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Sudden change of interferometric power for X-shape states

Dian Zhu(朱典)[1], Fu-Lin Zhang(张福林)[2,†], and Jing-Ling Chen(陈景灵)[1,‡]

[1] *Theoretical Physics Division, Chern Institute of Mathematics, Nankai University, Tianjin* 300071, *China*
[2] *Department of Physics, School of Science, Tianjin University, Tianjin* 300072, *China*

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Quantum interferometric power (IP), a discordlike measure, plays an important role in quantum metrology. We study the dynamics of IP for two-qubit X-shape states under different noisy environments. Our study shows that IP exhibits sudden change, and one side quantum channel is enough for the occurrence of a sudden change of IP. In particular, we show that the initial state having no sudden change of quantum discord exhibits a sudden change of IP under the dynamics of amplitude noise, but the converse is not true. Besides, we also investigate the dynamics of IP under two different kinds of composite noises. Our results also confirm that sudden change of IP occurs under such composite noises.

Keywords: quantum interferometric power, quantum noise, sudden change

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1. Introduction

Quantum correlations characterize the quantum feature of bipartite or multipartite system.[1] For a long time, entanglement was considered to be the only quantum correlation that was useful for quantum information processing. However, it is realized that there exists another quantum correlation weaker than entanglement called quantum discord,[2,3] which can be found in separable mixed states and may play an important role in quantum information processing. In the past two decades, a great deal of attentions have been received for quantum discord.[4–26] On the other hand, with the recent development of quantum metrology experimentally and theoretically, it has been realized that quantum correlations constitute an available resource in quantum metrology protocols.[27–40] In metrology, quantum Fisher information (QFI) determines the estimation precision of an unknown parameter and it can be considered as a quantifier quantifying the quantum correlation weaker than entanglement.[27–29,41–44] For a given bipartite probe state with only one part of the system undergoes a unitary transformation, Girolami *et al*. presented a discordlike quantifier, called interferometric power,[45] to determine the estimation accuracy of an unknown parameter in black-box settings. Based on QFI, the definition of IP naturally quantified the guaranteed sensitivity of the probe state in some interferometric devices. As a new tool of discord-like measure, IP plays an essential role in quantum metrology tasks.[45–48] Hence, a great deal of attention has been received for IP in recent studies, such as the dynamic of IP in multipartite coherent states,[49] dynamically producing asymmetric IP,[50] Gaussian IP,[46,51–53] and so on.

In realistic quantum information processing, the inevitable interaction between the quantum system and the environment destroys the quantum properties of the system, such as entanglement sudden death[54–56] and sudden change of quantum discord.[57,58] The study of these quantum characteristics of open quantum systems theoretically presents that quantum correlations have different dynamical behaviors compared with quantum coherence under the affection of the environment for a long time interval. On the other hand, these studies also help us to utilize quantum advantages in quantum information processing. Therefore, it is essential to investigate some specific dynamical behaviors of IP under the decoherence process. Sudden change[23,24] is a particular dynamical behavior of quantum discord, which arises from the dynamics of quantum discord for two-qubit systems passing through quantum noise channels. However, as the other discordlike measure, there are no investigations to show that the abrupt modification of the decay rate exists under different noisy channels for IP. In this paper, we investigate the dynamics of IP for two-qubit X-shape states under several different kinds of quantum noises. Our results show that compared to depolarizing noise, IP exhibits sudden change under amplitude noise and phase noise acting on the first qubit of the two-qubit system. In particular, our findings show that the condition of the sudden change of IP differs from quantum discord under amplitude noise acting on the first qubit of the two-qubit system. Our studies also show that IP exhibits sudden change under the two different kinds of dephasing noise acting on both two-qubit systems.

