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In-phase collective unconventional photon blockade and its stability in an asymmetrical cavity containing *N* bosonic atoms

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We present work on a cavity-driven QED system combining an asymmetrical Fabry–Perot cavity and *N* two-level atoms (TLAs) and show the convenience of simplifying from distinguishable atoms to undistinguishable bosons when the atoms are prepared in the same initial state. Such simplification is valid even when the atoms are not prepared in the in-phase condition, since any partial in-phase initial state will evolve into the ground state through a relaxation process. Thus, we get a reduced group of differential equations by introducing the Dicke states, and the under-zero Lyapunov exponents verify its stability. We also work out the collective unconventional photon blockade (UCPB) and get two kinds of giant nonreciprocal UCPBs (NUCPBs) in the weak-driving approximation. Results show that we can employ *N* noninteracting bosonic atoms to generate a collective UCPB instead of a monoatomic UCPB as the UCPB conditions do not vary with the number of atoms. Furthermore, the forward giant NUCPB only occurring for *N* larger than a certain number as well as the backward giant NUCPB are controllable by the cavity asymmetry and by the number of atoms. Our findings suggest a prospective approach to the generation of quantum nonreciprocity by *N* identical atoms.

Keywords: unconventional photon blockade, quantum nonreciprocity, asymmetrical cavity

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1. Introduction

Ordinary light is bunching since photons rarely interact with each other. However, quantum calculations and quantum communications require nonclassical antibunching photons. The generation of nonclassical light requires a strong indirect photon–photon interaction, which has prompted the study of the photon blockade effect.[1–11] Compared to the anharmonicity-induced conventional photon blockade, the quantum destructive interference-induced unconventional photon blockade (UCPB) provides a robust photon blockade effect beyond the strong coupling limitation, which has had much attention paid to its physical mechanism and potential applications.[12–24]

Recently, controllable UCPBs of a single atom[25] and of two atoms[26,27] in an asymmetrical cavity have been proposed and studied experimentally with few atoms in a bistable regime.[28] Due to the direction-dependent quantum statistics of the output field, the equal-time second-order correlation function $G^{(2)}(0)$ separates from each other when we change the driving direction. A giant nonreciprocal UCPB (NUCPB) appears that makes the photons bunch in the incident direction while fully antibunching in the opposite driving direction.

The diatomic UCPB conditions are the same as the monoatomic ones when the two atoms are undistinguishable.[25–27] This indicates that we cannot get the diatomic UCPB conditions by replacing the monoatomic ones with $\sqrt{2}$-times of atom–cavity coupling strength. This fact seemingly conflicts with the taken-for-granted experience: using a $\sqrt{N}$-times coupling strength of a monoatomic case to represent a polyatomic case in quantum optics.[29] The primary motivation of this work is to reveal the optimal conditions for the polyatomic UCPB and to show the physical mechanism behind the confliction.

Furthermore, capturing a single atom or two atoms in a single-mode cavity is still a challenge with current magneto-optical technology, which gives some trouble to the performance of a few-atoms UCPB. In general, manipulating atom clusters is much easier in a laboratory, and the number of atoms provides a new freedom to manipulate the UCPB effect. The second motivation of this work is to assess the feasibility of the collective UCPB instead of the monoatomic one.

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2. Theoretical model with distinguishable atoms

We introduce an asymmetric F-P cavity, assemble two walls with different reflectivity to couple to N atoms, and show the manipulation of a collective UCPB in this work. Recently, UCPBs in cavity systems with one or two TLAs have been discussed.[25–27,30] Here, we adopt the concise model by considering N TLAs with the same transition frequency $\omega_{\rm a}$ embedded in the center of a single-mode asymmetrical cavity with frequency $\omega_{\rm c}$. The cavity is driven by a continuous-wave (cw) coherent field with frequency $\omega_{\rm L}$, where $\delta_{\rm a}=\omega_{\rm a}-\omega_{\rm L}$ ($\delta_{\rm c}=\omega_{\rm c}-\omega_{\rm L}$) is the detuning between the atom (cavity) and the driving field. We note g and γ as, respectively, the monoatomic coupling strength and the spontaneous decay rate. The non-radiation dephasing effect is ignored by letting $\gamma_{\rm non}=0$ in this work.

