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Nonreciprocal mechanical entanglement in a spinning optomechanical system

Shan-Shan Chen(陈珊珊), Jing-Jing Zhang(张京京), Jia-Neng Li(李嘉能), Na-Na Zhang(张娜娜), Yong-Rui Guo(郭永瑞), and Huan Yang(杨桓)†

School of Optoelectronic Engineering, Chongqing University of Posts and Telecommunications, Chongqing 400065, *China*

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Quantum entanglement between distant massive mechanical oscillators is an important resource in sensitive measurements and quantum information processing. We achieve the nonreciprocal mechanical entanglement in a compound optomechanical device consisting of two mechanical oscillators and a spinning whispering-gallery mode (WGM) optical microresonator. It is found that obvious nonreciprocal mechanical entanglement emerges in this system in the presence of the Sagnac effect which is induced by the rotation of the WGM resonator, and the nonreciprocal region can be controlled by tuning the angular velocity of the rotation. The nonreciprocity originates from the breaking of the time-reversal symmetry of this multimode system due to the presence of the Sagnac effect. The optomechanical coupling and the mechanical interaction provide cooling channels for the first and second mechanical oscillators, respectively. Two mechanical oscillators can be cooled simultaneously. The simultaneous cooling and the mechanical coupling of two mechanical oscillators ensure the generation of mechanical entanglement. Furthermore, an optimal mechanical entanglement can be achieved when the moderate optical frequency detuning and the driving power are chosen. The thermal noise of the mechanical environment has a negative effect on mechanical entanglement. Our scheme provides promising opportunities for research of quantum information processing based on phonons and sensitive measurements.

Keywords: optomechanical system, quantum entanglement, nonreciprocity, Sagnac effect

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1. Introduction

Quantum entanglement is a valuable resource in quantum technologies, such as quantum computation, quantum communication, quantum sensing, and quantum information processing.[1–5] Nowadays, quantum entanglement has been prepared in many physical systems, including photons,[6] trapped ions,[7] atoms,[8] and superconducting qubits.[9] Mechanical oscillator, a solid-state system, serves as an effective carrier for examining quantum effects[10] and implementing quantum information processing.[11] Its tunable coupling to various physical systems, including the cavity optomechanical system (COMS)[12,13] and the nitrogen-vacancy center in diamond,[14] enhances its utility. The entanglement between mechanical oscillators is fundamental to the development of quantum networks based on phonons, a topic that has garnered significant attention. Nowadays, various methods for preparing mechanical entanglement have been proposed,[15–24] including use of radiation pressure of a photon hopping coupling between two cavities,[17] combination of processes of a cavity cooling and two-mode parametric interaction,[19] and utilization of reservoir engineering.[21–24] It can be seen that the optomechanical interaction between cavity field and mechanical oscillator plays an important role in the generation of mechanical entanglement.

The COMS,[25] where cavity field couples to mechanical oscillator via radiation-pressure force, provides a invaluable tool for manipulating the quantum properties of the mechanical oscillator, such as mechanical squeezing,[26–31] quantum entanglement[15–20,32,33] and ground-state cooling for mechanical oscillators.[34–37] The COMS is recognized as an ideal platform for preparing entangled states, with a particular emphasis on quantum entanglement between mechanical oscillators. Recently, research of the mechanical entanglement based on COM has made great achievements,[15–19,23,38–40] which paves the way towards preparation of entangled phonon networks and noise-resistant quantum resources.

Over the past several decades, nonreciprocal elements have proven indispensable for the establishment of noiseless and lossless quantum networks. In nonreciprocal devices, transmission signals are only feasible in one direction, while they are blocked in the opposite direction. Optical nonreciprocal devices have been initially studied extensively due to their potential applications in quantum information processing based on magnetooptical Faraday rotation effect,[41–44] the modulation of the refractive index,[45–47] parity-time-symmetric structures,[48–50] and so on. The concept of nonreciprocity is further extended to the other aspects, the phononic nonreciprocity can be observed in a wide class of COMS. For instance, nonreciprocal mechanical squeezing,[51] nonreciprocal phonon laser,[52] and nonreciprocal cooling of phonon modes.[36] Up to date, nonreciprocal quantum entanglement has attracted much attention due

