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Non-monotonic behavior of jam probability and stretched exponential distribution in pedestrian counterflow

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We adopt a floor field cellular automata model to study the statistical properties of bidirectional pedestrian flow moving in a straight corridor. We introduce a game-theoretic framework to deal with the conflict of multiple pedestrians trying to move to the same target location. By means of computer simulations, we show that the complementary cumulative distribution of the time interval between two consecutive pedestrians leaving the corridor can be fitted by a stretched exponential distribution, and surprisingly, the statistical properties of the two types of pedestrian flows are affected differently by the flow ratio, i.e., the ratio of the pedestrians walking toward different directions. We also find that the jam probability exhibits a non-monotonic behavior with the flow ratio, where the worst performance arises at an intermediate flow ratio of around 0.2. Our simulation results are consistent with some empirical observations, which suggest that the peculiar characteristics of the pedestrians may attributed to the anticipation mechanism of collision avoidance.

Keywords: traffic and crowd dynamics, evolutionary game theory

PACS: 02.50.Le, 45.70.Vn **DOI:** 10.1088/1674-1056/ad57ab

1. Introduction

Collective motion of many-body systems is an important research field in the fields of statistical physics and its neighboring disciplines. As a multidisciplinary subject, pedestrian dynamics have attracted the attention of researchers, due to the similarity between pedestrian flow and many classical physical systems such as granular materials and colloidal suspensions in external fields. Under certain scenes, the pedestrian flow may display some interesting self-organized phenomena, such as the arching effect near the exit or bottleneck,[1,2] lane formation in bidirectional flow,[3,4] jam/gridlock/deadlock,[5] the stop-and-go wave,[6,7] and so on.

Experimental study of pedestrian movement under special conditions is limited by several factors. First of all, for safety and moral considerations, it is impossible to especially reproduce the dangerous scenes. Secondly, the organizational burden of gathering hundreds of people is extremely huge, and experiments conducted for large groups with high density are still not abundant.[8,9] Finally, it is usually not easy to use images from surveillance cameras because of privacy violations and other issues. Therefore, animal experiments or computer simulations are usually used to study the behavior of people with high density. In order to study the dynamics of pedestrian flow, two kinds of representative models have been proposed. The first one is classified as force-based model,[10–15] which can reproduce successfully many empirical observations, but require relatively large amount of calculations.[16] The other one is the cellular automata model,[17–23] which can capture more easily the internal cognitive mechanism running at the individual level.

Unlike traditional physical system modeling, the difficulty in modeling pedestrian motion comes from the complex interactions between individuals. Some recent studies have shown that,[24–28] in order to fully understand the nature of pedestrian flow and reproduce phenomena such as lane formation, it is also necessary to account for psychological factors such as anticipation and cooperative behavior of the pedestrians. In doing so, researchers usually resort to the game theoretical framework, which is a powerful tool to quantify and analyze people's competitive and cooperative behavior.[29–40] In terms of game theory, crowds are divided into cooperators (C) and defectors (D), denoting the differences of their competitive willingness or ability in the face of spatial resource competition. The outcome of interaction between pedestrians is expressed by their obtained payoffs, which is mapped onto the probability of them moving to the target position. Decision makers have different decision-making tendencies in different types of games. A 2×2 payoff matrix is usually used to distinguish different types of games:

$$\begin{array}{c} \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}. \tag{1}$$

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The elements of the payoff matrix (R, S, T, and P) represent the payoff of a player choosing the strategy at the row when encountering another player choosing the strategy at the column. The relative magnitude of the four matrix elements determines the type of games. To the best of our knowledge, there is still little research on the role of game process in the dynamics of bidirectional pedestrian flow, though the cooperation between the pedestrians may be crucial in shaping the moving pattern of them when conflicts occur.

Perhaps, pedestrian counterflow in a straight corridor is the simplest type, where a high-density crowd state is prone to cause large-scale crowd crushes. For such a scenario, Kretz *et al.* conducted a series of supervision experiments,[41] and they observed that the total flow capacity in the bidirectional case is always greater than that observed in the unidirectional case, that is, balanced bidirectional flow performs better than unidirectional flow. However, a series of studies conducted by Zhang *et al.*[3] and Alhajyaseen *et al.*[42] have come to the opposite conclusion, and the differences in their results may come from the different initial conditions and boundary conditions. Unfortunately, no research has pointed out the potential mechanism leading to the difference in the presented experimental results, and the basic rules of the formation of congestion in pedestrian flow and the impact of different crowd parameters (such as total flow, density, and flow ratio) still have not been fully understood.

