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Bright soliton dynamics for resonant nonlinear Schrödinger equation with generalized cubic–quintic nonlinearity

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For systems modeled by the resonant nonlinear Schrödinger equation (RNLSE) with generalized cubic–quintic nonlinearity, we derive the bright soliton solution of the equation in (1+1) dimensions, using the modified F-expansion method along with the novel ansatz of F-base function. Furthermore, we extend the analytical study of soliton dynamics to higher (2+1) and (3+1) dimensions by using the self-similar method, and demonstrate the soliton behavior via graphical illustration. Moreover, we investigate the effect of the resonance term on bright soliton solution in (1+1) dimensions. Additionally, we consider the nonlinear equation models with perturbation terms and derive the bright soliton solutions for the one-dimensional (1D) to three-dimensional (3D) cases. The theoretical results derived can be used to guide the experimental studies and observations of bright solitons in systems described by RNLSE model.

Keywords: soliton, resonant nonlinear Schrödinger equation, F-expansion method

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1. Introduction

Nonlinear phenomena are hot research focus in contemporary physical science research within the past several decades, particularly in the investigation of solitons. There are many types of solitons, including bright solitons,[1] dark solitons,[2] kink solitons,[3] and so on, each exhibiting intriguing behaviors as highly interesting forms of nonlinear phenomena. There has recently been growing interest in models, such as the nonlinear Schrödinger equation (NLSE), along with their associated soliton behaviors.[4–9] However, in certain cases, the influence of resonance on the system becomes significant. Therefore, the introduction of the resonant nonlinear Schrödinger equation (RNLSE) to consider resonance effects is crucial for more accurate description of the nonlinear behavior of solitons.[10–13]

The RNLSE is an intermediate scheme between focusing and defocusing (intermodal dispersion). The NLSE can represent the propagation of a pulse through an optical fiber.[14] Different kinds of nonlinear physical systems can be represented by the general NLSE model. The NLSE has a very important role in applications, for example, superconductivity, optics and photonics, Bose–Einstein condensates, nonlinear systems, surface and water waves, light propagation in nonlinear optical fibers, mathematical biology, and quantum physics. Obtaining exact solutions to nonlinear models is one of the toughest challenges in applied mathematics, engineering and physics. Exact solutions provide a consistently comprehensive description of the behavior of the phenomenon under study.[15]

It is of critical importance to study the soliton solutions of the NLSE with distributed coefficients. The constant system NLSE with generalized cubic–quintic nonlinearity and its soliton solutions have been studied extensively in different fields, for example, Bose–Einstein condensates,[16] and particularly nonlinear optical systems.[17] In recent years, soliton propagation in nonlinear media has garnered significant attention due to its broad applications, particularly in optical communications and ultrafast all-optical switching devices.[18] And the NLSE with generalized cubic–quintic nonlinearity is a general model describing the propagation of pulses in the double-doped optical fibers.[19]

Recently, some prior works concentrated on the study of the exact soliton solutions of RNLSE in different forms with different methods. Ghazala Akram *et al*. used the auxiliary equation method to get soliton solutions of the RNLSE.[20] Hasan Bulut *et al*. constructed various optical soliton solutions of the RNLSE under Kerr law nonlinearity using an extended sine-Gordon equation expansion method.[21] The soliton solution of the RNLSE was obtained using the Sardar subequation method by Hadi Rezazadeh *et al*.[22] However, significant research gaps remain, such as the lack of exact soliton solutions for the RNLSE with generalized cubic–quintic nonlinearity.

In this work, we delve into the research of the RNLSE. Combining the F-expansion method[23–25] and the definition of F-base functions, we obtain the exact expression of bright soliton solutions for the (1+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity. Simultaneously, we extend the equation solutions to higher dimensions using the self-similar method,[26] and graphically illustrate the behavior of the bright soliton solutions. Moreover, we research on the effect of the resonance term on bright soliton solutions in (1+1) dimensions. Furthermore, we consider the RNLSE with

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perturbation terms and obtain its bright soliton solutions using the same method. Our analytical results can serve as guidance for experimental observation of the soliton behavior for systems modeled by the resonant nonlinear Schrödinger equation.

This work is organized as follows. The next section introduces the basic equation model and provides an overview of the F-expansion method applied to the RNLSE. Section 3 provides a detailed derivation process of bright soliton solutions for the RNLSE with generalized cubic–quintic terms using the F-expansion method in various dimensions. And the effect of the resonance term on the bright soliton solution in (1+1) dimensions is also discussed. Additionally, graphical representations are utilized to illustrate the soliton behavior. Section 4 considers the RNLSE with generalized cubic–quintic terms in presence of perturbation terms. By employing the same method as in the previous section, bright soliton solutions in different dimensions under this condition are obtained. The last section provides conclusive remarks.

2. Resonant nonlinear Schrödinger equation and F-expansion method

The resonant nonlinear Schrödinger equation[27] is given by

$$\mathrm{i}\psi_t + \alpha\nabla^2\psi + \beta F(|\psi|^2)\psi + \gamma\left(\frac{\nabla^2|\psi|}{|\psi|}\right)\psi = 0, \tag{1}$$

where the first term $\mathrm{i}\psi$ is the linear evolution term, α is the coefficient of group velocity dispersion, β is the coefficient of nonlinearity, and γ represents the coefficient of the resonant term. It comes from the scattering effects between interacting particles in the nonlinear medium.[28] Resonance usually refers to the frequency matching that occurs between the periodic forces inside or outside the system and the intrinsic frequency of the system. In Eq. (1), the coefficient of γ is often known as the quantum potential or Bohm potential. This term is often found in the study of chiral solitons in the quantum Hall effect. The RNLSE is applied in various contexts in quantum mechanics, notably in studying the Madelung fluid.[29] All the real parametric constants (α, β, γ) need to be determined through experimentation (α, β, $\gamma \neq 0$).

The F is a real-valued algebraic function, and it is essential for the complex function $F(|\psi|^2)\psi : C \to C$ to be smooth. Considering the complex plane C as a two-dimensional (2D) linear space R^2, the function $F(|\psi|^2)\psi$ is k times continuously differentiable, so that

$$F(|\psi|^2)\psi \in \bigcup_{m,n=1}^{\infty} C^k((-n,n)\times(-m,m);R^2). \tag{2}$$

In presence of perturbation terms, the resonant nonlinear Schrödinger equation[30] extends to

$$\begin{aligned}&\mathrm{i}\psi_t + \alpha\nabla^2\psi + \beta F(|\psi|^2)\psi + \gamma\left(\frac{\nabla^2|\psi|}{|\psi|}\right)\psi \\ &= \mathrm{i}(\delta\psi_x + \lambda(|\psi|^{2n}\psi)_x + \mu(|\psi|^{2n})_x\psi),\end{aligned} \tag{3}$$

where δ is the inter-modal dispersion, λ represents the coefficient of self-steepening for short pulses, and μ is the higher-order dispersion coefficient. The parameter n accounts for full nonlinearity. The perturbation form of the RNLSE provides a model that describes wave propagation in a medium, taking into account both nonlinear effects and potential perturbation influences.

In order to solve the aforementioned equations, we use the F-expansion method. F-expansion method is used to solve general partial differential equation[31] of the following form:

$$N(u, u_t, u_x, u_{xx}, \ldots) = 0, \tag{4}$$

where $u(x,t)$ represents the unknown function to be solved, and N denotes a polynomial involving $u(x,t)$ and its partial derivatives of different orders. Supposing that $u_(\xi)$ can be expressed as a finite power series of $G_(\xi)$ in the following form:

$$u(x,t) = \sum_{i=0}^{l} h_i G^i(\xi), \quad h_i \neq 0, \tag{5}$$

where l is the highest-order power index, and h_i is a constant. We can obtain the power index l of the highest order term of Eq. (5) by balancing the highest order nonlinear term in the ordinary differential equation with the highest order derivative term. $G_(\xi)$ is defined as

$$\begin{aligned}\left(\frac{\mathrm{d}G(\xi)}{\mathrm{d}\xi}\right)^2 &= a_n G^n(\xi) + a_{n-1}G^{n-1}(\xi) + \cdots \\ &\quad + a_2 G^2(\xi) + a_1 G(\xi) + a_0,\end{aligned} \tag{6}$$

where $\xi = px + qt$, and the constants p, q, a_n, a_{n-1}, $\ldots$ a_2, a_1, a_0 are parameters that need to be determined during the ensuing problem solving steps. By substituting Eq. (5) into Eq. (4), we can then utilize Eq. (6) to obtain the polynomial expression for $G(\xi)$. When we set all order coefficients of $G(\xi)$ in the polynomial to zero, we can derive these equations and the power index l of the highest order term of Eq. (5). We can determine these coefficients by solving the system of coefficient equations. To get the exact solution $u(x,t)$ of Eq. (4), we take the solution of $G(\xi)$ and the coefficients into Eq. (5).