†Corresponding author. E-mail: yanghuan@cqupt.edu.cn

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to its potential applications in quantum information processing, and many schemes of nonreciprocal entanglement[53–57] are implemented by using the spinning resonator where the counterpropagating modes undergo an opposite Fizeau drag caused by the optical Sagnac effect. By using the Fizeau frequency shift of optical mode, the optomechanical interaction between optical mode and mechanical mode can be controlled for different driving directions when the other parameters are unchanged, so nonreciprocal mechanical entanglement[53] or optomechanical entanglement[54] can be obtained. The realization of nonreciprocal mechanical entanglement motivates the growing body of experimental and theoretical works which devote to complex phonon-based quantum information processing and transmission.

In this paper, we study the nonreciprocal mechanical entanglement in a multimode COMS comprising of two mechanical oscillators and a spinning whispering-gallery mode (WGM) resonator. A spinning WGM resonator coupled to a mechanical oscillator, and this mechanical oscillator also mechanically coupled to another mechanical oscillator. An obvious nonreciprocal mechanical entanglement can be observed when the WGM resonator is spinning. We find that the nonreciprocal mechanical entanglement can be tuned by the angular velocity of the rotation and the driving power. The nonreciprocity can be destroyed by the thermal noise of the mechanical environment.

2. Optomechanical model

As shown in Fig. 1, we consider a compound system consisting of two mechanical oscillators and a spinning WGM microresonator. This WGM resonator spins clockwise at an angular velocity Ω. When a pump light with frequency $\omega_{\rm d}$ is coupled into and out of the resonator via a waveguide from the left or right hand side, the optical mode experiences a Fizeau shift, i.e., $\omega_{\rm c} \to \omega_{\rm c} + \Delta_{\rm F}$, with[58]

$$\Delta_{\rm F} = \pm \frac{nr\Omega\omega_{\rm c}}{c}\left(1 - \frac{1}{n^2} - \frac{\lambda}{n}\frac{{\rm d}n}{{\rm d}\lambda}\right), \tag{1}$$

where $\omega_{\rm c}$ is the resonance frequency of the resonator without spinning, n and r are the refractive index and the radius of the WGM resonator, respectively, c (λ) is the speed (wavelength) of light in vacuum. The dispersion term ${\rm d}n/{\rm d}\lambda$, characterizing the relativistic origin of the Sagnac effect, is relatively small in typical materials ($\sim 1\%$). When the resonator is driven from its left (right) side, $\Delta_{\rm F} > 0$ ($\Delta_{\rm F} < 0$).

When an optical driving field inputs from the left-hand side, the Hamiltonian of the system in the frame rotating at $H_0 = \omega_{\rm d} a^\dagger a$ can be expressed as ($\hbar = 1$)

$$H_{\rm s} = (\Delta_{\rm c} + \Delta_{\rm F})a^\dagger a + \sum_{j=1,2}\frac{\omega_{m_j}}{2}(q_j^2 + p_j^2)$$
$$- g_0 a^\dagger a q_1 - \eta q_1 q_2 + \varepsilon_{\rm d}(a - a^\dagger), \tag{2}$$

where a ($a^\dagger$) is the annihilation (creation) operators for the optical mode. $\Delta_{\rm c} = \omega_{\rm c} - \omega_{\rm d}$ is the frequency detuning between the cavity field and the pump field with amplitude $\varepsilon_{\rm d}$; q_j and p_j are the position and momentum operators of the mechanical oscillator j with frequency ω_{m_j}, respectively; $g_0 = \frac{\omega_{\rm c}}{r}\sqrt{\hbar/(\omega_{m_1} m_1)}$ is the single-photon coupling strength between the first mechanical resonator and the WGM resonator. There is the mechanical coupling between two mechanical oscillators via the so-called position–position interaction,[59] η denotes the mechanical coupling strength.

