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Optimization performance of quantum endoreversible Otto machines with dual-squeezed reservoirs

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We consider a quantum endoreversible Otto engine cycle and its inverse operation-Otto refrigeration cycle, employing two-level systems as the working substance and operating in dual-squeezed reservoirs. We demonstrate that the efficiency of heat engines at maximum work output and the coefficient of performance for refrigerators at the maximum χ criterion will degenerate to $\eta_{-}=\eta_{\rm C}/(2-\eta_{\rm C})$ and $\varepsilon_{-}=(\sqrt{9+8\varepsilon_{\rm C}}-3)/2$ when symmetric squeezing is satisfied, respectively. We also investigated the influences of squeezing degree on the performance optimization of quantum Otto heat engines at the maximum work output and refrigerators at the maximum χ criterion. These analytical results show that the efficiency of heat engines at maximum work output and the coefficient of performance for refrigerators at the maximum χ criterion can be improved, reduced or even inhibited in asymmetric squeezing. Furthermore, we also find that the efficiency of quantum Otto heat engines at maximum work output is lower than that obtained from the Otto heat engines based on a single harmonic oscillator system. However, the coefficient of performance of the corresponding refrigerator is higher.

Keywords: quantum Otto heat engine, quantum Otto refrigerator, optimization performance, dual-squeezed reservoirs

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1. Introduction

As early as in 1824, Carnot found that the efficiency of heat engine, which converts thermal resources into useful work, is bounded from above by the Carnot efficiency $\eta_{\rm C}=1-\beta_{\rm h}/\beta_{\rm c}$. For the refrigerator, which performs mechanical work to remove heat from a cold reservoir and discards it to a hot reservoir, the coefficient of performance is limited by $\varepsilon_{\rm C}=\beta_{\rm h}/(\beta_{\rm c}-\beta_{\rm h})$, where $\beta_{\rm h}$ and $\beta_{\rm c}$ are the inverse temperature of hot and cold reservoirs, respectively. In recent years, there is a growing interest in thermal machines based on quantum systems that exhibit novel thermodynamic properties, in addition to their potential applications in energy, these thermal machines based on quantum systems provide an excellent research platform for understanding the nonequilibrium thermodynamics of open systems and exploring the nature of the micro-field.[1–25]

In the field of quantum thermodynamics, the effects of reservoirs with different properties on the performance of quantum Otto machines have been widely used by researchers in theoretical and experimental study;[26–35] for instance, non-thermal reservoirs, squeezed reservoirs[36–41] and reservoirs at negative temperature.[42] Currently, squeezed reservoirs are widely used as quantum resources in the exploration of the optimization properties of quantum heat engines and quantum refrigerators.[38,43–45] Roßnagel *et al.* presented a theoretical study in which the quantum Otto heat engine was coupled to a hot-squeezed reservoir, while the cold reservoir was still a pure state. Their studies showed that the efficiency of quantum Otto heat engines at maximum power output can be dramatically improved with an increase in the squeezing parameter, even surpassing the classical Carnot limit when the squeezing parameter reaches a specific value.[36] Long *et al.* further studied the performance of a quantum Otto refrigerator, which is coupled to a cold-squeezed reservoir and the hot reservoir is in normal state, and also found that the squeezing can enhance the coefficient of performance for quantum Otto refrigerators.[37] Subsequently, Zhang[46] proposed a model of a quantum Otto heat engine and refrigerator with a single harmonic oscillator as working substance when the hot and cold reservoirs were in squeezed state, and the analysis showed that the efficiency of the heat engine at maximum work output and the coefficient of performance for refrigerator at the maximum χ may increase, decrease, or even be prohibited when the squeezing degree between the hot and cold reservoir has different relations. Therefore, two natural questions arise: How do different types of working substances affect the performance of the heat engine and refrigerator under dual squeezing? And, what is the difference between the performance of the heat engine and the refrigerator in the squeezing state with two-level systems and a single harmonic oscillator system as the working substance, respectively?

The purpose of this study is to address these problems. First, we establish a universal quantum Otto cycle for heat engines and refrigerators with two-level systems as working substance, where the quantum Otto cycle consists of two adiabatic processes and two isochoric processes, and operates in two squeezed reservoirs which will be assumed as a free re-

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source. Then, the influence of the squeezing degree on the performance optimization of quantum Otto heat engines at maximum work output and quantum Otto refrigerators at the maximum χ criterion has been fully investigated. Finally, we compared our results with those in Ref. [46]. The remainder of this paper is organized as follows. In Section 2, we briefly describe universal quantum Otto cycle for heat engines and refrigerators. In Sections 3 and 4, we investigate the efficiency of quantum Otto heat engines at maximum work output and the coefficient of performance for quantum refrigerators at the maximum χ criterion, respectively. The results are summarized in Section 5.