An asymmetrical cavity with different walls breaks the spatial symmetry. The cavity-loss rate is $\kappa_i=-c\ln R_i/(2L)$ ($i=1,2$), where c is the velocity of light, R_i is the reflectance of the cavity wall M_i, and L is the effective length of the optical cavity. A cavity is symmetrical only when $\kappa_1=\kappa_2$. We present the average cavity-loss rate as $\kappa_{\rm ave}=(\kappa_1+\kappa_2)/2$ and the total cavity-loss rate as $\kappa=\kappa_{\rm ave}+\kappa_{\rm loss,}$ where $\kappa_{\rm loss}$ represents the cavity mode dissipation, such as absorption induced by the cavity wall imperfections. We fix $\kappa_{\rm loss}/\kappa\ll 1$ as the high-quality and low-dissipation cavity discussed in this work. The schematic diagram of the present model is shown in Fig. 1.

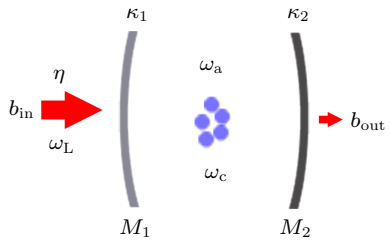


Fig. 1. Scheme of the asymmetrical cavity–atom coupling system driven by a cw on the left wall M_1 with driving strength η. Here, the cavity-loss rate κ_1 (κ_2) represents the decay of cavity mode through M_1 (M_2) and the frequency of the atom $\omega_{\rm a}$ ($\omega_{\rm c}$ and $\omega_{\rm L}$, cavity and light). The circles in the cavity represent N TLAs.

In a frame rotating with a frequency of input field $\omega_{\rm L}$, the Hamiltonian of the cavity-driven system is

$$H/\hbar=\Delta_{\rm a}\sum_{i=1}^{N}\sigma_i^{\dagger}\sigma_i+\Delta_{\rm c}a^{\dagger}a+g\sum_{i=1}^{N}\left(\sigma_i^{\dagger}a+a^{\dagger}\sigma_i\right)+H_{\rm d}. \quad (1)$$

Here, $\Delta_{\rm a}=\delta_{\rm a}-{\rm i}\gamma/2$ and $\Delta_{\rm c}=\delta_{\rm c}-{\rm i}\kappa/2$ represent the complex detuning. $\sigma_i=|g\rangle_i\,{}_i\langle e|$ ($i=1,2$) are the atomic pseudo-spin operators, and a is the annihilation operator of cavity mode. The first two items of the Hamiltonian are the energies of the bare atoms and the cavity field, the third item represents the atom–cavity interaction, and the last item $H_{\rm d}=\eta\left(a+a^{\dagger}\right)$ represents the cavity-driven interaction. $\eta=\sqrt{\kappa_i}b_{\rm in}$ ($i=1,2$) is the direction-dependent effective driving strength, and the index i refers to the incident mirror.

We define the forward case as an external light incoming on the left wall (M_1) and outgoing from the right wall (M_2). The backward case is the opposite one, in which light propagates from M_2 to M_1. $b_{\rm in}=\sqrt{n_{\rm in}}$ and $n_{\rm in}=P_{\rm in}/(\hbar\omega_{\rm L})$, respectively, characterize the normalized amplitude and intensity of incoming field with power $P_{\rm in}$. We focus on discussion of the UCPB in the forward case first; the backward case can be easily obtained by alternating κ_1 with κ_2.

We truncate the Hilbert space into the two-exciton manifold, which is allowed in the assumption of the weak-driving limitation. As a common means without loss of generality, we assume the atoms are distinguishable as they may be identified by initial conditions. The wave function is

$$\begin{aligned}\Psi=&\sum_{n=0}^{2}c_{g,n}\,|g,n\rangle+\sum_{i=1}^{N}\sum_{n=0}^{1}c_{i,n}\,|i,n\rangle\\&+\sum_{i<j}^{N}\sum_{j=1}^{N}c_{i,j,0}\,|i,j,0\rangle.\end{aligned} \quad (2)$$

Here, $|\alpha,\beta,n\rangle=|\alpha\rangle\otimes|\beta\rangle\otimes|\gamma\rangle$ is the product state of atoms and photons. Atomic states of $|g\rangle$, $|i\rangle$ and $|i,j\rangle$, respectively, represent the atomic ground state, the single excitation state of the i-th atom, and the dual excitation state of the i-th and j-th atoms. The photonic state of $|n\rangle$ states n cavity photons are excited.