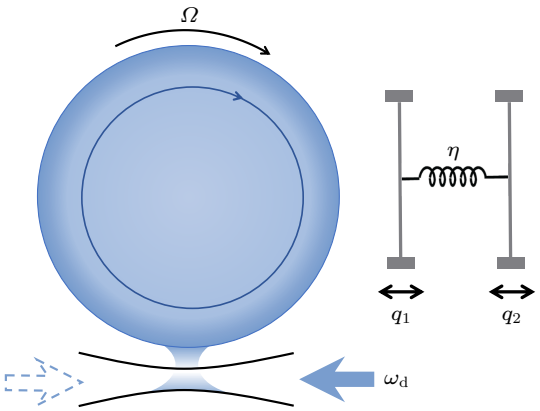


Fig. 1. Schematic of the spinning optomechanical system. A clockwise spinning microresonator with angular velocity Ω can be coupled to the first mechanical oscillator via the radiation-pressure force by driving the system from left or right hand side with the frequency $\omega_{\rm d}$. Two mechanical oscillators are coupled to each other via the "position–position" interaction.

In the strong-driving regime, the cavity and the mechanical modes will be in the large steady-state amplitudes, we can write the operators $a = \alpha + \delta a$, $q_j = q_{s_j} + \delta q_j$, and $p_j = p_{s_j} + \delta p_j$, where α and q_{s_j} (p_{s_j}) are the steady-state amplitudes of the cavity mode and the mechanical mode j, respectively. The equations of steady-state amplitudes can be derived as

$$\alpha = \frac{-{\rm i}\varepsilon_{\rm d}}{\kappa + {\rm i}\Delta_{\rm c}'}, \quad q_{s_1} = \frac{g_0\omega_{m_2}|\alpha|^2}{\omega_{m_1}\omega_{m_2} - \eta^2},$$
$$q_{s_2} = \frac{g_0\eta|\alpha|^2}{\omega_{m_1}\omega_{m_2} - \eta^2}, \quad p_{s_1} = p_{s_2} = 0, \tag{3}$$

where the effective optical frequency detuning $\Delta_{\rm c}' = \Delta_{\rm c} - g_0 q_{s_1}$; κ is the damping rate of the cavity mode. The quantum Langevin equations for the linearized system can be written as

$$\begin{aligned}
\delta\dot{a} &= [-{\rm i}(\Delta_{\rm c}' + \Delta_{\rm F}) - \kappa]\delta a + {\rm i}G\delta q_1 + \sqrt{2\kappa}\delta a_{\rm in},\\
\delta\dot{q}_j &= \omega_{m_j}\delta p_j,\\
\delta\dot{p}_1 &= -\omega_{m_1}\delta q_1 - \gamma_{m_1}\delta p_1 + \eta\delta q_2 + G^*\delta a + G\delta a^\dagger + \xi_1,\\
\delta\dot{p}_2 &= -\omega_{m_2}\delta q_2 - \gamma_{m_2}\delta p_2 + \eta\delta q_1 + \xi_2,
\end{aligned} \tag{4}$$

where $\delta a_{\rm in}$ and $\xi_{j=1,2}$ are the input zero-mean vacuum noise operator of the cavity mode and the zero-mean value Brownian noise operator of the j-th mechanical oscillator, respectively.

$G = g_0\alpha$ is the linearized optomechanical coupling strength. The correlation functions of the related noise operators are

$$\begin{aligned}\langle \delta a_{\rm in}^{\dagger}(t)\delta a_{\rm in}(t')\rangle &= \langle \delta a_{\rm in}(t)\delta a_{\rm in}(t')\rangle = 0,\\ \langle \delta a_{\rm in}(t)\delta a_{\rm in}^{\dagger}(t')\rangle &= \delta(t-t'),\\ \langle \xi_j(t)\xi_j(t')\rangle &\simeq \gamma_{m_j}(2n_{m_j}+1)\delta(t-t'),\\ &\text{for } \omega_{m_j}/\gamma_{m_j} \gg 1,\end{aligned} \tag{5}$$

where γ_{m_j} is damping rate of the mechanical mode, $n_{m_j} = [\exp(\hbar\omega_{m_j}/k_{\rm B}T)-1]^{-1}$ is the mean thermal phonon number, $k_{\rm B}$ is the Boltzmann constant, and T is the environmental temperature of the mechanical resonator bath. Introducing the amplitude and phase quadratures for the cavity mode,

$$\delta Q_a = (\delta a + \delta a^{\dagger})/\sqrt{2},\ \ \delta P_a = (\delta a - \delta a^{\dagger})/(\sqrt{2}{\rm i}), \tag{6}$$

and, accordingly, its input quantum noise quadratures

$$\delta Q^{\rm in} = (\delta a_{\rm in} + \delta a_{\rm in}^{\dagger})/\sqrt{2},\ \ \delta P^{\rm in} = (\delta a_{\rm in} - \delta a_{\rm in}^{\dagger})/(\sqrt{2}{\rm i}). \tag{7}$$