2. Universal quantum Otto cycle for heat engines and refrigerators

The quantum Otto cycle of heat engines and refrigerators with two-level systems as working substance is sketched in Fig. 1. The cycle consists of two quantum adiabatic processes where the hot and cold reservoirs are in squeezing degree $r_{\rm h}$ and $r_{\rm c}$, respectively, and two quantum isochoric processes where the frequency is time dependent and varies between $\omega_{\rm c}$ and $\omega_{\rm h}$ ($\omega_{\rm c} < \omega_{\rm h}$). The four consecutive thermodynamic cycle processes of quantum Otto heat engine and quantum Otto refrigerator are described as follows.[47,48]

(i) Isochoric cooling stroke. In the first stage, the two-level systems are weakly coupled to the cold-squeezed reservoir until the system reaches the steady squeezed state $\rho_1^{S_{\rm c}} = S_{\rm c}\rho_1^{\rm G}S_{\rm c}^{\dagger}$, where ρ is the operator representing the density matrix. The squeezing of cold reservoir changes the thermalized Gibbs state $\rho_1^{\rm G} = {\rm e}^{-\beta_{\rm c}H_{\rm c}}/{\rm Tr}({\rm e}^{-\beta_{\rm c}H_{\rm c}})$ according to squeezing operator $S_{\rm c} = (\mu_{\rm c}|-_x\rangle\langle+_x| + \nu_{\rm c}|+_x\rangle\langle-_x|)/\sqrt{\mu_{\rm c}^2+\nu_{\rm c}^2}$, where $\mu_{\rm c} = \cosh r_{\rm c}$ and $\nu_{\rm c} = \sinh r_{\rm c}$.[49] State $|-_x\rangle$ ($|+_x\rangle$) is the ground (excited) state of the two-level systems at this stage, and $r_{\rm c}$ is the squeezing parameter of cold reservoir. $H_{\rm c}$ is the Hamiltonian, $\beta_{\rm c} = 1/(k_{\rm B}T_{\rm c})$, where $\beta_{\rm c}$ is the inverse temperature of cold reservoir, $k_{\rm B}$ is the Boltzmann constant and $T_{\rm c}$ is the cold reservoir temperature. The Hamiltonian of the two-level systems remains unchanged during the thermalization process and has the form $H_{\rm c} = \frac{1}{2}\hbar\omega_{\rm c}\sigma_x$, where $\hbar$, $\omega_{\rm c}$ and σ_x are the reduced Planck constant, angular frequency, and x Pauli matrix, respectively.

(ii) Adiabatic expansion stroke. In the second stage, the two-level systems evolve unitarily from the state $\rho_1^{S_{\rm c}}$ (at time $t = 0$) to $\rho_2 = U\rho_1^{S_{\rm c}}U^{\dagger}$ (at time $t = \tau$), where U is the unitary operator accounting for the external drive of the Hamiltonian in two-level systems, which varies from $H_{\rm c} = \frac{1}{2}\hbar\omega_{\rm c}\sigma_x$ to $H_{\rm h} = \frac{1}{2}\hbar\omega_{\rm h}\sigma_y$, where $\omega_{\rm h}$ and σ_y are the angular frequency and the y Pauli matrix, respectively.

(iii) Isochoric heating stroke. In the third step, the two-level systems are weakly coupled to the hot-squeezed reservoir until they reach a steady squeezed state $\rho_3^{S_{\rm h}} = S_{\rm h}\rho_3^{\rm G}S_{\rm h}^{\dagger}$. According to operator $S_{\rm h} = (\mu_{\rm h}|-_y\rangle\langle+_y| + \nu_{\rm h}|+_y\rangle\langle-_y|)/\sqrt{\mu_{\rm h}^2+\nu_{\rm h}^2}$, where $\mu_{\rm h} = \cosh r_{\rm h}$ and $\nu_{\rm h} = \sinh r_{\rm h}$,[49] the squeezing for hot reservoir changes the thermalized ground state of the two-level systems $\rho_3^{\rm G} = {\rm e}^{-\beta_{\rm h}H_{\rm h}}/{\rm Tr}({\rm e}^{-\beta_{\rm h}H_{\rm h}})$. State $|-_y\rangle$ ($|+_y\rangle$) is the ground (excited) state of the two-level systems at this stage and $r_{\rm h}$ is the squeezing parameter of hot reservoir. As in the isochoric cooling stroke, the Hamiltonian $H_{\rm h} = \frac{1}{2}\hbar\omega_{\rm h}\sigma_y$ in two-level systems maintains its original value.

(iv) Adiabatic compression stroke. The final stage is the reverse of adiabatic expansion stroke (ii). Thus, the Hamiltonian of two-level systems goes back from $H_{\rm h} = \frac{1}{2}\hbar\omega_{\rm h}\sigma_y$ to $H_{\rm c} = \frac{1}{2}\hbar\omega_{\rm c}\sigma_x$ with the two-level systems state evolving unitarily from to $\rho_3^{S_{\rm h}}$ to $\rho_4 = U^{\dagger}\rho_3^{S_{\rm h}}U$.

