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# Effect of distribution of fines on evolution of cooperation in spatial public goods game

Xing-Ping Sun(孙兴平), Yan-Zheng Bi(毕研政), Hong-Wei Kang(康洪炜)[†], Yong Shen(沈勇), and Qing-Yi Chen(陈清毅)

*School of Software, Yunnan University, Kunming* 650000, *China*

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In the realm of public goods game, punishment, as a potent tool, stands out for fostering cooperation. While it effectively addresses the first-order free-rider problem, the associated costs can be substantial. Punishers incur expenses in imposing sanctions, while defectors face fines. Unfortunately, these monetary elements seemingly vanish into thin air, representing a loss to the system itself. However, by virtue of the redistribution of fines to cooperators and punishers, not only can we mitigate this loss, but the rewards for these cooperative individuals can be enhanced. Based upon this premise, this paper introduces a fine distribution mechanism to the traditional pool punishment model. Under identical parameter settings, by conducting a comparative experiment with the conventional punishment model, the paper aims to investigate the impact of fine distribution on the evolution of cooperation in spatial public goods game. The experimental results clearly demonstrate that, in instances where the punishment cost is prohibitively high, the cooperative strategies of the traditional pool punishment model may completely collapse. However, the model enriched with fine distribution manages to sustain a considerable number of cooperative strategies, thus highlighting its effectiveness in promoting and preserving cooperation, even in the face of substantial punishment cost.

**Keywords:** public goods game, fine distribution, cooperation

**PACS:** 02.50.Le, 02.50.Ng, 02.60.Cb, 02.70.Uu **DOI:** 10.1088/1674-1056/ad5aee

## 1. Introduction

Cooperation is widespread in nature.[1–4] However, according to Darwin's theory of evolution,[5] cooperation will eventually collapse under natural selection. Therefore, the explanation of the reasons for the emergence and maintenance of cooperation has become the focus of many scholars' research.[6,7] To address this phenomenon, various models have been established in recent years, such as the prisoner's dilemma game[8] and the public goods game.[9] In the public goods game, each player can choose whether to invest in the public pool or not. The sum of all players' investments is then multiplied by the synergy factor, and the resulting payoff is eventually distributed equally among all of them, regardless of their strategies. Obviously, as a rational player, one would choose to be the defector that is more favorable to oneself, and this is what leads to the tragedy of the commons.

Based on the previous work, Novak summarized five laws that promote cooperation: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection.[10] With the development of complex networks, attention has been focused on the evolution of cooperation in structured populations and their network reciprocity.[11–17] A number of mechanisms to promote cooperation have gradually emerged, such as rewards,[18–22] punishments,[23–28] voluntary participation,[29–31] environmental feedback,[32–34] reputation,[35–38] alliance,[39–41] exclusion,[42–45] and investment heterogeneity.[46–51]

The effectiveness of punishments has been compared with rewards,[19,35,52–57] and punishments are generally considered more effective than rewards.[35,54,58–64] However, punishments are usually considered expensive[65] because based on previous studies on punishment mechanisms, two parts of the money disappear from the system out of thin air as a loss: one part is the cost of punishment for the punishers, and the other part is the fine for the defectors. However, in the real world, the fine can be utilized to compensate for the loss of the victimized party. For example, when signing a contract, if one party violates the terms of the contract, it needs to pay liquidated damages to the victimized party, and these liquidated damages are used to compensate the victimized party's loss. Based on this idea, this paper adds the distribution of fine to the pool punishment model[66] and compares it with the pool punishment model without the distribution of fine under the same parameter settings. The ultimate goal is to observe the effect of the distribution of fine on the evolution of cooperation in the spatial public goods game.