Then, Eq. (4) can be written in the matrix form

$$\dot{f}(t) = Mf(t) + n(t), \tag{8}$$

where

$$\begin{aligned} f(t) &= [\delta Q_a, \delta P_a, \delta q_1, \delta p_1, \delta q_2, \delta p_2]^{\rm T},\\ n(t) &= [\sqrt{2\kappa}\delta Q^{\rm in}, \sqrt{2\kappa}\delta P^{\rm in}, 0, \xi_1, 0, \xi_2]^{\rm T},\end{aligned} \tag{9}$$

and the 6×6 time-independent matrix M is given by

$$M = \begin{bmatrix} -\kappa & \Delta_{\rm c}' & -G^y & 0 & 0 & 0\\ -\Delta_{\rm c}' & -\kappa & G^x & 0 & 0 & 0\\ 0 & 0 & 0 & \omega_{m_1} & 0 & 0\\ G^x & G^y & -\omega_{m_1} & -\gamma_{m_1} & \eta & 0\\ 0 & 0 & 0 & 0 & 0 & \omega_{m_2}\\ 0 & 0 & \eta & 0 & -\omega_{m_2} & -\gamma_{m_2}\end{bmatrix}. \tag{10}$$

Here, G^x and G^y are the real and imaginary parts of the effective optomechanical coupling rate $G_1 \equiv \sqrt{2}g_0\alpha = G^x + {\rm i}G^y$, respectively. The stability of the system can be ensured when the real parts of all eigenvalues of M are negative. In all subsequent calculations, the stability of the system has been checked.

The zero-mean Gaussian nature of the vacuum input noise and thermal noise, and the linearized Hamiltonian ensure that the evolved state of the system will remain the Gaussian nature at all times, the dynamics of the system can be characterized by a 6×6 covariance matrix (CM) V with elements

$$V_{kl} = \langle f_k(t)f_l(t) + f_l(t)f_k(t)\rangle/2,\ \ k,l = 1,2,3,4,5,6, \tag{11}$$

the motion equations in Eq. (4) can be expressed as

$$\dot{V}(t) = MV(t) + V(t)M^{\rm T} + D, \tag{12}$$

where $D = {\rm Diag}[\kappa,\kappa,0,\gamma_{m_1}(2n_{m_1}+1),0,\gamma_{m_2}(2n_{m_2}+1)]$ is the diagonal noise correlation matrix.

When the stability condition is fulfilled, the equation of motion for the CM V in Eq. (12) can be reduced as the Lyapunov equation

$$MV + VM^{\rm T} = -D. \tag{13}$$

In order to calculate the quantum entanglement between two mechanical modes, we adopt the logarithmic negativity, which is defined as[60]

$$E_{\rm N} = \max[0, -\ln(2\nu^-)], \tag{14}$$

where $\nu^- = 2^{-1/2}\{\sum(V_{\mu\nu}) - [\sum(V_{\mu\nu})^2 - 4\det V_{\mu\nu}]^{1/2}\}^{1/2}$, and $\sum(V_{\mu\nu}) = \det A + \det B - 2\det C$ is the 4×4 CM associated with the two mechanical modes, with μ and ν describing the two mechanical modes and expressed as