In addition, the bidirectional pedestrian flow in actual situations is usually discontinuous and may be unequal in number towarding different directions. Lam *et al.*[43,44] used video recordings to study the pedestrian behavior at crosswalks in different places in Hong Kong. They found that the relationship between flow and flow ratio can be closely fitted with a third-order polynomial function, while the limited pedestrian counterflow will reduce the overall flow capacity, but it will form a stable lane under balanced conditions, and thus reduce the friction between individuals and increase the capacity of pedestrian flow. Several relevant studies pointed out that the flow ratio has an important impact on the flow capacity of bidirectional pedestrian flow, and confirmed that there is a non-monotonic relationship between pedestrian flow capacity and flow ratio.[3,5,42,45]

From an application perspective, the data obtained from the automated pedestrian tracking and detection system may be limited to very simple information (i.e., typically the situations where both the walking speed and the density of the pedestrians are relatively low). Karamouzas *et al.*[46] and Cristín *et al.*[47] pointed out that the average quantity is not enough to describe the behavior of the pedestrian, and the use of statistical distribution may be instead more conducive to revealing its dynamical properties. Therefore, to predict the occurrence of congestion (or even crushes in extreme cases) by means of space size, crowd density, and other observational measurements, it is very important to understand the statistical laws behind the collective phenomena of pedestrians.

In order to better understand the jamming of pedestrian counterflow and the statistical law behind it, and clarify the mechanism that cannot be directly revealed by empirical research, we study in this paper the statistical characteristics of pedestrian counterflow in a straight corridor by combining cellular automata model and the evolutionary game theory. We compare our simulation results with those obtained by several empirical studies. Remarkably, we find that these interesting phenomena may be originated from the anticipation collision avoidance mechanism by the moving pedestrians.

The remainder of this paper is organized as follows: Section 2 introduce our model in detail. Section 3 summarizes the related concepts and definitions. Numerical simulations and analyses are performed in Section 4. Finally, conclusions and outlook is given in Section 5.

2. Model

Two different species of particles (denoting pedestrians) are used to model bidirectional pedestrian flow moving in a straight corridor. In this paper, particles of type A are directed towards the right, while particles of type B are directed towards the left. They move in their desired walking directions until they reach the boundary, then absorbed and removed from the system (i.e., the open boundary condition is mainly used in our current study). Note that we do not introduce additional parameters to control the removal process that is similar to the asymmetric simple exclusion process (ASEP). Initially, N particles located in a straight corridor with width W and length L. A subset of L neighboring cells along the walking direction is defined as a row, and a subset of W neighboring cells perpendicular to the walking direction is defined as a column. Then the initial average density of the pedestrians is defined as $\rho = N/(L \times W)$.

2.1. Floor field model of the pedestrian flow

At each time step, each particle left in the corridor tries to move to one of the von Neumann neighbor sites (except walls or obstacles) or stays at its current position. The position located at row i and column j is assigned a coordinate (i, j), and the probability of choosing this site by a pedestrian p_{ij} is calculated by the floor field:[17,18,48]

$$p_{ij} = \frac{1}{Z}\xi_{ij}\exp(k_{\mathrm{S}}S_{ij} + k_{\mathrm{A}}A_{ij}), \tag{2}$$

where $Z = \sum_{(i,j)\in\Omega}\xi_{ij}\exp(k_{\mathrm{S}}S_{ij} + k_{\mathrm{A}}A_{ij})$ is the normalization factor. Ω represents the set of the von Neumann neighbors of the focal particle and the position it occupied. If position (i, j) is currently occupied by another pedestrian or by an obstacle or by the wall, $\xi_{ij} = 0$, otherwise $\xi_{ij} = 1$. k_{S} (k_{A}) is the

coupling strength of the static (anticipation) floor field, respectively. S_{ij} represents the static floor field (SFF):

$$S_{ij}^{\mathrm{A}} = \begin{cases} +1, & \text{if position } (i,j) \text{ is to the right,} \\ -1, & \text{if position } (i,j) \text{ is to the left,} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