By substituting the wave function in the Schrödinger equation, one can get a set of differential equations

$${\rm i}\dot{c}_{g,0}=\eta c_{g,1}, \quad (3a)$$

$${\rm i}\dot{c}_{g,1}=\Delta_{\rm c}c_{g,1}+\sqrt{2}\eta c_{g,2}+g\sum_{i=1}^{N}c_{i,0}+\eta c_{g,0}, \quad (3b)$$

$${\rm i}\dot{c}_{g,2}=2\Delta_{\rm c}c_{g,2}+\sqrt{2}\eta c_{g,1}+\sqrt{2}g\sum_{i=1}^{N}c_{i,1}, \quad (3c)$$

$${\rm i}\dot{c}_{i,0}=\Delta_{\rm a}c_{i,0}+gc_{g,1}+\eta c_{i,1}, \quad (3d)$$

$${\rm i}\dot{c}_{i,1}=\Delta_{\rm ac}c_{i,1}+\eta c_{i,0}+\sqrt{2}gc_{g,2}+g\sum_{j(\neq i)}^{N}c_{i,j,0}, \quad (3e)$$

$${\rm i}\dot{c}_{i,j,0}=2\Delta_{\rm a}c_{i,j,0}+gc_{i,1}+gc_{j,1}. \quad (3f)$$

Here, we rewrite $\Delta_{\rm ac}=\Delta_{\rm a}+\Delta_{\rm c}$. In general, the differential group can be numerically solved using the Runge–Kutta method after a given initial condition. However, such numerical solution needs a great amount of calculation when N is huge since the dimension of the equations increases with N^2.

3. Collective UCPB of in-phase bosonic atoms

In this section, we discuss a perfect interference case where the atoms are treated as undistinguishable bosons. Generally speaking, atoms in a cavity can be identified by their species and positions. In this work, we place the same TLAs

in the center of a single-mode asymmetrical cavity. Therefore, the atoms can be treated as undistinguishable when their locations are indistinguishable inherently. According to the dynamical equation, the atoms will have the same probability amplitudes all the time if we prepare the atoms in the same initial states no matter whether the atoms can be identified by their locations or not. In this case, the atoms are undistinguishable bosons practically.

From the differential group of Eq. (3), we find that if atoms are prepared in the same initial ground states ($c_{g,0}=1$, $c_{g,1}=c_{g,2}=c_{i,0}=c_{i,1}=c_{i,j,0}=0$), they will stay in phase all the time. We call this the in-phase UCPB since the atoms are in phase and they will interfere perfectly. The distinguishable atoms evolve into the undistinguishable bosonic particles in the in-phase UCPB and we can deal with them as collective states or Dicke states.

Following the procedure to deal with collective states,[29,31,32] we define a direct product of atomic state and photonic state $|N,i,j\rangle=|N,i\rangle\otimes|j\rangle$ as the atom–photon Fock state. The atomic state $|N,i\rangle$ is the so-called Dicke state. It presents a collective state of atoms where only i of N atoms are excited. For instance, $|N,0\rangle=|g\rangle$, $|N,1\rangle=\sum_{i=1}^{N}|i\rangle/\sqrt{N}$, and $|N,2\rangle=\sum_{i<j}^{N}\sum_{j=1}^{N}|i,j\rangle/\sqrt{C_N^2}$ where the permutation $C_N^i=N!/i!\,(N-i)!$. Then, the wave function in the collective picture reads

$$\Psi=\sum_{n=0}^{2}C_{0,n}\,|N,0,n\rangle+\sum_{n=0}^{1}C_{1,n}\,|N,1,n\rangle+C_{2,0}\,|N,2,0\rangle\,. \quad (4)$$

