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Influence of quasi-electrostatic support on amplification of space charge waves in amplification section of a superheterodyne free electron laser

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A theoretical study of the influence of a quasi-electrostatic support on the amplification level of the slow space charge wave (SCW) in the amplification section of a superheterodyne free electron laser (FEL) was carried out. One of the ways to significantly increase the saturation level of the slow SCW is maintaining the conditions of a three-wave parametric resonance between the slow, fast SCWs and the resulting pump electric field. This can be done by introducing the quasi-electrostatic support in the superheterodyne FEL amplification section. Also, it was found that the generated pump electric field significantly influences the maintenance of parametric resonance conditions. As a result, this increases the saturation level of the slow SCW by 70%. Finally, the quasi-electrostatic support significantly reduces the maximum value of the electrostatic undulator pump field strength, which is necessary to achieve the maximum saturation level of the slow SCW.

Keywords: superheterodyne free-electron lasers, space charge waves, electrostatic undulator, quasi-electrostatic support

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1. Introduction

The most powerful sources of electromagnetic radiation in the terahertz range are free electron lasers (FELs).[1,2] Such devices have wide applications in various fields, such as biomedical imaging, spectroscopy, materials science,[1–3] *etc*. One of the directions of terahertz FELs research is to increase their efficiency and analyze new schemes of their operation.[1,2,4–8] Among the promising concepts of terahertz FELs is a superheterodyne free electron laser with an electrostatic undulator.[9–11]

The main feature of superheterodyne FELs is using Raman scattering as an additional mechanism for amplifying input electromagnetic signal.[12] This physical process occurs as a three-wave parametric resonance between the slow and fast space charge waves (SCWs) and the longitudinal periodically reversing pump electric field created by the electrostatic undulator. As a result, this creates a parametric instability, causing an amplification of the slow SCW, which then feeds the input electromagnetic signal. A similar interaction is realized in the amplification section of the slow SCW in the klystron or single-section model of a superheterodyne FEL.[9] In this work, we limit ourselves to the study of physical processes in this section.

The analysis in Refs. [10,11] showed that the generated pump electric field significantly affects the amplification processes of the slow SCW in the amplification section. This field is created in the REB under the influence of the electrostatic undulator pump field. The generated pump electric field allows us to increase the slow SCW growth increment by 33% and, as a consequence, reduce the slow SCW saturation length by 22.5%.[10,11] However, its saturation level increases slightly.

The presented work, in contrast to Refs. [10,11], is devoted to finding ways to increase the saturation level of the slow SCW. For this purpose, we use a new model of the amplification section that contains a quasi-electrostatic support. This quasi-electrostatic support can compensate for the loss of kinetic energy of the relativistic electron beam during the growth of the slow and fast SCWs. It allows us to maintain the conditions of the three-wave parametric resonance between SCWs and the pump electric field for longer, achieving higher saturation levels. As a result of such studies, we found the optimal parameters of the quasi-electrostatic support field and demonstrated that the saturation level of the slow SCW can increase by 70%.

2. Model

In the current work, we consider processes in a superheterodyne free electron laser's space charge wave amplification section. The amplification section diagram is shown in Fig. 1. Models of the transit section may be different,[9] but we choose the simplest one. In this section, the relativistic electron beam 1 (Fig. 1) moves in a longitudinal periodically

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reversing field 3 created by the electrodes of the electrostatic undulator 2.

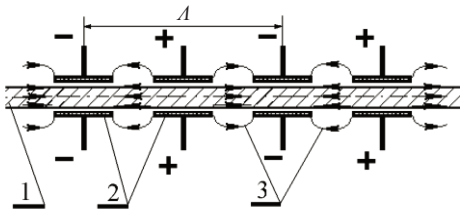


Fig. 1. Amplification section scheme. In the figure, 1: relativistic electron beam; 2: undulator's electrodes; 3: periodic reversible pump electric field lines of strength.