$$V_{\mu\nu} = \begin{pmatrix} A & C\\ C^{\rm T} & B\end{pmatrix}. \tag{15}$$

3. Nonreciprocal mechanical entanglement

In the following discussion, we study the realization of the nonreciprocal mechanical entanglement. As shown in Fig. 2(a), we plot the logarithmic negativity $E_{\rm N}$ as a function of the optical detuning $\Delta_{\rm c}/\omega_{m_1}$. In our numerical simulations, experimentally feasible values[61–63] are employed: $n = 1.44$, $r = 0.3$ mm, $\lambda = 1064$ nm, and $P = 20$ mW. We choose the rotation speed $\Omega = 5$ kHz. Nowadays, Ω can be increased even up to GHz values based on a levitated OM system,[64,65] so $\Omega = 5$ kHz is experimentally feasible. The other parameters are chosen as $\omega_{m_1} = \omega_{m_2} = 2\pi\times 10$ MHz, $m_1 = m_2 =$ 250 ng, $\gamma_{m_1} = \gamma_{m_2} = 10^{-5}\omega_{m_1}$, $n_{m_1} = n_{m_2} = n_m = 100$, and $\eta = 0.3\omega_{m_1}$. Moreover, the frequency shift induced by radiation pressure is very small compared to the detuning of the cavity field, i.e., $g_0 q_{s_1} \ll \Delta_{\rm c}$, thus we set $\Delta_{\rm c}' = \Delta_{\rm c}$ for simplicity in the following calculation. It can been seen from Fig. 2(a) that nonreciprocal mechanical entanglement can be realized. There are two nonreciprocal mechanical entanglement segments (shadowed blue bottom region) on the $\Delta_{\rm c}$ axis. When the WGM resonator is spinning and the optical detuning $\Delta_{\rm c}$ is within the left (right) shadowed blue bottom region, the mechanical entanglement emerges (vanishes), i.e., $E_{\rm N} > 0$ ($E_{\rm N} = 0$) for $\Delta_{\rm F} > 0$ ($\Delta_{\rm F} < 0$), whereas it vanishes (emerges), i.e., $E_{\rm N} = 0$ ($E_{\rm N} > 0$) for $\Delta_{\rm F} < 0$ ($\Delta_{\rm F} > 0$). Note that the nonreciprocal mechanical entanglement vanishes for a stationary WGM resonator, the degree of the mechanical entanglement remains constant regardless of driving directions. The physical mechanism for the generation of the nonreciprocal mechanical entanglement can be explained as follows. The optomechanical interaction between the cavity mode and the first mechanical oscillator, and the mechanical coupling between two mechanical oscillators links three modes together. The

cavity provides a direct cooling channel for the first mechanical oscillator, while the mechanical interaction provides the channel to extract the thermal excitations from the second mechanical resonator. Thus, two mechanical oscillators can be cooled simultaneously, which can be verified from Fig. 2(b), we plot the the final mean phonon numbers $n_1^{\rm f}$ and $n_2^{\rm f}$ of two mechanical oscillators as a function of the optical detuning $\Delta_{\rm c}/\omega_{m_1}$, the optimal cooling and the optimal mechanical entanglement emerge at the same $\Delta_{\rm c}/\omega_{m_1}$, and the cooling efficiency of two mechanical oscillators is direction-dependent, the effective cooling of the mechanical oscillators is basic for the generation of mechanical entanglement. The nonreciprocity originates from the breaking of the time-reversal symmetry of this compound system, which is induced by the Sagnac effect.