## 2. Model and method

A public goods game is played on a square lattice with periodic boundaries, where initially each player is designated as a cooperator, defector, or punisher with equal probability. During each iteration, each player participates in five groups of the game, which are centered on themselves and their four neighbors. In this model, if a player is a cooperator, they need

[†]Corresponding author. E-mail: hwkang@ynu.edu.cn

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to contribute a cooperation cost ($c$) to the public pool. If there are both punishers and defectors in the group, a portion of the total fines collected from the defectors, based on the $\alpha$ ratio, is allocated to be evenly distributed among the cooperators. If a player is a defector, no investment into the public pool is required, but a fine is paid when punishers are present. If a player is a punisher, in addition to investing into the public pool, they need to pay a fixed punishment cost ($\beta$). If there are both defectors and cooperators in the group, a portion of the total fines collected from the defectors, corresponding to the $(1-\alpha)$ ratio, is allocated to be evenly distributed among the punishers. If only defectors are present in the group with no cooperators, then the sum of the fines of all the defectors is distributed equally among all the punishers, ensuring that the fines are not lost and are fully utilized. The payoff formula for this model is as follows:

$$\begin{aligned}
\pi_{\mathrm{C}} &= \frac{r(N_{\mathrm{C}}+N_{\mathrm{PC}})c}{N} - c + \frac{\alpha N_{\mathrm{D}} \cdot \mathrm{fine} \cdot H(N_{\mathrm{PC}})}{N_{\mathrm{C}}}, \\
\pi_{\mathrm{D}} &= \frac{r(N_{\mathrm{C}}+N_{\mathrm{PC}})c}{N} - H(N_{\mathrm{PC}}) \cdot \mathrm{fine}, \\
\pi_{\mathrm{PC}} &= \frac{r(N_{\mathrm{C}}+N_{\mathrm{PC}})c}{N} - c - \beta + \frac{(1-\alpha \mathrm{H}(N_{\mathrm{C}}))N_{\mathrm{D}} \cdot \mathrm{fine}}{N_{\mathrm{PC}}},
\end{aligned} \tag{1}$$

where $H(x)$ refers to the Heaviside function as follows:

$$H(x) = \begin{cases} 0, & x \leqslant 0, \\ 1, & x > 0. \end{cases} \tag{2}$$

After an iteration of the game, each player $x$ has a cumulative payoff $P_x$ in the five groups. Then, the player randomly chooses a neighbor $y$. The cumulative payoff of the neighbor player $y$ is denoted as $P_y$. Based on the payoffs of both players, a learning probability $W$ is computed, representing the probability that player $x$ adopts the strategy of player $y$. $W$ is computed as follows:

$$W = \frac{1}{1+\exp[\frac{P_x - P_y}{k}]}. \tag{3}$$

Here, $k$ denotes the noise, which is set to 0.5. The reason for setting $k$ is that individuals in the real world are bounded rational, and with random noise $k$, players with high payoffs have a certain probability of adopting the strategies of players with low payoffs. However, in general, it is still the strategies of players with higher payoffs that are more likely to be learned. In this paper, we employ Monte Carlo simulation to synchronously update the player strategies in the system. The size of the square lattice network used is $L = 100$, and all the simulation results require at most $10^4$ iterations. To minimize the effect of randomness, all the simulation results are averaged by repeating the process 10 times.

## 3. Simulation results

Figure 1 shows the change in the cooperation level of the two models with the change in the punishment cost $\beta$ under the same parameter settings. Firstly, as a whole, the level of cooperation of the two models tends to decrease as the punishment cost $\beta$ increases, except for individual points. Secondly, for most values of punishment cost $\beta$, the cooperation level of the pool punishment model with distribution of fines is higher than the cooperation level of the pool punishment model without distribution of fines. For example, when the punishment cost $\beta = 0.40$, the cooperation level of the pool punishment model without distribution of fines is 0, while the cooperation level of the pool punishment model with distribution of fines can still be maintained at a relatively high level. However, for a few values of punishment cost $\beta$, the cooperation level of the pool punishment model without distribution of fines is instead higher than the cooperation level of the pool punishment model with distribution of fines. For example, when the punishment cost $\beta = 0.15$, the cooperation level of the pool punishment model without distribution of fines reaches 1, while the cooperation level of the pool punishment model with distribution of fines is less than 0.8.