3. Bright soliton solution for RNLSE with generalized cubic–quintic nonlinearity based on the F-expansion method

Firstly, we consider the (1+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity,[32] which is expressed in the following form:

$$\mathrm{i}\psi_t + \alpha\psi_{xx} + \beta_1|\psi|^{2n}\psi + \beta_2|\psi|^{4n}\psi + \gamma\left(\frac{|\psi|_{xx}}{|\psi|}\right)\psi = 0, \tag{7}$$

where the function $F(|\psi|^2)$ in Eq. (1) is defined as $F(|\psi|^2) = \frac{1}{\beta}(\beta_1|\psi|^{2n} + \beta_2|\psi|^{4n})$, β_1 and β_2 are experimentally determined constants. To obtain an exact solution for Eq. (7), we apply the traveling wave ansatz to the wave function $\psi(x,t)$, expressed as

$$\psi(x,t) = \varphi^{1/n}(\xi)\,\mathrm{e}^{\mathrm{i}(kx+wt)}, \tag{8}$$

where the variable ξ is defined as

$$\xi = px + qt, \tag{9}$$

while k, w, p, q are constant parameters to be determined. Additionally, the modulus of $\psi(x,t)$ can be expressed as

$$|\psi| = \varphi^{1/n}(\xi). \tag{10}$$

By employing Eqs. (8)–(10), all the partial derivatives involved in Eq. (7) are expressed as

$$\psi_t = \left(\frac{q}{n}\varphi^{1/n-1}\varphi' + \mathrm{i}w\varphi^{1/n}\right)\mathrm{e}^{\mathrm{i}(kx+wt)}, \tag{11a}$$

$$\psi_x = \left(\frac{p}{n}\varphi^{1/n-1}\varphi' + \mathrm{i}k\varphi^{1/n}\right)\mathrm{e}^{\mathrm{i}(kx+wt)}, \tag{11b}$$

$$\psi_{xx} = \left(\frac{p^2}{n}\left(\frac{1}{n}-1\right)\varphi^{1/n-2}(\varphi')^2 + \frac{p^2}{n}\varphi^{1/n-1}\varphi'' + 2\mathrm{i}k\frac{p}{n}\varphi^{1/n-1}\varphi' - k^2\varphi^{1/n}\right)\mathrm{e}^{\mathrm{i}(kx+wt)}, \tag{11c}$$

$$|\psi|_x = \frac{p}{n}\varphi^{1/n-1}\varphi', \tag{11d}$$

$$|\psi|_{xx} = \frac{p^2}{n}\left(\frac{1}{n}-1\right)\varphi^{1/n-2}(\varphi')^2 + \frac{p^2}{n}\varphi^{1/n-1}\varphi''. \tag{11e}$$

Substituting the above Eq. (11) into Eq. (7) and simplify the equation by multiplying $\varphi^{-1/n}$, we derive the ordinary differential equation concerning $\varphi(\xi)$ as

$$\begin{aligned}&\mathrm{i}\frac{q}{n}\varphi^{-1}\varphi' + 2\mathrm{i}\alpha k\frac{p}{n}\varphi^{-1}\varphi' + (\alpha+\gamma)\\&\times\frac{p^2}{n}\left(\frac{1}{n}-1\right)\varphi^{-2}(\varphi')^2 + (\alpha+\gamma)\frac{p^2}{n}\varphi^{-1}\varphi''\\&+\beta_1\varphi^2 + \beta_2\varphi^4 - (\alpha k^2 + w) = 0.\end{aligned} \tag{12}$$

We can get the following system of equations by separating the imaginary part and real part of Eq. (12)

$$\mathrm{i}\frac{q}{n}\varphi^{-1}\varphi' + 2\mathrm{i}\alpha k\frac{p}{n}\varphi^{-1}\varphi' = 0, \tag{13a}$$

$$\begin{aligned}&(\alpha+\gamma)\frac{p^2}{n}\left(\frac{1}{n}-1\right)\varphi^{-2}(\varphi')^2 + (\alpha+\gamma)\frac{p^2}{n}\varphi^{-1}\varphi''\\&+\beta_1\varphi^2 + \beta_2\varphi^4 - (\alpha k^2 + w) = 0.\end{aligned} \tag{13b}$$

From Eq. (13a), we obtain the following expression:

$$q = -2\alpha kp. \tag{14}$$

To obtain the solution of $\varphi(x,t)$ in Eqs. (13) by utilizing the F-expansion method, we employ the following ansatz:

$$\varphi(\xi) = G(\xi). \tag{15}$$

We define $G(\xi)$ as follows:[33]

$$\left(\frac{\mathrm{d}G(\xi)}{\mathrm{d}\xi}\right)^2 = a_6G^6 + a_4G^4 + a_2G^2. \tag{16}$$

The transformed equation for φ and ξ is as follows:

$$\frac{\mathrm{d}\varphi}{\sqrt{a_6\varphi^6 + a_4\varphi^4 + a_2\varphi^2}} = \mathrm{d}\xi. \tag{17}$$

If we define $\phi = \varphi^2$ and then multiply both the numerator and denominator of the left side of Eq. (17) by $\phi = \varphi$, we can rewrite Eq. (17) in the following form:

$$\begin{aligned}&\frac{\varphi\,\mathrm{d}\varphi}{\varphi\sqrt{a_6\varphi^6 + a_4\varphi^4 + a_2\varphi^2}}\\&= \frac{1}{2}\frac{\mathrm{d}\phi}{\sqrt{a_6(\phi)^4 + a_4(\phi)^3 + a_2(\phi)^2}} = \mathrm{d}\xi.\end{aligned} \tag{18}$$

So we can obtain the analytical format of ϕ as follows:[34]

$$\phi = \frac{1}{\sqrt{a_4^2/4a_2^2 - a_6/a_2}\cosh(2a_2^{1/2}\xi) - a_4/2a_2}. \tag{19}$$

Based on Eq. (19), we can define $c = -a_4/2a_2$ and $d = a_4^2/4a_2^2 - a_6/a_2$. Then the derivatives of various orders of φ can be expressed as[35]

$$(\varphi')^2 = a_6\varphi^6 + a_4\varphi^4 + a_2\varphi^2, \tag{20a}$$

$$\varphi'' = \frac{\mathrm{d}\varphi'}{\mathrm{d}\varphi}\frac{\mathrm{d}\varphi}{\mathrm{d}\xi} = \frac{1}{2}\frac{\mathrm{d}(\varphi')^2}{\mathrm{d}\varphi} = 3a_6\varphi^5 + 2a_4\varphi^3 + a_2\varphi. \tag{20b}$$

Substituting Eq. (20) into the real part of Eq. (13), we get

$$\varphi^4 : \beta_2 + a_6(\alpha+\gamma)\frac{p^2}{n}\left(\frac{1}{n}+2\right) = 0, \tag{21a}$$

$$\varphi^2 : \beta_1 + a_4(\alpha+\gamma)\frac{p^2}{n}\left(\frac{1}{n}+1\right) = 0, \tag{21b}$$

$$\varphi^0 : a_2(\alpha+\gamma)\left(\frac{p}{n}\right)^2 - (\alpha k^2 + w) = 0. \tag{21c}$$