†Corresponding author. E-mail: flzhang@tju.edu.cn
‡Corresponding author. E-mail: chenjl@nankai.edu.cn

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2. Quantum IP as the discordlike measure of X-shape states

The concept of IP of quantum states has been introduced explicitly in Ref. [45]. As one of the measure of discordlike quantum correlations, IP provides a computable way of quantum discord quantification. For a bipartite system $\rho_{\rm AB}$, the IP is defined as the minimum of the QFI by taking over all the local Hamiltonians $H_{\rm A}$ acted only on the subsystem A,

$$\mathcal{IP}(\rho_{\rm AB}) = \frac{1}{4}\min_{H_{\rm A}} \mathcal{F}(\rho_{\rm AB}, H_{\rm A}), \tag{1}$$

where

$$\begin{aligned}&\mathcal{F}(\rho_{\rm AB}, H_{\rm A})\\ &= 4\sum_{i\neq j; q_i+q_j\neq 0} \frac{(q_i-q_j)^2}{q_i+q_j} \left|\langle\psi_1|H_{\rm A}\otimes\mathbb{1}_{\rm B}|\psi_j\rangle\right|^2\end{aligned} \tag{2}$$

with $\{q_i, |\psi_i\rangle\}$ being the eigenvalues and the eigenvectors of $\rho_{\rm AB}$, respectively. For the case of subsystem A being a qubit, one can take the set of Hamiltonians $H_{\rm A} = \vec{n}\cdot\vec{\sigma}$ with $\vec{n}=1$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. Then equation (1) can be reduced to a closed formula

$$\mathcal{IP}(\rho_{\rm AB}) = \zeta_{\min}[M], \tag{3}$$

where $\zeta_{\min}[M]$ is the smallest eigenvalue of the matrix M with elements[45]

$$\begin{aligned}&M_{m,n}\\ &= \frac{1}{2}\sum_{i,l;q_i+q_l\neq 0} \frac{(q_i-q_l)^2}{q_i+q_l}\langle\psi_i|\sigma_m\otimes\mathbb{1}|\psi_l\rangle\langle\psi_l|\sigma_n\otimes\mathbb{1}|\psi_i\rangle.\end{aligned} \tag{4}$$

One can note that IP has the following properties: (i) $\mathcal{IP}(\rho_{\rm AB})$ equals zero if $\rho_{\rm AB}$ is a classical state (with respect to A); (ii) $\mathcal{IP}(\rho_{\rm AB})$ is invariant under local unitary operation; (iii) $\mathcal{IP}(\rho_{\rm AB})$ is monotonically decreasing under local completely positive and trace-preserving maps on subsystem B; (iv) $\mathcal{IP}(\rho_{\rm AB})$ reduces to a measure of entanglement if $\rho_{\rm AB}$ is a pure state.[45] These properties imply that $\mathcal{IP}(\rho_{\rm AB})$ is a proper measure of discord-type correlation.

Let us consider the following X-shape states of two-qubit system,[25]

$$\rho_{\rm AB} = \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + \vec{r}\cdot\vec{\sigma}\otimes\mathbb{1} + \mathbb{1}\otimes\vec{s}\cdot\vec{\sigma} + \sum_{j=1}^{3} c_j\sigma_j\otimes\sigma_j), \tag{5}$$

where $\mathbb{1}$ is the 2×2 identity matrix, $\vec{r} = (0,0,r)$, $\vec{s} = (0,0,s)$, $c_j\in\mathbb{R}$, and σ_j $(j=1,2,3)$ are the Pauli matrices. It is easy to see that equation (5) reduces to Bell-diagonal states when $r=s=0$.