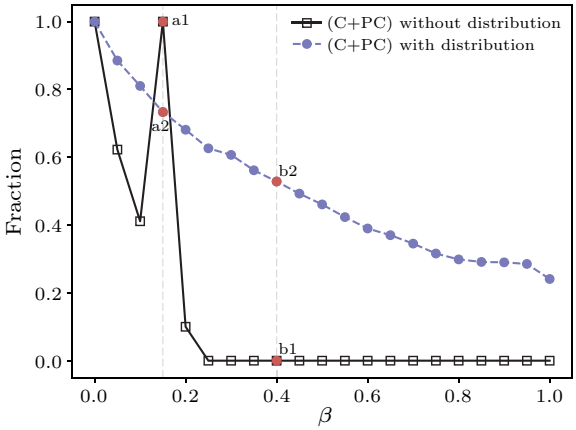



**Fig. 1.** Level of cooperation depicted between the pool punishment model without the distribution of fines and the pool punishment model with the distribution of fines, as a function of the punishment cost $\beta$. The synergy factor $r$ is set to 2.0, the fine is 0.95, the distribution coefficient $\alpha$ is 0.5, and the investment cost $c$ is 1. The two dashed lines at $\beta = 0.15$ and $\beta = 0.4$ intersect the two curves at four points a1, a2, b1, and b2, respectively.

The strategy evolution corresponding to the four points a1, a2, b1, and b2 in Fig. 1 is illustrated in Fig. 2. Figures 2(a1) and 2(b1) depict the strategy evolution of the pool punishment model without distribution of fines, while Figs. 2(a2) and 2(b2) show the strategy evolution of the pool punishment model with distribution of fines. It can be observed that all players in Fig. 2(a1) eventually become punishers, while in Fig. 2(b1) all players eventually become defectors. The reason for this stark contrast can be attributed to the lower punishment cost in Fig. 2(a1), resulting in a lighter burden on the punishers and enabling the eventual elimination of defectors. Conversely, in Fig. 2(b1), the higher punishment cost and heavier burden on the punishers lead to their eventual disappearance under the joint aggression of the first-order free rider (D) and the second-order free rider (C). Consequently, the cooperators

are left unprotected, and due to the small synergy factor $r$, they cannot survive solely through network reciprocity, ultimately resulting in the transformation of all players into defectors. Figures 2(a2) and 2(b2) depict the pool punishment model with distribution of fines. After adding the distribution of fines, both graphs exhibit a clear oscillating trend, with similar curve trends. Initially, the number of cooperators in both graphs increases rapidly, while simultaneously the number of defectors and punishers decreases. This is because the punishers penalize the defectors and allocate a portion of the fine to the cooperators, resulting in relatively higher payoffs for the cooperators. However, as the number of cooperators reaches a certain level, the number of defectors begins to rise. This occurs because the number of punishers at this stage is too small, allowing defectors to expand their territory by invading cooperators. As the number of defectors reaches a certain level, the punishers can retaliate against the defectors, leading to an increase in the number of punishers. With a sufficient number of punishers, the number of cooperators increases once more, and this cyclic pattern continues, with all three strategies exhibiting an oscillating trend. Eventually, a dynamic equilibrium is reached, evident from the subgraphs in Figs. 3(a2) and 3(b2). In the late stages of evolution, the slopes of the curves representing the transformation between the three strategies are nearly identical, indicating that their transition rates are similar. This explains why the three-strategy share stabilizes in the late stages of evolution. Interestingly, upon comparing Figs. 2(a1) and 2(a2), it is evident that the cooperation level (C + PC) of the pool punishment model with distribution of fines is not as high as that of the pool punishment model without distribution of fines. This is because the final punishers can completely dominate the network in the pool punishment model without distribution of fines. However, with the inclusion of distribution of fines, cooperators are enabled to survive, consequently allowing the survival of defectors as well. Hence, the pool punishment model with distribution of fines is not as effective as the one without distribution of fines in promoting cooperation under in this circumstance. Furthermore, upon the comparison between Figs. 2(b1) and 2(b2), while all players become defectors in Fig. 2(b1), the cooperation level (C + PC) in Fig. 2(b2) can still be maintained at a high level. This difference arises from the excessively high punishment cost in Fig. 2(b1), resulting in a heavier burden on the punishers in the pool punishment model without distribution of fines, ultimately leading to the disappearance of punishers first, followed by the transformation of all players into defectors. However, with the distribution of fine in Fig. 2(b2), the burden on the punishers is reduced, allowing cooperators and punishers to survive. Thus, in this circumstance, the pool punishment model with distribution of fines proves more effective than the one without distribution of fines in promoting cooperation.

