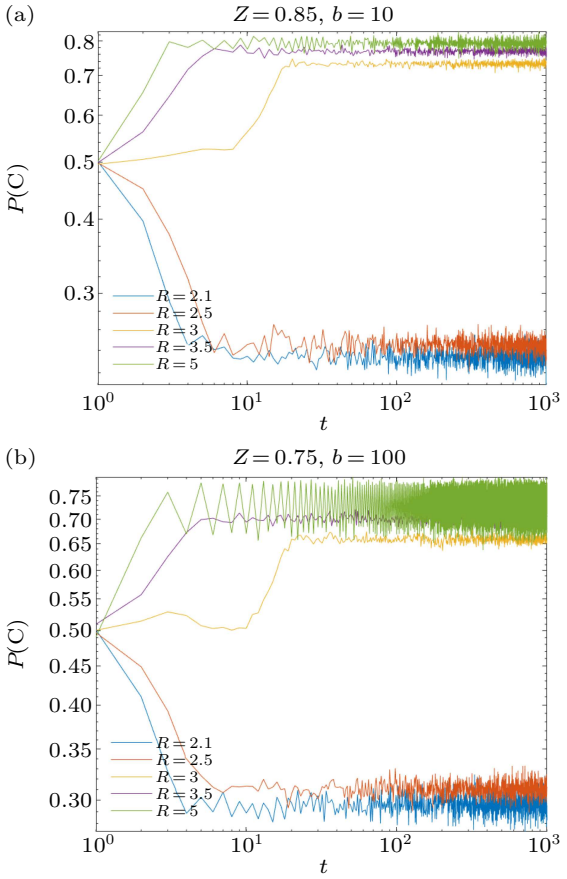


Fig. 5. Changes in the proportion of cooperation over time step t under different R ($Z = 0.85$, $b = 10$; $Z = 0.75$, $b = 100$).

It is worth noting that the trends shown in the parts of Fig. 5 are similar, indicating that the results are robust. It can be seen that when R is greater than a value of about 3 the proportion of cooperation increases with time until it stabilizes, while when R is less than a value of about 3 the proportion of cooperation decreases with time until it stabilizes. More specifically, when R decreases from 2.5 to 2.1, the final cooperation level also decreases, and the speed of reaching the evolutionary stable state becomes faster; when R increases from 3 to 5, the final cooperation level also increases, and the speed of reaching the evolutionary stable state becomes faster. That is to say, the larger the value of R, the more conducive it is to promote cooperation, which can not only increase the cooperation level but also increase the speed of finally converging to a stable state where most people cooperate.

This result means that an increase in the payoffs of cooperation will make agents more willing to take cooperative ac-

tions. From the perspective of social choice theory, a higher R enhances the attractiveness of cooperative behavior, promotes the formation of collective consistency and rapid stability and also reflects the strong driving effect of payoffs on individual decision-making. It can be seen that for a given Z and b there is a threshold; making R greater than this threshold will make the population evolve towards cooperation and making R less than this threshold will make the population evolve towards defection. The closer to this threshold, the slower the evolution to the final stable state, and the further away from this threshold, the faster the group will evolve to the final state. Therefore, in the follow-up simulations, in order to study the influence of conformity threshold Z and sample size b on the evolution results, larger and smaller R will also be set respectively to test the robustness of the conclusions.

In the following simulation, $R = 3$ and $R = 2.5$ were set to further test the robustness of the subsequent conclusions in the cases where the population eventually tends towards cooperation and defection. Then we studied how the proportion of cooperation evolves over time step t under different conformity thresholds. When $b = 10$, the relationship between $P(\mathrm{C})$ and time step t under different conformity thresholds Z is as shown in Fig. 6.