Here, $C_{0,n}=c_{g,n}$, $C_{1,n}=\sum_{i=1}^{N}c_{i,n}/\sqrt{C_N^1}$, and $C_{2,0}=\sum_{i<j}^{N}\sum_{j=1}^{N}c_{i,j,0}/\sqrt{C_N^2}$. By summing over the atom index in Eq. (3), we get the evolution of the collective wave functions in the time domain

$$\mathrm{i}\dot{C}_{0,0}=\eta C_{0,1}, \quad (5a)$$

$$\mathrm{i}\dot{C}_{0,1}=\Delta_{\mathrm{c}}C_{0,1}+\sqrt{2}\eta C_{0,2}+\sqrt{N}gC_{1,0}+\eta C_{0,0}, \quad (5b)$$

$$\mathrm{i}\dot{C}_{0,2}=2\Delta_{\mathrm{c}}C_{0,2}+\sqrt{2}\eta C_{0,1}+\sqrt{2N}gC_{1,1}, \quad (5c)$$

$$\mathrm{i}\dot{C}_{1,0}=\Delta_{\mathrm{a}}C_{1,0}+\sqrt{N}gC_{0,1}+\eta C_{1,1}, \quad (5d)$$

$$\mathrm{i}\dot{C}_{1,1}=\Delta_{\mathrm{ac}}C_{1,1}+\eta C_{1,0}+\sqrt{2N}gC_{0,2}+\sqrt{2\,(N-1)}gC_{2,0}, \quad (5e)$$

$$\mathrm{i}\dot{C}_{2,0}=2\Delta_{\mathrm{a}}C_{2,0}+\sqrt{2\,(N-1)}gC_{1,1}. \quad (5f)$$

The collective picture brings us two benefits. Firstly, it reduces the calculation greatly when N is a very large number and the calculation does not increase with atom numbers anymore. Such a collective picture is valid only when the atoms are undistinguishable bosons and keep in-phase in the dynamic evolution process.

Secondly, it brings convenience to exploring the physical mechanism of the collective UCPB. A sketch of energy level with possible interferential channels is plotted in Fig. 2. There are two quantum transitions to produce the bunching state. One pathway is $|N,0,1\rangle\leftrightarrow|N,0,2\rangle$ through the pumping effect, and the other is $|N,0,1\rangle\leftrightarrow|N,1,0\rangle\leftrightarrow|N,1,1\rangle\leftrightarrow|N,0,2\rangle$ which combines the pumping effect and the cavity–atom interaction together. The collective UCPB occurs when the interference between two quantum channels is destructive completely.

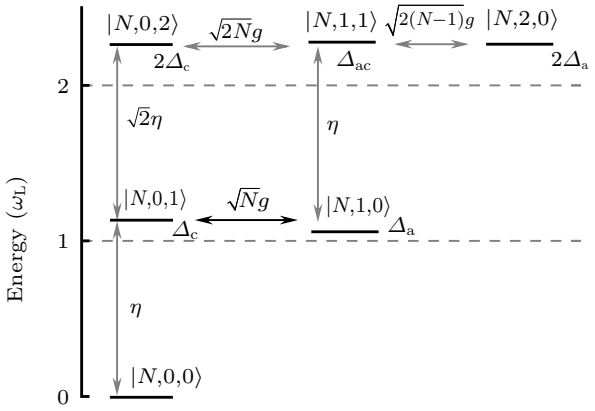


Fig. 2. Scheme of energy level with possible interferential channels. The gray lines present the transitions between layers through bumping, as well as the couplings within the same layer.

Now, we work out the collective UCPB analytically in the weak-driving approximation, where the probability of a higher-order excited state is much smaller than the lower-order one. Taking the ground state as the initial state ($C_{0,0}=1$, $C_{0,1}=C_{0,2}=C_{1,n}=C_{2,0}=0$), the driving light is too weak to excite the system to higher levels, and most of atoms will stay in the ground state all the time. Therefore, we set $C_{0,0}\equiv 1$ and set column vectors as $C=(C_{0,1}\ C_{0,2}\ C_{1,0}\ C_{1,1}\ C_{2,0})^{\mathrm{T}}$, $A=(-\eta C_{0,0}\ 0\ 0\ 0\ 0)^{\mathrm{T}}$. The dynamical Eq. (5) can be simplified as $\mathrm{i}\,\mathrm{d}C/\mathrm{d}t=MC-A$, where the 5×5 non-Hermitian matrix M is the truncated effective Hamiltonian in the two-exciton framework