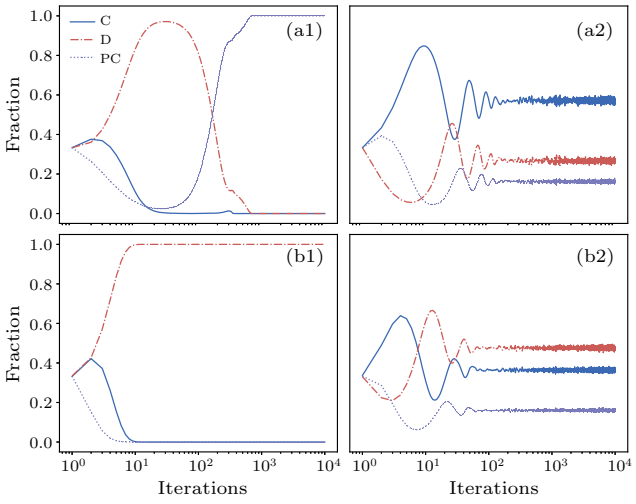



**Fig. 2.** Curves of strategy fraction evolution over time: (a) corresponding to the punishment cost $\beta = 0.15$, (b) corresponding to the punishment cost $\beta = 0.40$. Here $r = 2.0$, fine = 0.95, $\alpha = 0.5$, and $c = 1$.

For a deeper understanding of Fig. 2, figure 3 illustrates the number of strategy transitions in the system. Upon the observation of Fig. 3(a1), it is evident that initially, the transition of other players to defectors grows rapidly, resulting in an increasing number of defectors. However, this trend gradually begins to stabilize, while simultaneously, the transition of defectors to punishers grows rapidly, leading to an increase in the number of punishers, ultimately dominating the network. In contrast, Fig. 3(b1) shows a similar initial strategy transition to Fig. 3(a1), with a rapid growth in the transition of other players to defectors, resulting in the domination of defectors in the network. Comparing Figs. 3(a2) and 3(b2), it is evident that the strategy transitions in both the figures follow a similar pattern: Initially, there is a rapid growth in the transition of other players to cooperators, leading to an increasing number of cooperators. However, this growth tends to level off, and simultaneously, the transition of other players to defectors grows rapidly, followed by a leveling off in the transition rate. Concurrently, there is a rapid growth in the transition of other players to punishers, resulting in an increase in the number of punishers. This pattern continues, ultimately resulting in the highest number of transitions from other players to cooperators in Fig. 3(a2), leading to the largest fraction of cooperators in the later stages of evolution in Fig. 2(a2). Conversely, Fig. 3(b2) exhibits the highest number of transitions from other players to defectors, resulting in the largest fraction of late evolutionary defectors in Fig. 2(b2). Furthermore, it can be observed that the slopes of the curves representing the transition among the three strategies in the late stage of evolution are nearly identical in both Figs. 3(a2) and 3(b2). This indicates that the three strategies transition at almost the same rate, explaining why the fraction of the late evolutionary stage in the three strategies reaches dynamic equilibrium in Figs. 2(a2) and 2(b2).

