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# Sliding-mode-based preassigned-time control of a class of memristor chaotic systems

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This paper addresses the preassigned-time chaos control problem of memristor chaotic systems with time delays. Since the introduction of memristor, the presented models are nonlinear systems with chaotic dynamics. First, the TS fuzzy method is adopted to describe the chaotic systems. Then, a sliding-model-based control approach is proposed to achieve the preassigned-time stabilization of the presented models, where the upper bound of stabilization time can be arbitrarily specified in advance. Finally, simulation results demonstrate the validity of presented control approach and theoretic results.

**Keywords:** memristor chaotic system, TS fuzzy method, preassigned-time stabilization, sliding-mode control

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## 1. Introduction

Memristors are electronic components that exhibit memory of resistance, a property that allows them to remember the amount of charge that has passed through them. They are pivotal in mimicking the behavior of biological neurons and synapses, and are increasingly utilized in neuromorphic engineering for applications such as brain-inspired computing, artificial intelligence, and adaptive learning systems.[1–3] As a class of nonlinear systems, memristor chaotic systems (MCSs) have shown unique advantages and broad application prospects in fields such as information security, cryptography, communication systems, and chaotic computing.[4–6] Scholars have devoted extensive research efforts to MCSs, focusing intently on dynamic analyses and control, in order to gain a deeper comprehension and exploit the full potential of the spacial systems.[7–10]

The primary challenge in ensuring system stability stems from the presence of nonlinear terms, which are particularly pronounced in memristor-based systems. The intricate nonlinearities introduced by memristors present novel obstacles in the control of MCSs. To address these challenges, the TS fuzzy method, a powerful tool known for its efficacy in managing nonlinearities, has been employed. The TS fuzzy method was firstly developed by Takagi and Sugeno,[11] and it can transform a complicated nonlinear model to the combination of several linear systems by using IF–THEN rules.[12,13] Compared with the original systems, the transformed systems can not only reduce the computational complexity and the energy consumption in the controller design, but also they are easy to be implemented in practice. Nowadays, TS fuzzy method has played an essential role in researches of MCSs due to its advantages of high accuracy and simple modeling strategy.[14,15]

The stabilization control of MCSs is crucial for their successful implementation in practical scenarios.[16–20] Theses papers delved into the stabilization and synchronization of MCSs, which are particularly relevant for applications where rapid stabilization is crucial, such as secure communication,[16] bio-inspired computing,[17] highly interconnected and coordinated computational platforms,[18] data security,[19] and reconfigurable computing and memory devices.[20] In reality applications, it is expected to clear off the instability of MCSs in a finite time. Compared with asymptotic stability, finite-time and fixed-time stability has a faster convergence speed, higher steady-state precision and better anti-jamming performance features.[21–23] However, both the finite-time and fixed-time control still have the drawbacks as the settling-time is related to the initial values or the system parameters, which may restrict their practical applications in cases that the initial values or the system parameters are difficult to get or even unknown. In practice, it is required that the stability or control of chaotic systems be realized in a time which is not only independent to the both the initial values and the system parameters, but also can be arbitrarily set in advance.

Inspired by above analyses, this paper aims to discuss the preassigned-time stability of MCSs with delays based on TS fuzzy method, where the stable time can be set in advance. The main contributions are summarized below.

(i) The nonlinear model of MCSs with time delays as well as the chaotic behaviors is presented. The TS fuzzy method is

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adopted to deal with the nonlinear terms of MCSs. Therefore, the nonlinear delayed system is transformed into the combination of several linear models, which makes the dynamical analysis and stability problem simple and convenient.

(ii) A novel sliding-mode controller is proposed to realize the preassigned-time stabilization of MCSs with time delays. Compared to the finite-time and fixed-time results in Ref. [22], where the settling time are constrained by the system and controller parameters, the settling time in this paper can be arbitrarily set in advance.

This paper includes five aspects. The second part introduces some preliminaries and chaotic systems model based on TS fuzzy method, and the third part gives the control method and main results. In the fourth part, numerical simulations verify the effectiveness and feasibility of derived results. Finally, the work of this paper is summarized.