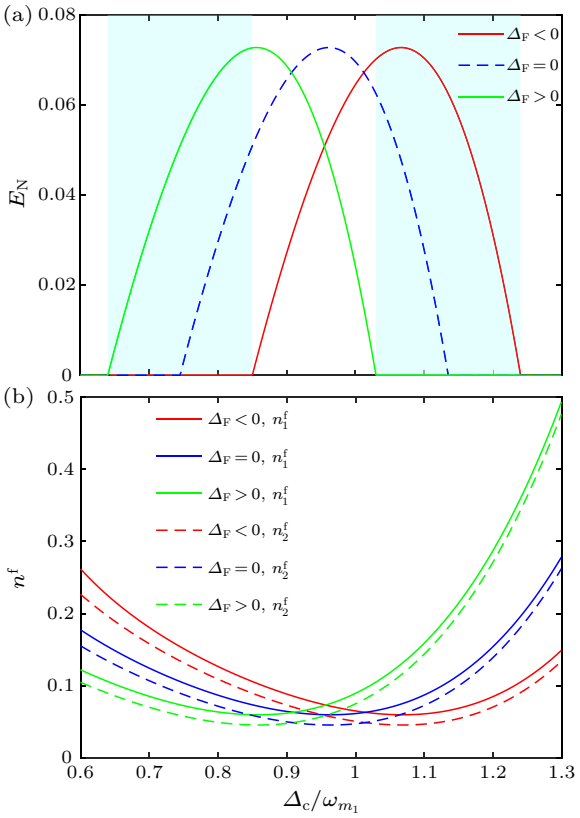


Fig. 2. (a) Logarithmic negativity $E_{\rm N}$ and (b) the final mean phonon numbers $n_1^{\rm f}$ and $n_2^{\rm f}$ versus optical detuning $\Delta_{\rm c}/\omega_{m_1}$ for different driving directions. The shadowed blue bottom region corresponds to the region where the nonreciprocal mechanical entanglement exists.

According to Fig. 2, rotation of the WGM resonator is necessary for the emergence of the non-reciprocal mechanical entanglement, so angular velocity of the rotation is an important physical quantity in the investigation of the non-reciprocal mechanical entanglement. To this end, logarithmic negativity $E_{\rm N}$ is shown in Fig. 3 as a function of the optical detuning $\Delta_{\rm c}$ at various angular velocities upon driving the WGM resonator from its left-hand side $\Delta_{\rm F}>0$ or right-hand side $\Delta_{\rm F}<0$. From Fig. 3, it is shown that logarithmic negativity $E_{\rm N}$ shifts linearly with Ω, a red shift [see Fig. 3(a)] and blue shift [see Fig. 3(b)] are generated for $\Delta_{\rm F}<0$ and $\Delta_{\rm F}>0$, respectively. Obviously, non-reciprocal mechanical entanglement can be modulated by controlling Ω. The frequency region where the non-reciprocal mechanical entanglement exists also increases with increasing Ω.

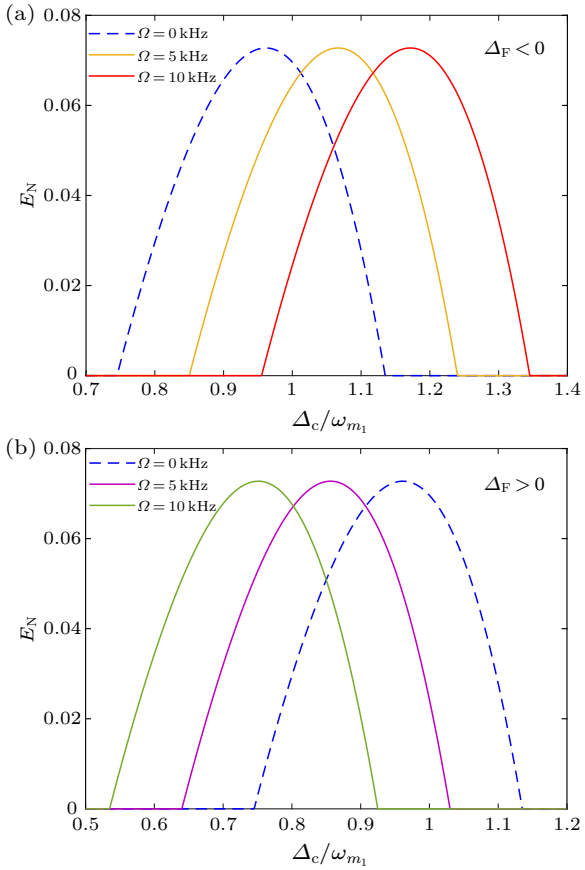


Fig. 3. Logarithmic negativity $E_{\rm N}$ versus optical detuning $\Delta_{\rm c}/\omega_{m_1}$ at various angular velocities Ω when the WGM resonator is driven from its (a) right-hand side $\Delta_{\rm F}<0$ or (b) left-hand side $\Delta_{\rm F}>0$. All other parameters and the meanings of three color lines are the same as those in Fig. 2.