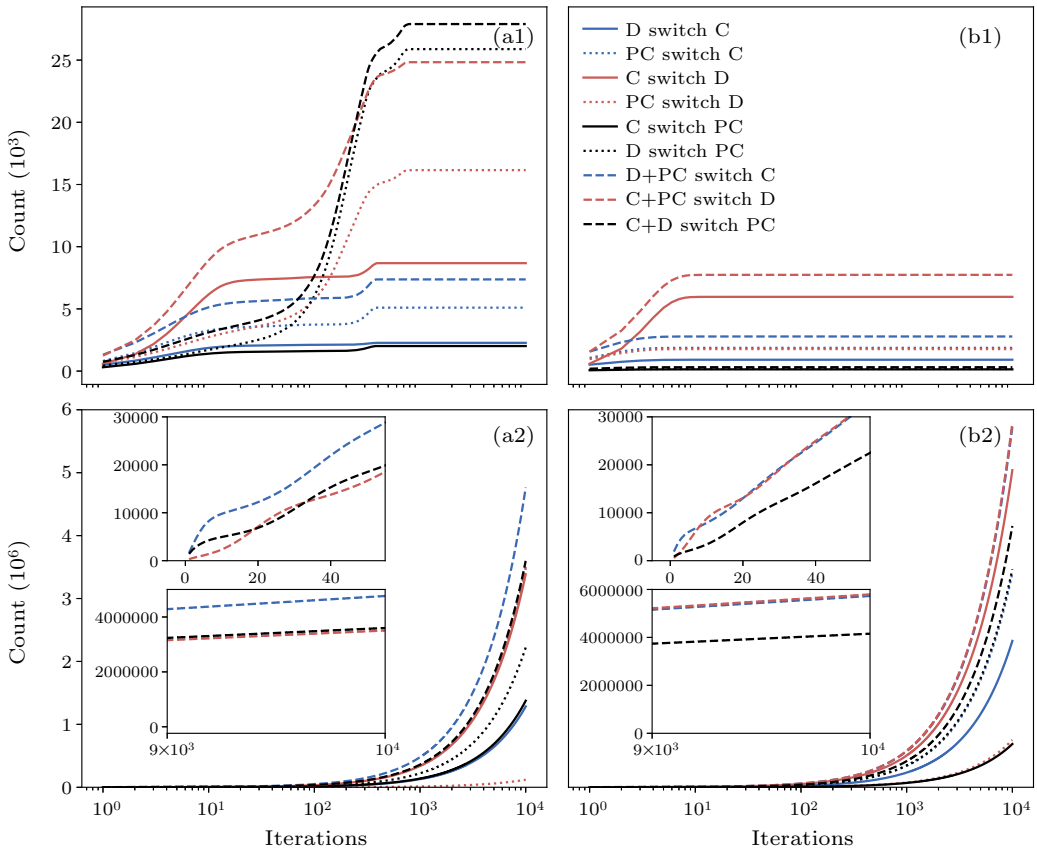



**Fig. 3.** Comparative curves for the number of strategy transitions: (a) corresponding to the punishment cost $\beta = 0.15$, (b) corresponding to the punishment cost $\beta = 0.4$. Here $r = 2.0$, fine = 0.95, $\alpha = 0.5$, and $c = 1$.

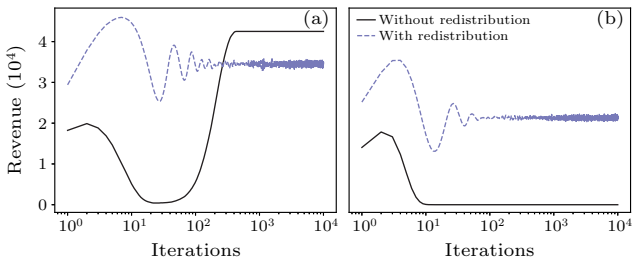



**Fig. 4.** Comparative curves depicting the evolution of the total system payoff: (a) corresponding to the punishment cost $\beta = 0.15$, (b) corresponding to $\beta = 0.4$. Here $r = 2.0$, fine = 0.95, $\alpha = 0.5$, and $c = 1$.