By solving the systems of Eq. (21), we get the coefficients expressed as

$$a_6 = -\frac{n^2\beta_2}{(\alpha+\gamma)p^2(1+2n)}, \tag{22a}$$

$$a_4 = -\frac{n^2\beta_1}{(\alpha+\gamma)p^2(1+n)}, \tag{22b}$$

$$a_2 = \frac{n^2(\alpha k^2 + w)}{(\alpha+\gamma)p^2}, \tag{22c}$$

$$c = \frac{\beta_1}{2(1+n)(\alpha k^2 + w)}, \tag{22d}$$

$$d = \frac{\beta_1^2}{4(1+n)^2(\alpha k^2 + w)^2} + \frac{\beta_2}{(1+2n)(\alpha k^2 + w)}. \tag{22e}$$

For $d > 0$ and based on Eq. (19), we can get the analytical format of $\varphi(\xi)$ expressed as

$$\varphi(\xi) = G(\xi) = \sqrt{\frac{1}{d^{1/2}\cosh(2a_2^{1/2}\xi) + c}}. \tag{23}$$

Plugging the coefficients of Eqs. (22) and (14) into Eq. (23), we get the exact solution of $\varphi(\xi)$ as

$$\varphi(x,t)=\sqrt{\frac{1}{\sqrt{\dfrac{\beta_1^2}{4(1+n)^2(\alpha k^2+w)^2}+\dfrac{\beta_2}{(1+2n)(\alpha k^2+w)}}\cosh\left[2\sqrt{\dfrac{n^2(\alpha k^2+w)}{(\alpha+\gamma)}}(x-2\alpha kt)\right]+\dfrac{\beta_1}{2(1+n)(\alpha k^2+w)}}}. \tag{24}$$

Substituting Eq. (24) into Eq. (8), we get the bright soliton solution of Eq. (7) as

$$\psi(x,t)=\mathrm{e}^{\mathrm{i}(kx+wt)}\left(\frac{1}{\sqrt{\dfrac{\beta_1^2}{4(1+n)^2(\alpha k^2+w)^2}+\dfrac{\beta_2}{(1+2n)(\alpha k^2+w)}}\cosh\left[2\sqrt{\dfrac{n^2(\alpha k^2+w)}{(\alpha+\gamma)}}(x-2\alpha kt)\right]+\dfrac{\beta_1}{2(1+n)(\alpha k^2+w)}}\right)^{1/2n}. \tag{25}$$

Figure 1 shows the bright soliton solution (25) in the cases where n ranges from 1 to 5 with appropriate coefficients under the static condition of $t=0$. Figures 2 and 3 depict the bright soliton solution (25) under time diffusion with appropriate coefficients.

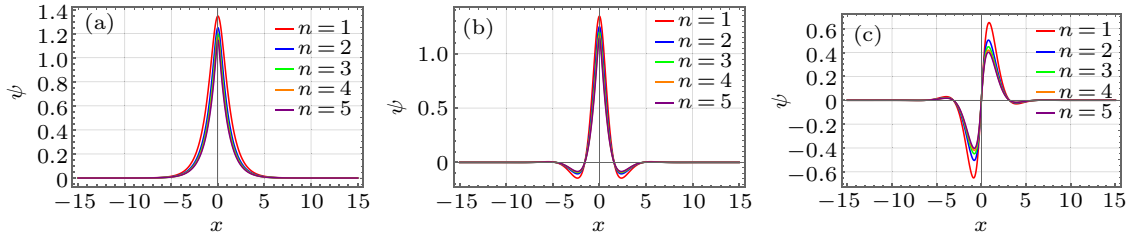


Fig. 1. Waveform plot of bright soliton ψ for various power indexes n in the (1+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k=1$, $w=1$ when $t=0$ for (a) modulas, (b) real part, and (c) imaginary part.

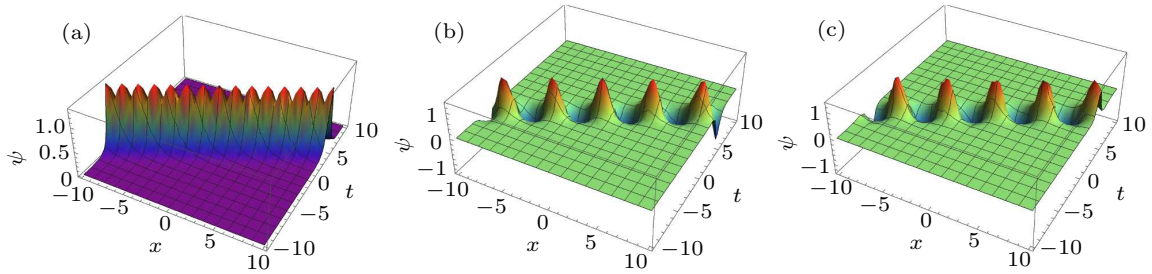


Fig. 2. Waveform plot of bright soliton ψ for (1+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k=1$, $w=1$, $n=1$ under time diffusion for (a) modulus, (b) real part, and (c) imaginary part.

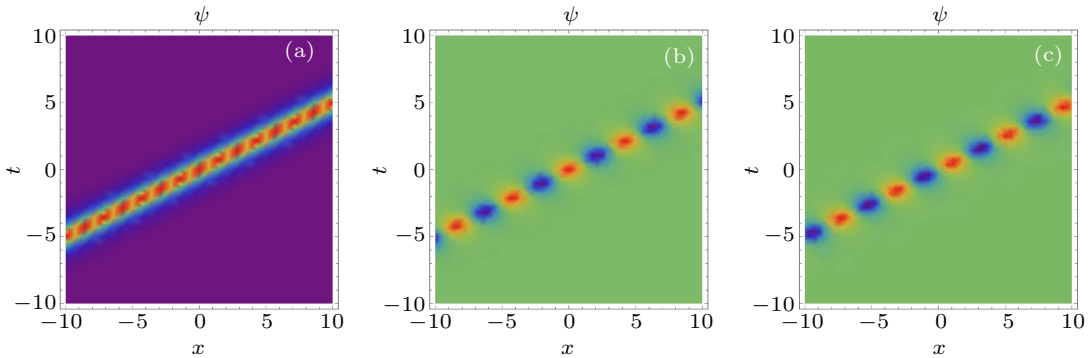


Fig. 3. Density plot of bright solution ψ for (1+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k=1$, $w=1$, $n=1$ under time diffusion for (a) modulus, (b) real part, and (c) imaginary part.

By analyzing the above results we find that the pulse changes its width periodically, so the solution is also call as breathing soliton solution. Such phenomena have been studied in related work.[36–38]

Subsequently, we extend our exploration to the higher-dimensional case, delving into the (2+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity. Employing the self-similar method, we commence by implementing a transformation analogous to Eq. (8) as

$$\psi(x,y,t)=\varphi^{1/n}(\xi)\,\mathrm{e}^{\mathrm{i}(k_1x+k_2y+wt)}, \tag{26}$$

where the ξ assumes the (2+1)-dimensional form as follows:

$$\xi=p_1x+p_2y+qt, \tag{27}$$

while k_1, k_2, w, p_1, p_2, q are constant parameters to be determined in the ensuing problem solving steps. Following the procedure similar to the previous steps, by employing Eqs. (26), (27), and (10), we express all the partial derivatives involved in Eq. (7) as

$$\psi_t = \left(\frac{q}{n}\varphi^{1/n-1} + \mathrm{i}w\varphi^{1/n}\right)\mathrm{e}^{\mathrm{i}(k_1x+k_2y+wt)}, \tag{28a}$$

$$\psi_x + \psi_y = \left(\frac{p_1+p_2}{n}\right)\varphi^{1/n-1}\varphi' + \mathrm{i}(k_1+k_2)\varphi^{1/n})\mathrm{e}^{\mathrm{i}(k_1x+k_2y+wt)}, \tag{28b}$$

$$\nabla^2\psi = \left(\frac{p_1^2+p_2^2}{n}\left[\left(\frac{1}{n}-1\right)\varphi^{1/n-2}(\varphi')^2 + \varphi^{1/n-1}\varphi''\right] + 2\mathrm{i}(k_1+k_2)\frac{p_1+p_2}{n}\varphi^{1/n-1}\varphi' - (k_1^2+k_2^2)\varphi^{1/n}\right)\mathrm{e}^{\mathrm{i}(k_1x+k_2y+wt)}, \tag{28c}$$

$$|\psi|_x + |\psi|_y = \frac{p_1+p_2}{n}\varphi^{1/n-1}\varphi', \tag{28d}$$

$$\nabla^2|\psi| = \frac{p_1^2+p_2^2}{n}\left(\frac{1}{n}-1\right)\varphi^{1/n-2}(\varphi')^2 + \frac{p_1^2+p_2^2}{n}\varphi^{1/n-1}\varphi''. \tag{28e}$$