for particles of type A, and $S_{ij}^{\mathrm{B}} = -S_{ij}^{\mathrm{A}}$ for particles of type B (Note that in our current setting, the particles of type A move from the left to the right direction in the straight corridor, and the particles of type B behave just oppositely, please see Fig. 1(a) for reference). By contrast, the term related to A_{ij} in Eq. (2) can be regarded as the anticipation floor field (AFF),[49] which for particles of type A is defined as[50]

$$A_{ij}^{\mathrm{A}} = \sum_{(i,j')\in\Gamma} \lambda^{|j-j'|}\tau_{ij'}, \tag{4}$$

where the summation only takes into account those cells toward the desired walking direction of the focal pedestrian. The quantity τ_{ij} is defined as

$$\tau_{ij} = \begin{cases} +1, & \text{if there is a particle of type A,} \\ -1, & \text{if there is a particle of type B,} \\ 0, & \text{if there is no particle,} \end{cases} \tag{5}$$

and $A_{ij}^{\mathrm{B}} = -A_{ij}^{\mathrm{A}}$ for particles of type B. Parameter $\lambda \in (0,1)$ can be regarded as the "interaction radius" of AFF.

2.2. Games played by the pedestrians

In this paper, synchronous update rule is adopted, i.e., multiple individuals attempting to move to the same position is allowed, then we say a conflict occurs. Following our previous work,[34,51] a game-theoretical approach is used to solve the conflict, after that, at most one pedestrian can move to the chosen position.

Specifically, we first give a weight to cooperators and defectors, respectively, to calculate their payoffs. Here, the cooperators refer to pedestrians who do not collide or push others when facing conflicts (i.e., they are more willing to yield to other pedestrians), while defectors are those performing the opposite behavior. Thus, it is reasonable to set the weight of cooperators to 1, and the weight of defectors to $q > 1$, indicating the competitive advantage of defectors over cooperators in the conflict situation. Whenever a conflict involves n pedestrians, assuming that m of them are cooperators (thus $m \le n$), the payoff of a focal pedestrian is determined by

$$p_{\mathrm{c}} = \frac{1}{m+(n-m)q}, \tag{6}$$

$$p_{\mathrm{d}} = \frac{q\delta^{n-m-1}}{m+(n-m)q}. \tag{7}$$

Here, the effective weights (or payoffs) p_{c} and p_{d} denote, respectively, the probabilities of a cooperator and a defector doing a jump to the competed vacant cell. A reduced payoff for defectors due to their rat race is introduced by parameter $\delta \in [0,1)$ (i.e., when two or more defectors compete for one position, they may continuously push and shove each other so that no one can reach the position at a certain time step). Therefore, the 2×2 payoff matrix can be written as

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} 1/2 & 1/(1+q) \\ q/(1+q) & \delta/2 \end{pmatrix} \end{array}. \tag{8}$$

Obviously, by adjusting the parameters q and δ, the pedestrians may play different types of games, either a Prisoner's dilemma (PD, $T>R>P>S$) or a Hawk-dove (HD, $T>R>S>P$).[34] There is a significant difference in the decision-making tendencies of the crowd between the two types of games. For HD, due to $T>R>S>P$, people usually choose strategies that are opposite to their opponents to ensure greater payoffs for themselves. However, for PD, due to $T>R>P>S$, no matter what strategy the opponent chooses, choosing betrayal always yields significant benefits.

In case a conflict happens, the pedestrians involved are also allowed to change their strategies according to the Fermi rule[52]

$$p_{\mathrm{ch}} = \frac{1}{1+\exp(-\frac{\Delta P}{\kappa})}, \tag{9}$$

where ΔP is the virtual change of the payoff by assuming that if one has changed his/her strategy while all the others do not. Parameter $\kappa > 0$ is the quantization of noise effect in the decision-making or a characterization of the self-interest degree of the pedestrians. The larger the κ, the more selfless an individual is, and the smaller the κ, the more selfish an individual is.

3. Concepts and definitions

In a bidirectional flow, the number of pedestrians walking toward different directions are not necessarily equal. For convenience, we call the pedestrian flow with lower (greater) number of individuals as minor (major) flow, respectively. The pedestrian flow in a certain direction can be directly described by the number of pedestrians of this type initially.[53] Denote the flow ratio as r, the definition is

$$r = \frac{\text{minor flow}}{\text{minor flow} + \text{major flow}}. \tag{10}$$

In this study, r is controlled by reducing the number of type-A particles at the beginning, that is, the definition of flow ratio can be alternatively written as

$$r = \frac{\text{number of particles of type A initially}}{\text{total number of particles initially}}, \tag{11}$$

and obviously $r \in [0, 0.5]$.