## 2. Preliminaries

We consider a memristor chaotic system shown in Fig. 1, which consists of a memristor $M$, two inductors $L_1$, $L_2$, a capacitor $C$ and a resistor $R$. Based on Kirchhoff principle and circuit shown below, the following model is obtained:

$$\begin{cases} \dfrac{\mathrm{d}u_c(t)}{\mathrm{d}t} = \dfrac{1}{c}(i_1(t) - i_2(t)), \\ \dfrac{\mathrm{d}i_1(t)}{\mathrm{d}t} = \dfrac{1}{l_1}[ri_1(t) - u_c(t)], \\ \dfrac{\mathrm{d}i_2(t)}{\mathrm{d}t} = \dfrac{1}{l_2}[u_c(t) - W[q(t)]i_2(t)], \\ \dfrac{\mathrm{d}q(t)}{\mathrm{d}t} = i_2(t), \end{cases} \tag{1}$$

where $c$ means the capacitance of the capacitor $C$, $l_1$, and $l_2$ represent the inductance of the inductors $L_1$ and $L_2$ respectively. $r$ is the resistance of the resistor $R$. $u_c(t)$ represents the voltage of the capacitor. The $i_1(t)$ and $i_2(t)$ are the electrical current of the inductors respectively.

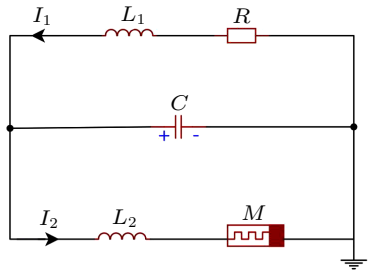



**Fig. 1.** The memristor chaotic circuit.

A piecewise model is chosen to describe the memristor.[24] According to this model, the flux flowing through the memristor $\Phi(q(t))$ is defined as

$$\Phi(q(t)) = \varkappa_2 q(t) + 0.5(\varkappa_1 - \varkappa_2)(|q(t)+1| - |q(t)-1|),$$

where $\varkappa_1$ and $\varkappa_2$ are constants decided by the physical characteristic of the memristor. Then, $W[q(t)]$ is the memristance of the memristor and $q(t)$ is the current flowing through the memristor.

$$W[q(t)] = \frac{\mathrm{d}\Phi(q(t))}{\mathrm{d}q(t)} = \begin{cases} \varkappa_1, & |q(t)| \le 1, \\ \varkappa_2, & |q(t)| > 1. \end{cases}$$

To simplify the derivation, the following substitution is made: $x_1(t) = u_c(t)$, $x_2(t) = i_1(t)$, $x_3(t) = i_2(t)$, $x_4(t) = q(t)$. Then substitute $a_1$ for $1/c$, $a_2$ for $r/l_1$, $a_3$ for $1/l_2$, $a_4$ for $1/l_2$, system (1) can be rearranged as

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = a_1(x_2(t) - x_3(t)), \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = a_2 x_2(t) - a_3 x_1(t), \\ \dfrac{\mathrm{d}x_3(t)}{\mathrm{d}t} = a_4(x_1(t) - W[x_4(t)]x_3(t)), \\ \dfrac{\mathrm{d}x_4(t)}{\mathrm{d}t} = x_3(t), \end{cases} \tag{2}$$

the $W[x_4(t)]$ is given as

$$W[x_4(t)] = \begin{cases} w_1, & |x_4(t)| \le 1, \\ w_2, & |x_4(t)| > 1, \end{cases} \tag{3}$$

where $w_1$ and $w_2$ are constants decided by the physical characteristic of the memristor.

Note that under the given structure of system (2), chaos phenomena will emerged if the system parameters and initial values meet specific conditions. Consider the system (2) with $a_1 = 0.15$, $a_2 = 0.03$, $a_3 = 1.8$, $a_4 = 0.8$, $w_1 = 10$, $w_2 = 0.2$, and the initial values as $x_1(0) = 0.25$, $x_2(0) = -0.21$, $x_3(0) = -0.1$, $x_4(0) = 0.9$. The chaotic behaviors are depicted in Fig. 2.