By substituting the above Eq. (28) into Eq. (7) and simplifying the equation through multiplication by $\varphi^{-1/n}$, we obtain the ordinary differential equation involving $\varphi(\xi)$ as

$$\frac{\mathrm{i}q}{n}\varphi^{-1}\varphi' + 2\mathrm{i}\alpha(k_1+k_2)\frac{p_1+p_2}{n}\varphi^{-1}\varphi' + (\alpha+\gamma)(p_1^2+p_2^2) \times\left(\frac{1}{n}-1\right)\frac{1}{n}\varphi^{-2}(\varphi')^2 + (\alpha+\gamma)(p_1^2+p_2^2)\frac{1}{n}\varphi^{-1}\varphi'' + \beta_1\varphi^2 + \beta_2\varphi^4 - (\alpha(k_1^2+k_2^2)+w) = 0. \tag{29}$$

We can get the following system of equations by separating the imaginary part and real part of Eq. (29):

$$\mathrm{i}\frac{q}{n}\varphi^{-1}\varphi' + 2\mathrm{i}\alpha(k_1+k_2)\frac{p_1+p_2}{n}\varphi^{-1}\varphi' = 0, \tag{30a}$$

$$(\alpha+\gamma)(p_1^2+p_2^2)\left(\frac{1}{n}-1\right)\frac{1}{n}\varphi^{-2}(\varphi')^2 + (\alpha+\gamma)(p_1^2+p_2^2)\frac{1}{n}\varphi^{-1}\varphi'' + \beta_1\varphi^2 + \beta_2\varphi^4 - (\alpha(k_1^2+k_2^2)+w) = 0. \tag{30b}$$

From Eq. (30a), we get

$$q = -2\alpha(k_1+k_2)(p_1+p_2). \tag{31}$$

Similar to the previous procedure, employing the F-expansion method as Eq. (16) and substituting Eq. (20) into Eq. (30b), we get

$$\varphi^4 : \beta_2 + a_6(\alpha+\gamma)\frac{p_1^2+p_2^2}{n}\left(\frac{1}{n}+2\right) = 0, \tag{32a}$$

$$\varphi^2 : \beta_1 + a_4(\alpha+\gamma)\frac{p_1^2+p_2^2}{n}\left(\frac{1}{n}+1\right) = 0, \tag{32b}$$

$$\varphi^0 : a_2(\alpha+\gamma)\frac{p_1^2+p_2^2}{n^2} - (\alpha(k_1^2+k_2^2)+w) = 0. \tag{32c}$$

Solving the systems of Eq. (32), we obtain the coefficients expressed as

$$a_6 = -\frac{n^2\beta_2}{(\alpha+\gamma)(p_1^2+p_2^2)(1+2n)}, \tag{33a}$$

$$a_4 = -\frac{n^2\beta_1}{(\alpha+\gamma)(p_1^2+p_2^2)(1+n)}, \tag{33b}$$

$$a_2 = \frac{n^2(\alpha(k_1^2+k_2^2)+w)}{(\alpha+\gamma)(p_1^2+p_2^2)}, \tag{33c}$$

$$c = \frac{\beta_1}{2(1+n)(\alpha(k_1^2+k_2^2)+w)}, \tag{33d}$$

$$d = \frac{\beta_1^2}{4(1+n)^2(\alpha(k_1^2+k_2^2)+w)^2} + \frac{\beta_2}{(1+2n)(\alpha(k_1^2+k_2^2)+w)}, \tag{33e}$$

$$p_1^2+p_2^2 = \frac{(\alpha(k_1^2+k_2^2)+w)n^2}{(\alpha+\gamma)a_2}. \tag{33f}$$

Substituting the coefficients of Eqs. (33) and (31) into Eq. (23), we get the exact solution of $\varphi(\xi)$ as

$$\varphi(x,y,t) = \sqrt{\frac{1}{A\cosh\left[2\sqrt{\frac{n^2(\alpha(k_1^2+k_2^2)+w)}{(\alpha+\gamma)\left(1+p_2^2/p_1^2\right)}}\left(x+\frac{p_2}{p_1}y-2\alpha(k_1+k_2)\left(1+\frac{p_2}{p_1}\right)t\right)\right] + \frac{\beta_1}{2(1+n)(\alpha(k_1^2+k_2^2)+w)}}}, \tag{34}$$

where

$$A = \sqrt{\frac{\beta_1^2}{4(1+n)^2(\alpha(k_1^2+k_2^2)+w)^2} + \frac{\beta_2}{(1+2n)(\alpha(k_1^2+k_2^2)+w)}}.$$

We derive the bright soliton solution of Eq. (7) by substituting Eq. (34) into Eq. (26), which is expressed as

$$\psi(x,y,t) = \mathrm{e}^{\mathrm{i}(k_1x+k_2y+wt)}$$

$$\times\left(\frac{1}{A\cosh\left[2\sqrt{\frac{n^2(\alpha(k_1^2+k_2^2)+w)}{(\alpha+\gamma)\left(1+p_2^2/p_1^2\right)}}\left(x+\frac{p_2}{p_1}y-2\alpha(k_1+k_2)\left(1+\frac{p_2}{p_1}\right)t\right)\right]+\frac{\beta_1}{2(1+n)(\alpha(k_1^2+k_2^2)+w)}}\right)^{1/2n}, \tag{35}$$

where

$$A=\sqrt{\frac{\beta_1^2}{4(1+n)^2(\alpha(k_1^2+k_2^2)+w)^2}+\frac{\beta_2}{(1+2n)(\alpha(k_1^2+k_2^2)+w)}}.$$

Figures 4 and 5 show the plots of bright soliton solution (35) when $t=0$ with appropriate coefficients.

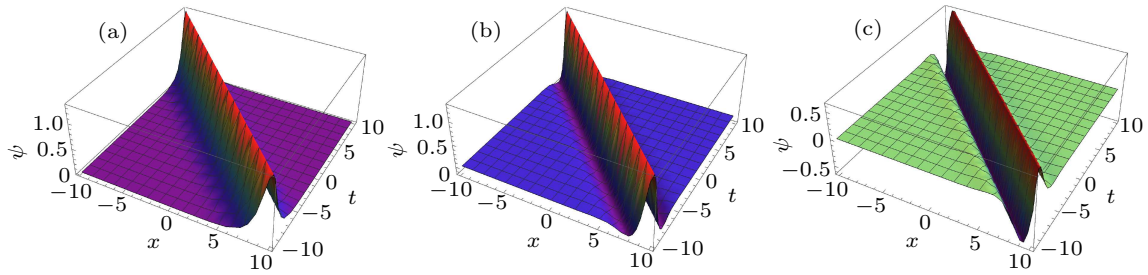


Fig. 4. Waveform plot of bright soliton ψ for (2+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k_1=1$, $k_2=1$, $w=1$, $p_1=1$, $p_2=1$, $n=1$ when $t=0$ for (a) modulus, (b) real part, and (c) imaginary part.