The form of IP of $\rho_{\rm AB}$ can be given by Eq. (4), that is

$$\mathcal{IP}(\rho_{\rm AB}) = \min\{M_{11}, M_{22}, M_{33}\}, \tag{6}$$

where

$$\begin{aligned}
M_{11} &= \frac{(\lambda_1-\lambda_3)^2}{\lambda_1+\lambda_3}\cdot\frac{(x_1+y_1)^2}{(1+x_1^2)(1+y_1^2)}\\
&+ \frac{(\lambda_1-\lambda_4)^2}{\lambda_1+\lambda_4}\cdot\frac{(x_1+y_2)^2}{(1+x_1^2)(1+y_2^2)}\\
&+ \frac{(\lambda_2-\lambda_3)^2}{\lambda_2+\lambda_3}\cdot\frac{(x_2+y_1)^2}{(1+x_2^2)(1+y_1^2)}\\
&+ \frac{(\lambda_2-\lambda_4)^2}{\lambda_2+\lambda_4}\cdot\frac{(x_2+y_2)^2}{(1+x_2^2)(1+y_2^2)},\\
M_{22} &= \frac{(\lambda_1-\lambda_3)^2}{\lambda_1+\lambda_3}\cdot\frac{(x_1-y_1)^2}{(1+x_1^2)(1+y_1^2)}\\
&+ \frac{(\lambda_1-\lambda_4)^2}{\lambda_1+\lambda_4}\cdot\frac{(x_1-y_2)^2}{(1+x_1^2)(1+y_2^2)}\\
&+ \frac{(\lambda_2-\lambda_3)^2}{\lambda_2+\lambda_3}\cdot\frac{(x_2-y_1)^2}{(1+x_2^2)(1+y_1^2)}\\
&+ \frac{(\lambda_2-\lambda_4)^2}{\lambda_2+\lambda_4}\cdot\frac{(x_2-y_2)^2}{(1+x_2^2)(1+y_2^2)},\\
M_{33} &= \frac{(\lambda_1-\lambda_2)^2}{\lambda_1+\lambda_2}\cdot\frac{(x_1x_2-1)^2}{(1+x_1^2)(1+x_2^2)}\\
&+ \frac{(\lambda_3-\lambda_4)^2}{\lambda_3+\lambda_4}\cdot\frac{(y_1y_2-1)^2}{(1+y_1^2)(1+y_2^2)}.
\end{aligned} \tag{7}$$

Here $\lambda_i\geq 0$ are eigenvalues of $\rho_{\rm AB}$,

$$\begin{aligned}
x_1 &= \frac{r-s-2(\lambda_2-\lambda_1)}{c_1+c_2}, \quad x_2 = \frac{r-s+2(\lambda_2-\lambda_1)}{c_1+c_2},\\
y_1 &= \frac{r+s-2(\lambda_4-\lambda_3)}{c_1-c_2}, \quad y_2 = \frac{r+s+2(\lambda_4-\lambda_3)}{c_1-c_2}.
\end{aligned}$$

From the above expressions, one can find that $M_{11}\geq M_{22}$ (or $M_{11}<M_{22}$) if $|c_1|\leq|c_2|$ (or $|c_1|>|c_2|$). Hence, the IP of $\rho_{\rm AB}$ can be rewritten as

$$\mathcal{IP}(\rho_{\rm AB}) = \begin{cases}\min\{M_{22}, M_{33}\}, & \text{if } |c_1|<|c_2|,\\ \min\{M_{11}, M_{33}\}, & \text{if } |c_1|>|c_2|.\end{cases} \tag{8}$$

3. Sudden change of IP under one side quantum channel

In this section, based on the analytical formula of IP given above, we can study the dynamics of IP for a two-qubit system over three kinds of quantum noises acting on the first qubit: amplitude noise, phase noise, and depolarizing noise. Decoherence in quantum systems within thermal environments involves amplitude and phase noise. When amplitude noise predominates, it can be approximately assumed that only amplitude noise affects the system; similarly, when phase noise dominates, only phase noise is considered. Additionally, depolarizing noise represents a simple form of an ideal noise model, and the dynamical properties of quantum correlations under this type of noise merit investigation. For simplicity, we

consider the Bell-diagonal states as the initial states of two-qubit system,[26]

$$\rho = \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + \sum_{i=1}^{3} c_i\sigma_i\otimes\sigma_i). \tag{9}$$

3.1. Amplitude noise

Amplitude damping, or amplitude noise, describes the energy dissipation from a quantum system. One can consider a two-qubit system where the first qubit is through this quantum channel. Then the Kraus operators for the whole system are given by[54]

$$K_{1a} = \begin{pmatrix} \eta & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$K_{2a} = \begin{pmatrix} 0 & 0 \\ \sqrt{1-\eta^2} & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{10}$$

where $\eta = \mathrm{e}^{-\tau t/2}$, τ is the amplitude decay rate, t is time. The evolution of the initial states Eq. (9) under this quantum channel can be described by

$$\mathcal{E}_a(\rho) = K_{1a}\rho K_{1a}^\dagger + K_{2a}\rho K_{2a}^\dagger = \frac{1}{4}\begin{pmatrix} \eta^2(1+c_3) & 0 & 0 & \eta(c_1-c_2) \\ 0 & \eta^2(1-c_3) & \eta(c_1+c_2) & 0 \\ 0 & \eta(c_1+c_2) & 2-\eta^2-\eta^2 c_3 & 0 \\ \eta(c_1-c_2) & 0 & 0 & 2-\eta^2+\eta^2 c_3 \end{pmatrix}$$
$$= \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + r(t)\sigma_3\otimes\mathbb{1} + s(t)\mathbb{1}\otimes\sigma_3 + \sum_{j=1}^{3} c_j(t)\sigma_j\otimes\sigma_j), \tag{11}$$