Considering the effect of time delay, this paper considers a class of chaotic systems with delays

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = a_1(x_2(t) - x_3(t)) + \lambda_1 x_1(t - \rho_1(t)), \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = a_2 x_2(t) - a_3 x_1(t) + \lambda_2 x_2(t - \rho_2(t)), \\ \dfrac{\mathrm{d}x_3(t)}{\mathrm{d}t} = a_4(x_1(t) - W[x_4(t)]x_3(t)) + \lambda_3 x_3(t - \rho_3(t)), \\ \dfrac{\mathrm{d}x_4(t)}{\mathrm{d}t} = x_3(t) + \lambda_4 x_4(t - \rho_4(t)), \end{cases} \tag{4}$$

where $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$ are constants and $\rho_1(t), \rho_2(t), \rho_3(t), \rho_4(t)$ are the time-varying delays which satisfy $0 \le \rho_s(t) \le \rho$ for $s = 1,2,3,4$. If $x_4(t)$ belongs to different fuzzy sets $M_1, M_2$, system (4) with control can be expressed below.

Rule 1: IF $x_4(t)$ is $M_1$,
THEN $\dot{x}(t) = A_1 x(t) + Bx(t - \rho(t)) + Cu(t)$;

Rule 2: IF $x_4(t)$ is $M_2$,
THEN $\dot{x}(t) = A_2 x(t) + Bx(t - \rho(t)) + Cu(t)$;
where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^{\mathrm{T}}$, $x(t - \rho(t)) = (x_1(t - \rho_1(t)), x_2(t - \rho_2(t)), x_3(t - \rho_3(t)), x_4(t - \rho_4(t)))^{\mathrm{T}}$, $u(t)$ is the

controller with the gain matrix $C = \mathrm{diag}\{c_1, c_2, c_3, c_4\}$ to be designed,

$$A_1 = \begin{bmatrix} 0 & a_1 & -a_1 & 0 \\ -a_3 & a_2 & 0 & 0 \\ a_4 & 0 & -a_4 w_1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & a_1 & -a_1 & 0 \\ -a_3 & a_2 & 0 & 0 \\ a_4 & 0 & -a_4 w_2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$B = \mathrm{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$, and the membership functions are

$$M_1(x_4(t)) = \begin{cases} 1, & |x_4(t)| \le 1, \\ 0, & |x_4(t)| > 1, \end{cases}$$
$$M_2(x_4(t)) = \begin{cases} 0, & |x_4(t)| \le 1, \\ 1, & |x_4(t)| > 1. \end{cases}$$

Thus,

$$\dot{x}(t) = \sum_{r=1}^{2} M_r(x_4(t)) A_r x(t) + Bx(t - \rho(t)) + Cu(t), \tag{5}$$

with the equivalent form

$$\begin{aligned} \dot{x}_s(t) = {} & \textstyle\sum_{r=1}^{2} M_r(x_4(t)) \sum_{k=1}^{4} a_{rsk} x_k(t) \\ & + \lambda_s x_s(t - \rho_s(t)) + c_s u_s(t), \end{aligned} \tag{6}$$

where $A_r = \{a_{rsk}\}_{4\times 4}$, $r = 1,2$, $s,k = 1,2,3,4$.