The modulated REB is fed to the input of the amplification section. In this section the parameters of the electrostatic undulator are chosen so that the conditions of the three-wave resonance between the slow $\boldsymbol{E}_\alpha$, fast $\boldsymbol{E}_\beta$ SCWs, and the pump electric field of the electrostatic undulator $\boldsymbol{E}_{20}$ are satisfied. Due to the three-wave parametric resonance, $\boldsymbol{E}_\alpha$ is amplified, which then amplifies the superheterodyne FEL's input electromagnetic signal. Monochromatic field strengths $\boldsymbol{E}_\alpha$, $\boldsymbol{E}_\beta$, and $\boldsymbol{E}_{20}$ have the following forms:

$$\boldsymbol{E}_\alpha = [E_\alpha \exp(\mathrm{i}p_\alpha) + \text{c.c.}]\,\boldsymbol{e}_z, \tag{1}$$

$$\boldsymbol{E}_\beta = \left[E_\beta \exp\left(\mathrm{i}p_\beta\right) + \text{c.c.}\right]\boldsymbol{e}_z, \tag{2}$$

$$\boldsymbol{E}_{20} = [E_{20} \exp(\mathrm{i}p_2) + \text{c.c.}]\,\boldsymbol{e}_z. \tag{3}$$

In these formulas $p_\alpha = \omega t - k_\alpha z$ and $p_\beta = \omega t - k_\beta z$ are phases of the slow and fast SCWs, k_α and k_β are their wavenumbers, ω is their wave frequency, E_α and E_β are the complex electric strength amplitudes of the corresponding SCWs, $\boldsymbol{e}_z$ is the unit vector of the longitudinal axis Z. E_{20} is the complex electric strength amplitude of the pump electric field, $p_2 = k_2 z$ is its static phase, $k_2 = 2\pi/\Lambda$ is its phase wave number, Λ is the undulation period (Fig. 1). The condition for the three-wave parametric resonance between the fast, slow SCWs and the electrostatic undulator pump field can be represented as the following relationship between their phases:

$$p_2 = p_\alpha - p_\beta. \tag{4}$$

The above-described amplification process is significantly influenced by the effect of creating a generated pump electric field $\boldsymbol{E}_2^{\mathrm{g}}$:[10,11]

$$\boldsymbol{E}_2^{\mathrm{g}} = \left[E_2^{\mathrm{g}} \exp(\mathrm{i}p_2) + \text{c.c.}\right]\boldsymbol{e}_z. \tag{5}$$

In this equation E_2^{g} is the complex electric strength amplitude of the generated pump electric field.

In this work, to increase the saturation level of the slow SCW significantly, we propose to use the quasi-electrostatic support $E_0(z)$, which acts on the electron beam in the direction of its movement. This field compensates for the loss of kinetic energy of the electron beam occurring due to the amplification of the slow and fast SCWs. This increases the duration of the three-wave parametric resonance, which ultimately increases the slow SCW saturation level. Please note that this field is not periodic, and its strength varies depending on the longitudinal coordinate z. Therefore, our goal is to find the type of this dependence with optimal parameters. We also noted that to create the quasi-electrostatic support field, it is not necessary to change the design of the SCW amplification section. It is enough to apply a certain voltage to electrodes 2 (Fig. 1), which allows them to create a special external field equal to the sum of the external undulator pump field and the quasi-electrostatic support $\boldsymbol{E}_{\text{ext}} = (\boldsymbol{E}_{20} + \boldsymbol{E}_0(z))$.

3. Basic equations

To study the physical processes in the amplification section numerically, we use the quasi-hydrodynamic equation, the continuity equation and one of the Maxwell equations as output. Let us consider the case when the collision of electrons and their thermal spread in velocities can be neglected. Additionally, we study a model in which all values depend only on the longitudinal coordinate z and time t.

To obtain solutions to this system of equations, we use methods of the hierarchical theory of oscillations and waves.[12,13] This approach is based on the Krylov–Bogolyubov asymptotic integration method.[14,15] This method allows us to separate the variables of the system of equations into slow and fast ones. We find solutions for the fast variables analytically using the asymptotic integration procedure and for the slow variables using standard numerical integration methods.

As a result of this approach, we obtain differential equations for the slowly variable amplitudes of the slow and fast SCWs, the amplitude of the generated pump electric field, and the constant terms of the velocity and concentration of the relativistic electron beam in the cubic nonlinear approximation.

$$C_{2,\alpha}\frac{\mathrm{d}^2E_\alpha}{\mathrm{d}z^2} + C_{1,\alpha}\frac{\mathrm{d}E_\alpha}{\mathrm{d}z} + D_\alpha E_\alpha = C_{3,\alpha}E_\beta E_2^* + F_\alpha, \tag{6}$$

$$C_{2,\beta}\frac{\mathrm{d}^2E_\beta}{\mathrm{d}z^2} + C_{1,\beta}\frac{\mathrm{d}E_\beta}{\mathrm{d}z} + D_\beta E_\beta = C_{3,\beta}E_\alpha E_2 + F_\beta, \tag{7}$$