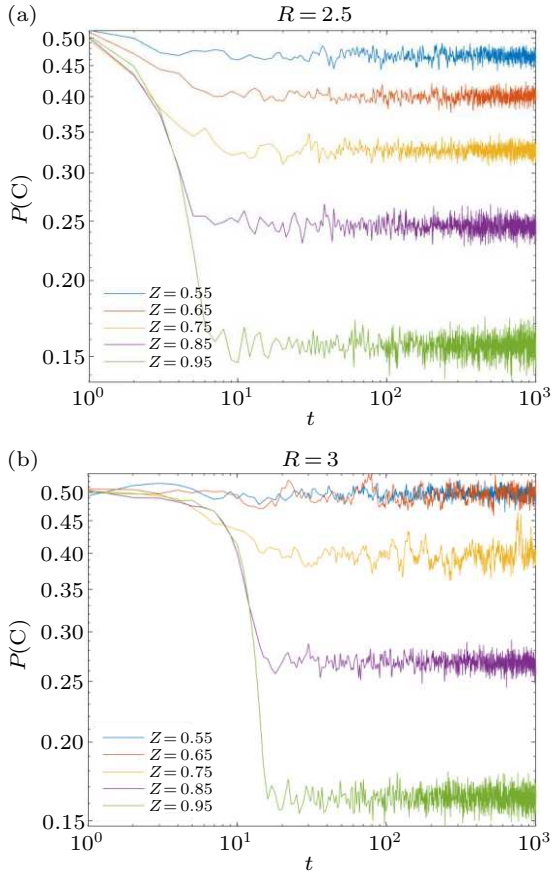


Fig. 6. Changes in the proportion of cooperation over time step t under different conformity thresholds Z ($R = 2.5$, $b = 10$; $R = 3$, $b = 10$).

The results in Fig. 6 show that when the conformity threshold Z increases, that is, conformity decreases, the cooperation level in the population will significantly decrease. For a smaller Z, such as $Z = 0.55$, the cooperation level is close to 0.5, which means that the number of cooperators and defectors in the population is not significantly different, and neither strategy occupies the main body of the group. For a larger Z, such as $Z = 0.95$, the cooperation level is close to 0.15, which means that most agents in the population have become defectors, and only a small proportion of the population are cooperators. However, the size of the conformity threshold does not affect the speed at which the cooperation level evolves to the stable state, indicating that an increase in conformity does not make the population evolve faster but only affects the final cooperation level.

From a microscopic mechanism analysis, when agents exhibit high conformity, they will blindly imitate the strategy of the majority of people around them rather than infer rationally, thus falling into a continuous cycle of mutual conformity imitation, resulting in a long-term presence of a large number of different strategies in the entire population and the inability to achieve consistency in the beliefs of agents' behavioral choices. From the perspective of social psychology, this result reveals the effect of conformity on group behavior: higher conformity increases strategic similarity between groups through imitative behavior, but may also lead agents to fall into an "imitation cycle" and fail to achieve high consistency in overall behavioral beliefs. This discovery has practical significance. For example, in actual society, guiding conformity behavior reasonably can promote group coordination and cooperation while maintaining diversity.

In addition, by comparing the two parts of Fig. 6, some results about R can also be found. The performance of cooperation when $R = 3$ is better than when $R = 2.5$. When R increases, the increase in the cooperation level is greater when the conformity threshold is low, and the cooperation level is almost unchanged when the conformity threshold is high. This result indicates that when R does not exceed the threshold that can change the direction of evolution, an increase in the payoffs of cooperation can enhance the final proportion of cooperation in the population, and this effect is mainly significant when conformity is high, because agents with low conformity will infer the best strategy based on the sample, and are less affected by other factors.

Next, we studied how the proportion of cooperation evolves over time step t under different sample sizes. When $Z = 0.75$, the relationship between $P(\mathrm{C})$ and time step t under different sample sizes b is shown in Fig. 7.

Figure 7 shows that when the number of samples increases, the cooperation level in the population will significantly decrease. When the number of samples is small, the cooperation level is close to 0.5, with about the same number of cooperators and defectors in the population, because a smaller

sample size cannot well reflect the true situation of the neighbors and has a certain impact on the inference or conformity of agents; when the number of samples is large, the cooperation level is close to 0.3, with most agents in the population becoming defectors, and only a small number of cooperators exist because a larger sample size can reflect the true situation of the neighbors, enabling agents to make accurate decisions based on real information, and making most agents in the population evolve towards a certain strategy in unison.