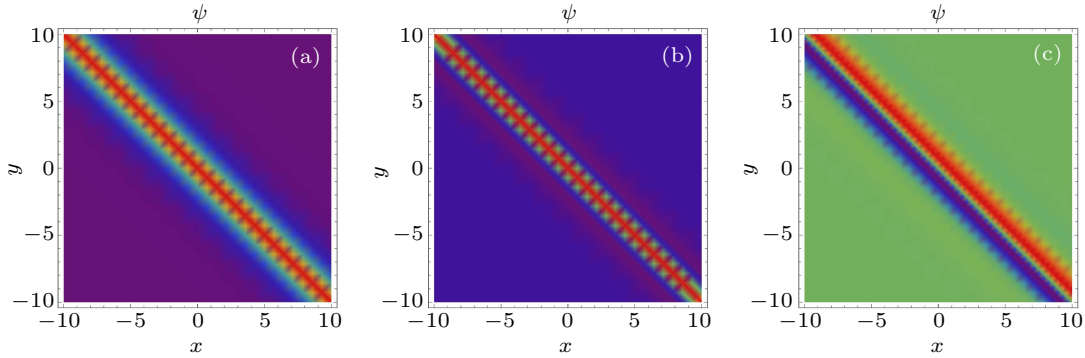


Fig. 5. Density plot of bright solution ψ for (2+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k_1=1$, $k_2=1$, $w=1$, $p_1=1$, $p_2=1$, $n=1$ for (a) modulus, (b) real part, and (c) imaginary part.

Similarly, for the (3+1)-dimensional equation, utilizing the self-similar method, we assume

$$\psi(x,y,z,t)=\varphi^{1/n}(\xi)\,\mathrm{e}^{\mathrm{i}(k_1x+k_2y+k_3z+wt)}, \tag{36}$$

where the variable $\xi=p_1x+p_2y+p_3z+qt$. Then similar to the previous steps, by applying the F-expanshion method and deriving the expressions of the coefficients, substituting them into Eqs. (23) and (36), the solution for (3+1)-dimensional equation can be expressed as

$$\psi(x,y,z,t)=\mathrm{e}^{\mathrm{i}(k_1x+k_2y+k_3z+wt)}\left(\frac{1}{B\cosh\left[C\left(x+\frac{p_2}{p_1}y+\frac{p_3}{p_1}z-2\alpha(k_1+k_2+k_3)\left(1+\frac{p_2}{p_1}+\frac{p_3}{p_1}\right)t\right)\right]+D}\right)^{1/2n}, \tag{37}$$

where

$$B=\sqrt{\frac{\beta_1^2}{4(1+n)^2(\alpha(k_1^2+k_2^2+k_3^2)+w)^2}+\frac{\beta_2}{(1+2n)(\alpha(k_1^2+k_2^2+k_3^2)+w)}},$$

$$C=2\sqrt{\frac{n^2(\alpha(k_1^2+k_2^2+k_3^2)+w)}{(\alpha+\gamma)\left(1+p_2^2/p_1^2+p_3^2/p_1^2\right)}},\quad D=\frac{\beta_1}{2(1+n)(\alpha(k_1^2+k_2^2+k_3^2)+w)}.$$

Figure 6 shows the plot of bright soliton solution (37) when $t=0$ with appropriate coefficients.

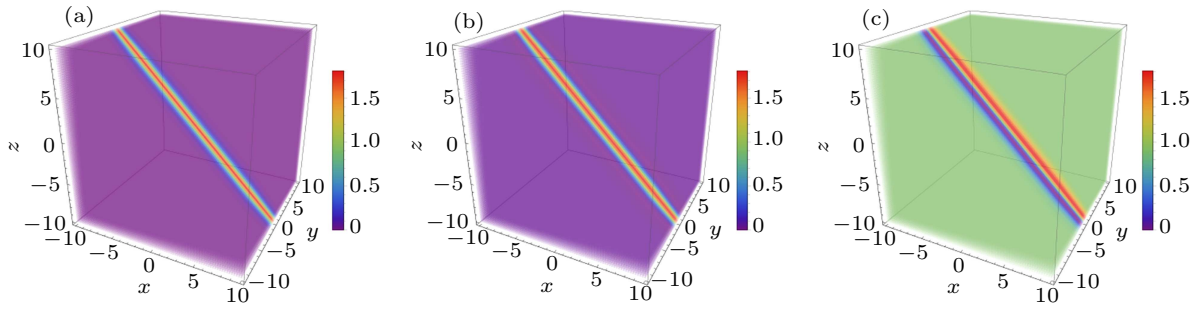


Fig. 6. Density plot of bright soliton ψ for (3+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k_1=1$, $k_2=1$, $k_3=1$, $w=1$, $p_1=1$, $p_2=1$, $p_3=1$, $n=1$ when $t=0$ for (a) modulus, (b) real part, and (c) imaginary part.

Next, we research on the effect of the resonance term on bright soliton in (1+1) dimensions. For Eq. (7), we get the following form by removing the resonance term which is expressed as

$$\mathrm{i}\psi_t+\alpha\psi_{xx}+\beta_1|\psi|^{2n}\psi+\beta_2|\psi|^{4n}\psi=0. \tag{38}$$

We do not present much details of calculation for this scenario, and directly give the bright soliton solution obtained through the F-expansion method as follows:

$$\psi(x,t)=\mathrm{e}^{\mathrm{i}(kx+wt)}\left(\frac{1}{\sqrt{\dfrac{\beta_1^2}{4(1+n)^2(\alpha k^2+w)^2}+\dfrac{\beta_2}{(1+2n)(\alpha k^2+w)}}\cosh\left[2\sqrt{\dfrac{n^2(\alpha k^2+w)}{\alpha}}(x-2\alpha kt)\right]+\dfrac{\beta_1}{2(1+n)(\alpha k^2+w)}}\right)^{1/2n}. \tag{39}$$

Figure 7 shows the plot of bright soliton solutions (25) and (39) obtained for the cases of $n=2$ under the static condition of $t=0$ with appropriate coefficients.

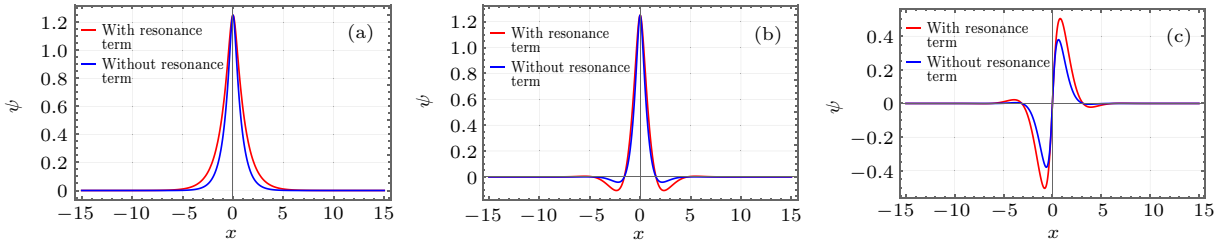


Fig. 7. Waveform plot of bright soliton ψ for power index $n=2$ in the (1+1)-dimensional RNLSE and NLSE with generalized cubic–quintic nonlinearity with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k=1$, $w=1$ when $t=0$ for (a) modulus, (b) real part, and (c) imaginary part.

We can see the increase in the pulse width of the bright soliton with the addition of the resonance term.

The flow velocity of the 3D system is

$$\boldsymbol{v}=\frac{\hbar}{2m\mathrm{i}}(\psi^*\nabla\psi-\psi\nabla\psi^*)=\frac{\hbar}{m}[k_1\boldsymbol{e}_x+k_2\boldsymbol{e}_y+k_3\boldsymbol{e}_z], \tag{40}$$

which is constant, from which we can see that $\boldsymbol{v}$ and soliton shape orientation vector $(1,p_2/p_1,p_3/p_1)$ are not in the same direction generally, so the solution (37) is truly 3D solution.