As shown in Fig. 1(a), initially, there are 100 type-A particles, and the total number of particles is 300, thus $r = 100/300 \approx 0.333$. In this way, unidirectional flow refers to $r = 0$, balanced flow refers to $r = 0.5$ and unbalanced flow refers to $r \neq 0.5$.

One of the most important phenomenon found in the bidirectional flow is the emergence of jam/gridlock/deadlock, as shown in Fig. 1(b). If the pedestrians fail to form lanes in the corridor in time or the lane is damaged due to random fluctuations, the two types of pedestrians block each other and have no space to cross, then the pedestrians cannot leave the corridor for a quite long time, we say that the (global) jam happens. In practice, we implement 2000 independent simulations, and for each run, if the pedestrians fail to leave the corridor totally and the final snapshot shows characteristics like the typical configuration in Fig. 1(b), we judge that a global jam emerges in this run.[51]

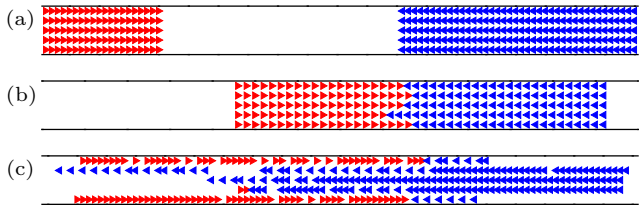


Fig. 1. Typical snapshots: (a) initial configuration, $r \approx 0.333$; (b) (global) jam; (c) lanes formation.

We investigate the statistical distribution of the time interval between two consecutive pedestrians leaving the system successively (denoting as Δt) from the boundary (two types of pedestrians are counted separately). Finally, in order to further explain the non-monotonic behavior of the jam probability with the flow ratio, it is necessary to introduce an indicator n_c, defined as the ratio of the number of conflicts in the system at that time to the number of pedestrians currently left in the corridor.

4. Results and discussion

Unless otherwise specified, the parameters in our model are chosen and fixed as follows:

$$q = 3, \quad \kappa = 0.01, \quad L = 100, \quad \rho = 0.6, \tag{12}$$

$$\lambda = 0.8, \quad k_S = 2.5, \quad k_A = 2.5 \quad \rho_c(0) = 0.5, \tag{13}$$

where $\rho_c(0)$ denotes the initial fraction of cooperators among the pedestrians. Note that with $q = 3$, the games played by the pedestrians belong to the PD for $\delta > 0.5$ and to the HD for $\delta < 0.5$. We select $\lambda = 0.8$ to ensure that the pedestrians can gain information far enough away, and the pedestrians far away have less impact, guaranteeing the results of the model are of physical meaning.[50,51] Finally, κ is set to 0.01 so that the pedestrians can make highly accurate response during the game.

As shown in Fig. 2, the complementary cumulative distribution function (CCDF) of Δt shows the stretched exponential law[54–56]

$$f(x) \sim \exp\left[-\left(\frac{x}{x_0}\right)^k\right]. \tag{14}$$

Note that we also tried to fitting the data by other types of distributions including power-law and log-normal distributions, and we find that stretched exponential distributions result in the best performance (see supplementary material for more details). There is no statistical difference between the two types of pedestrians with $r = 0.5$. However, when the number of pedestrians on both sides is not equal, the statistical nature of the bidirectional pedestrian flows changes greatly. We show in Fig. 3 the decay exponent k of the distribution of the CCDF(Δt) for the two types of particles as a function of r. We observe that the values of k vary significantly for different r, although the CCDF(Δt) of two types of particles still satisfy the stretched exponential law. Unexpectedly, the minor and the major pedestrian flows are affected differently by varying r. As the r value decreases, the decay exponent k of the minor flow increases continuously, while k of the major flow shows a downward trend instead. Moreover, the decreasing range of k for the major flow is relatively narrower than the increasing range for the minor flow, indicating that the minor pedestrian flow is more readily affected by the unbalanced flow ratio. It is worth pointing out that similar phenomena have also been observed in some empirical studies,[3,43,57] which suggests that our current game-theoretical model serves as a good candidate to explain the dynamic behavior of the pedestrian counterflow in real systems. Finally, we also find that the way of dealing with conflicts (different game types, controlled by δ) seems to have only quantitative impact on k value.[50,51]