where $r(t) = \eta^2 - 1$, $s(t) = 0$, $c_1(t) = \eta c_1$, $c_2(t) = \eta c_2$, and $c_3(t) = \eta^2 c_3$. By Eq. (8), the IP of $\mathcal{E}_a(\rho)$ is

$$\mathcal{IP}(\mathcal{E}_a(\rho)) = \begin{cases} \min\{M_{22}(t), M_{33}(t)\}, & \text{if } |c_1(t)| < |c_2(t)|, \\ \min\{M_{11}(t), M_{33}(t)\}, & \text{if } |c_1(t)| > |c_2(t)|. \end{cases} \tag{12}$$

In particular, equation (7) can be reduced to the following simple form when $t = 0$:

$$M_{11}(0) = \frac{c_2^2 + c_3^3 + 2c_1c_2c_3}{1-c_1^2},$$
$$M_{22}(0) = \frac{c_1^2 + c_3^3 + 2c_1c_2c_3}{1-c_2^2},$$
$$M_{33}(0) = \frac{c_2^2 + c_2^3 + 2c_1c_2c_3}{1-c_3^2}. \tag{13}$$

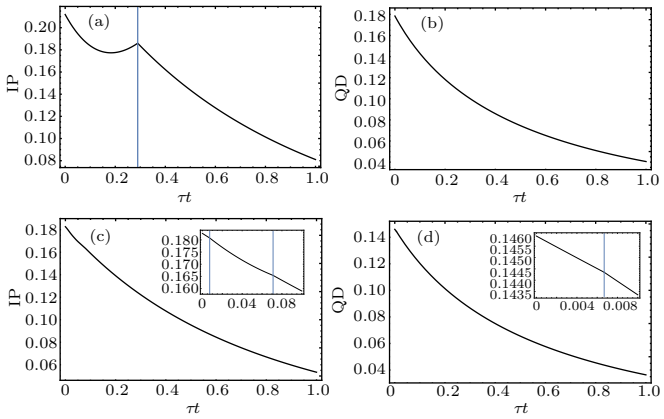


Fig. 1. The evolution of IP ((a) and (c)) versus the evolution of quantum discord ((b) and (d)) under amplitude noise acting on the first qubit of the bipartite system. Considering the initial state not satisfying the condition of a sudden change of quantum discord ($c_1 = 0.4$, $c_2 = 0.2$, $c_3 = 0.3$), we can see that IP exhibits a sudden change with the same situation. On the other hand, if we consider the initial state satisfying the condition of a sudden change of quantum discord ($c_1 = 0.3$, $c_2 = 0.2$, $c_3 = 0.301$), one can see that a sudden change of IP still occurs with the same situation.

It is easy to find that $M_{33}(0) > M_{11}(0)$ (or $M_{33}(0) > M_{22}(0)$) if $|c_3| < |c_1|$ (or $|c_3| < |c_2|$), and when $t \to \infty$, $M_{11}(\infty) = M_{22}(\infty) > M_{33}(\infty)$. Hence, a sudden change of IP occurs when $|c_3| < \max\{|c_1|, |c_2|\}$ in the initial Bell-diagonal state ρ with this kind of amplitude noisy channel. However, a sudden change of quantum discord occurs when $|c_3| > \max\{|c_1|, |c_2|\}$ in the initial state ρ with the same noise environment.[57] We find that the initial state satisfying the condition of a sudden change of quantum discord also exhibits a sudden change of IP, which means that the conditions of the sudden change of IP and quantum discord are not complementary. These results have been shown in Fig. 1.