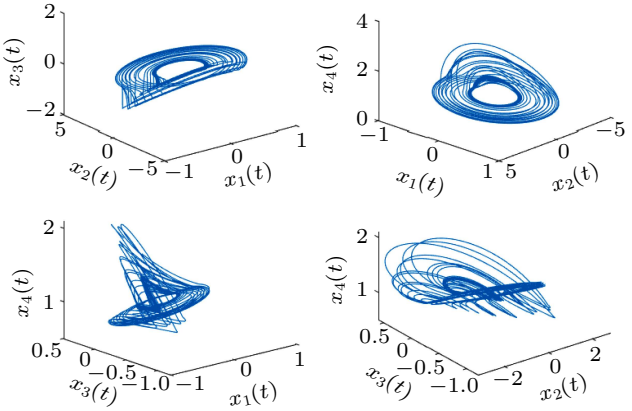



**Fig. 2.** Chaotic behaviors of system (2) with initial condition $x(0) = [0.25, -0.21, -0.1, 0.9]^{\mathrm{T}}$.

**Remark 1** Several memristor models are known to exist. In Ref. [25], the memristor was replaced by circuit and mathematically derived with cubic model. Though cubic model can depict the the characteristics of memristor well, the introduction of high-order nonlinear terms complicates the system and controller design. As for the piecewise model, the system contained switch terms rather than high-order terms. Surprisingly, the switch terms can be described perfectly by the membership functions which is chosen based on model properties rather than experience. It is based on this that the TS fuzzy system is carried out.

**Remark 2** Memristors, as nonlinear passive circuit elements, exhibit behavior that is not easily described by traditional linear models. The TS fuzzy model can capture the nonlinear characteristics of memristors by dividing the behavior into a set of local linear models, each associated with a specific fuzzy set. In this paper, the TS fuzzy method is adopted to deal with the switching characteristic of piecewise memristor model. In fact, TS fuzzy model method can deal with other memristor model in Ref. [26] as long as there is nonlinear term.

Then, we present a definition and a lemma which are useful in the following section.

**Definition 1** System (5) or system (6) is said to be preassigned-time stable with the designed controller $u(t)$, if there exists a pre-given time $T$ such that

$$\lim_{t\to T} |x_s(t)| = 0, |x_s(t)| \equiv 0 \quad (t \ge T). \tag{7}$$

**Lemma 1**[22] For any real numbers $y_s$ $(s = 1,2,\ldots,m)$ and $0 < \gamma_1 < 1$, $\gamma_2 > 1$, two inequalities below hold:

$$\sum_{s=1}^{m} |y_s|^{\gamma_1} \ge \left( \sum_{s=1}^{m} |y_s| \right)^{\gamma_1}, \tag{8}$$

$$\sum_{s=1}^{m} |y_s|^{\gamma_2} \ge m^{1-\gamma_2} \left( \sum_{s=1}^{m} |y_s| \right)^{\gamma_2}. \tag{9}$$

## 3. Main results

In this section, we will design a sliding-mode controller to realize the preassigned-time stabilization of chaotic systems with delays. The sliding-mode surface is defined below:

$$\Gamma = \{x(t) | \Delta(t) = (\Delta_1(t), \Delta_2(t), \Delta_3(t), \Delta_4(t))^{\mathrm{T}} = 0\}, \tag{10}$$

$$\begin{aligned} \Delta_s(t) = {} & \delta_s x_s(t) + \int_0^t \frac{\pi \check{\delta}}{2\gamma T_2} (|x_s(\tau)|^{1-\gamma} \\ & + \xi |x_s(\tau)|^{1+\gamma}) \mathrm{sign}(x_s(\tau)) \mathrm{d}\tau, \end{aligned} \tag{11}$$

where $\xi = 3^{\gamma/2}$, $\delta_s > 0$ $(s = 1,2,3,4)$, $0 < \gamma < 1$, $T_2 > 0$, $\check{\delta} = \max_s\{\delta_s\}$, $\mathrm{sign}(\cdot)$ is the standard signum function.

Thus, the time derivative of Eq. (11) is

$$\begin{aligned} \dot{\Delta}_s(t) = {} & \delta_s \dot{x}_s(t) + \frac{\pi \check{\delta}}{2\gamma T_2} (|x_s(t)|^{1-\gamma} \\ & + \xi |x_s(t)|^{1+\gamma}) \mathrm{sign}(x_s(t)). \end{aligned} \tag{12}$$

Assume the sliding-mode be reached in $T_1$, then

$$\dot{\Delta}_s(t) = \Delta_s(t) = 0, \tag{13}$$

for all $t \ge T_1$. Further, assume the states of the sliding-mode reach the equilibrium point within $T_2$. Hence, it follows that

$$\dot{x}_s(t) = -\frac{\pi \check{\delta}}{2\gamma \delta_s T_2} (|x_s(t)|^{1-\gamma} + \xi |x_s(t)|^{1+\gamma}) \mathrm{sign}(x_s(t)). \tag{14}$$