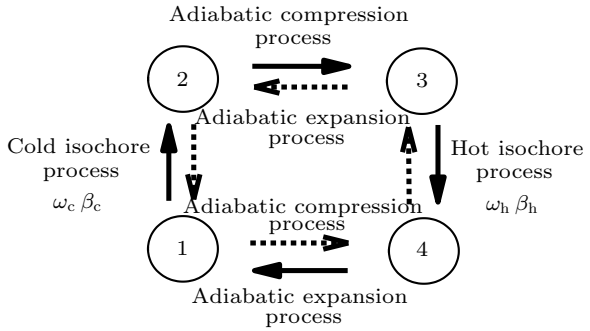


Fig. 1. Schematic of quantum Otto heat engines (dotted arrow, counterclockwise) and refrigerators (solid arrow, clockwise) cycle with hot and cold-squeezed reservoirs.

After a complete cycle, the heat engines and refrigerators return to their original states.

In the first law of thermodynamics we know that the change of the internal energy E for a given system during an entire thermodynamic process is made up of work W and heat Q. In quantum thermodynamics the form of the first law thus can be written as $\Delta E = Q + W$, where ΔE represents the average change in the system internal energy, which is given by $E = {\rm Tr}(\rho H)$, heat is $Q = \int {\rm d}t{\rm Tr}[({\rm d}\rho/{\rm d}t)H]{\rm d}t$ and work averages is $W = \int {\rm d}t{\rm Tr}[\rho({\rm d}H/{\rm d}t)]$.[50,51] According to the description of the process in the quantum Otto cycle, we can easily draw the conclusion that $W = 0$ and $\Delta E = Q$ in the isochoric heating and cooling strokes and $Q = 0$ and $\Delta E = W$ in the adiabatic expansion and compression strokes. Applying the information provided in (i)–(iv) strokes and combining the definitions of work and heat as given above, the expressions of the average heats exchanged with the cold and hot reservoirs as well as the average net work can be obtained as follows:[48]

$$Q_{\rm c} = -\frac{1}{2}\hbar\omega_{\rm c}(\zeta_{\rm c}\tanh\theta_{\rm c} - \zeta_{\rm h}\tanh\theta_{\rm h}), \tag{1}$$

$$Q_{\rm h} = \frac{1}{2}\hbar\omega_{\rm h}(\zeta_{\rm c}\tanh\theta_{\rm c} - \zeta_{\rm h}\tanh\theta_{\rm h}), \tag{2}$$

$$W = -\frac{1}{2}\hbar(\omega_{\rm h} - \omega_{\rm c})(\zeta_{\rm c}\tanh\theta_{\rm c} - \zeta_{\rm h}\tanh\theta_{\rm h}), \tag{3}$$

where $\theta_{c(h)} = \frac{1}{2}\beta_{c(h)}\hbar\omega_{c(h)}$ and the squeezing term $\zeta_{c(h)} = 1/(\mu^2_{c(h)} + \nu^2_{c(h)})^2$.

In order to facilitate our further research on the efficiency of heat engines and the performance coefficient of refrigerators, we consider the condition of high-temperature limit with $\omega_i\beta_i \ll 1$, thus $\tanh(\omega_i\beta_i) \approx \omega_i\beta_i$. Just for convenience, we assume $\hbar = 1$, and in such a case, Eqs. (1)–(3) can be approximated as follows:

$$Q_c = -\frac{1}{4}\omega_c(\zeta_c\beta_c\omega_c - \zeta_h\beta_h\omega_h), \tag{4}$$

$$Q_h = \frac{1}{4}\omega_h(\zeta_c\beta_c\omega_c - \zeta_h\beta_h\omega_h), \tag{5}$$

$$W = -\frac{1}{4}(\omega_h - \omega_c)(\zeta_c\beta_c\omega_c - \zeta_h\beta_h\omega_h). \tag{6}$$

For heat engines, the physical quantity on which we need to focus is the efficiency η. In thermodynamics, the efficiency is defined as the ratio of the total work output per cycle and the heat absorption from the hot-squeezed reservoir, $\eta = -(Q_h + Q_c)/Q_h = -W/Q_h$, in combination with Eqs. (5) and (6), thus

$$\eta = 1 - \omega_c/\omega_h, \tag{7}$$

the result is consistent with that in Refs. [26,36,52–54] for the adiabatic processes.

For refrigerators, there is an important physical quantity, i.e., the coefficient of performance $\varepsilon = Q_c/(Q_h - Q_c) = Q_c/W$, where Q_c is the average heat absorbed from the cold-squeezed reservoir by performing an average net work W_{net} on the two-level systems and Q_h is the average heat released to the hot-squeezed reservoir. Considering Eqs. (5) and (6), the expression of the coefficient of performance for refrigerators can be rewritten as

$$\varepsilon = \omega_c/(\omega_h - \omega_c). \tag{8}$$

3. The efficiency of quantum Otto heat engines at maximum work output

For a quantum Otto heat engine, the thermodynamic cycle follows a counterclockwise direction, i.e., $2 \to 1 \to 4 \to 3 \to 2$, as indicated by the dotted arrow in Fig. 1. We can then describe the cycle of quantum Otto heat engines using the following processes.