$$C_{2,2}\frac{\mathrm{d}^2E_2^{\mathrm{g}}}{\mathrm{d}z^2} + C_{1,2}\frac{\mathrm{d}E_2^{\mathrm{g}}}{\mathrm{d}z} + D_2E_2^{\mathrm{g}} = C_0E_{20} + C_{3,2}E_\alpha^*E_\beta + F_2, \tag{8}$$

$$\frac{\mathrm{d}\upsilon_0}{\mathrm{d}z} = \frac{eE_0(z)}{m_{\mathrm{e}}\upsilon_0\gamma_0^3} + F_\upsilon, \qquad \frac{\mathrm{d}n_0}{\mathrm{d}z} = -\frac{n_0eE_0(z)}{m_{\mathrm{e}}\upsilon_0^2\gamma_0^3} + F_n. \tag{9}$$

In these equations,

$$F_\chi = F_\chi(\boldsymbol{E}_0, \boldsymbol{E}_\alpha, \boldsymbol{E}_\beta, \boldsymbol{E}_2), \quad \chi \in \{\alpha, \beta, 2\},$$
$$F_\upsilon = F_\upsilon(\boldsymbol{E}_0, \boldsymbol{E}_\alpha, \boldsymbol{E}_\beta, \boldsymbol{E}_2),$$
$$F_n = F_n(\boldsymbol{E}_0, \boldsymbol{E}_\alpha, \boldsymbol{E}_\beta, \boldsymbol{E}_2)$$

are cubic nonlinear terms depending on the amplitudes of the fields of the system under study.

The coefficients of the resulting system of differential equations look like this:

$$
\begin{aligned}
&C_{1,\chi} = \partial D_\chi / \partial(-\mathrm{i}k_\chi),\\
&C_{2,\chi} = \partial^2 D_\chi / \partial(-\mathrm{i}k_\chi)^2/2,\\
&D_\chi = -\mathrm{i}k_\chi\left(1-\frac{\omega_\mathrm{p}^2}{(\omega_\chi - k_\chi \upsilon_0)^2\gamma_0^3}\right);
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
&C_{3,\alpha} = \frac{k_\alpha \cdot \omega_\mathrm{p}^2 e/m_\mathrm{e}}{\Omega_\alpha \Omega_\beta k_2 \upsilon_0 \gamma_0^6}\cdot\left(\frac{k_\alpha}{\Omega_\alpha}+\frac{k_\beta}{\Omega_\beta}-\frac{k_2}{k_2\upsilon_0}-\frac{3\upsilon_0\gamma_0^2}{c^2}\right),\\
&C_{3,\beta} = -k_\beta C_{3,\alpha}/k_\alpha,\\
&C_0 = -\mathrm{i}k_2\omega_\mathrm{p}^2/(\Omega_2^2\gamma_0^2),\\
&\Omega_\chi = \omega_\chi - k_\chi\upsilon_0.
\end{aligned}
\tag{11}
$$

In these equations $\omega_\alpha = \omega_\beta = \omega$, $\omega_2 = 0$, e and m_e are values of the charge and mass of the electron, c is the speed of light in vacuum; γ_0 is the constant term of the Lorentz factor; ω_p is the plasma frequency of the electron beam.

We should note that in these equations the dispersion functions for the slow and fast SCWs $D_\alpha(\omega_\alpha, k_\alpha) = 0$ and $D_\beta(\omega_\beta, k_\beta) = 0$, respectively, because they are the natural beam waves, but $D_2(k_2) \neq 0$.

4. Analysis

As we mentioned above, this work aims to include the quasi-electrostatic support $E_0(z)$ in the superheterodyne FEL amplification section to increase the saturation level of the slow SCW during the three-wave parametric resonance. Let's calculate, analyze and find out how this can be achieved.

First, let us find the optimal shape of quasi-electrostatic support $E_0(z)$. That is, we need to find a field that would allow us to achieve the maximum saturation level of the slow SCW and which can be easily implemented in an experiment. The quasi-electrostatic support must maintain the conditions of three-wave parametric resonance, which depends on the longitudinal velocity of the relativistic electron beam. Therefore, we numerically calculated the quasi-electrostatic support, at which the constant term of the REB velocity remained constant. The field is nonlinear, but it can be fit by the following quasilinear function:

$$
E_0(z) = \begin{cases} 0, & z < z_{E_0},\\ \tau_{E0}(z - z_{E0}), & z \geq z_{E0}. \end{cases}
\tag{12}
$$

In other words, the field $E_0(z)$ is specified by a linear function that depends on two parameters: z_{E0} and τ_{E0}. The physical meaning of these characteristics is as follows: z_{E0} is the beginning of the slow SCW saturation, τ_{E0} is the gradient of the increase in quasi-electrostatic support strength in the saturation region. Equation (12) has only two parameters, which allows us to perform mathematical modeling of the system under study conveniently.