### 3.1. Reachability of sliding-mode surface within a preassigned time

In this part, the designed controller is given as

$$\begin{aligned} u_s(t) = {} & -\frac{\pi \check{\delta}}{2\gamma c_s \delta_s T_2} (|x_s(t)|^{1-\gamma} + \xi |x_s(t)|^{1+\gamma}) \mathrm{sign}(x_s(t)) \\ & - \alpha_s(t) \mathrm{sign}(\Delta_s(t)), \end{aligned} \tag{15}$$

where

$$\alpha_s(t) = \sum_{k=1}^{4} \varpi_{sk}|x_k(t)| + \zeta_s|x_s(t-\rho_s(t))| + \frac{\pi}{2\gamma T_1 \beta}(|\Delta_s(t)|^{1-\gamma} + \xi|\Delta_s(t)|^{1+\gamma})$$

with the parameters

$$\beta = \min_s\{\delta_s c_s\}, \quad \varpi_{sk} > \frac{1}{c_s}|a_{rsk}|, \quad \zeta_s > \frac{\lambda_s}{c_s}.$$

**Theorem 1** By using the controller (15), the state variables of system (6) will approach the sliding-mode surface (10) and reach it within a preassigned time $T_1$.

**Proof** The Lyapunov function is selected as

$$V_1(t) = \sum_{s=1}^{4} \Delta_s^2(t). \tag{16}$$

Then, the time derivative of Eq. (16) is

$$\begin{aligned}\dot{V}_1(t) &= \sum_{s=1}^{4} 2\Delta_s(t)\dot{\Delta}_s(t) \\ &= \sum_{s=1}^{4} 2\Delta_s(t)[\delta_s\dot{x}_s(t) + \frac{\pi\check{\delta}}{2\gamma T_2}(|x_s(t)|^{1-\gamma} \\ &\quad + \xi|x_s(t)|^{1+\gamma})\mathrm{sign}(x_s(t))]. \end{aligned} \tag{17}$$

By substituting Eqs. (6) and (15) into Eq. (17), we can get

$$\begin{aligned}\dot{V}_1(t) &= \sum_{r=1}^{2}\sum_{s=1}^{4} 2\Delta_s(t)\delta_s M_r(x_4(t))\Big[\sum_{k=1}^{4} a_{rsk}x_k(t) \\ &\quad + \lambda_s x_s(t-\rho_s(t)) - c_s\alpha_s(t)\mathrm{sign}(\Delta_s(t))\Big] \\ &\le \sum_{r=1}^{2}\sum_{s=1}^{4} 2\delta_s M_r(x_4(t))\Big[\sum_{k=1}^{4}|a_{rsk}||\Delta_s(t)||x_k(t)| \\ &\quad + \lambda_s|\Delta_s(t)||x_s(t-\rho_s(t))| - c_s\Big(\sum_{k=1}^{4}\varpi_{sk}|x_k(t)| \\ &\quad + \zeta_s|x_s(t-\rho_s(t))| + \frac{\pi}{2\gamma T_1\beta}(|\Delta_s(t)|^{1-\gamma} \\ &\quad + \xi|\Delta_s(t)|^{1+\gamma})\Big)|\Delta_s(t)|\Big] \\ &\le -\sum_{s=1}^{4}\frac{\delta_s c_s\pi}{\gamma T_1\beta}(|\Delta_s(t)|^{2-\gamma} + \xi|\Delta_s(t)|^{2+\gamma}). \end{aligned} \tag{18}$$

Based on Lemma 1, it yields

$$\dot{V}_1(t) \le -\frac{\pi}{\gamma T_1}\left(V_1^{1-\gamma/2} + V_1^{1+\gamma/2}\right). \tag{19}$$

From inequality (19), we can obtain that

$$\begin{aligned} T &\le -\frac{\gamma T_1}{\pi}\int_{+\infty}^{0}\frac{\mathrm{d}V}{V^{1-\gamma/2}+V^{1+\gamma/2}} \\ &= \frac{\gamma T_1}{\pi}\int_{0}^{+\infty}\frac{\mathrm{d}V}{V^{1-\gamma/2}+V^{1+\gamma/2}} \\ &= T_1\left(\frac{2}{\pi}\arctan(+\infty)\right) = T_1. \end{aligned} \tag{20}$$

The proof is completed.