The comparative curves illustrating the evolution of the overall payoff for the four points a1, a2, b1, and b2 in Fig. 1 are presented in Fig. 4. In Fig. 4(a), during the early stages of evolution, the overall payoff of the pool punishment model with distribution of fines surpasses that of the model without distribution of fines. Behind this, not only the direct impact of fine distribution on the overall payoff contribute, but also the higher fraction of cooperative strategies (C + PC) in the model with distribution of fines helps. Consequently, more players invest in the public pool, resulting in higher payoffs. However, as the evolution progresses, the overall payoff of the model without distribution of fines exceeds that of the model with distribution of fines. This is attributed to the higher prevalence of cooperative strategies (C + PC) in the model without distribution of fines during the later stages of evolution, leading to more players investing in the public pool. Despite the presence of fine distribution in the model with distribution of fines, its aggregate effect is overshadowed by the larger number of players investing in the public pool. In Fig. 4(b), the overall payoff of the pool punishment model with distribution of fines consistently surpasses that of the model without distribution of fines, in both the early and late stages of the evolution. During the early stages, the model with distribution of fines benefits from a higher prevalence of cooperative strategies (C + PC) and the additional effect of the fine distribution, resulting in a higher overall payoff. In the late stages, while the cooperative strategies (C + PC) in the model without distribution of fines disappear, leading to a zero overall payoff, a significant fraction of cooperative strategies (C + PC) persists in the model with distribution of fines, allowing the overall payoff to remain relatively high.

A snapshot of the strategy evolution for the four points a1, a2, b1, and b2 in Fig. 1 is depicted in Fig. 5. In Fig. 5(a1), during the early stage of evolution, defectors swiftly infiltrate cooperators and punishers as the various strategies are relatively evenly distributed and no significant clusters have been formed. However, as punishers begin to form larger clusters, they can counter-invade defectors through network reciprocity, ultimately dominating the network. Conversely, in Fig. 5(b1), due to the substantial punishment cost burdening punishers, even if clusters form, they struggle to resist defector invasions through network reciprocity. Without the protection of punish-

ers, cooperators vanish, leading to the complete dominance of defectors in the network. In Figs. 5(a2) and 5(b2), early in the evolution, cooperators expand rapidly, aided by fines provided by punishers. However, as time progresses, defectors also expand again, as cooperators previously invaded both defectors and punishers. Consequently, after the decline in the number of punishers, cooperators lose protection and succumb to invasions by defectors. Subsequently, punishers can expand again, facilitated by increased interaction with defectors due to their larger presence. This symbiotic relationship among the three strategies leads to a dynamic equilibrium.

Figure 6 illustrates the impact of different parameter combinations on the level of cooperation (C + PC) from a macroscopic perspective. Across various synergy factor $r$, the pool punishment model with distribution of fines appears to be more conducive to cooperation (C + PC) overall. Specifically, under similar fine conditions, the pool punishment model with distribution of fines enables a significant fraction of cooperative strategies to persist despite the substantial punishment cost. This phenomenon arises because, in the absence of fine distribution, heavy punishment cost overloads the punishers in the pool punishment model without distribution of fines, leading to their eventual disappearance. Consequently, without the protection of punishers, cooperators cannot sustain, ultimately, leading to a scenario where all players eventually adopt the defector strategy. In contrast, in the pool punishment model with distribution of fines, punishers can redistribute collected fines from defectors to both cooperators and punishers, thereby enhancing the survivability of cooperative strategies (C + PC) and maintaining a certain level of cooperation despite heavy punishment cost. However, within a relatively small range of fine values and with an appropriately set punishment cost $\beta$, defectors persist in the pool punishment model with distribution of fines while being eliminated in the model without distribution of fines. This occurrence is attributed to the eventual domination of the network by punishers in the absence of fine distribution, whereas the inclusion of fine distribution in the pool punishment model prevents the complete removal of defectors, resulting in a mutually beneficial symbiosis among the three strategies. Consequently, at this juncture, the cooperation level (C + PC) of the pool punishment model with distribution of fines is not as high as that of the model without distribution of fines. Another noteworthy observation across these six heatmaps is the reduction in stabilization points as the synergy factor $r$ increases.