3.2. Phase noise

Next we investigate the dynamics of IP over phase noise channel. Phase noise describes the loss of quantum information without loss of energy. The Kraus operators of phase noise for the whole system can be read as[1,59]

$$K_{1p} = \sqrt{\alpha}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$K_{2p} = \sqrt{1-\alpha}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{14}$$

where

$$\alpha = \frac{1}{2}(1+\sqrt{1-\gamma}), \quad \gamma = \mathrm{e}^{(-\tau t/2)}$$

and τ denotes transversal decay rate. The output states from this quantum channel is

$$\mathcal{E}_p(\rho) = K_{1p}\rho K_{1p}^\dagger + K_{2p}\rho K_{2p}^\dagger$$

$$= \frac{1}{4}\begin{pmatrix} 1+c_3 & 0 & 0 & \gamma(c_1-c_2) \\ 0 & 1-c_3 & \gamma(c_1+c_2) & 0 \\ 0 & \gamma(c_1+c_2) & 1-c_3 & 0 \\ \gamma(c_1-c_2) & 0 & 0 & 1+c_3 \end{pmatrix}$$
$$= \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + \sum_{i=1}^{3} c_i(t)\sigma_i\otimes\sigma_i), \quad (15)$$

where $c_1(t) = \gamma c_1$, $c_2(t) = \gamma c_2$, and $c_3(t) = c_3$. By Eq. (8), the expression of IP of $\mathcal{E}_p(\rho)$ can be obtained, that is

$$\mathcal{IP}(\mathcal{E}_p(\rho)) = \begin{cases} \min\{M_{22}(t), M_{33}(t)\}, & \text{if } |c_1(t)| < |c_2(t)|, \\ \min\{M_{11}(t), M_{33}(t)\}, & \text{if } |c_1(t)| > |c_2(t)|, \end{cases} \quad (16)$$

where

$$M_{11}(t) = \frac{\gamma^2 c_2^2(t) + c_3^2(t) + 2\gamma^2 c_1(t)c_2(t)c_3(t)}{1-\gamma^2 c_1^2(t)},$$
$$M_{22}(t) = \frac{\gamma^2 c_1^2(t) + c_3^2(t) + 2\gamma^2 c_1(t)c_2(t)c_3(t)}{1-\gamma^2 c_2^2(t)},$$
$$M_{33}(t) = \frac{\gamma^2 c_1^2(t) + \gamma^2 c_2^2(t) + 2\gamma^2 c_1(t)c_2(t)c_3(t)}{1-c_3^2(t)}. \quad (17)$$

In Ref. [45], it is already known that the IP of $\mathcal{E}_p(\rho)$ can be rewritten as the following form

$$\mathcal{IP}(\mathcal{E}_p(\rho)) = \frac{\|C(t)\|^2 - \|C(t)\|_\infty^2 + 2\det C(t)}{1-\|C(t)\|_\infty^2}, \quad (18)$$

where

$$\|C(t)\|^2 = \mathrm{Tr}[C^{\mathrm{T}} C] = c_1^2(t) + c_2^2(t) + c_3^2(t),$$

the square Hilbert–Schmidt of $C(t)$ and $\|C(t)\|_\infty^2 = \max\{c_1^2(t), c_2^2(t), c_3^2(t)\}$ the operator norm of $C(t)$. If $|c_3| \geq \max\{|c_1|, |c_2|\}$, $\|C(t)\|_\infty^2$ will reduce to $c_3^2(t)$ since the decay rates of $\max\{|c_1(t)|, |c_2(t)|\}$ and $|c_3(t)|$ are different, then the IP $\mathcal{IP}(\mathcal{E}_p(\rho))$ decays monotonically. If $|c_3| < \max\{|c_1|, |c_2|\}$ and $c_3 \neq 0$, the decay rate of IP $\mathcal{IP}(\mathcal{E}_p(\rho))$ sudden changes at

$$t_0 = -\frac{2}{\tau}\ln\left|\frac{c_3}{\max\{|c_1|, |c_2|\}}\right|.$$

In Fig. 2, we choose three different $\{c_i\}$ to represent the dynamics of IP of ρ under single qubit phase noise. As a result, a sudden change of IP may occur under this kind of noise environment.