### 3.2. Preassigned-time stability of sliding-mode dynamics

In this part, while on the surface (10), the state variables of system (14) will further reach the equilibrium point within a pre-given time $T_2$.

**Theorem 2** System (14) is preassigned-time stable within time $T_2$.

**Proof** Choose the Lyapunov function

$$V_2(t) = \sum_{s=1}^{4} x_s^2(t). \tag{21}$$

Then, the time derivative of Eq. (21) is

$$\begin{aligned}\dot{V}_2(t) &= \sum_{s=1}^{4} 2x_s(t)\dot{x}_s(t) \\ &\le -\sum_{s=1}^{4}\frac{\pi\check{\delta}}{\gamma\delta_s T_2}(|x_s(t)|^{2-\gamma} + \xi|x_s(t)|^{2+\gamma}). \end{aligned} \tag{22}$$

Based on Lemma 1, it thus results in

$$\dot{V}_2(t) \le -\frac{\pi}{\gamma T_2}(V_2^{1-\gamma/2} + V_2^{1+\gamma/2}). \tag{23}$$

Similar to the proof of Theorem 1, if follows that

$$T \le -\frac{\gamma T_2}{\pi}\int_{+\infty}^{0}\frac{\mathrm{d}V}{V^{1-\gamma/2}+V^{1+\gamma/2}} = T_2. \tag{24}$$

The proof is completed.

**Remark 3** The state variables of system (5) or system (6) tend to approach and reach the designed surface (10) within a preassigned time $T_1$ firstly. After reaching it, the state variables of equivalent system (14) converge to zero in a further preassigned time $T_2$. As a consequence, the whole preassigned time $T$ satisfies $T = T_1 + T_2$.

**Remark 4** From the main results, it is concluded that the estimation stability time $T$ is uncorrelated with the initial values and parameters of system, which can be arbitrarily set in advance based on actual application requirements. The good feature of preassigned-time control makes it an ideal control approach to analyze the dynamical behaviors of chaotic systems compared to the finite-time and fixed-time control.

**Remark 5** In this paper, the sliding-mode surface design incorporates the signum function, playing a pivotal role in the control strategy by ensuring that the system stabilizes within a predetermined time frame. This function offers rapid feedback, aiding the system in resisting disturbances and swiftly achieving equilibrium. On the other hand, it also has its limitations. Notably, the signum function's discontinuity can cause high-frequency oscillations or chattering, which may damage the actuator and degrade control performance. Furthermore, the utilization of the signum function complicates controller design, increasing the challenge of practical implementation.

**Remark 6** Compared with traditional PID control and adaptive control,[27,28] the sliding-mode control possess higher robustness (high capability to track system inputs and reject unwanted disturbances) and faster response speed due to the presence of sliding-mode surfaces. Besides, the sliding-mode control also experiences chattering phenomena due to

the rapid switching of the control signal on both sides of the sliding-mode surfaces, which makes it challenging to implement in practice.

**Remark 7** Compared to asymptotic stability, both finite-time and fixed-time stability offer faster convergence, improved steady-state accuracy, and superior resistance to disturbances.[21–23] However, these methods rely on initial conditions or system parameters, which will limit their applicability when such values are hard to determine or unknown. Preassigned-time control stands out by being independent of initial values and system parameters, togethering with the advantage of allowing the stabilization time to be set arbitrarily in advance.