Entanglement is closely related to the environment temperature. To investigate the influence of the environment temperature on nonreciprocal entanglement, in Fig. 4, we plot $E_{\rm N}$ as a function of the thermal phonon numbers n_m. The maximal value of mechanical entanglement decreases rapidly with n_m, which means that the thermal noise has a negative impact on the entanglement characteristics of mechanical oscillators. Nonreciprocal mechanical entanglement still can be achieved even when $n_m=200$. A low temperature is beneficial to prepare nonreciprocal mechanical entanglement.

The optomechnaical coupling determined by the driving power provides a crucial bridge for the realization of the nonreciprocal entanglement. In order to investigate the dependence of the mechanical entanglement on driving power, we show the contour plots of $E_{\rm N}$ as a function of P and $\Delta_{\rm c}$ in Fig. 5 when the WGM resonator is stationary. The optimal mechanical entanglement emerges at the optical frequency $\Delta_{\rm c}\approx\omega_{m_1}$ and the driving power $P=62.88$ mW. The reason is that the

optimal cavity-assisted cooling efficiency of first mechanical oscillator is maximum in the red-detuned regime where the production of a cavity photon is accompanied by the disappearance of a phonon, and the counter-rotating-wave term in the linearized optomechanical coupling causes the small deviation of ω_{m_1}. At first, with the increase of P, the average number of photons in the cavity increases according to Eq. (3), the optomechanical interaction is enhanced, the cooling effect of two mechanical oscillator is improved, so the mechanical entanglement gradually becomes stronger. However, when P is strong enough, with the increase in P, the rotating wave approximation in the linearized interaction between the cavity mode and the first mechanical mode is not well satisfied, the cooling effect of the first mechanical oscillator decreases, so an optimal mechanical entanglement is obtained when a proper P and Δ_c are chosen. By spinning the resonator, an opposite Fizeau frequency shift of the counterpropagating modes results in the difference in the effective frequency of the optical mode for different driving directions, but does not change the relationship between the driving power and the mechanical entanglement. Therefore, the driving power can be used to modulate the maximum mechanical entanglement in the nonreciprocal region on Δ_c-axis. Hence, the modulation of the driving power can be an important tool used to control the nonreciprocal mechanical entanglement.

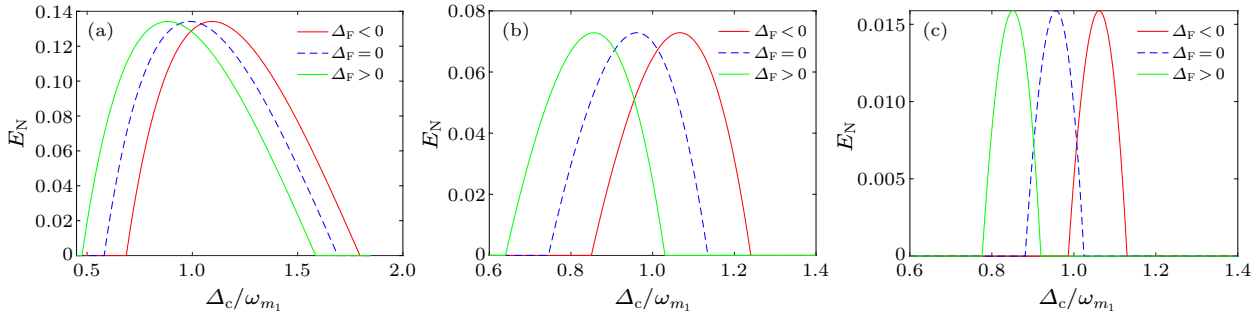


Fig. 4. Logarithmic negativity E_N as a function of optical detuning Δ_c/ω_{m_1} for different thermal phonon numbers n_m with (a) $n_m = 0$, (b) $n_m = 100$, and (c) $n_m = 200$. All other parameters and the meanings of three color lines are the same as those in Fig. 2.