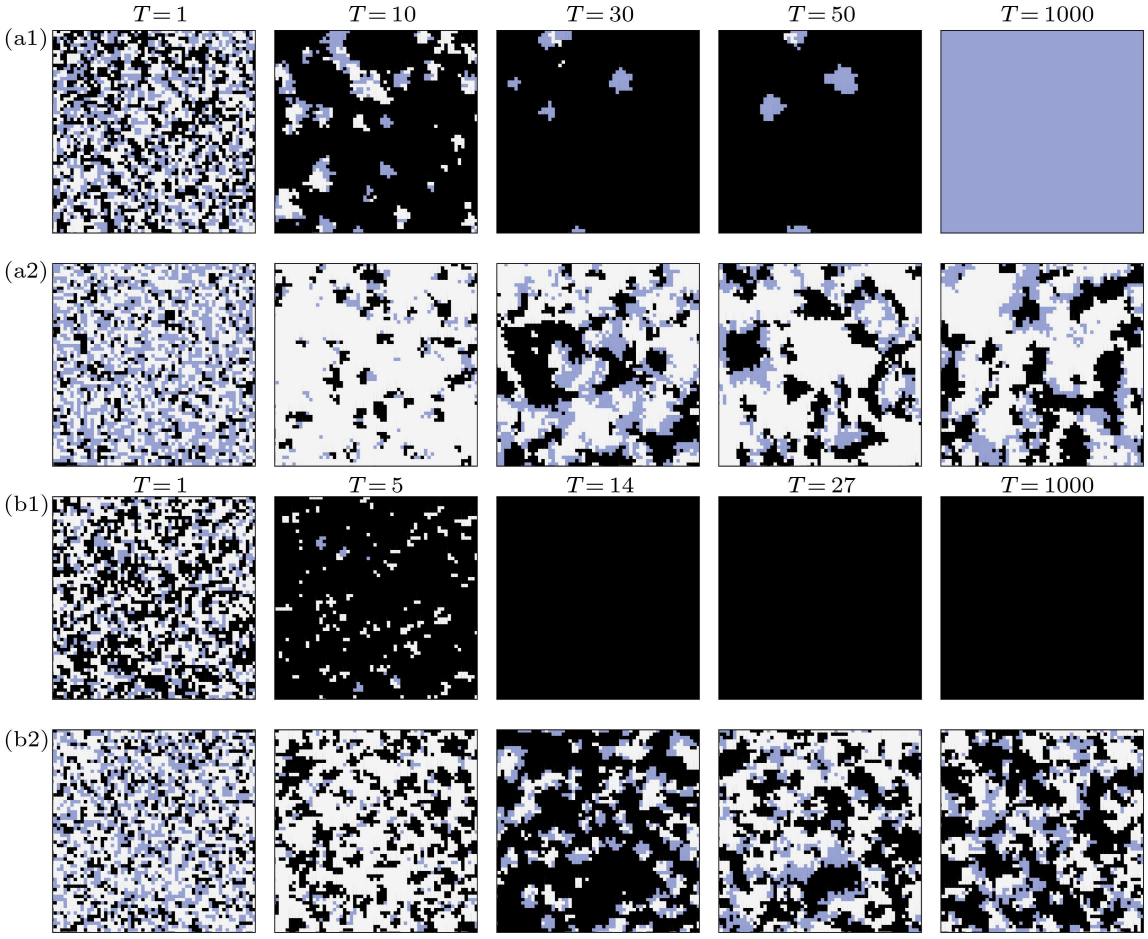



**Fig. 5.** Snapshots of the different Monte Carlo times are shown from left to right. White represents cooperators (C), black represents defectors (D), and gray represents punishers (PC). (a) Corresponding to punishment cost $\beta = 0.15$. (b) Corresponding to punishment cost $\beta = 0.4$. Here $r = 2.0$, fine = 0.95, $\alpha = 0.5$, and $c = 1$.