## 4. Numerical simulations

In this part, we first give the following delayed MCSs:

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = 0.15(x_2(t)-x_3(t))+0.0005x_1(t-\rho_1(t)), \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = 0.03x_2(t)-1.8x_1(t)+0.0005x_2(t-\rho_2(t)), \\ \dfrac{\mathrm{d}x_3(t)}{\mathrm{d}t} = 0.8(x_1(t)-W[x_4(t)]x_3(t)) \\ \qquad\quad +0.0005x_3(t-\rho_3(t)), \\ \dfrac{\mathrm{d}x_4(t)}{\mathrm{d}t} = x_3(t)+0.0005x_4(t-\rho_4(t)), \end{cases} \tag{25}$$

where

$$W[x_4(t)] = \begin{cases} 10, & |x_4(t)| \le 1, \\ 0.2, & |x_4(t)| > 1, \end{cases} \tag{26}$$

and $\rho_s(t) = \exp(t)/(1+\exp(t))$, $s = 1,2,3,4$, the initial condition is chosen as $x(\varepsilon) = [0.5,-0.2,-0.1,0.9]^{\mathrm{T}}$ (which is called case 1). The trajectories of state variables without control are given in Fig. 3, which implies that the states are unstable.

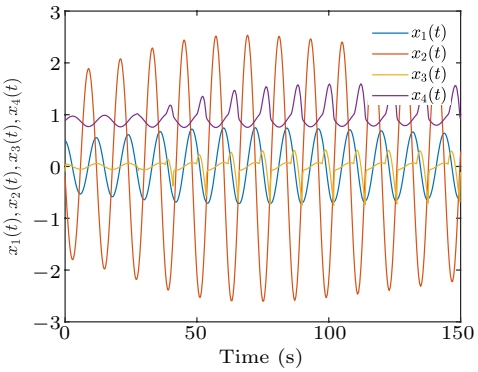



**Fig. 3.** The state trajectories of system (25) without control.

Then, system (25) with control is transformed into two linear systems via the TS fuzzy method, where $u(t)$ is designed as Eq. (15), and

$$A_1 = \begin{bmatrix} 0 & 0.15 & -0.15 & 0 \\ -1.8 & 0.03 & 0 & 0 \\ 0.8 & 0 & -8 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0.15 & -0.15 & 0 \\ -1.8 & 0.03 & 0 & 0 \\ 0.8 & 0 & -0.16 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$B = \mathrm{diag}\{0.0005, 0.0005, 0.0005, 0.0005\}$.

Choose the parameters $\delta_s = c_s = 1$, $s = 1,2,3,4$, $\gamma = 0.2$. Then, we set $T = 1$ s with $T_1 = T_2 = 0.5$ s. Then, based on the results of Theorems 1 and 2, the state variables of the system (5) or system (6) will move toward and reach the designed surface (10) within a preassigned time 0.5 s. After reaching the surface, system (14) will be active and its state variables approach to zero in a further preassigned time 0.5 s. As a consequence, the trajectories of states will converge to and reach zero within the preassigned time $T = 1$ s, which are displayed in Fig. 4. Moreover, we choose $T = 0.1$ s with $T_1 = T_2 = 0.05$ s, and the similar results are also shown in Fig. 5.