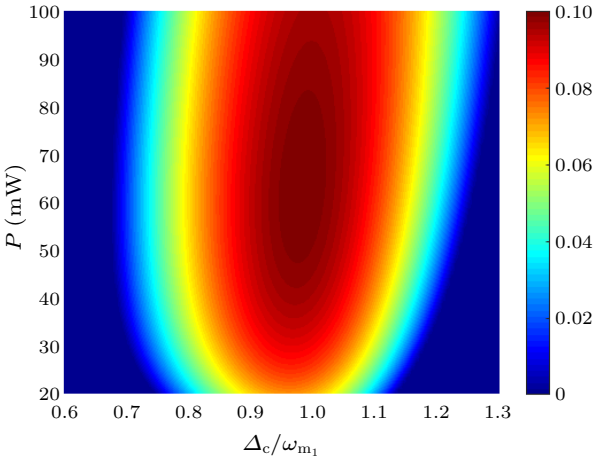


Fig. 5. Logarithmic negativity E_N versus optical detuning Δ_c/ω_{m_1} and driving power P. Here we choose $\Delta_F = 0$, the other parameters remain the same as those in Fig. 2.

In fact, many works have shown the nonreciprocity of quantum effects in spinning optomechanical systems,[53,54,66,67] such as the nonreciprocal enhancement of remote mechanical entanglement,[53] the nonreciprocal optomechanical entanglement,[54] and the nonreciprocal mechanical squeezing.[67] In these schemes and in our scheme, the rotation-induced Sagnac frequency shift affects the resonance condition of the optomechanical interaction, then the quantum properties of the cavity field and mechanical oscillator can be controlled in different directions. For example, the work for the realization of the nonreciprocal remote entanglement between two spatially separated mechanical oscillators[53] was proposed based on two spinning optomechanical resonators which are indirectly coupled through a telecommunication fiber, and two mechanical oscillators do not need to interact directly. The rotation-induced Sagnac frequency shift of the cavity mode causes the difference in the optomechanical coupling for different driving directions, so the mechanical entanglement which is obtained through the radiation-pressure-induced interactions is nonreciprocal. Unlike this scheme,[53] our scheme only consider a three-mode system and two mechanical oscillators coupled to each other, the origin of the nonreciprocity of mechanical entanglement is similar to Ref. [53], where the breaking of the time reversal symmetry of compound system due to Sagnac effect causes the Fizeau frequency shift of the cavity mode, while the generation of mechanical entanglement comes from the direct interaction between two mechanical oscillators and the effective cooling of two mechanical oscillators by using the optomechanical coupling and mechanical interaction. Similarly, the preparation of the nonreciprocal optomechanical entanglement in Ref. [54] has also utilized the rotation-induced Sagnac–Fizeau frequency shift to make that the cavity mode is only resonant with the Stokes or anti-Stokes sidebands in one driving direction, which is the same as our scheme, then the directional optomechanical entanglemnent is generated. Moreover, the relationship between the optical backscattering losses and the nonreciprocity was investigated comprehensively in Ref. [54]. In addition, the physical mechanism for

the generation of the nonreciprocal mechanical squeezing in Ref. [67] is partially similar to that of our scheme, both of which use the opposite Sagnac–Fizeau frequency shift for two counterpropagating optical modes in the spinning resonator to realize the resonance between the optical mode in a specific direction and the anti-Stokes sideband scattered by the mechanical mode, so the different cooling efficiencies of the mechanical oscillator in different directions can be achieved. Our ultimate goal is to achieve nonreciprocal mechanical entanglement, and it is designed to generate nonreciprocal mechanical squeezing.

4. Conclusion

In summary, we present a scheme for preparing the nonreciprocal mechanical entanglement based on a spinning optomechanical system comprising of two mechanical oscillators and a spinning optical resonator. The optical Sagnac effect induced by the rotation of the optical resonator leads to the nonreciprocity. The optomechanical coupling provides the direct cooling channel for the first mechanical oscillator, and the mechanical interaction provides the channel to extract thermal excitation from the second mechanical oscillator, the mechanical entanglement is induced by the simultaneous cooling and the mechanical coupling of two mechanical oscillators. The frequency region where nonreciprocal mechanical entanglement emerges can be broadened with the increase of the angular velocity of the rotation of the optical WGM resonator. The nonreciprocal mechanical entanglement can be modulated by the driving power and the optical frequency detuning. A low environment temperature is beneficial to the preparation of the nonreciprocal mechanical entanglement.

Acknowledgments

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