(1) Isochoric heat release ($2 \to 1$) and absorption processes ($4 \to 3$). The isochoric process is realized by coupling the working system with fixed frequency ω_c (ω_h) to a cold (hot)-squeezed reservoir at inverse temperature β_c ($> \beta_h$). During the isochoric processes, the system is alternatingly coupled to a cold-squeezed reservoir with squeezing parameter r_c and a hot-squeezed reservoir with squeezing parameter r_h. In general, the durations of two isochoric processes are assumed to be much shorter than those of two adiabatic processes. Thus, the frequency remains constant at a fixed ω_c and ω_h in these processes when the systems are coupled to two squeezed reservoirs and the energy transfer is realized merely via heat released (absorbed) from the squeezed reservoir during the heat exchange period. As a result, the systems relax to the initial squeezed thermal state $1(\beta_c, r_c)$ when the systems are coupled to a cold-squeezed reservoir and release heat to it, and to the state $3(\beta_h, r_h)$ when the systems are coupled to a hot-squeezed reservoir and absorb heat from it. After the entire cycle, the systems recover its initial state.

(2) Adiabatic compression process ($1 \to 4$) and expansion process ($3 \to 2$). During the isentropic processes (quasistatic, quantum adiabatic), the frequency of two-level system is modulated between ω_c and ω_h while the systems are isolated from cold and hot squeezed reservoirs. Thus, this transformation is unitary, the von Neumann entropy keeps a constant, and the mean work has been done by the system during the two processes. the states of system change from $1(\beta_c, r_c)$ to $4(\beta_c\omega_c/\omega_h, r_c)$ when the frequency increases from ω_c to ω_h in adiabatic compression process and from $3(\beta_h, r_h)$ to $2(\beta_h\omega_h/\omega_c, r_h)$ when the frequency decreases from ω_h to ω_c in adiabatic expansion process, where we have used the relation $\beta_h\omega_h = \beta_c\omega_c$, which were obtained in two quantum adiabatic strokes. In the adiabatic process, the state of the system remains unchanged and thus the final state is identical to the initial one.

One of the main purposes of this study is to explore the characteristics of the efficiency of heat engines at maximum work output. The maximum work output can be obtained using Eq. (6) with respect to the frequency ω_c and ω_h. In this sudden limit where the time taken for the two adiabatic strokes is negligible, by performing $\partial W/\partial\omega_c = 0$ and $\partial W/\partial\omega_h = 0$, we find that the work output is maximum when the frequency ratio satisfies the following relation:

$$\frac{\omega_c}{\omega_h} = \frac{2\alpha(1-\eta_C)}{1+\alpha(1-\eta_C)}. \tag{9}$$

It follows that by inserting these optimal values of Eq. (9) into to Eq.(7) the efficiency of heat engines at maximum work output reads

$$\eta^* = \frac{1-\alpha(1-\eta_C)}{1+\alpha(1-\eta_C)}, \tag{10}$$

where $\alpha = \zeta_h/\zeta_c$ and $\zeta_{c(h)} = \mathrm{sech}^2[2r_{c(h)}]$.

Equation (10) clearly shows that the efficiency of heat engines at maximum work output depends explicitly on the squeezing degree for the two squeezed reservoirs. By calculating Eq. (10), we can easily see that the efficiency of heat engines at maximum work output will degenerate to the limit value $\eta_- = \eta_C/(2-\eta_C)$ when the squeezing parameters of

the hot and cold reservoirs are equal ($r_{\rm h} = r_{\rm c}$) or when the hot and cold reservoirs are in a non-squeezed state ($r = 0$). It also means, at this point, that the efficiency of heat engines at maximum work output is independent of the squeezing parameter r. At this time, the efficiency corresponds to Eq. (9) in Ref. [40] where both the adiabaticity and squeezing parameters are equal to 0. It is worth mentioning here that for an engine with two-level systems as the working substance, the result is particularly interesting when the heat transport law is a linear phenomenological law in thermodynamics at the high-temperature limit. It is identical to the upper bound obtained in a minimally irreversible heat engine model and coincides with a reported universal upper bound with a low-dissipation assumption. Therefore, when the heat transport law is linear phenomenological law in the high-temperature case, the results show that our heat engine model reproduces the same upper bound as one derived from the low-dissipation heat engine at the extremely asymmetric limit. This indicates that the same limit might be taken for both the heat engine with the linear phenomenological law and the low-dissipation heat engine in the asymmetric dissipation limit.[6,55–58]

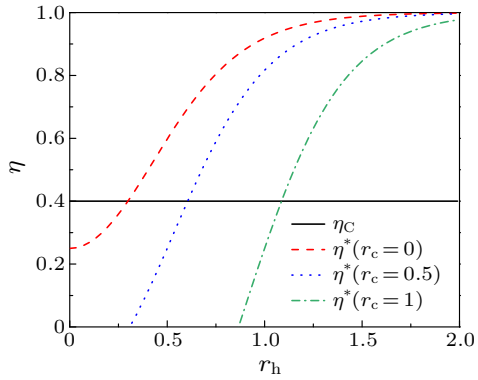


Fig. 2. Efficiency of the heat engine at maximum work output as a function of the squeezing parameter $r_{\rm h}$ of the hot reservoir for different squeezing parameters $r_{\rm c}$ of the cold reservoir.