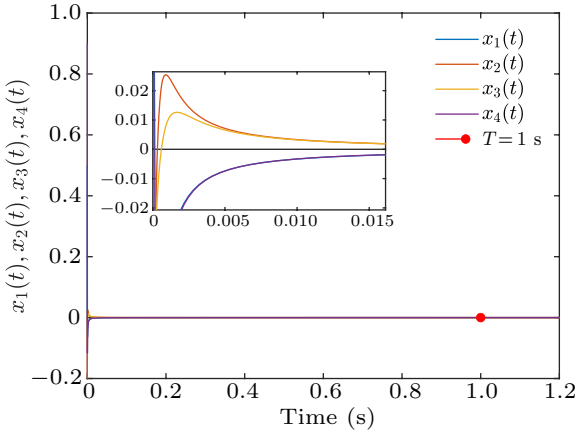



**Fig. 4.** The state trajectories with control and $x(\varepsilon)$ is in case 1, and $T = 1$ s.

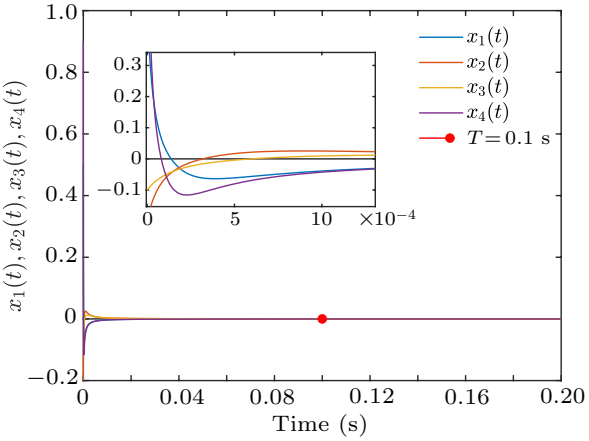



**Fig. 5.** The state trajectories with control and $x(\varepsilon)$ is in case 1, and $T = 0.1$ s.

To show the irrelevance between the preassigned time and the initial values, we give another kinds of simulation results by magnifying the initial values to five times as $x(\varepsilon) = [2.5,-1,-0.5,4.5]^{\mathrm{T}}$ (which is called case 2). Then, the corresponding simulation results are illustrated in Figs. 6 and 7. Note that, the state trajectories converge to zero within the preassigned time and they are not influenced by the different initial values, which verifies the superiority of preassigned-time control method.

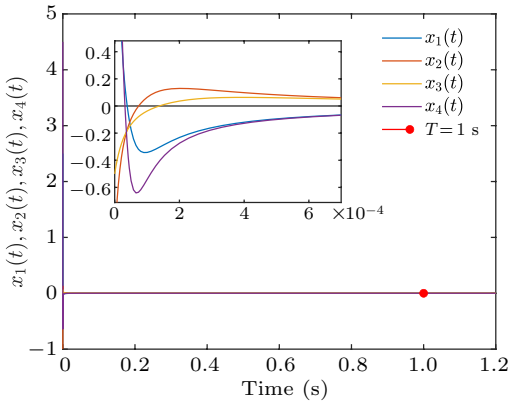



**Fig. 6.** The state trajectories with control and $x(\varepsilon)$ is in case 2, and $T = 1$ s.

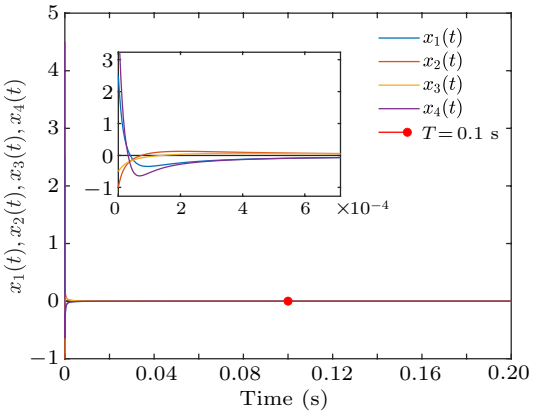



**Fig. 7.** The state trajectories with control and $x(\varepsilon)$ is in case 2, and $T = 0.1$ s.

**Remark 8** In numerical simulations, high-order saturation functions are utilized in lieu of signum functions. This approximation, however, marginally compromises the accuracy of the simulation outcomes. Notably, the system's state convergence is not uniformly smooth, particularly at the onset of controller activation. This phenomenon underscores a key trait of sliding-mode control: the propensity for high-frequency oscillations arising from the signum function's switching. Mitigating such oscillations and achieving their elimination is a pivotal area of ongoing research in sliding-mode control.

## 5. Conclusions

In this paper, we have proposed a sliding-mode control for chaotic systems with time delays. The presented control schemes have been implemented based on the TS fuzzy method considering nonlinear terms and time delays in chaotic systems, which guarantees the stabilization within an arbitrary preassigned time. Besides, the sufficient and simple conditions have been derived. Finally, the control approach and theoretical results have been verified by numerical simulations. While our methodology demonstrates promise, it is prudent to recognize areas for potential enhancement. The sliding-mode control, as implemented, may encounter challenges such as chattering, which could impact its practical deployment. Additionally, the presence of distributed delays and stochastic disturbances, prevalent in real-world scenarios, suggests a need for further investigation. Future research will thus concentrate on the stability of chaotic systems with distributed delays and stochastic disturbances.

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