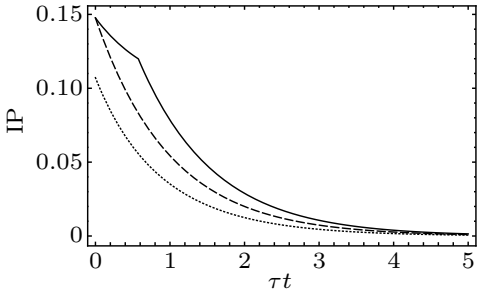


Fig. 2. IP of the initial Bell-diagonal states under phase noise acting on the first qubit of the quantum system. The solid line for $c_1 = 0.4$, $c_2 = 0.1$, $c_3 = 0.3$; the dashed line for $c_1 = 0.1$, $c_2 = 0.3$, $c_3 = 0.4$; the dotted line for $c_1 = 0.4$, $c_2 = 0.3$, $c_3 = 0$. The sudden change only happens at situation (solid line).

3.3. Depolarizing noise

As one of the important types of quantum noise, the depolarizing noise describes a process that takes a state into completely mixed state $\mathbb{1}/2$ with probability p and the state being left untouched with probability $1-p$. The operation elements for depolarizing noise are shown as $\{\sqrt{1-3p/4}\mathbb{1}, \sqrt{p}\sigma_x/2, \sqrt{p}\sigma_y/2, \sqrt{p}\sigma_z/2\}$ and the Kraus operators for the whole system are given by[1]

$$K_{1\mathrm{d}} = \sqrt{1-\frac{3p}{4}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\otimes\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$K_{2\mathrm{d}} = \frac{\sqrt{p}}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\otimes\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$K_{3\mathrm{d}} = \frac{\sqrt{p}}{2}\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}\otimes\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$K_{4d} = \frac{\sqrt{p}}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\otimes\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (19)$$

where $p = 1 - \mathrm{e}^{-\tau t}$. The Bell-diagonal states after this noise is

$$\mathcal{E}_{\mathrm{d}}(\rho) = \sum_{j=1}^{4} K_{j\mathrm{d}}\rho K_{j\mathrm{d}}^{\dagger} = \frac{1}{4}\begin{pmatrix} 1+(1-p)c_3 & 0 & 0 & (1-p)(c_1-c_2) \\ 0 & 1-(1-p)c_3 & (1-p)(c_1+c_2) & 0 \\ 0 & (1-p)(c_1+c_2) & 1-(1-p)c_3 & 0 \\ (1-p)(c_1-c_2) & 0 & 0 & 1+(1-p)c_3 \end{pmatrix}$$
$$= \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + \sum_{i=1}^{3} c_i(t)\sigma_i\otimes\sigma_i), \quad (20)$$

where $c_i(t) = (1-p)c_i$, $i = 1,2,3$. Similarly, the form of the IP of $\mathcal{E}_{\mathrm{d}}(\rho)$ is Eq. (18) but $c_i(t) = (1-p)c_i$, $i = 1,2,3$. The dynamics of IP for X-shape states depend on the evolution of the matrix elements of the density matrix. Since the anisotropy of amplitude noise and phase noise, these elements have different decay rates and cause a sudden change of the minimum in Eq. (6). In comparison, the isotropy of depolarizing noise results in the identical decay rate of $c_i(t)$ $(i = 1,2,3)$, and the

other elements remain zero. Therefore, the isotropy of depolarizing noise ensures no sudden change of IP occurs, while the anisotropy of amplitude noise and phase noise leads to the sudden change in IP. We show some cases in Fig. 3.

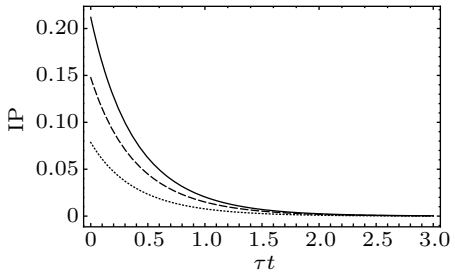


Fig. 3. IP of the initial Bell-diagonal states under depolarizing noise acting on the first qubit of the quantum system. The solid line for $c_1 = 0.4$, $c_2 = 0.3$, $c_3 = 0.2$; the dashed line for $c_1 = 0.1$, $c_2 = 0.3$, $c_3 = 0.4$; the dotted line for $c_1 = 0.1$, $c_2 = 0.4$, $c_3 = 0.2$.

4. Sudden change of IP under two-qubit dephasing model

In this section, we investigate the dynamics of IP of Bell-diagonal states that are independently interacting with identical colored dephasing environments or are interacting with a common dephasing bath.