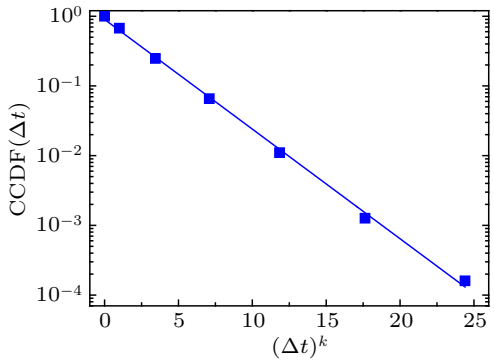


Fig. 2. The CCDF(Δt) shows the stretched exponential law, the solid lines are guides for the eyes. $k \approx 1.783$. Other parameters: $W = 10$, $\delta = 0.1$, $r = 0.5$.

Since Δt is defined as the time interval between two consecutive pedestrians leaving the system successively, the distribution index (the k value) is associated with the processes of lane formation and dissolution. These process involves the decision of choosing a target position to move (i.e., affected by local density and the flow ratio) and the resolution of a large

number of conflicts (i.e., the game processes). For bidirectional pedestrian flow, lane formation and dissolution could happen frequently, especially under high-density pedestrians in the case of narrow areas. The stochastic factor contained in the model (and also in reality) may also greatly affect the observed results. These factors have complicated impacts on the statistics of Δt as a function of the flow ratio. For such an open and nonlinear interactive system, (whose theoretical solutions are usually notoriously difficult to yield), the authors currently failed to provide a solid theoretical analysis to its dynamical behavior.

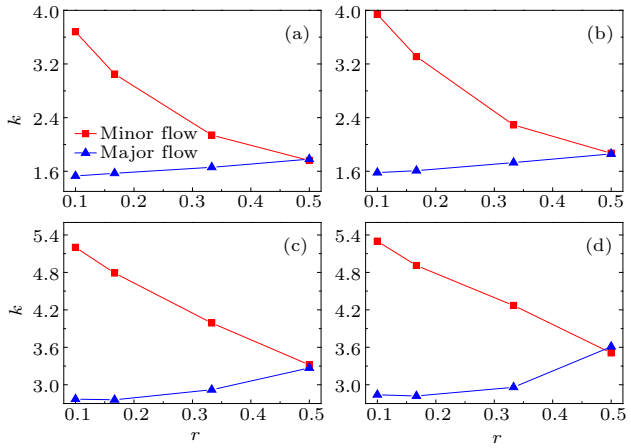


Fig. 3. The k value as a function of r, W, and δ: (a) $W = 10$, $\delta = 0.1$; (b) $W = 10$, $\delta = 0.7$; (c) $W = 5$, $\delta = 0.1$; (d) $W = 5$, $\delta = 0.7$.

A few previous empirical studies focused on the impact of the flow ratio on the efficiency of passing through the corridor boundary of pedestrians (characterized by the specific flow, i.e., the number of individuals crossing the boundary per unit time and per unit width). It was found that the efficiency of crossing boundary shows a non-monotonic behavior as a function of the flow ratio r. Note that our current model involves long-distance interaction (due to the introduction of the anticipation mechanism), which makes it inconvenient to adopt the periodic boundary conditions to calculate the efficiency of the pedestrian flow as in some previous studies.[19,20] Alternatively, we here check how the jam probability is affected by the flow ratio r in the bidirectional pedestrian flow.

Interestingly, we find that the jam probability displays a non-monotonous behavior with varying r, as shown in Fig. 4. The maximal jam probability appears at a moderate value around $r = 0.2$, which can be explained as follows. When $r \leq 0.1$, the situation is similar to the case of unidirectional flow, where the pedestrians in the minor flow can behave very flexibly such that they can move back and forth to reduce the conflict of movement, as already revealed by some experiments.[57] In such condition, the global jam is less likely to occur since conflicts among the pedestrians are effectively reduced. When $r \approx 0.5$, the magnitude of the pedestrian flow in both directions is nearly equal, which allows the anticipation mechanism to work well. As such, the lane formation in the system becomes relatively easy so that the global jam is also unlikely to happen as long as the width of the corridor is not too small.[51] By contrast, for $r \approx 0.2$, the number of pedestrians in the minor flow is not sufficiently small to enable them move flexibly, as compared to the case of $r \leq 0.1$. Moreover, the anticipation effect of pedestrians in the major flow is also relatively weak as compared to the case of $r \approx 0.5$, since there are not enough pedestrians walking toward the opposite direction. Taken together, the combined effect of the above two factors leads to the emergence of maximal jam probability in the region of $r \approx 0.2$.