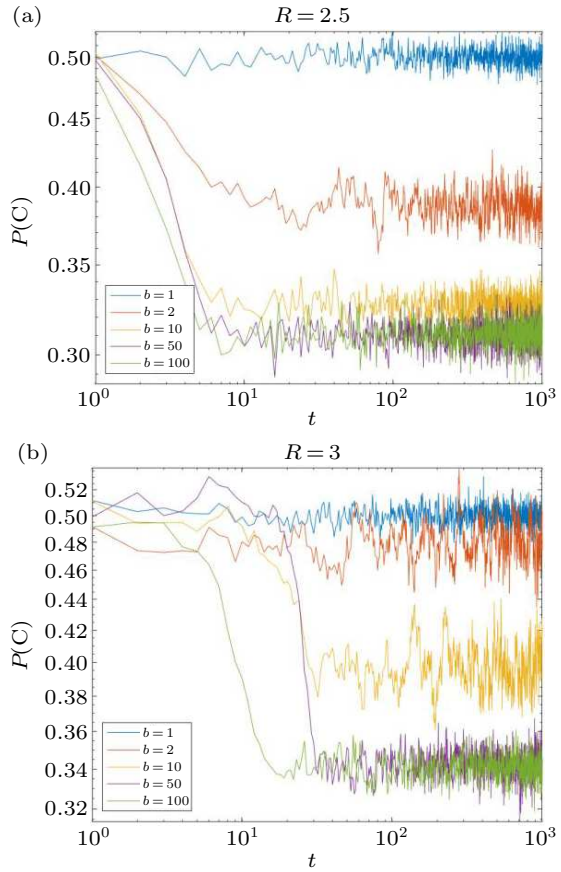


Fig. 7. Changes in the proportion of cooperation over time step t under different sample sizes b ($R = 2.5$, $Z = 0.75$; $R = 3$, $Z = 0.75$).

From a microscopic perspective, the change in sample size affects the depth of agents' acquisition of local information and the accuracy of decision-making. When the sample size is small, agents' perceptions of neighbor strategies are easily affected by randomness, leading to greater uncertainty in strategy selection and a more balanced distribution of strategies in the population. When the sample size is large, agents can obtain more authentic neighbor information, enhancing the advantage of the defection strategy and suppressing the scalability of the cooperation strategy. Besides that, the analysis results also indicate that when the sample size increases to a certain extent, a larger sample size will no longer improve the cooperation level of the stable state in the population: because there are a few high-degree nodes and a majority of low-degree nodes in the network, and when the sample size is large enough increasing the sample size only affects a small number of nodes and the change in the final cooperation level is minimal. The cooperation level in the population is mainly determined by nodes with lower degrees, as they make up the majority of the population.

In addition, by comparing the two parts of Fig. 7, some results about R can also be found. The performance of cooperation when $R = 3$ is better than when $R = 2.5$, and the fluctuation level when $R = 3$ is greater than when $R = 2.5$. This result indicates that when R does not exceed the threshold that can change the direction of evolution, an increase in the payoffs of cooperation can enhance the final proportion of cooperation in the population, and will also increase the fluctuation level during the evolution process.

In order to compare the impact of high-degree nodes and low-degree nodes on the cooperation level in the population, we studied how the proportion of cooperation in high-degree nodes and low-degree nodes in the population evolves over time step t. When $Z = 0.75$ and $b = 100$, the relationship between the proportion of cooperation in high-degree nodes and low-degree nodes and time step t is shown in Fig. 8.

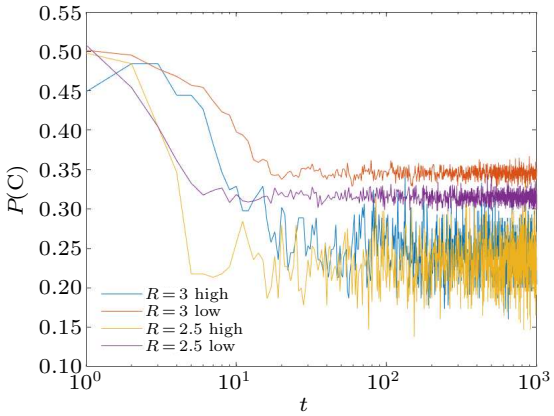


Fig. 8. Changes in the proportion of cooperation over time step t in different nodes.