4. Bright soliton solution for RNLSE with generalized cubic–quintic nonlinearity in presence of perturbation terms

Additionally, we focus on the RNlSE with generalized cubic–quintic nonlinearity in the presence of perturbation terms,[39] which takes the following form:

$$\mathrm{i}\psi_t+\alpha\nabla^2\psi+\beta_1|\psi|^{2n}\psi+\beta_2|\psi|^{4n}\psi+\gamma\left(\frac{\nabla^2|\psi|}{|\psi|}\right)\psi$$
$$=\mathrm{i}(\delta\psi_x+\lambda(|\psi|^{2m}\psi)_x+\mu(|\psi|^{2m})_x\psi). \tag{41}$$

Following the previous steps, for the (1+1)-dimensional equation, we apply the traveling wave ansatz as Eqs. (8) and (9). And then substitute the ansatz into Eq. (41) gives

$$\begin{aligned}&\mathrm{i}\frac{q}{n}\varphi^{-1}\varphi'+2\mathrm{i}\alpha k\frac{p}{n}\varphi^{-1}\varphi'+(\alpha+\gamma)\frac{p^2}{n}\left(\frac{1}{n}-1\right)\varphi^{-2}(\varphi')^2\\&+(\alpha+\gamma)\frac{p^2}{n}\varphi^{-1}\varphi''+\beta_1\varphi^2+\beta_2\varphi^4-(\alpha k^2+w)\\&=\mathrm{i}\frac{\delta p}{n}\varphi^{-1}\varphi'-k\delta+\mathrm{i}\lambda p\frac{2m}{n}\varphi^{2m/n-1}\varphi'\\&-k\lambda\varphi^{2m/n}+\mathrm{i}\mu p\frac{2m}{n}\varphi^{2m/n-1}\varphi'.\end{aligned} \tag{42}$$

We can get the following system of equations by separating the imaginary and real parts:

$$\begin{aligned}&\mathrm{i}\frac{q}{n}\varphi^{-1}\varphi'+2\mathrm{i}\alpha k\frac{p}{n}\varphi^{-1}\varphi'-\mathrm{i}\frac{\delta p}{n}\varphi^{-1}\varphi'\\&-\mathrm{i}\lambda p\frac{1+4n}{n}\varphi^3\varphi'-\mathrm{i}4\mu p\varphi^3\varphi'=0,\end{aligned} \tag{43a}$$

$$(\alpha+\gamma)\frac{p^2}{n}\left(\frac{1}{n}-1\right)\varphi^{-2}(\varphi')^2+(\alpha+\gamma)\frac{p^2}{n}\varphi^{-1}\varphi''$$
$$+\beta_1\varphi^2+(\beta_2+k\lambda)\varphi^4-(\alpha k^2+w-k\delta)=0. \quad (43b)$$

By achieving a balance between the highest derivative term φ^4 and the nonlinear term $\varphi^{2m/n}$, the parameter m can be determined. The required equilibrium is met by the balancing formula $2m/n=4$, leading to $m=2n$. Simultaneously, applying the F-expansion method as Eq. (16) to substituting Eq. (20) into Eq. (43), and extracting the coefficients of each term, we obtain

$$\varphi^{-1}\varphi' : q+2\alpha kp\varphi^{-1}\varphi'-\delta p\varphi^{-1}\varphi'=0, \quad (44a)$$
$$\varphi^3\varphi' : \lambda\frac{1+4n}{n}\varphi^3\varphi'+4\mu\varphi^3\varphi'=0, \quad (44b)$$
$$\varphi^4 : \beta_2+k\lambda+a_6(\alpha+\gamma)\frac{p^2}{n}\left(\frac{1}{n}+2\right)=0, \quad (44c)$$
$$\varphi^2 : \beta_1+a_4(\alpha+\gamma)\frac{p^2}{n}\left(\frac{1}{n}+1\right)=0, \quad (44d)$$
$$\varphi^0 : a_2(\alpha+\gamma)\left(\frac{p}{n}\right)^2-(\alpha k^2+w-k\delta)=0. \quad (44e)$$

So the coefficients can be solved by Eq. (44) as

$$a_6=-\frac{n^2(\beta_2+k\lambda)}{(\alpha+\gamma)p^2(1+2n)}, \quad (45a)$$
$$a_4=-\frac{n^2\beta_1}{(\alpha+\gamma)p^2(1+n)}, \quad (45b)$$
$$a_2=\frac{n^2(\alpha k^2+w-k\delta)}{(\alpha+\gamma)p^2}, \quad (45c)$$
$$c=\frac{\beta_1}{2(1+n)(\alpha k^2+w-k\delta)}, \quad (45d)$$
$$d=\frac{\beta_1^2}{4(1+n)^2(\alpha k^2+w-k\delta)^2}+\frac{\beta_2+k\lambda}{(1+2n)(\alpha k^2+w-k\delta)}, \quad (45e)$$
$$p=\sqrt{\frac{n^2(\alpha k^2+w-k\delta)}{(\alpha+\gamma)a_2}}, \quad (45f)$$
$$q=-2\alpha kp+\delta p, \quad (45g)$$
$$\lambda=-\frac{4n\mu}{1+4n}. \quad (45h)$$

By substituting the coefficients of Eq. (45) into Eq. (23) and plugging into Eq. (8) , we can get the exact solution of $\psi(x,t)$ as follows:

$$\psi(x,t)=\mathrm{e}^{\mathrm{i}(kx+wt)}\left(\frac{1}{E\cosh\left[2\sqrt{\frac{n^2(\alpha k^2+w-k\delta)}{(\alpha+\gamma)}}(x-(2\alpha k-\delta)t)\right]+\frac{\beta_1}{2(1+n)(\alpha k^2+w-k\delta)}}\right)^{1/2n}, \quad (46)$$

where

$$E=\sqrt{\frac{\beta_1^2}{4(1+n)^2(\alpha k^2+w-k\delta)^2}+\frac{\beta_2+k\lambda}{(1+2n)(\alpha k^2+w-k\delta)}}.$$

Figure 8 shows the bright soliton solution (46) in the cases where n ranges from 1 to 5 with appropriate coefficients under the static condition of $t=0$. Figures 9 and 10 depict the bright soliton solution (46) under time diffusion with appropriate coefficients.

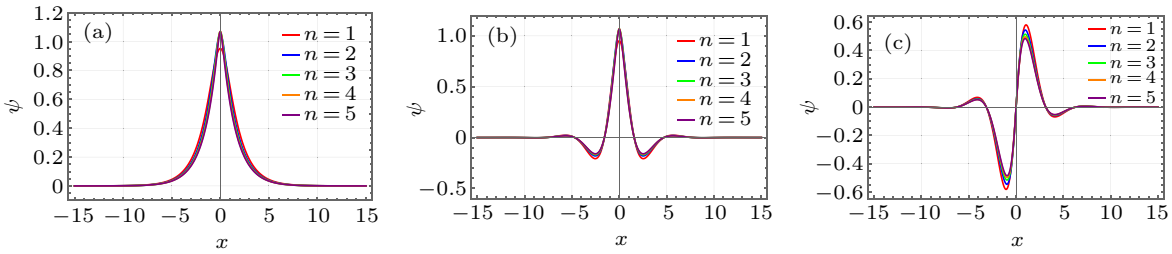


Fig. 8. Waveform plot of bright soliton ψ for various power indexes n in the (1+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity in presence of perturbation terms with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k=1$, $w=1$, $\lambda=1$, $\delta=1$ when $t=0$ for (a) modulus, (b) real part, and (c) imaginary part.

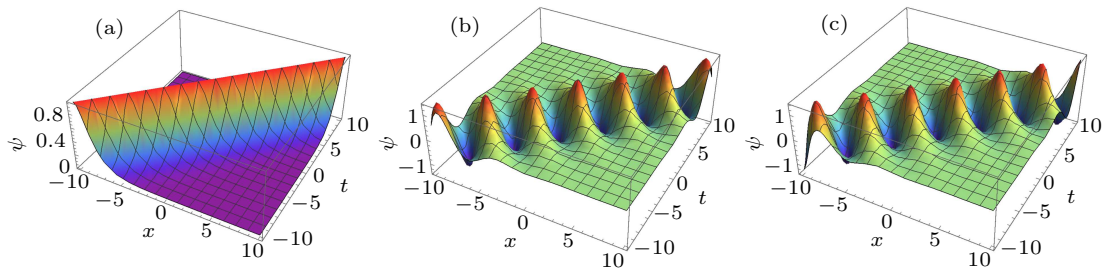


Fig. 9. Waveform plot of bright soliton ψ for (1+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity in presence of perturbation terms with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k=1$, $w=1$, $n=1$, $\lambda=1$, $\delta=1$ under time diffusion for (a) modulus, (b) real part, and (c) imaginary part.