Further studies have shown that the saturation level of the slow SCW when using a quasi-linear quasi-electrostatic support with optimal parameters and the saturation level of the slow SCW with a non-linear quasi-electrostatic support, which ensures the constancy of the longitudinal velocity, are the same (Fig. 2, graphs 2).

To carry out numeric analysis, let us consider the SCW amplification section with the following parameters: the plasma frequency of the electron beam $\omega_\mathrm{p} = 3\times10^{11}$ s^{-1}, the constant term of the relativistic factor $\gamma_0 = 3.5$. The cyclic frequency of the slow SCW is $\omega = 0.63\times10^{12}$ s^{-1}. According to Eq. (4), the period of the pump electric field is $\Lambda = 2.0$ cm.

Solving Eqs. (6)–(9) by standard optimization procedures, we found the optimal parameters for $\tau_{E0}^{\mathrm{opt}} = -1.6$ MV/m^2 and $z_{E0}^{\mathrm{opt}} = 143$ cm. To make sure that these parameters are acceptable, let us see the series of graphs of the dependences of the electric strength amplitude modulus of the slow SCW $|E_\alpha|$ on the longitudinal coordinate z at different τ_{E0} and with the common parameter $z_{E0}^{\mathrm{opt}} = 143$ cm (Fig. 2). In Fig. 2, 1 corresponds to the graph at $\tau_{E0} = -1.8$ MV/m^2, 2 to $\tau_{E0} = \tau_{E0}^{\mathrm{opt}} = -1.6$ MV/m^2, 3 to $\tau_{E0} = -1.3$ MV/m^2, and 4 to $\tau_{E0} = 0$.

Figure 2 demonstrates the maximum saturation level of E_α at $\tau_{E0} = \tau_{E0}^{\mathrm{opt}}$ (graph 2). Let us note, that the dependence $|E_\alpha(z)|$ with parameters $z_{E0} = z_{E0}^{\mathrm{opt}} = 143$ cm and $\tau_{E0} = \tau_{E0}^{\mathrm{opt}} = -1.6$ MV/m^2 (graph 2) coincides with $|E_\alpha(z)|$ with numerically calculated the quasi-electrostatic support, at which the constant term of the velocity remained constant.

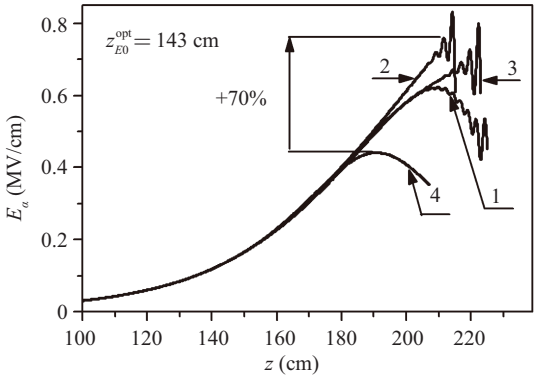


Fig. 2. Dependences of the slow SCW electric strength amplitude modulus $|E_\alpha|$ on the longitudinal coordinate z at different values of the parameter τ_{E0}. Graph 1 corresponds to $\tau_{E0} = -1.8$ MV/m^2, 2 to $\tau_{E0} = \tau_{E0}^{\mathrm{opt}} = -1.6$ MV/m^2, 3 to $\tau_{E0} = -1.3$ MV/m^2, and 4 to $\tau_{E0} = 0$. The parameter $z_{E0} = z_{E0}^{\mathrm{opt}} = 143$ cm is the same for all dependencies.

The existence of optimal parameters for the quasi-electrostatic support is illustrated more clearly in Fig. 3. It shows the dependence of the slow SCW electric strength saturation level modules $|E_\alpha^{\mathrm{sat}}|$ on the parameter τ_{E0}. On the graph, point 1 corresponds to the maximum of E_α at $\tau_{E0} = -1.8$ MV/m^2, point 2 to $\tau_{E0} = \tau_{E0}^{\mathrm{opt}} = -1.6$ MV/m^2, point 3 to $\tau_{E0} = -1.3$ MV/m^2, and point 4 to $\tau_{E0} = 0$. The graph maximum is at $\tau_{E0} = \tau_{E0}^{\mathrm{opt}} = -1.6$ MV/m^2.