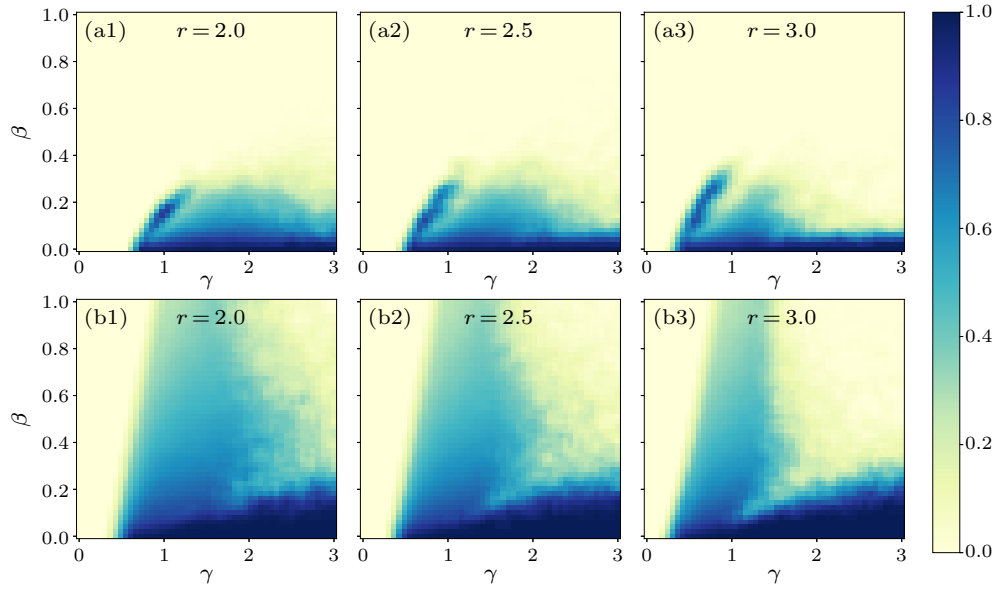



**Fig. 6.** Fraction of cooperative strategies (C + PC) as a function of ($\beta$, fine) from $r = 2.0$ to $r = 3.0$: (a) the pool punishment model without distribution of fines, and (b) the pool punishment model with distribution of fines. Here $\alpha = 0.5$ and $c = 1$.

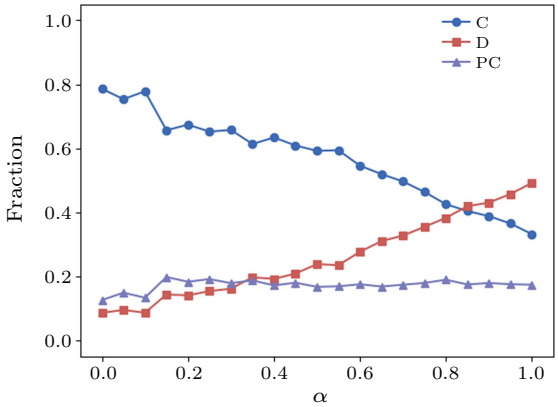



**Fig. 7.** Curve of strategy fraction evolution with different values of $\alpha$, for $r = 2.0$, $c = 1$, fine = 0.95, and $\beta = 0.15$.

In order to investigate the effect of $\alpha$ on the evolution of cooperation, we plot Fig. 7. Figure 7 shows that as $\alpha$ increases, the fraction of defectors tends to grow. This is because as $\alpha$ increases, more fines are distributed to the cooperators and the survivability of the punishers is decreased, then the punishment of the defectors decreases, which conduces to more of their survivals, leading to growth tendency of the fraction of defectors. Therefore, to increase the level of cooperation, more fines should be distributed to punishers, corresponding to the use of smaller $\alpha$ values.

## 4. Conclusion

This paper explores the impact of fine distribution on cooperation evolution within a spatial public goods game, observing that under high punishment cost, the cooperative strategy in the punishment model without distribution of fines diminishes completely, whereas in the punishment model with distribution of fines, cooperation persists, eventually leading to a mutually beneficial equilibrium among the three strategies. However, within a certain range of punishment costs and fines, the cooperation level peaks in the punishment model without distribution of fines, with no defectors surviving. In contrast, cooperation level decreases in the pool punishment model with distribution of fines. This is attributed to the ability of the punishment model without distribution of fines to enable punishers to dominate the network entirely, whereas the punishment model with distribution of fines fosters symbiosis among the strategies, resulting in a lower cooperation level compared to the former. Therefore, while fine distribution generally enhances cooperation, it may also hinder it in specific cases.

We find that the more fines are distributed to the punishers, the higher the level of cooperation, which is due to the fact that more punishers survive to punish the defectors, thus increasing the level of cooperation.

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