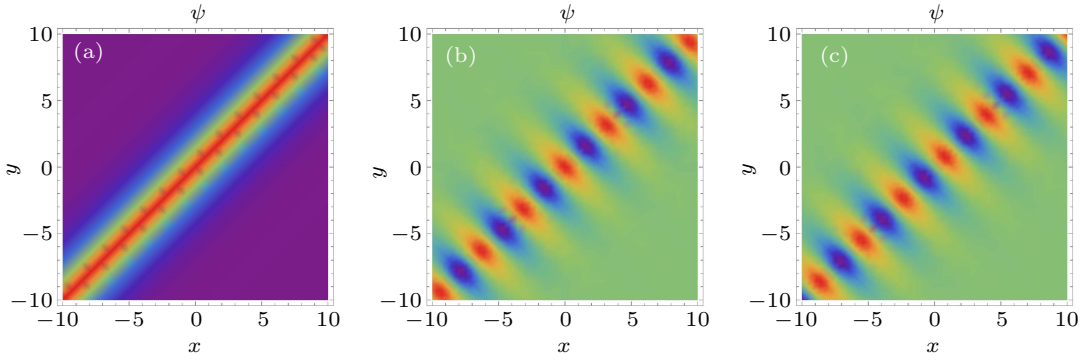


Fig. 10. Density plot of bright solution ψ for (1+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity in presence of perturbation terms with parameters $\alpha=1$, $\beta_1=1$, $\beta_2=1$, $\gamma=1$, $k=1$, $w=1$, $n=1$, $\lambda=1$, $\delta=1$ under time diffusion for (a) modulus, (b) real part, and (c) imaginary part.

Also, we extend our exploration to the higher-dimensional case by delving into the (2+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity in the presence of perturbation terms. Similarly, we apply the assumption as Eqs. (26) and (27). Then we substitute these equations and bring $m=2n$ into Eq. (41), we get the imaginary and real parts as

$$\begin{aligned}&\mathrm{i}\frac{q}{n}\varphi^{-1}\varphi'+2\mathrm{i}\alpha k\frac{p_1+p_2}{n}\varphi^{-1}\varphi'-\mathrm{i}\frac{\delta p_1}{n}\varphi^{-1}\varphi'\\&-\mathrm{i}\lambda p_1\frac{1+4n}{n}\varphi^3\varphi'-4\mathrm{i}\mu p_1\varphi^3\varphi'=0,\end{aligned}\tag{47a}$$

$$\begin{aligned}&(\alpha+\gamma)\frac{p_1^2+p_2^2}{n}\left(\frac{1}{n}-1\right)\varphi^{-2}(\varphi')^2+(\alpha+\gamma)\\&\times\frac{p_1^2+p_2^2}{n}\varphi^{-1}\varphi''+\beta_1\varphi^2+(\beta_2+k_1\lambda)\varphi^4\\&-(\alpha(k_1^2+k_2^2)+w-k_1\delta)=0.\end{aligned}\tag{47b}$$

Similar to the previous steps and extracting the coefficients of each term from Eqs. (47a), we get

$$\varphi^{-1}\varphi':q+2\alpha(k_1+k_2)(p_1+p_2)\varphi^{-1}\varphi'-\delta p_1\varphi^{-1}\varphi'=0,\tag{48a}$$

$$\varphi^3\varphi':\lambda\frac{1+4n}{n}\varphi^3\varphi'+4\mu\varphi^3\varphi'=0,\tag{48b}$$

$$\varphi^4:\beta_2+k_1\lambda+a_6(\alpha+\gamma)\frac{p_1^2+p_2^2}{n}\left(\frac{1}{n}+2\right)=0,\tag{48c}$$

$$\varphi^2:\beta_1+a_4(\alpha+\gamma)\frac{p_1^2+p_2^2}{n}\left(\frac{1}{n}+1\right)=0,\tag{48d}$$

$$\varphi^0:a_2(\alpha+\gamma)\frac{p_1^2+p_2^2}{n^2}-(\alpha(k_1^2+k_2^2)+w-k_1\delta).\tag{48e}$$

Solving the above systems of Eq. (48), we get

$$a_6=-\frac{n^2(\beta_2+k_1\lambda)}{(\alpha+\gamma)(p_1^2+p_2^2)(1+2n)},\tag{49a}$$

$$a_4=-\frac{n^2\beta_1}{(\alpha+\gamma)(p_1^2+p_2^2)(1+n)},\tag{49b}$$

$$a_2=\frac{n^2(\alpha(k_1^2+k_2^2)+w-k_1\delta)}{(\alpha+\gamma)(p_1^2+p_2^2)},\tag{49c}$$

$$p_1^2+p_2^2=\frac{n^2(\alpha(k_1^2+k_2^2)+w-k_1\delta)}{(\alpha+\gamma)a_2^2},\tag{49d}$$

$$q=-2\alpha(k_1+k_2)(p_1+p_2)+\delta p_1,\tag{49e}$$

$$\lambda=-\frac{4n\mu}{1+4n}.\tag{49f}$$

So the exact solution is expressed as

$$\begin{aligned}\psi(x,y,t)\;=\;&\mathrm{e}^{\mathrm{i}(k_1x+k_2y+wt)}\\&\times\left(\frac{1}{F\cosh\left[2\sqrt{\frac{n^2(\alpha(k_1^2+k_2^2)+w-k_1\delta)}{(\alpha+\gamma)\left(1+\frac{p_2^2}{p_1^2}\right)}}\left(x-\left(2\alpha(k_1+k_2)\left(1+\frac{p_2}{p_1}\right)-\delta\right)t\right)\right]+\frac{\beta_1}{2(1+n)(\alpha(k_1^2+k_2^2)+w-k_1\delta)}}\right)^{1/2n},\end{aligned}\tag{50}$$

where

$$F=\sqrt{\frac{\beta_1^2}{4(1+n)^2(\alpha(k_1^2+k_2^2)+w-k_1\delta)^2}+\frac{\beta_2+k_1\lambda}{(1+2n)(\alpha(k_1^2+k_2^2)+w-k_1\delta)}}.$$

Figures 11 and 12 show the plots of bright soliton solution (50) when $t=0$ with appropriate coefficients.

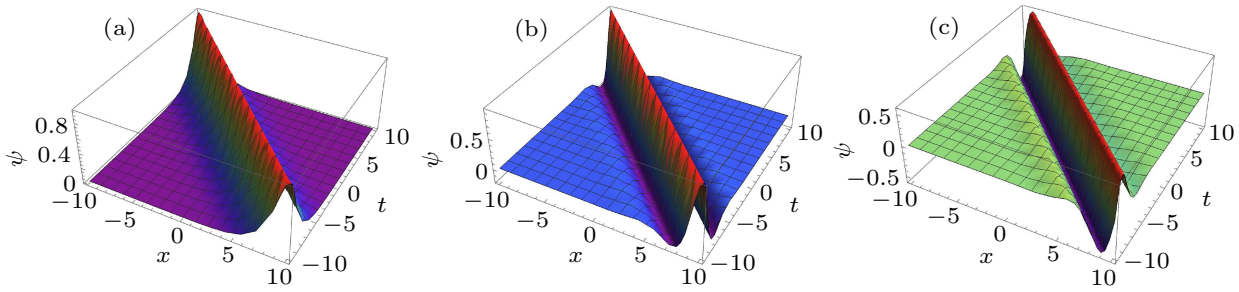


Fig. 11. Waveform plot of bright soliton ψ for (2+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity in presence of perturbation terms with parameters $\alpha = 1$, $\beta_1 = 1$, $\beta_2 = 1$, $\gamma = 1$, $k_1 = 1$, $k_2 = 1$, $w = 1$, $p_1 = 1$, $p_2 = 1$, $n = 1$, $\lambda = 1$, $\delta = 1$ when $t = 0$ for (a) modulus, (b) real part, and (c) imaginary part.