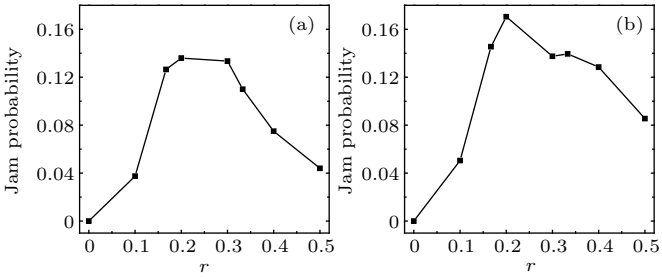


Fig. 4. Jam probability as a function of r and δ with $W = 5$: (a) $\delta = 0.1$, (b) $\delta = 0.7$.

As for the impact of δ, HD games are played under $\delta = 0.1$. Even if the best response in the HD game is to behave differently with the opponents, mutual cooperation can result in a greater collective payoff than mutual defection. Such a sufficient small reduced payoff δ will just severely impede the movement of defectors. Thus, pedestrians tend to become cooperators (especially when pedestrians have a high level of rationality $\kappa = 0.01$), which makes conflict-solving easier. While PD games are played around $\delta = 0.7$, more than one defectors in a conflict are expected to be observed, leading to the lowest probability of solving a conflict, thus the formation of lanes is more difficult, so the jam probability increases.[34,51]

Here we would like to remind that our above results can be quantitatively influenced by the other details of the proposed model, though the qualitative properties of our presented results are preserved. For example, as the system length decreases, the k value [the fitting exponent in Eq. (14)] of minor flow increases, while the k value of major flow decreases. As for the initial density, we also find that the difference in k value between major and minor flows increases with the pedestrian density. Moreover, if a random initial configuration is used, the change in k values is similar to the effect caused by density due to the fact that the individuals have, on average, more available space. Since thorough discussions on the effect of these parameters (system length, initial density, and random initial configuration) slightly go beyond the scope of our current work, we just summarize our further results of these issues in the SM to better serve the readers.

Figure 5 shows the time series of n_c for different combinations of parameters r and δ. In the case of balanced flow

$r = 0.5$, the anticipation mechanism can play a useful role before the two types of pedestrians encounter, where n_c rises to a relatively high level in a short time. After the conflicts begin to be resolved consecutively, lanes are formed in the system, and n_c will naturally decline to a low level. By comparison, for $r \approx 0.2$, as discussed above, the anticipation effect of the pedestrians in the major flow is relatively weak, conflicts occur more frequently when the distance between the two types of pedestrians is very close, thus there is no enough space and time to form the lanes. For this reason, n_c remaining at larger values lasts for a longer term, in comparison to the case of $r = 0.5$, which is also in line with the result of jam probability for $r \approx 0.2$.

From Figs. 5(a) and 5(b), we also note that n_c for $\delta = 0.1$ is always lower than that in the case of $\delta = 0.7$ for each value of r. Since $\delta = 0.7$ gives rise to the weakest ability of conflict resolution of the pedestrians,[51] our results indicate explicitly that collision avoidance mechanism (due to the anticipation of the pedestrians) works more efficiently when the frequency of cooperation in the two type pedestrians is high.

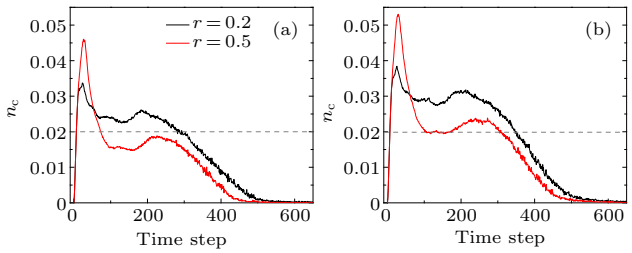


Fig. 5. Time series of n_c with $r = 0.2$ and $r = 0.5$ for $W = 5$: (a) $\delta = 0.1$, (b) $\delta = 0.7$. The gray dash lines are guides for the eyes.