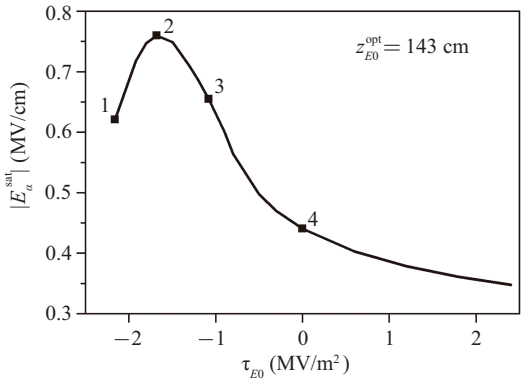


Fig. 3. Dependence of the slow SCW electric strength saturation level modules $|E_\alpha^{\rm sat}|$ on τ_{E0} at $z_{E0} = z_{E0}^{\rm opt} = 143$ cm. Point 1 corresponds to $\tau_{E0} = -1.8$ MV/m^2, point 2 to $\tau_{E0} = \tau_{E0}^{\rm opt} = -1.6$ MV/m^2, point 3 to $\tau_{E0} = -1.3$ MV/m^2, and point 4 to $\tau_{E0} = 0$.

Figure 4 shows graphs of the dependence of the constant term of the electron beam velocity υ_0, normalized to the speed of light in vacuum c, on the longitudinal coordinate z without the optimal quasi-electrostatic support $E_0(z)$ (graph 1) and with it (graph 2).

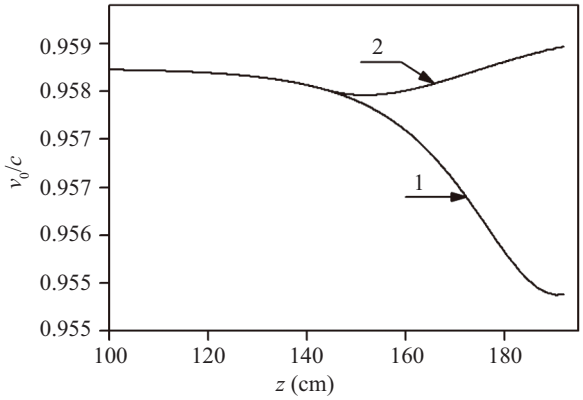


Fig. 4. Dependence of the constant term of the electron beam velocity υ_0, normalized to the speed of light in vacuum c, on the longitudinal coordinate z without the optimal quasi-electrostatic support $E_0(z)$ (graph 1) and with it (graph 2) at optimal parameters $\tau_{E0} = \tau_{E0}^{\rm opt} = -1.6$ MV/m^2 at $z_{E0} = z_{E0}^{\rm opt} = 143$ cm.

We see that the REB's constant velocity υ_0 change in the presence of acceptable quasi-electrostatic support $E_0(z)$ (graph 2) is much less than in the absence of the field (graph 1).

Thus, the quasi-electrostatic support $E_0(z)$ with optimal parameters allows us to maintain resonance conditions between the slow and fast SCWs and the electrostatic undulator pump field that finally increases the saturation levels of the slow SCW. This is clearly visible in Fig. 2, comparing graphs 2 and 4 and observing an increase in the saturation level by approximately $\sim$70%.

One of the factors that significantly affects the processes of the SCW amplification in the device under study is the creation of the generated pump electric field $\boldsymbol{E}_2^{\rm g}$.[10,11] This field is characterized by the phase p_2 and is generated due to the two following mechanisms. The first mechanism is the electron beam modulation under the influence of the electrostatic undulator pump field $\boldsymbol{E}_{20}$.[10,11] The second mechanism is the resonant interaction of the slow and fast SCWs, which causes generated pump electric field $\boldsymbol{E}_2^{\rm g}$. The latter three-wave resonance mechanism significantly reduces $\boldsymbol{E}_2^{\rm g}$ when the amplitudes of the slow and fast SCWs become large enough in the saturation region, and the REB constant velocity is reduced.[10]

The optimal quasi-electrostatic support $E_0(z)$ expands the region of the three-wave parametric resonant interaction of the electron beam with electric fields. Therefore, we expect a significant influence of $E_0(z)$ on the generated pump electric field strength amplitude $E_2^{\rm g}$ as well.

Figure 5 shows the dependence of the generated pump electric field strength amplitude modulus $|E_2^{\rm g}|$ on the longitudinal coordinate z without (graph 1) and with (graph 2) the optimal quasi-electrostatic support $E_0(z)$ at optimal parameters.