$$M=\begin{pmatrix}\Delta_{\mathrm{c}} & \sqrt{2}\eta & \sqrt{N}g & 0 & 0\\ \sqrt{2}\eta & 2\Delta_{\mathrm{c}} & 0 & \sqrt{2N}g & 0\\ \sqrt{N}g & 0 & \Delta_{\mathrm{a}} & \eta & 0\\ 0 & \sqrt{2N}g & \eta & \Delta_{\mathrm{ac}} & \sqrt{2\,(N-1)}g\\ 0 & 0 & 0 & \sqrt{2\,(N-1)}g & 2\Delta_{\mathrm{a}}\end{pmatrix}. \quad (6)$$

By letting $\mathrm{d}C/\mathrm{d}t=0$ we obtain the steady solution. According to Cramer's rule, the amplitude of the wave function is $C_i=D_i/D$, where C_i is the i-th component of vector C. $D=\det(M)$ and $D_i=\det(M_i)$ are the determinants of M and M_i, where M_i is the transformation of matrix M by replacing the i-th column with column vector A.

Then, the cavity field intensity is $n_{\mathrm{c}}=|C_{0,1}|^2+|C_{1,1}|^2+2|C_{0,2}|^2$, which indicates that the field is composed of both bunching and antibunching components. By introducing quantum input–output theory,[33] we get the output field as $b_{\mathrm{out}}=-\mathrm{i}\sqrt{\kappa_2}a$. The output power is $P_{\mathrm{out}}=n_{\mathrm{out}}\hbar\omega_{\mathrm{L}}$, where $n_{\mathrm{out}}=\langle b_{\mathrm{out}}^{\dagger}b_{\mathrm{out}}\rangle=\kappa_2 n_{\mathrm{c}}$ is the normalized output intensity. Finally,

we get the equal-time second-order correlation function for the output photons $G^{(2)}(0) = 2|C_{0,2}|^2/n_c^2$.

Perfect antibunching light without any bunching component can be obtained by solving $D_2 = 0$, which provides two optimal matching conditions. One is the frequency-matching condition, i.e., $(2\gamma+\kappa)\delta_a + \gamma\delta_c = 0$, and the other one is the intensity-matching condition, i.e., $n_{in} = \left(g^2 - (\gamma+\kappa)\delta_a^2/\gamma - \gamma(\gamma+\kappa)/4\right)/\kappa_1$. Comparing the collective UCPB conditions to the conditions of a monoatomic case,[25] the matching conditions for the collective UCPB and for the monoatomic UCPB are the same. This conclusion is verified in the discussion of the diatomic UCPB.[26,27] The physics under this phenomenon originates from the quantum interference among N atoms. Each atom produces antibunching photons only when the monoatomic UCPB conditions do match. As the N undistinguishable atoms are in phase, the cluster produces antibunching photons only when all atoms match with the monoatomic UCPB conditions.

4. Controllable collective UCPB and giant NUCPB

The intensity condition clearly shows that the optimal incident light intensity to produce the collective UCPB relies on the incident direction since n_{in} is inversely proportional to κ_1. We define the direction-dependent optimal atom–cavity coupling strengths, i.e., $g_{opt-for}$ for the forward incident case and $g_{opt-back}$ for the backward incident case, as the coupling strengths to get the fully antibunching transmitted light:

$$g_{opt-for/back} = \sqrt{\frac{\gamma+\kappa}{\gamma}\delta_a^2 + \frac{\gamma(\gamma+\kappa)}{4} + \kappa_i n_{in}}, \quad (i = 1,2). \tag{7}$$

For the sake of simplicity but without loss of generality, we focus on an asymmetrical cavity with $\kappa_1 < \kappa_2$ to discuss the phenomenon of the collective UCPB. In the asymmetrical cavity, $g_{opt-for} < g_{opt-back}$ as we set $\kappa_1 < \kappa_2$. From now on, we normalize all the parameters by setting $\kappa \equiv 1$ and $\kappa_{loss} = 0.02$ for simplicity and set the asymmetrical cavity with $\kappa_1 = 0.36$ and $\kappa_2 = 1.60$.