It is worth noting that some previous studies have pointed out that the dynamics of pedestrian counterflow is quite special for $r \approx 0.2$, where the direction change, collision avoidance, and the speed decrease of the pedestrians show large fluctuations.[43,53,57] Murakami *et al.* also implied that avoidance maneuvers are normally a cooperative process and that mutual anticipation between pedestrians facilitates the pattern formation among the moving pedestrians.[58] Our simulation results can reproduce the properties of the empirical observations qualitatively, which can be basically explained by the collision avoidance mechanism due to anticipation of the pedestrians. It should also be pointed out that the non-monotonic behavior of jam probability with the flow ratio r is related to the specific parameters we choose here. If the anticipation mechanism is very strong or the corridor width is large enough (such as $k_A = 5$, $W = 5$), jam is almost impossible to occur.[51] Nonetheless, in a very narrow corridor with dense pedestrians, jam is almost an inevitable event. Although the decrease of the jam probability can be observed under low flow ratio r and large k_A value, it does not display a non-monotonic behavior with r, as shown in Fig. 6.

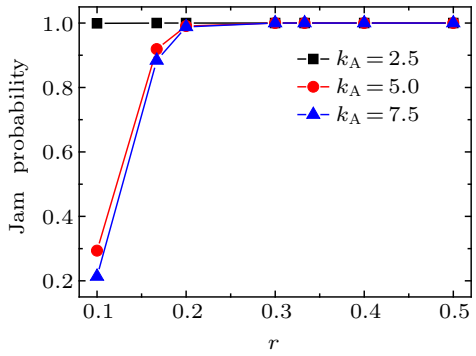


Fig. 6. Jam probability as a function of flow ratio. Parameters: $W = 2$, $\rho = 0.9$, $\delta = 0.7$.

In order to further verify the above explanations, we present in Fig. 7(a) the jam probability as a function of the flow ratio r for $\lambda = 0.65$ and $\lambda = 0.95$. It is shown that when the pedestrians cannot acquire information far enough away (i.e., small λ, similar to the experiments conducted by Murakami *et al.*[58]), the jam probability is significantly increased. Meanwhile, for larger λ value, the quantity n_c is lower, see Fig. 7(b), because in such situation the lanes are easily to form, and once formed, overtaking behavior of the pedestrians in the subsequent movements is highly suppressed.

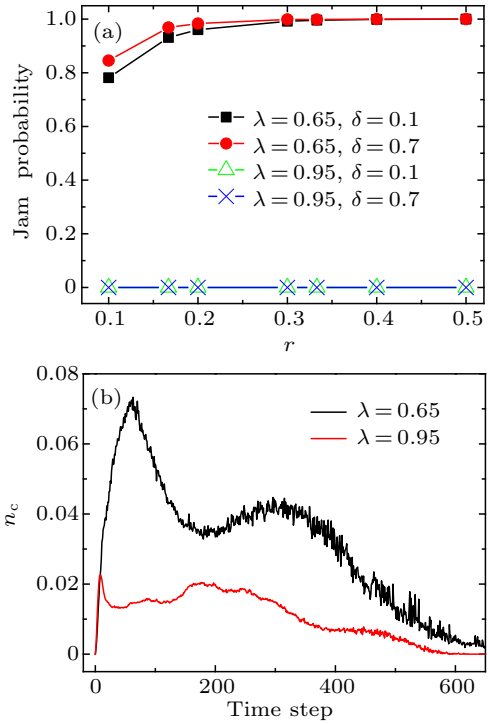


Fig. 7. (a) For $W = 5$, the jam probability as a function of r for $\lambda = 0.65$ and $\lambda = 0.95$; (b) n_c for the two λ values. Other parameters: $r = 0.2$, $\delta = 0.1$.

In Fig. 8, we plot the jam probability as a function of the parameter λ. It can be seen that the non-monotonic behavior can only be observed within a specific range of λ values (the jam probability for $r = 0.2$ is higher than that for $r = 0.5$). Thus, if we consider this non-monotonic phenomenon to be one of the characteristics of real pedestrian flow, $\lambda \in (0.75, 0.9)$ should be the reasonable range of values in this model.