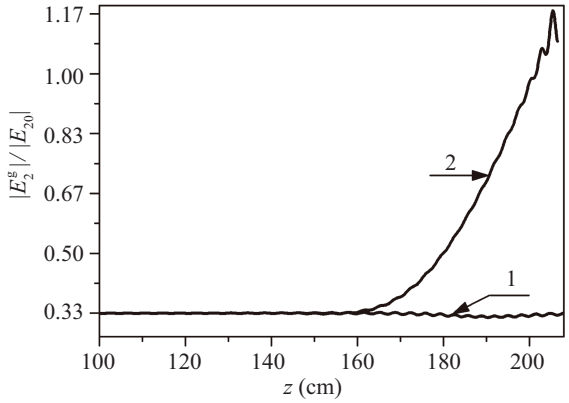


Fig. 5. Dependences of the generated pump electric field strength amplitude modulus $|E_2^{\rm g}|$, normalized to the electrostatic undulator field amplitude modulus $|E_{20}|$ ($|E_2^{\rm g}|/|E_{20}|$) on the z coordinate without the optimal quasi-electrostatic support $E_0(z)$ (graph 1) and with it (graph 2) at optimal parameters $\tau_{E0}^{\rm opt} = -1.6$ MV/m^2 at $z_{E0}^{\rm opt} = 143$ cm.

In Fig. 5 we see that the quasi-electrostatic support $E_0(z)$ significantly affects the generated pump electric field amplitude $E_2^{\rm g}$, increasing its modulus almost four times in the saturation region (graph 2). At the same time, in the absence of $E_0(z)$, the generated pump electric field decreases slightly (graph 1).

Please note that the amplification processes of SCWs are directly influenced not by the generated pump electric field itself but by the resulting pump electric field $\boldsymbol{E}_2 = \boldsymbol{E}_{20} + \boldsymbol{E}_2^{\rm g}$. This is due to the fact that it depends not only on the generated pump electric field strength amplitude modulus $|E_2^{\rm g}|$ but also on its argument arg $E_2^{\rm g}$.

Figure 6 shows the dependence of the resulting pump electric field strength amplitude modulus $|E_2| = |E_{20} + E_2^{\rm g}|$ on the longitudinal coordinate z without (graph 1) and with (graph 2) the optimal quasi-electrostatic support $E_0(z)$ with optimal parameters. Graph 3 shows the value of the electrostatic undulator field strength amplitude modulus $|E_{20}|$, normalized to itself along the z axis.

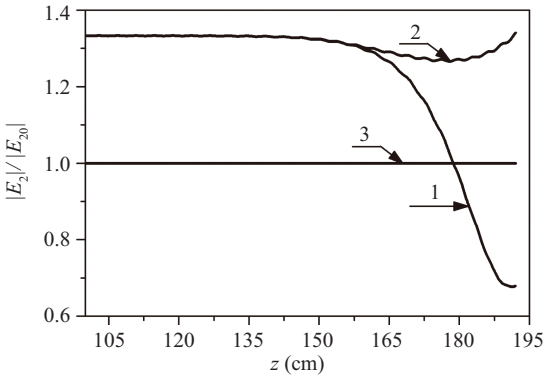


Fig. 6. Dependences of the resulting periodic pump electric field strength amplitude modulus $|E_2|$, normalized to the electrostatic undulator field amplitude $|E_{20}|$ ($|E_2|/|E_{20}|$) on the longitudinal coordinate z without (graph 1) and with (graph 2) the quasi-electrostatic support $E_0(z)$ with optimal parameters $\tau_{E0}^{\rm opt} = -1.6$ MV/m^2 and $z_{E0}^{\rm opt} = 143$ cm. Graph 3 shows the value of the electrostatic undulator field strength amplitude modulus $|E_{20}|$, normalized to itself along the z axis.

Despite a relatively strong increase in the generated pump electric field amplitude under the influence of the optimal quasi-electrostatic support $E_0(z)$ (Fig. 5, graph 2), the resulting pump electric field changes insignificantly (Fig. 6, graph 2). At the same time, a slight change in the generated pump electric field in the absence of the quasi-electrostatic support $E_0(z)$ (Fig. 5, graph 1) decreases the resulting pump electric field by almost two times (Fig. 6, graph 1).

To understand these interesting processes, let us look at Fig. 7 showing the behaviors of the generated pump electric field amplitude argument arg $E_2^{\rm g}$ on the longitudinal coordinate z without the optimal quasi-electrostatic support $E_0(z)$ (graph 1) and with it (graph 2). The argument of the electrostatic undulator pump field amplitude E_{20} is taken equal to zero along the entire device.