In Fig. 8, it can be seen that the consistency of high-degree nodes is higher than that of low-degree nodes, and the speed of evolution of high-degree nodes is faster than that of low-degree nodes, indicating that high-degree nodes can more easily influence the surrounding agents to choose the same strategy as them, guiding the evolution of the population and playing a leading role. The fluctuation level of high-degree nodes is greater, mainly because there are fewer high-degree nodes. When R increases, low-degree nodes are more easily affected and the proportion of cooperators increases to a certain extent, while the cooperation level of high-degree nodes changes less. However, the cooperation level in the population is mainly determined by low-degree nodes. This is because nodes with lower degrees occupy the majority in the network, and their strategies are more directly influenced by local neighbors. Therefore, as R increases, they are more inclined to adopt cooperation strategies.

Nodes with higher degrees, due to their wide connection range, can quickly propagate their chosen strategies, promoting local network consistency. From the perspective of social psychology, nodes with higher degrees can be likened to opinion leaders in society, who spread influence through broader social connections and guide the direction of group behavior. However, this dominant role may also be limited by its small number, and the final level of population cooperation is more determined by nodes with lower degrees. A small number of high-impact individuals can guide a group, but the overall characteristics of the group are still constrained by the majority of low-impact individuals.

Figure 9 shows the changes in the cooperation level in the population as the sample size b and conformity threshold Z vary when $R = 2.5$. When the sample size is about 10 or less, the cooperation level is close to 0.5, and the size of the conformity threshold has almost no effect on the cooperation level; when the sample size is larger, the larger the conformity threshold, the lower the cooperation level, and the higher the consistency of the population. This result indicates that when the sample is small, the decision-making of agents is more random, and the population does not easily evolve to the same strategy; when the sample is large, the higher the conformity, the less likely it is for the agents in the population to evolve to the same strategy. Therefore, conformity does not necessarily lead most agents to choose the same strategy.

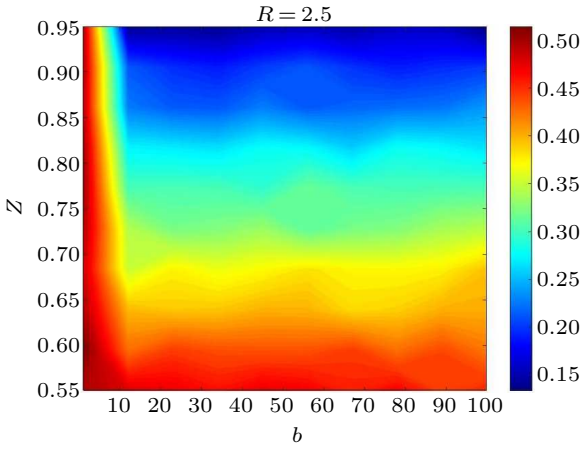


Fig. 9. Changes of the cooperation level with sample size b and conformity threshold Z ($R = 2.5$).

This phenomenon reveals the dual role of conformity behavior in promoting group consistency and influencing the direction of group decision-making. A lower conformity threshold can promote rapid convergence of local neighbor strategies, thereby forming small-scale strategy consistency within a local area. However, excessive conformity may lead to blind imitation, resulting in individuals within the population being unable to effectively integrate global information and hindering the consistency of the overall strategy. This is consistent with the group polarization theory, indicating that individuals may be more inclined to develop along their neighbors' mainstream strategies under high-conformity conditions, suppressing global diversity and balance. This indicates that simply improving the ability to acquire information is not enough to achieve high-level cooperation in social networks. It is also necessary to design a reasonable conformity mechanism to prevent the negative impact of excessive imitation behavior on cooperation strategies.

Figure 10 shows the changes in the cooperation level in the population as R and conformity threshold Z vary when $b = 100$. When the conformity threshold is less than about 0.65, the cooperation level is about 0.5, and R has little effect on the cooperation level; when the conformity threshold is larger, the population has higher consistency and the size of R affects the cooperation level in the population. Specifically, when R is less than about 3 the cooperation level is very low, and the population comprises almost entirely defectors; when R is greater than about 3 the cooperation level is very high, and the population comprises almost entirely cooperators.