In order to show the collective UCPB in an asymmetrical cavity intuitively, we plot the functions of $G^{(2)}(0)$ as a function of atom–cavity coupling strength g in both the forward and the backward directions, as shown in Fig. 3(a). We focus on the resonant case with $\delta_a = \delta_c = 0$ to match the frequency restriction naturally, $\gamma = 0.1$ as well as $n_{in} = 0.04$ in the present numerical simulation. We chose these parameters as they have been introduced in previous UCPB works.[25–27,30] As expected, $g_{opt-for} \approx 0.2047$ separates from $g_{opt-back} \approx 0.3025$ in the monoatomic, diatomic, and polyatomic cases, and the optimal coupling strengths do not vary with atom number.

The collective UCPB in an asymmetrical cavity opens windows to perform two kinds of giant NUCPB, as shown in Fig. 3(a) by the two vertical black dotted lines. The first one is the forward giant NUCPB, where the transmitted light is antibunching in the forward incident case and is bunching in the backward incident case, fulfilling $G^{(2)}_{for}(0) \approx 0$ and $G^{(2)}_{back}(0) > 1$. Here, we rewrite $G^{(2)}_{for}(0)$ and $G^{(2)}_{back}(0)$ as $G^{(2)}(0)$ for the forward and the backward incident cases, respectively. The forward giant NUCPB exists in the case of $N = 10$ but not in the case of $N = 1$ or 2. The absence of the forward NUCPB in the case of $N = 1$ or 2 is consistent with previous work on monoatomic and diatomic NUCPBs.[25–27]

In order to show the controllable forward NUCPB by varying the atom number, we fix $g = g_{opt-for} = 0.2047$ and plot $G^{(2)}(0)$ for both incident cases versus atom number N, as shown in Fig. 3(b). The transmitted field from M_2 is fully antibunching ($G^{(2)}_{for}(0) \approx 0$) no matter how many atoms there are. However, $G^{(2)}_{back}(0)$ increases with atom number N. It is less than 1 when $N < 5$, while it is larger than 1 when $N \geq 5$, which indicates that the forward giant NUCPB occurs when $N \geq 5$. That is why we could not obtain the forward giant NUCPB in previous work on monoatomic and diatomic NUCPBs.

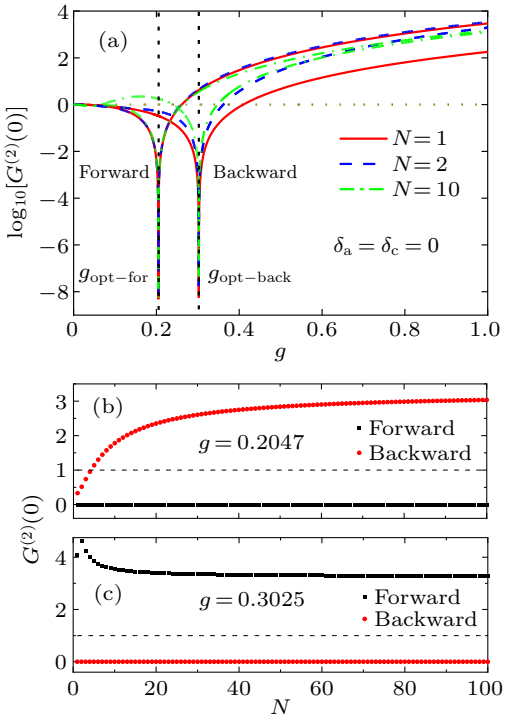


Fig. 3. Collective UCPB and NUCPB in an asymmetrical cavity. In panel (a), the red solid curves are $G^{(2)}(0)$ for the monoatomic case, while the blue dashed curves are for the diatomic case and the green dash-dotted ones for the 10-atom case with $\delta_a = \delta_c = 0$. The two vertical black dotted lines present two kinds of giant NUCPBs. Panels (b) and (c) show $G^{(2)}(0)$ of giant NUCPBs for both forward and backward cases versus atom number N for fixed $\delta_a = \delta_c$. In all panels, the average cavity-loss rate is fixed and normalized as unit, the asymmetrical cavity is $\kappa_1 = 0.36$ and $\kappa_2 = 1.60$, and the other parameters are $\kappa_{loss} = 0.36$, $\gamma = 0.1$, and $n_{in} = 0.04$.