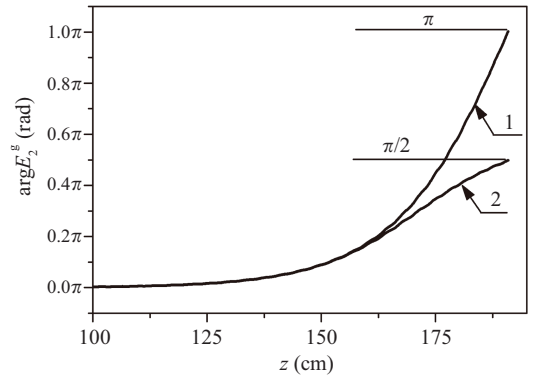


Fig. 7. Dependences of the generated pump electric field amplitude argument arg $E_2^{\rm g}$ on the longitudinal coordinate z without the quasi-electrostatic support $E_0(z)$ (graph 1) and with it (graph 2) at optimal parameters $\tau_{E0}^{\rm opt} = -1.6$ MV/m^2 at $z_{E0}^{\rm opt} = 143$ cm.

From Fig. 7 it follows that at the initial region of the amplification process, the generated pump electric field in both cases has an argument that practically equals 0, i.e., coincides with the argument of the electrostatic undulator pump field amplitude E_{20}. Therefore, at the initial region of the SCWs amplification the amplitude modulus of the resulting pump electric field equals $|E_{20}| + |E_2^{\rm g}| = 1.33|E_{20}|$ (compare graphs 3 with 1 or 2 in Fig. 6). However, in the saturation region, the phase of the generated pump electric field in the absence of the quasi-electrostatic support $E_0(z)$ changes by π rad (graph 1 in Fig. 7). Consequently, the resulting amplitude modulus is equal to the difference of the corresponding amplitude modules and decreases by almost two times ($|E_{20}| - |E_2^{\rm g}| = 0.67|E_{20}|$). So, such resulting pump electric field behavior significantly impacts the saturation level of the space charge waves.

In the case of using the optimal quasi-electrostatic support $E_0(z)$, the behavior of the generated pump electric field argument is significantly different: in the saturation region, it takes a value close to $\pi/2$ rad (Fig. 7, graph 2). Thus, the resulting pump electric field strength amplitude modulus remains noticeably larger than the electrostatic undulator pump field strength amplitude despite a slight decrease in the saturation region (Fig. 6, graph 2).

So, let us consider the change in the electric strength amplitude initial phases (arguments) of the slow SCW in terms of electron bunching processes in a relativistic electron beam. It was shown in Ref. [10] that the electrostatic undulator pump field causes REB modulation, i.e., the appearance of static bunches of electrons in the REB that create their own collective field, which is called the generated pump electric field $\boldsymbol{E}_2^{\rm g}$. At the initial stage of the SCWs amplification, electron bunches create $\boldsymbol{E}_2^{\rm g}$, which is in phase with the electrostatic undulator pump field $\boldsymbol{E}_{20}$. In this work, we found that in the saturation region, without the quasi-electrostatic support $\boldsymbol{E}_0(z)$, the argument of $\boldsymbol{E}_2^{\rm g}$ gradually changes to the opposite one. This means the electron bunches gradually shift relative to the electrostatic undulator by half the undulation period. $E_0(z)$ significantly helps these bunches to resist their shift. As a result, the decrease in the resulting pump electric field in the saturation region becomes much smaller (Fig. 6, graphs 1 and 2 in the saturation region).

Thus, the quasi-electrostatic support, on the one hand, increases the length at which the conditions of the three-wave parametric resonance are satisfied and ensures the energy transfer from the relativistic electron beam into the space charge waves. On the other hand, $E_0(z)$ significantly increases the resulting pump electric field in the saturation region. As a result of these processes, the saturation level of the slow SCW in the studied amplification section increases by 70% (Fig. 2).

Finally, let us determine what maximum value of optimal $E_0(z)$ is required to achieve the maximum saturation level of the slow SCW field and compare it with the value of the E_{20} to achieve the same maximum saturation level of the slow SCW. Figure 8 shows a graph of the dependence of the slow SCW electric strength amplitude modulus $|E_\alpha|$ on the longitudinal coordinate z. Graph 1 corresponds to the case when

the amplitude of the undulator pump field amplitude modulus equals $|E_{20}|$, and the optimal quasi-electrostatic support $E_0(z)$ is turned on. The maximum value of $E_0(z)$ achieves at the saturation coordinate and equals 10% of the $|E_{20}|$ and, as a result, the external electrostatic field maximum amplitude modulus equals $|E_{\text{ext1}}^{\max}| = 1.1|E_{20}|$. Graph 2 corresponds to the case when the undulator pump field strength amplitude modulus increases by 3 times ($3|E_{20}|$) and quasi-electrostatic support $E_0(z)$ is absent, which means that the external electrostatic field maximum amplitude modulus equals $|E_{\text{ext2}}^{\max}| = 3|E_{20}|$.