Figure 2 depicts the variation in the efficiency of heat engines at maximum work output with the change in the squeezing parameter $r_{\rm h}$ of the hot reservoir when the squeezing parameter $r_{\rm c}$ of the cold reservoir is fixed. It can be easily seen from Fig. 2 that when the squeezing parameter $r_{\rm h}$ of the hot reservoir is constant, the higher the squeezing parameter $r_{\rm c}$ of the cold reservoir, the lower the corresponding efficiency of heat engines at maximum work output. We can understand this conclusion from the theoretical analysis that when the squeezing parameter $r_{\rm h}$ of the hot reservoir is greater than the squeezing parameter $r_{\rm c}$ of the cold reservoir (i.e., $r_{\rm h}/r_{\rm c} > 1$ or $\zeta_{\rm h}/\zeta_{\rm c} < 1$), the heat engine is in a real squeezed state and the efficiency of heat engines at maximum work output can be actually improved. We find that the efficiency of heat engines at maximum work output is equal to the Carnot efficiency $\eta_{\rm C}$ when the condition $r_{\rm h} = \frac{1}{2}\text{arc}\,\text{sech}\left[\sqrt{\frac{1}{1+\eta_{\rm C}}}\,\text{sech}(2r_{\rm c})\right]$ is satisfied. The efficiency of heat engines at maximum work output will be greater than the Carnot efficiency $\eta_{\rm C}$ with an increase in $r_{\rm h}$ and eventually approaches the maximum value $\eta = 1$ when the ratio of $r_{\rm h}/r_{\rm c}$ approaches infinity (it also means that the ratio of $\zeta_{\rm h}/\zeta_{\rm c}$ approaches 0).

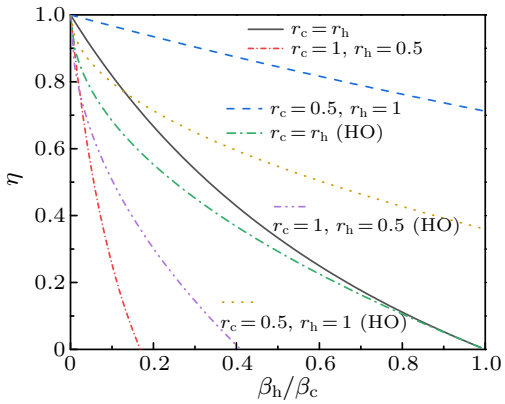


Fig. 3. Comparison diagram of the efficiency η of the heat engine with different working substances at maximum work output as a function of the inverse temperature ratio $\beta_{\rm h}/\beta_{\rm c}$ for different squeezing parameters.

Figure 3 shows the efficiency of the heat engine at maximum work output as a function of the inverse temperature ratio $\beta_{\rm h}/\beta_{\rm c}$ for different squeezing parameters. Here, the solid black line in the middle indicates the efficiency $\eta_- = \eta_{\rm C}/(2-\eta_{\rm C})$, where the squeezing parameter $r_{\rm c} = r_{\rm h}$. From Fig. 3, we can get the conclusion that when $r_{\rm c} = 1$ and $r_{\rm h} = 0.5$, the efficiency of the heat engine at maximum work output is significantly less than η_- and decreases rapidly as the inverse temperature ratio increases. However, when the condition of $r_{\rm c} = 0.5$ and $r_{\rm h} = 1$ is satisfied, the efficiency of the heat engine at maximum work output is obviously greater than η_-, even greater than zero at inverse temperature ratio $\beta_{\rm h}/\beta_{\rm c} = 1$, which does not violate the second law of thermodynamics because of the quantum resource, i.e., squeezed reservoir. While the three curves in the middle in Fig. 3 are the conclusions obtained in Ref. [46], in that work the authors address optimal performance of quantum Otto machines based on a single harmonic oscillator (HO) with time-dependent frequency ω and operating with a hot and cold-squeezed reservoir. The optimized efficiency obtained by them in Ref. [46] takes the form $\eta_{\rm HO} = 1 - \sqrt{\frac{\beta_{\rm h}}{\beta_{\rm c}}\frac{\text{sech}(2r_{\rm h})}{\text{sech}(2r_{\rm c})}}$, and it will simplify to $\eta = \eta_{\rm CA} = 1 - \sqrt{1-\eta_{\rm C}}$ when the squeezing parameter $r_{\rm h} = r_{\rm c}$, which corresponds to the CA efficiency ($\eta_{\rm CA}$) in the CA model. We compare our result with that obtained in Ref. [46] and can draw the following three conclusions from Fig. 3. First, the efficiency of heat engines with two-level systems and a single harmonic oscillator system as working substance increases monotonically as the ratio $\beta_{\rm h}/\beta_{\rm c}$ decreases. Second, for the heat engine with two-level systems as the working substance, the efficiency of $r_{\rm c} < r_{\rm h}$ is greater than that of $r_{\rm c} > r_{\rm h}$ under the premise of the ratio $\beta_{\rm h}/\beta_{\rm c}$ being determined. A similar conclusion can be obtained for the heat engine with a single harmonic oscillator system as the working substance. Finally,