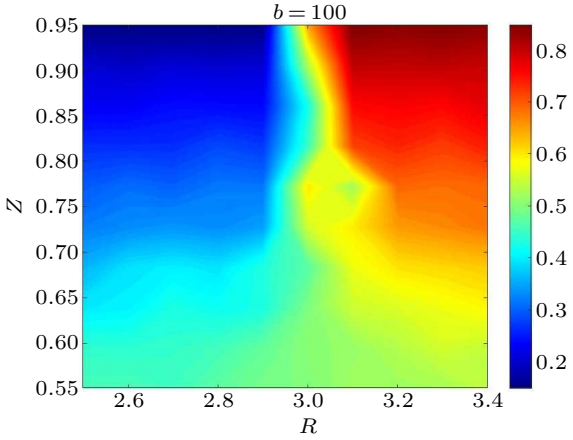


Fig. 10. Changes in the cooperation level with R and conformity threshold Z ($b = 100$).

From a micromechanism perspective, low conformity makes agents more inclined to make independent decisions rather than imitating neighbors' strategies. However, when conformity is high, agents lack careful consideration due to blind imitation, resulting in strategy selection being greatly affected by the randomness of neighbor distribution, making it difficult to achieve global consistency. In addition, the size of the payoff R in the game plays a decisive role in the final state of the population: higher payoffs encourage agents to adopt cooperation strategies, thereby promoting the overall cooperation of the population, and lower payoffs lead to the advantage of defection strategy. A high conformity can be seen as an excessive dependence on social norms, which may hinder individuals' rational behavioral choices under uncertain conditions, leading to group behavior falling into local equilibrium rather than the global optimum.

5. Conclusion

In this paper we have studied the influence of conformity on the evolution of cooperation in the stag hunt game with sampling inference on networks. In our model, agents need to play games with neighbors and obtain average payoffs at each time step, then sample among neighbors and obtain the number of the two strategies in the sample of neighbors; then agents update strategies driven by conformity, or infer the payoffs of neighbors based on the sample, and then update strategies driven by payoffs.

Through a large number of simulations, we have obtained the relationship between variables such as conformity and sample size with the cooperation level, and have obtained a series of conclusions with practical significance. The most important conclusion is that rational inference through samples is more conducive to the evolution of the same strategy among all agents in the population than blind conformity.

On the lattice network, different strategies often appear in the form of clusters, resisting the invasion of other strategies. An increase in the payoff R obtained from cooperation will enhance the cooperation level in the population at the evolutionary stable state and affect the speed of evolution. Conformity can also determine the ability of different strategies to gather and the size of clusters, thereby affecting the ability to resist the invasion of other strategies. When the sample size is large, the population has higher consistency, and the sample size will affect the ability of clusters in the population to resist invasion.

On a scale-free network, higher payoff R obtained from cooperation will make agents more willing to take cooperative actions, with a threshold determining evolution: R above the threshold leads to cooperation, below it to defection and the closer to it, the slower the evolution to the final stable state. When agents have a high conformity, they will blindly imitate the strategy of most people around them, rather than infer rationally, thus falling into a continuous cycle of mutual conformity imitation, resulting in the inability to achieve consistency in the behavior of agents in the entire population. In addition, a smaller sample size cannot reflect the true situation of the neighbors, and has a certain impact on the inference or conformity of agents. On a scale-free network, the cooperation level in the population is mainly determined by nodes with lower degrees, while nodes with higher degrees can more easily influence the surrounding agents and play a leading role in the evolution of the population. Authenticity of information from neighbors only exists when the sample size is large. Conformity does not necessarily lead most agents to choose the same strategy, and due to the blind imitation of agents it is difficult for the agents in the population to reach a consensus.

In summary, this work provides some new insights into how conformity can promote the evolution of cooperation. The size of the payoffs obtained from cooperation determines the direction of the entire population, so managers in organizations should pay attention to establishing a positive cooperative culture and use reward and punishment mechanisms to encourage cooperative behavior. Agents making decisions through rational inference can improve the overall quality of decision-making in the population. At the same time, the authenticity and comprehensiveness of information should be valued, because a smaller sample has a larger degree of randomness. In management practice, it is necessary to ensure that the information obtained is sufficiently authentic and comprehensive to avoid making decisions based on inaccurate or incomplete information, leading to incorrect results. However, this work does not consider the heterogeneous conformity thresholds of agents, nor does it consider more complex strategy update rules beyond the Fermi rule; these will be further considered in future research.

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