4.1. Identical colored dephasing noise

The study of the dynamics quantum open system based on the Markov approximation contains the majority of physical situation but lacking the case of system–environment interactions with memory. In Ref. [60], a model describing system–environment interactions with memory without using Born–Markov approximation has been presented, and the conditions for system evolution to satisfy the complete positive trace-preserving map have been given.

The Kraus operators describing the above dynamics are given as follows:[58]

$$M_1 = \sqrt{\beta}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_2 = \sqrt{1-\beta}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{21}$$

where the operators satisfy $\sum_k^2 M_k^\dagger M_k = \mathbb{1}$, $\beta = (1+\Lambda(\nu))/2$, and $\Lambda(\nu) = \mathrm{e}^{-\nu}[\cos(\mu\nu) + \sin(\mu\nu)/\mu]$, $\mu = \sqrt{(4a\tau)^2 - 1}$, and a is a coin-flip random variable, $\nu = t/2\tau$ is the dimensionless time. After the evolution of Bell-diagonal states under the independent interaction with identical colored dephasing environment, the output density matrix can be obtained as

$$\mathcal{E}_{ic}(\rho) = \sum_{i,j}(M_i \otimes M_j)\rho(M_i \otimes M_j)^\dagger, \tag{22}$$

where the operators M_i and M_j act on the first and second qubits, respectively. Actually, this is a completely positive trace-preserving map.

After the straightforward calculation, one can obtain that

$$\begin{aligned}\mathcal{E}_{ic}(\rho) &= \frac{1}{4}\begin{pmatrix} 1+c_3 & 0 & 0 & \Lambda^2(\nu)(c_1-c_2) \\ 0 & 1-c_3 & \Lambda^2(\nu)(c_1+c_2) & 0 \\ 0 & \Lambda^2(\nu)(c_1+c_2) & 1-c_3 & 0 \\ \Lambda^2(\nu)(c_1-c_2) & 0 & 0 & 1+c_3 \end{pmatrix} \\ &= \frac{1}{4}(\mathbb{1}\otimes\mathbb{1} + \sum_{i=1}^{3} c_i(t)\sigma_i \otimes \sigma_i),\end{aligned} \tag{23}$$

where $c_1(t) = \Lambda^2(\nu)c_1$, $c_2(t) = \Lambda^2(\nu)c_2$, and $c_3(t) = c_3$. By Eq. (18), we get the form of the IP of $\mathcal{E}_{ic}(\rho)$. Since $\Lambda^2(\nu)$ is monotonically decreasing as similar as γ, the sudden change of IP for $\mathcal{E}_{ic}(\rho)$ can occur with the proper $\{c_i\}$, see Fig. 4.

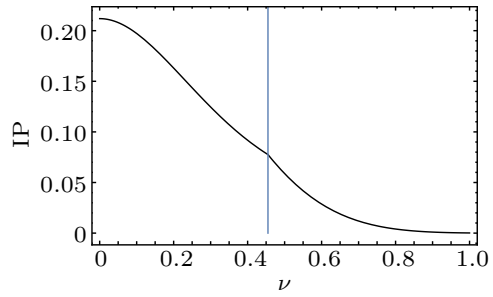


Fig. 4. The dynamics of the IP of the initial Bell-diagonal state ρ, described by parameters $c_1 = 0.3$, $c_2 = 0.4$, $c_3 = 0.2$, independently interacting with identical colored dephasing noise having $a = 1s$, $\tau = 0.5s$. The decay rate of IP sudden changes at $\nu = 0.455$.

4.2. Two qubits coupling to common bath

Considering the two-qubit system coupling to a same bosonic environment, the total Hamiltonian can be written as[61]

$$\begin{aligned}H_{\mathrm{c}} &= \frac{1}{2}\sum_{j=1,2}\sigma_z^j + \sum_k \omega_k b_k^\dagger b_k \\ &\quad + \sum_{j=1,2}\sum_k \sigma_z^j(g_k b_k^\dagger + \mathrm{H.c.}).\end{aligned} \tag{24}$$