The second one is the backward giant NUCPB, where the transmitted light is antibunching in the backward incident case and is bunching in the forward incident case, fulfilling $G^{(2)}_{back}(0) \approx 0$ and $G^{(2)}_{for}(0) > 1$. In order to show the control-

lable backward NUCPB by varying the atom number, we fix $g = g_{\rm opt-back} = 0.3025$ and plot $G^{(2)}(0)$ versus atom number N, as shown in Fig. 3(c). The transmitted field from M_1 is fully antibunching ($G^{(2)}_{\rm back}(0) \approx 0$), while the transmitted light from M_2 is bunching ($G^{(2)}_{\rm for}(0) > 1$) no matter how many atoms there are. $G^{(2)}_{\rm for}(0)$ increases from 4.322 to the maximum 4.898 when the atom number changes from 1 to 2, and it decreases with N when $N > 2$. It indicates that we can manipulate quantum nonreciprocity by atom number.

Therefore, the collective UCPB and NUCPB are controllable by the cavity asymmetry and the atom number. Collective UCPB conditions do not vary with atom number, which indicates that we can produce a collective UPCB by replacing a single atom with the N non-interaction bosonic atoms, which will greatly reduce the difficulty of experimental realization. The results of the collective UCPB and NUCPB indicate that we should not take it for granted that using $\sqrt{N}$-times of coupling strength of a monoatomic case represents a polyatomic case in quantum optics.[29]

5. Stability of collective UCPB beyond the in-phase case

We have shown that the atoms can be treated as undistinguishable bosons in the in-phase case, where the same atoms are prepared in the initial ground state. The in-phase atoms are fully undistinguishable bosons. However, if all of the atoms are not prepared in the ground state, such as the i-th atom is excited, they may be partially identified by the initial condition, and all of the atoms will not stay in phase in the following dynamical evolution. In contrast to the in-phase case, we call it the partial in-phase case.

Fortunately, the partial in-phase initial conditions will evolve into the in-phase ground state through a relaxation process no matter how partial the initial condition is. As we know, any excited atom or excited cavity mode will degenerate to a deexcited state through dissipation channels if we turn off the driving light for a time interval much longer than the relaxation time. This indicates that we can always transfer the partial in-phase initial state to the ground state. In fact, as we only care about the stationary solution of the system in the weak-driving approximation, any deviation induced by the initial condition will disappear when $t \to \infty$.

We also note that the stationary solution of the collective UCPB solution is stable. We estimate the Lyapunov spectrum of Eq. (5) using the standard method.[34] We calculate the dynamic of the Lyapunov exponents of the $N = 13$ case by setting the initial condition as all ground atoms and zero cavitary photon. For the forward NUCPB ($g = 0.2047$), the maximum of the Lyapunov exponents is $\lambda_{\rm max} = -0.0013$ in the forward incident case and it is $\lambda_{\rm max} = -0.0059$ in the backward incident case. For the backward NUCPB case ($g = 0.3025$), the maximum exponent is $\lambda_{\rm max} = -0.0006$ in the forward incident case, and it is $\lambda_{\rm max} = -0.0028$ in the backward incident case. We also change the initial conditions and perform the calculation again. As expected, all of the Lyapunov exponents are less than zero even when we change the initial conditions greatly. It indicates that the evolution of the collective states differential equations in the weak-driving condition is stable.