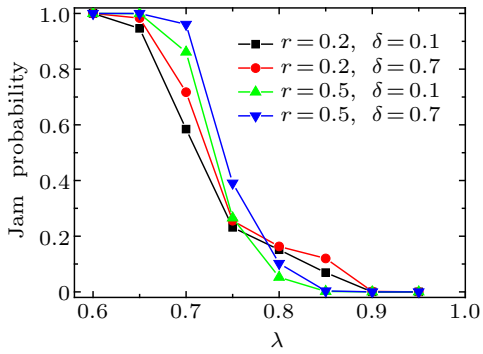


Fig. 8. For several combinations of r and δ, jam probability as a function of λ, $W = 5$.

5. Conclusions and outlook

In summary, by combining the floor field model and the game theory, we have studied the counterflow dynamics of pedestrians moving in a straight corridor. The pedestrians choose their next moving positions according to both the static and the anticipation floor field. Whenever competing for a same target cell during the walking process, an one-shot game is played among the involved pedestrians, and the payoffs reaped by them determine directly their probabilities to occupy the preferred vacant site.

We have investigated the statistical distribution of the time interval between two consecutive pedestrians leaving the system successively (denoted as Δt). We found that the CCDF(Δt) satisfies the stretched exponential law, and the decay exponent k index of the minor and major pedestrian flows is inversely affected by the flow ratio r. Unfortunately, it is difficult for us to find a satisfactory explanation of the intuitive physical meaning of this result at this time. We hope that this problem can be addressed in a future work. Still, it is worth mentioning that, the stretched exponential distribution indicates the deviations from a power law description, i.e., the corresponding phenomenon is not only caused by a finite-size correction. Sometimes it is associated with some multiplicative processes, which often constitute zeroth-order descriptions of a large variety of physical systems, exhibiting anomalous probability distribution functions and relaxation behaviors. In addition to describing the discharge of capacitors and the dielectric spectrum of polymers, such stretched exponential distribution law can also be applied in some defect diffusion models, and attention attracting phenomena on complex networks.[59] As for the field of pedestrian motion, it was found that the traffic segregation exhibits structural instabilities characterized by the alternation of organized and disorganized states, where the lifetime of well-organized clusters of pedestrians follows a stretched exponential relaxation process.[60]

Interestingly, we also found a non-monotonic behavior of the jam probability with the flow ratio, where the worst jam appears around $r = 0.2$. This result is in agreement with some existing empirical observations, and can be explained by the anticipation-induced collision avoidance mechanism. The non-monotonic behavior of jam probability and stretched exponential distribution are related to the conflict resolution process (i.e., the game process). In addition to game parameters, the geometric factors of this model, such as corridor width and the flow ratio, will affect the local density of the crowd and the number of pedestrians involved in a conflict, thereby affecting the game played by pedestrians and these statistical results. Therefore, the findings of the non-monotonic behavior of jam probability and stretched exponential distribution are relevant to the introduction of game theory. One should also note that our model did not fully reproduce all experimental phenomena such as rotation range of pedestrians during collision.[53] It may be due to two main reasons. Firstly, for CA models based on discrete space and time, some crucial details of real motions of the pedestrians may not be included appropriately. Secondly, our current model might have oversimplified the interaction process and cognitive mechanism of the crowd. Under realistic conditions (assuming that our model can describe the real situation well), these factors may have potential correlations. For example, if the number of participants in a conflict (parameter n) is small, the degree of competition between defectors may decrease, i.e., $\delta = \delta(n)$. Besides, pedestrians may experience significant changes in their behavior patterns during their movement, such as gradually becoming restless when walking at a slower speed, which means parameters q, δ, and κ may all be some kind of function of time and the moving speed.

In the field of pedestrian collective motion, pedestrian counterflow in a straight corridor is one of the simplest scenes, and it is also a common scene in daily life, such as bidirectional traffic through crosswalks, indoor corridors, and going up and down stairs. In order to avoid crushes accidents when the pedestrian density is high and improve the efficiency of crowd traffic, we must understand in depth the mechanism of pedestrian flow movement. Since the surrounding environment and individuals' cognitive ability determine largely the results of pedestrians' behavior, the interplay between the two aspects plays a pivotal role in characterizing the dynamic behavior of the pedestrian counterflow. For the pedestrian counterflow scenario, our proposed model, which takes into account explicitly the cognitive capacity of the pedestrians (that is, the game process), might provide a promising way to depict the crowd traffic in a more systematic way.[45] We hope this study is helpful to link the different experimental observations on pedestrian counterflow to game theoretical models, and also helps to link the pedestrian movement system with other typical physical systems.

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