The dynamics of Bell-diagonal states under such common dephasing bath can be expressed by the following Kraus operators:

$$K_{1cb} = \begin{pmatrix} \sqrt{\chi} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \sqrt{\chi} \end{pmatrix},$$

$$K_{2cb}=\begin{pmatrix}\sqrt{1-\chi} & 0 & 0 & 0\\ 0 & \frac{1}{\sqrt{2}} & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & 0\\ 0 & 0 & 0 & -\sqrt{1-\chi}\end{pmatrix}, \tag{25}$$

where $\chi=(\xi^4+1)/2$, $\xi(t)=\exp[-\Gamma(t)]$, and $\Gamma(t)$ is the decoherence function with the form

$$\Gamma(t)=\int_0^{\infty}\frac{1-\cos(\omega t)}{\omega^2}J(\omega)\mathrm{d}(\omega). \tag{26}$$

By considering the spectral density as

$$J(\omega)=\frac{\omega^s}{\omega_{\mathrm{c}}^{s-1}}\exp\left(-\frac{\omega}{\omega_{\mathrm{c}}}\right)$$

with the cut-off frequency $\omega_{\mathrm{c}}=1$ and the positive parameter $s=4$, we can investigate the dynamics of the Bell-diagonal states coupling to a same bosonic environment.

The matrix form of $\mathcal{E}_{\mathrm{cb}}(\rho)$ with the initial Bell-diagonal state is as follows:

$$\begin{aligned}&\mathcal{E}_{\mathrm{cb}}(\rho)\\ &=K_{1\mathrm{cb}}\rho K_{1\mathrm{cb}}^{\dagger}+K_{2\mathrm{cb}}\rho K_{2\mathrm{cb}}^{\dagger}\\ &=\frac{1}{4}\begin{pmatrix}1+c_3 & 0 & 0 & \xi^4(t)(c_1-c_2)\\ 0 & 1-c_3 & c_1+c_2 & 0\\ 0 & c_1+c_2 & 1-c_3 & 0\\ \xi^4(t)(c_1-c_2) & 0 & 0 & 1+c_3\end{pmatrix}\\ &=\frac{1}{4}(\mathbb{1}\otimes\mathbb{1}+\sum_{i=1}^{3}c_i(t)\sigma_i\otimes\sigma_i),\end{aligned} \tag{27}$$

where $c_1(t)=[(1+\xi^4(t))c_1+(1-\xi^4(t))c_2]/2$, $c_2(t)=[(1-\xi^4(t))c_1+(1+\xi^4(t))c_2]/2$, and $c_3(t)=c_3$. By Eq. (18), we find that if $c_1=c_2$, $c_1(t)$, and $c_2(t)$ will equal c_1, then the IP of $\mathcal{E}_{\mathrm{cb}}(\rho)$ will reduce to a constant. And if $c_1\neq c_2$, the sudden change of IP happens when $c_3>(c_1+c_2)/2$ with this kind of noise environment. We show these in Fig. 5.

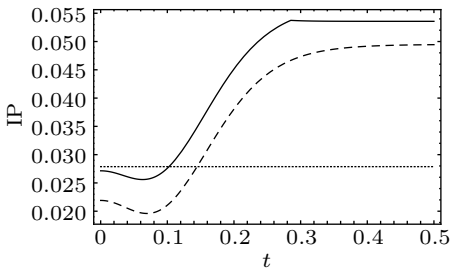


Fig. 5. IP of the initial Bell-diagonal states under the two-qubit dephasing model with colored noise with $c_1=c_2=0.15, c_3=0.05$ (dotted line); $c_1=0.4, c_2=-0.1, c_3=0.16$ (solid line); $c_1=0.4, c_2=-0.1, c_3=0.14$ (dashed line); respectively. Sudden change of IP happens only at situation (solid line).

5. Conclusion

In summary, we have investigated the dynamics of IP for the X class of quantum states under several different kinds of noise channels, such as amplitude noise, phase noise, and depolarizing noise acting only on one qubit of the quantum system, and two different types of phase noise acting on both two qubits of the quantum system. Our results show that, as one of the discordlike measures, IP exhibits sudden change behavior. Compared with the dynamics of IP under depolarizing noise, the sudden change of IP occurs under amplitude noise and phase noise if chosen proper initial states. Similar to quantum discord,[57] the sudden change of IP occurs when the quantum noise acts only on one qubit of the quantum system, which means that composite noise is not the necessary condition for the sudden change of IP. Furthermore, our results show that the initial state ρ having no sudden change of quantum discord exhibits a sudden change of IP under the dynamics of amplitude noise, but the converse is not true.

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