the comparison shows that when all other conditions are identical, the efficiency in Ref. [46] is significantly greater than that in our paper when the squeezing parameter meets $r_h < r_c$. However, the conclusion is opposite when the squeezing parameter is $r_h > r_c$. This means that the efficiency of the heat engine at maximum work output is directly related to the type of working substance when the hot and cold reservoirs of the heat engine are in the squeezed state.

The variation of the efficiency of the heat engine at maximum work output with the squeezing parameters r_c and r_h for fixed inverse temperature ratio $\beta_h/\beta_c = 0.25$ is explained in detail in Fig. 4. The diagonal line (black line), where $r_h = r_c$ means that the hot and cold reservoirs are in the same squeezing degree, corresponds to $\eta_- = \eta_C/(2-\eta_C)$. The conclusion that $\eta^* > \eta_-$ when $r_h > r_c$, while $\eta^* < \eta_-$ when $r_h < r_c$ can be easily drawn. This shows that the efficiency of the heat engine at maximum work output can be enhanced only when the condition of the squeezing degree of hot reservoir being greater than that of the cold reservoir is satisfied, otherwise it will be reduced or even inhibited.

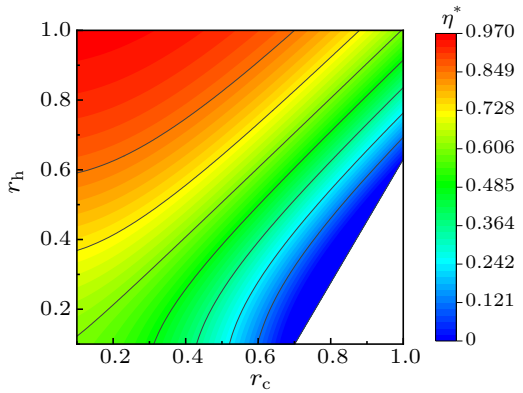


Fig. 4. The efficiency of the heat engine at the maximum work output as a function of the squeezing parameters r_c and r_h for a fixed inverse temperature ratio $\beta_h/\beta_c = 0.25$.

4. The coefficient of performance of quantum Otto refrigerators at maximum χ criterion

As shown by the solid arrow(clockwise) in Fig. 1, the quantum Otto refrigerator cycle is a clockwise cycle process, i.e., $1 \to 2 \to 3 \to 4 \to 1$. In this cycle, the heat exchange and work processes are completely the opposite of those in the heat engine cycle. Therefore, the heat absorbed is from the cold-squeezed reservoir and the heat released is from the hot-squeezed reservoir. The total input work in a cycle is performed after the entire refrigeration cycle. After the entire cycle, the system recovers its initial state.

In research on performance optimization for refrigerators, the figure of merit $\chi = \varepsilon\dot{Q}_c$ can be regarded as a relatively appropriate criterion. Then, based on Eqs. (4) and (5), the figure of merit χ can be expressed as follows:

$$\chi = -\frac{\omega_c^2(\zeta_c\beta_c\omega_c - \zeta_h\beta_h\omega_h)}{4(\omega_h - \omega_c)\tau}. \tag{11}$$

where τ indicates the duration in a complete cycle. When the time of the two adiabatic strokes is ignored, by setting the derivatives of χ with respect to frequency ω_c and ω_h equal to zero, i.e., $\partial\chi/\partial\omega_c = 0$ and $\partial\chi/\partial/\omega_h = 0$, one can obtain that the frequency ratio satisfies the relation

$$\frac{\omega_c}{\omega_h} = \frac{3\beta_c\zeta_c + \beta_h\zeta_h - \sqrt{9\beta_c^2\zeta_c^2 - 10\beta_c\beta_h\zeta_c\zeta_h + \beta_h^2\zeta_h^2}}{4\beta_c\zeta_c}. \tag{12}$$

As a result, according to Eq. (8), the coefficient of performance at maximum χ criterion gives rise to

$$\varepsilon^* = \frac{3\kappa + \sqrt{(1-\kappa)(9-\kappa)} - 3}{2(1-\kappa)}, \tag{13}$$

where $\kappa = \frac{\beta_h\zeta_h}{\beta_c\zeta_c}$.

Equation (13) explicitly indicates that the coefficient of performance of the refrigerator at the maximum χ criterion is directly related to the squeezing degree of the two squeezed reservoirs. When the hot and cold reservoirs are in their pure state which means no squeezing ($r_c = r_h = 0$) or symmetric squeezing state ($r_c = r_h$), we easily get that the coefficient of performance ε^* at the maximum χ criterion will be rewritten as $\varepsilon_- = (\sqrt{9+8\varepsilon_C} - 3)/2$. At this time, the coefficient of performance of the refrigerator at the maximum χ criterion is not related to the squeezing parameter and has the characteristics of extreme asymmetric or low-dissipation limits.[56,59] The same explanation can be applied to the analysis of refrigerators for heat engines.