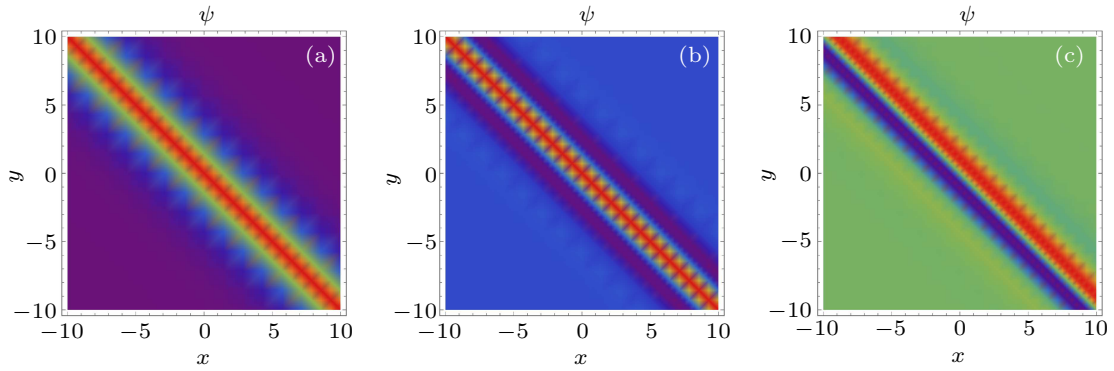


Fig. 12. Density plot of bright solution ψ for (2+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity in presence of perturbation terms with parameters $\alpha = 1$, $\beta_1 = 1$, $\beta_2 = 1$, $\gamma = 1$, $k_1 = 1$, $k_2 = 1$, $w = 1$, $p_1 = 1$, $p_2 = 1$, $n = 1$, $\lambda = 1$, $\delta = 1$ for (a) modulus, (b) real part, and (c) imaginary part.

For the (3+1)-dimensional equation, employing the transformation as Eq. (36) and following previous steps of F-expansion method, we finally obtain the bright soliton solution of (3+1)-dimensional RNSLE with cubic–quintic nonlinearity in presence of perturbation terms as

$$\psi(x,y,z,t) = \mathrm{e}^{\mathrm{i}(k_1x+k_2y+k_3z+wt)} \times \left(\frac{1}{H\cosh\left[L(x-(2\alpha(k_1+k_2+k_3)\left(1+\frac{p_2}{p_1}+\frac{p_3}{p_1}\right)-\delta)t)\right]+\frac{\beta_1}{2(1+n)(\alpha(k_1^2+k_2^2+k_3^2)+w-k_1\delta)}} \right)^{1/2n}, \tag{51}$$

where

$$H = \sqrt{\frac{\beta_1^2}{4(1+n)^2(\alpha(k_1^2+k_2^2+k_3^2)+w-k_1\delta)^2}+\frac{\beta_2+k_1\lambda}{(1+2n)(\alpha(k_1^2+k_2^2+k_3^2)+w-k_1\delta)}},\quad L = 2\sqrt{\frac{n^2(\alpha(k_1^2+k_2^2+k_3^2)+w-k_1\delta)}{(\alpha+\gamma)\left(1+p_2^2/p_1^2+p_3^2/p_1^2\right)}}.$$

Figure 13 shows the plot of bright soliton solution (51) when $t = 0$ with appropriate coefficients.

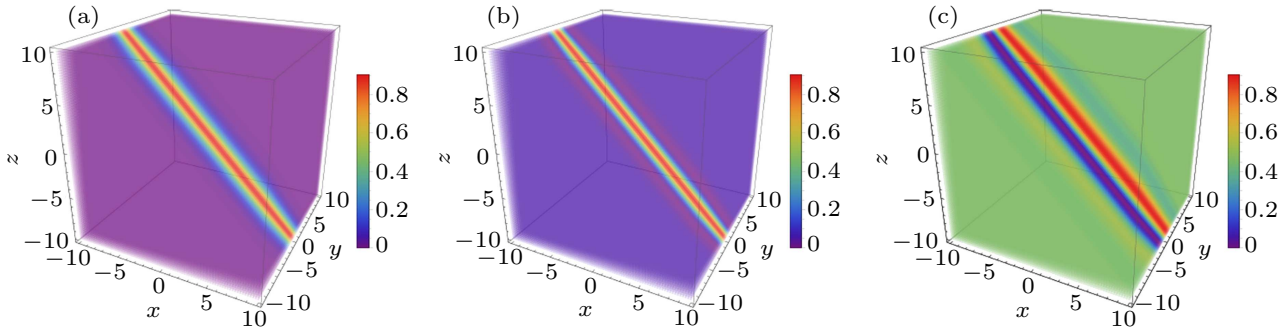


Fig. 13. Density plot of bright soliton ψ for (3+1)-dimensional RNLSE with generalized cubic–quintic nonlinearity in presence of perturbation terms with parameters $\alpha = 1$, $\beta_1 = 1$, $\beta_2 = 1$, $\gamma = 1$, $k_1 = 1$, $k_2 = 1$, $k_3 = 1$, $w = 1$, $p_1 = 1$, $p_2 = 1$, $p_3 = 1$, $n = 1$, $\lambda = 1$, $\delta = 1$ when $t = 0$ for (a) modulus, (b) real part, and (c) imaginary part.

We now perform stability analysis for the derived soliton solution ψ. Since higher dimensional soliton are derived based on self-similar ansatz which has basis from one-dimensional scenario, we only need to analyze the stability feature of the one dimensional case of Eq. (41), suppose that the perturbed solution for $\psi(x,t)$ is

$$\psi(\tilde{x},t) = [\psi(x,t) + \tilde{U}(x,t)]\,\mathrm{e}^{\mathrm{i}\mu t}. \tag{52}$$

Substituting Eq. (52) into the one-dimensional form of Eq. (41), we get the equation for $\tilde{U}(x,t)$ as (in the limit $\psi \to 0$ as $x \to \infty$)

$$\mathrm{i}\tilde{U}(x,t) + \alpha \frac{\mathrm{d}^2}{\mathrm{d}x^2}\tilde{U}(x,t) = \mathrm{i}\delta \frac{\mathrm{d}}{\mathrm{d}x}\tilde{U}(x,t). \tag{53}$$

When considering the normal modes of $\tilde{U}$, we have

$$\tilde{U}_{\mathrm{mode}}(x,t) = f(x)\,\mathrm{e}^{\lambda t} + g^*(x)\,\mathrm{e}^{\lambda^* t}. \tag{54}$$

Considering the behavior of $\tilde{U}_{\mathrm{mode}}(x,t)$ when $x \to \pm\infty$, that is $f(x) \sim c_1\,\mathrm{e}^{\mathrm{i}kx}$, $g(x) \sim c_2\,\mathrm{e}^{\mathrm{i}kx}$, and substituting Eq. (54) in Eq. (53), we get the algebraic equation for λ,[40]

$$\lambda = \pm\mathrm{i}(\alpha k^2 - k + \mu). \tag{55}$$

Since k, μ, and α are of real values, so λ is pure imaginary, which make solution ψ we derived for Eq. (41) stable.

Bright soliton solution of the RNLSE with generalized cubic–quintic nonlinearity, which are derived here, have application in the study of the propagation of nonlinear waves (NLW) in media with dispersive properties[15] (for example, lithium niobate crystals[29]), explanation of the dynamics of madelung fluids in certain nonlinear processes,[41] and description of the propagation of high-intensity pulses in optical fibers in fiber-optic communication systems with nonuniform media.[42]

5. Conclusion

In this work, we focused on exploring the soliton dynamics of RNLSE with generalized cubic–quintic nonlinearity. We obtained the bright soliton solutions of the equation in different dimensions via the improved F-expansion method and the self-similar method, and we develop our study to the equation in presence of perturbation terms. We also research on the effect of the resonance term on bright soliton solution in (1+1) dimensions. Through graphical representations, we visually depicted the behavior of the bright soliton solutions. The theoretical results obtained in our work can be used as guidance for investigation of solitons in systems described by the RNSLE model with generalized cubic–quintic nonlinearity.

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