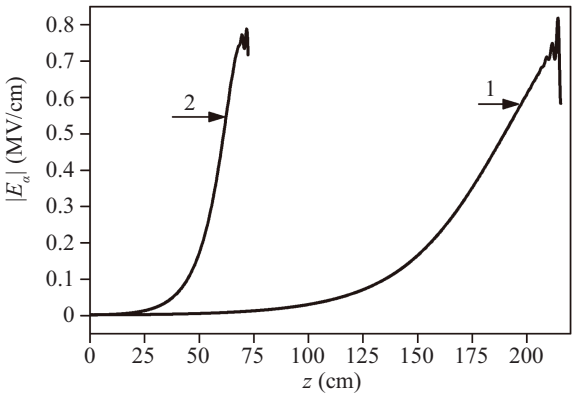


Fig. 8. Graphs of the dependence of the slow SCW electric strength amplitude E_α on the longitudinal coordinate z. Graph 1 corresponds to the case when the amplitude of the undulator pump field equals E_{20}, and the maximum of the optimal quasi-electrostatic support $E_0(z)$ equals 10% of $|E_{20}|$. Graph 2 corresponds to the case when the undulator pump field strength amplitude modulus equals $3|E_{20}|$, and quasi-electrostatic support $E_0(z)$ is absent.

Comparing graphs 1 and 2 in Fig. 8 we conclude that to obtain the same slow SCW saturation level value we can either increase the electrostatic undulator pump field amplitude E_{20} by three times ($|E_{\text{ext2}}^{\max}| = 3|E_{20}|$ graph 2) or turn on the quasi-electrostatic support $E(z)$ with optimal parameters τ_E^{opt} and z_E^{opt} and as a consequence, with its maximum electric strength value of 10% of the E_{20} ($|E_{\text{ext1}}^{\max}| = 1.1|E_{20}|$, graph 1). In other words, to achieve the same significant increase in the SCW saturation level, $E(z)$ allows us to reduces the external electrostatic field maximum amplitude modulus by around $|E_{\text{ext2}}^{\max}|/|E_{\text{ext1}}^{\max}| = 2.73$.

5. Conclusions

In this paper, for the first time we demonstrate the possibility of significantly increasing the saturation level of the slow SCW (by 70%) by maintaining the conditions of the three-wave parametric resonance between the slow, fast SCWs and the resulting pump electric field in the amplification section of the superheterodyne FEL. This is possible by introducing a new component of an external electrostatic field into the system called quasi-electrostatic support $E_0(z)$.

We found the simplest form of the quasi-electrostatic support $E_0(z)$ that describes a linear dependence of the electric field strength on the longitudinal coordinate z, which begins at the coordinate z_{E0} and has a constant electric field strength gradient τ_{E0}. This field, on the one hand, allows us to analyze nonlinear processes at saturation region easily. On the other hand, it can be easily experimentally implemented in an electrostatic undulator.

For the superheterodyne FEL amplification section with the considered parameters, the optimal parameters of $E_0(z)$ are calculated.

For the first time we found that the quasi-electrostatic support $E_0(z)$ increases the electric strength amplitude modulus of the generated pump field E_2^{g} by four times in the saturation region. Moreover, $E_0(z)$ reduces the difference between the arguments of the electrostatic undulator and generated pump electric fields by $\pi/2$ rad in the saturation region. Thus, improvement of E_2^{g} by the $E_0(z)$ significantly enhance the resulting pump electric field in the amplification section of the superheterodyne FEL resulting in an increase in SCW amplification.

We compare the influence of the electrostatic undulator pump field E_{20} and $E_0(z)$ on the saturation level of the slow SCW. Figure 8 shows that increasing the amplitude modulus of E_{20} three times relative to its initial level allows the slow SCW to achieve the maximum saturation level. However, applying the quasi-electrostatic support with a maximum strength of 10% of the initial E_{20} can achieve the same maximum saturation level of the slow SCW. Thus, to achieve a significant increase in the saturation level of the SCWs, $E_0(z)$ reduces the external electrostatic field maximum amplitude modulus by around 2.73.

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