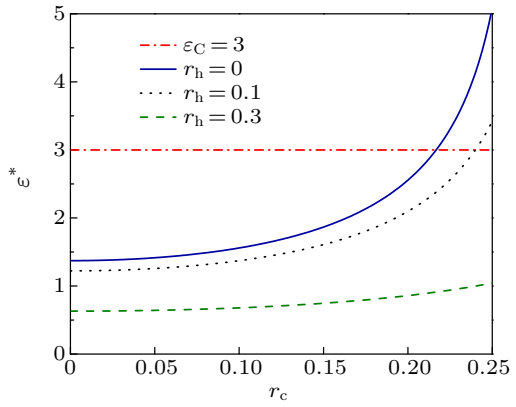


Fig. 5. Coefficient of performance at the maximum χ criterion as a function of the squeezing parameter r_c of the cold reservoir for different squeezing parameters r_h of the hot reservoir.

It is easy to determine from Fig. 5 that when the squeezing parameter r_c of the cold reservoir is fixed, the higher the squeezing parameter r_h of the hot reservoir is, the lower the corresponding coefficient of performance of the refrigerator at the maximum χ criterion is. When the condition $r_c = \frac{1}{2}\text{arc sech}[\frac{\sqrt{2+\varepsilon_C}}{\sqrt{3+\varepsilon_C}}\text{sech}(2r_h)]$ is satisfied, the coefficient of performance of the refrigerator at the maximum χ criterion is equal to that of performance ε_C. Subsequently, the coefficient of performance of the refrigerator at the maximum χ criterion will be greater than the Carnot coefficient ε_C with an increase

in r_c. We can understand this conclusion from the theoretical analysis, Fig. 5 shows that the coefficient of performance of the refrigerator at the maximum χ criterion will decrease when the ratio of r_h/r_c increases.

The variation curves of ε^* and ε_{HO} are plotted as a function of the inverse temperature ratio β_h/β_c for different squeezing parameters, as shown in Fig. 6, where $\varepsilon_{HO} = 1/\sqrt{1-\frac{\beta_h \operatorname{sech}(2r_h)}{\beta_c \operatorname{sech}(2r_c)}} - 1$ is the result of Ref. [46] with a single harmonic oscillator (HO) system as working substance and operating with two squeezed reservoirs. In our study, it is obvious that the coefficient of performance at the maximum χ criterion is less than $\varepsilon_-(r_h = r_c)$ when $r_c = 0.25$ and $r_h = 0.5$ and is larger than ε_- when $r_c = 0.5$ and $r_h = 0.25$ for fixed $\beta_h/\beta_c = 0.75$. Figure 6 further illustrates that only when the squeezing parameter r_c of cold reservoir is greater than the squeezing parameter r_h of the hot reservoir (i.e., $r_c/r_h > 1$ or $\zeta_c/\zeta_h < 1$), the refrigerator is in a true sense of squeezed state and the coefficient of performance can be actually enhanced for the fixed ratio of β_h/β_c. We compare our result with that obtained in Ref. [46]. The following results, similar to those in the heat engine, can easily be drawn from Fig. 6. First, when both the hot and cold reservoirs are in a squeezed state, the variation trend of the coefficient of performance curve is roughly the same, i.e., the value of the coefficient of performance increases with an increase in the inverse temperature ratio of β_h/β_c. Second, for a refrigerator with two-level systems as the working substance, given the same ratio of β_h/β_c, the performance coefficient of the refrigerator when $r_c < r_h$ is lower than that when $r_c > r_h$. A similar conclusion can be drawn for a refrigerator with a single-harmonic oscillator system as the working substance. Finally, when all other conditions (e.g., r_c, r_h and β_h/β_c) are equal, the coefficient of performance of the refrigerator at the maximum χ criterion in our study is greater than that in Ref. [46]. This also suggests that the coefficient of performance of a refrigerator at the maximum χ criterion is strongly related to the type of working substance.

For the quantum Otto heat engine described in Section 3, the variation of the coefficient of performance of quantum Otto refrigerator with the squeezing parameters r_c and r_h for fixed inverse temperature ratio $\beta_h/\beta_c = 0.5$ is explained in detail in Fig. 7. The diagonal line (black line) corresponds to $\varepsilon_-(r_h = r_c)$. It is shown that the coefficient of performance can be enhanced when $r_c > r_h$, whereas it is suppressed when $r_c < r_h$. This indicates that the coefficient of performance of refrigerators at the maximum χ criterion can be enhanced only when the squeezing degree of the cold reservoir is greater than that of the hot reservoir; otherwise, it will be reduced or even inhibited.