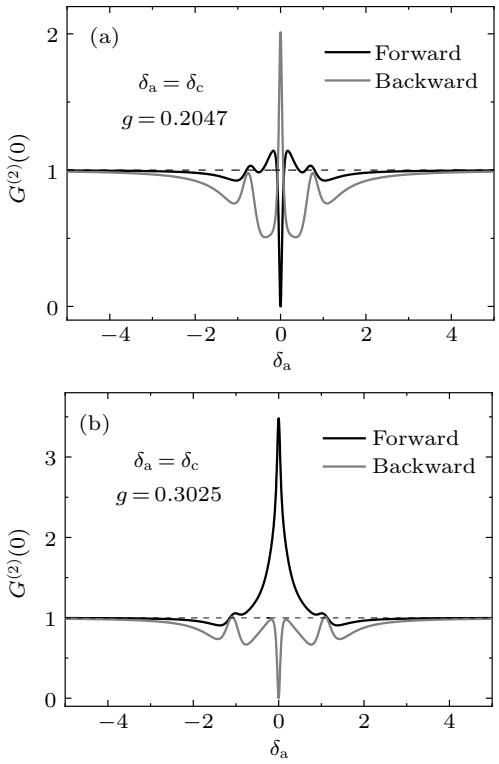


Fig. 4. Forward and backward NUCPBs. Panel (a) shows the forward giant NUCPB and panel (b) shows the backward one when UCPB conditions match. In both panels, the average cavity-loss rate is fixed and normalized as unit, the asymmetrical cavity is $\kappa_1 = 0.36$ and $\kappa_2 = 1.60$, and the other parameters are $\kappa_{\rm loss} = 0.02$, $\gamma = 0.1$, and $n_{\rm in} = 0.04$.

Finally, we show how to test the prediction of two such kinds of collective NUCPBs within an asymmetrical atoms–cavity coupling system. After fabricating an asymmetrical cavity, all the cavity parameters ($\omega_c, \kappa_1, \kappa_2, \kappa_{\rm loss}$) can be measured. Then, loading identical atoms into the cavity, the atomic parameters (ω_a, γ, N) and the atom–cavity coupling strength (g) can be measured by vacuum-induced Rabi splitting technology. One may be curious about the consistency of the atom–cavity coupling strength of all atoms when N is a large number. Technically, we can use an effective coupling strength to replace the real coupling strength as atoms are undistinguishable. Furthermore, we can enlarge the number of atoms using optical well array technology when we extend the single-mode cavity to a multi-mode cavity. If the same atoms in ground state are located in the optical well array along with the same coupling strength, they will have the same probability amplitudes all the time as they perform the same dynamical evolution. The atoms also can be treated as undistinguishable bosons since they are indistinguishable practically. According

to the frequency- and intensity-matching conditions, we can calculate the proper operating light frequency and intensity $(\omega_{\rm L}, n_{\rm in})$ to generate the forward or backward giant NUCPBs. Now, we can fix light intensity and vary light detuning, and measure the second-order correlation functions of output field in the forward and the backward incident cases to examine the existence of giant NUCPBs.

In order to show the forward and the backward giant NUCPBs clearly, we plot $G^{(2)}(0)$ as a function of atom-light detuning $\delta_{\rm a}$ in the forward and the backward giant NUCPB cases with $\omega_{\rm a} = \omega_{\rm c}$, $N = 13$, $\gamma = 0.1$ and $n_{\rm in} = 0.04$, as shown in Figs. 4(a) and 4(b). The curves of $G^{(2)}_{\rm for}(0)$ and $G^{(2)}_{\rm back}(0)$ apparently deviate from each other when $\delta_{\rm a} = \delta_{\rm c} = 0$, which is clearly shown as the forward giant NUCPB occurs at $g = 0.2047$ and the backward giant NUCPB occurs at $g = 0.3025$.

6. Conclusion

In conclusion, we propose a cavity-driven QED model combined with an asymmetrical Fabry–Perot cavity and N TLAs and show how to simplify the system from distinguishable atoms to undistinguishable bosons when atoms are prepared in the ground state. Furthermore, such simplification is valid even when the atoms are not prepared in the in-phase condition, as any partial in-phase initial state will evolve into the ground state through a relaxation process. We calculate the Lyapunov exponents, which show that the simplified differential equations of collective states are stable as the maximum of the Lyapunov exponents is less than zero. Finally, we work out the collective UCPB and get two kinds of giant NUCPBs in the weak-driving approximation. Results show that we can employ N noninteracting bosonic atoms to generate the collective UCPB instead of the monoatomic UCPB as polyatomic UCPB conditions do not vary with the number of atoms. Furthermore, two kinds of giant NUCPBs, i.e., the forward giant NUCPB only occurring for $N \geq 5$ and the backward giant NUCPB, are controllable by the cavity asymmetry and by the atom number. Our findings suggest a prospective approach to the generation of quantum nonreciprocity by N identical atoms.

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