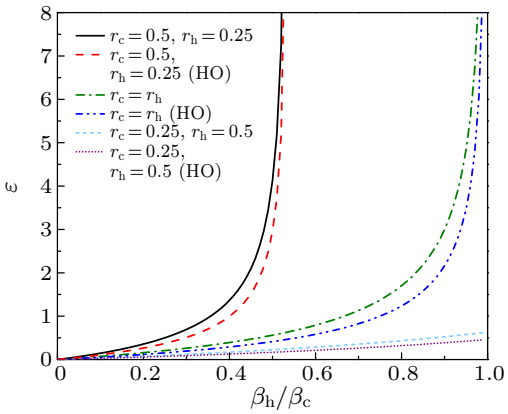


Fig. 6. Comparison diagram of the coefficient of performance of refrigerators with different working substances at the maximum χ criterion as a function of inverse temperature ratio β_h/β_c for different squeezing parameters.

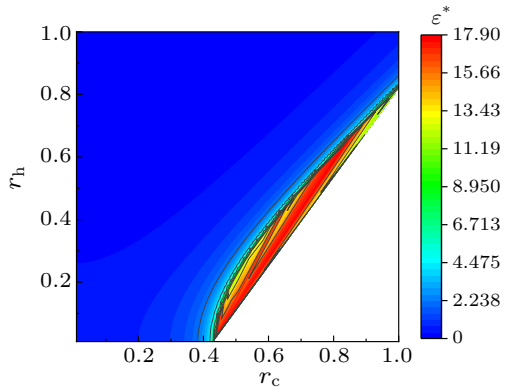


Fig. 7. Coefficient of performance of refrigerators at the maximum χ criterion as a function of the squeezing parameters r_c and r_h for a fixed temperature ratio $\beta_h/\beta_c = 0.5$.

It should be noted that the refrigerator proposed here is not a simple reverse operation of a heat engine when the refrigerator is in a squeezed state. When the heat engine and refrigerator are in a squeezed state, the heat absorption and heat release processes in the cycle can be directly affected by the external environment or other factors, and the influences of squeezing on the optimization performance of the refrigerator and heat engine are completely different.

5. Conclusions

In this study, a cycle of a quantum Otto machine with two-level systems as the working substance and operating between two squeezed reservoirs was established. The cycle includes two quantum adiabatic and two quantum isochoric processes. The influence of the squeezing degree on the optimization performance of quantum Otto heat engines at the maximum work output and quantum Otto refrigerators at the maximum χ criterion has been fully studied. These analyses have shown that under a symmetric squeezing state ($r_h = r_c$) or non-squeezing ($r_h = r_c = 0$), the efficiency of the heat engine at maximum work and the coefficient of performance of quantum Otto refrigerators at maximum χ criterion are always equal to $\eta_- = \eta_C/(2-\eta_C)$ and $\varepsilon_- = (\sqrt{9+8\varepsilon_c}-3)/2$, respectively,

and independent of the squeezing parameters r_h and r_c. However, under an asymmetric squeezing state ($r_h \neq r_c$), the efficiency of the heat engine at maximum work can be improved only when the condition of $r_h > r_c$ is satisfied; otherwise, it will be reduced or even suppressed in the case of $r_h < r_c$, and the coefficient of performance of quantum Otto refrigerators at the maximum χ criterion can be enhanced only when the condition of $r_c > r_h$ is satisfied, while it is decreased or even inhibited when $r_c < r_h$.

In addition, when the other conditions are equal, by comparison, we conclude that the efficiency of the heat engine at maximum work with two-level systems as the working substance is smaller than that with the harmonic oscillator as the working substance when the squeezing parameter $r_h < r_c$, which is contrary to the conclusion when the squeezing parameter $r_h > r_c$. However, for the refrigerator, the coefficient of performance of the refrigerator at the maximum χ criterion with two-level systems as the working substance is higher than that of the refrigerator with a single harmonic oscillator system as the working substance.

In particular, in the two adiabatic processes of the quantum Otto cycle, there is a very important physical quantity, i.e., the adiabatic parameter, which is a measure of degree of adiabatic processes and plays an important role in the quantum Otto cycle.[33,42] The adiabatic parameter provides the probability of transition between the two levels of the two-level systems and allows us to study the efficiency and performance coefficients of the quantum Otto engine in any time frame. This so-called adiabatic parameter is the transition probability induced by the unitary evolution; the faster the unitary process, the greater the adiabatic parameter. When the adiabatic parameter is equal to zero, the process is quasi-static. In this study, the performance of a quantum Otto heat engine and refrigerator was optimized under quasi-static conditions, where the adiabatic parameter was null. It should be clear that when adiabatic parameters are considered, many interesting and novel conclusions are generated here. For a universal quantum Otto machine, we can make the machine work as a thermal engine, refrigerator, or heater by changing the possibility of the control parameters, such as the squeezing and adiabatic parameters and different adiabatic parameters, which will greatly affect the performance of the heat engine and refrigerator.[40,48]

With further research on the performance of heat engines and refrigerators in the squeezing state, we believe that there will be more scientific and novel results in the future.

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