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Evolutionary dynamics of tax-based strong altruistic reward and punishment in a public goods game

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In public goods games, punishments and rewards have been shown to be effective mechanisms for maintaining individual cooperation. However, punishments and rewards are costly to incentivize cooperation. Therefore, the generation of costly penalties and rewards has been a complex problem in promoting the development of cooperation. In real society, specialized institutions exist to punish evil people or reward good people by collecting taxes. We propose a strong altruistic punishment or reward strategy in the public goods game through this phenomenon. Through theoretical analysis and numerical calculation, we can get that tax-based strong altruistic punishment (reward) has more evolutionary advantages than traditional strong altruistic punishment (reward) in maintaining cooperation and tax-based strong altruistic reward leads to a higher level of cooperation than tax-based strong altruistic punishment.

Keywords: evolutionary game theory, strong altruism, punishment, reward

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1. Introduction

How cooperative behavior develops has always been a question that has plagued the human race.[1–5] When self-interest conflicts with collective interest, some individuals will cause the collective goods to be damaged to maximize their interests, thus resulting in the "tragedy of the commons",[6,7] which is the so-called social dilemma.[8] Undoubtedly, individuals in a population will always choose to defect without other factors interfering, thus making cooperation and not natural selection. In recent decades, many scholars have studied what enables cooperation to emerge and be maintained among some individuals. Evolutionary game theory is commonly used to explore this question and has been widely applied in fields such as biology, physics, and economics.[9–19] Scholars typically study public goods games,[20–24] which is the classic multi-player prisoner's dilemma, and point out that incentives and punishments are effective mechanisms, thus enabling humans to overcome these challenges because the lure of rewards and the cost of punishments promotes cooperation among selfish individuals.[25–35] However, punishments and rewards come at a high cost to incentivize cooperation, so that second-order free-riding can occur. Therefore, finding effective incentives to promote cooperative behavior is necessary. To address social dilemmas, researchers have proposed several mechanisms, such as imitation,[36,37] Moran,[38,39] aspiration-driven,[40,41] reward,[42,43] and punishment.[44,45]

Sasaki and Unemi[46] suggest that a reward fund for cooperative behavior can avoid conflicts between contributors and second-order free-riders. Cooperation with free-riders can be well maintained under certain conditions when a strong altruistic rewarder exists in a public goods game (PGG). As an alternative strategy, pool punishment is considered as a potential way to avoid exploiting free-riders, and such a punishment pool contributes to both the public and punishment pools. Based on this research, we can propose a strong altruistic punisher that punishes not only the defector but also the cooperator. To the best of our knowledge, few scholars have so far studied the evolutionary dynamics of cooperation in strongly altruistic punishing groups.

In real life, governments set up specialized punishment and reward agencies, and collect a fixed share of the tax value to punish evil people or reward people of good character, such as mercenaries.[47] Previous scholars have examined several models that contain taxes in PPGs. For example, Wang *et al.*[48] introduced tax factors in pure punishment and pure reward, and their study proved that tax-based pure punishment (reward) is more effective than traditional pure punishment (reward) in solving cooperation problems. Shen *et al.*[49] proposed a tax-based model for rewarding cooperators, and comprehensively analyzed strong and weak altruism strategies. Their results proved that strong altruism rewards cooperators more frequently than weak altruism rewards cooperators. This paper proposes a tax-based model of punishers and rewarders, and introduces strong altruistic strategies to analyze cooperation.

Therefore, we introduce the tax factor into the strong altruistic punishment strategy and the strong altruistic reward strategy. We argue that each participant must provide a fixed tax amount per capita incentive to incentivize strong altruistic punishment and strong altruistic reward enforcement. The rest of this paper is organized as follows. In Section 2, we present two mathematical models based on the PGG game: tax-based

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strong altruistic punisher and tax-based strong altruistic rewarder, and analyze them theoretically utilizing replicating equations. Then, in Section 3, we provide the main theoretical results and verify them numerically. Finally, in Section 4, we summarize our findings.

2. Models and methods

2.1. Public goods game

This paper considers a group of individuals in an infinite and well-mixed population, where the group with population size M $(M>1)$ participates in the PGG. Each individual has two choices in this game, where the individual either chooses to cooperate by contributing a fixed cost c $(c>0)$ to the public pool or chooses to defect by contributing nothing to the public pool. The total contribution from cooperators in the population is multiplied by the enhancement factor r $(1<r<N)$ and then distributed equally among all participants in the population. Therefore, if every participant chooses to cooperate, everyone in the population receives the maximum benefit $(r-1)c$. On the contrary, cooperators incur costs, so the defect choice is optimal for participants seeking higher payoffs. If everyone does it, then they do not get the payoff.

2.2. Tax-based strong altruistic punishment

Next, we consider another strategy, tax-based strong altruistic punishment. Tax-based strong altruistic punishers must also contribute c to the common pool. They also have to contribute to the punishment pool a certain amount of H_{SP} $(H_{SP}>0)$, which makes every defector impose a penalty ω $(\omega>0)$. Under the strong altruism condition, they must charge penalty ε $(\varepsilon>0)$ on second-order free-riders (cooperators). For simplicity of calculation, we assume $\omega>\varepsilon$. To increase the intensity of cooperation, we introduce a tax factor into the model to improve the punisher's punishment. Specifically, all participants in the population must pay a certain amount of tax A into the tax pool. The total tax in the tax pool is divided equally among all punishers. Based on this description, the desired payoffs of the C, D, and SP strategies in the population are

$$\begin{cases} \Pi_C = \dfrac{rc\,(M_C+M_{SP}+1)}{M} - c - A - \varepsilon M_{SP}, \\ \Pi_D = \dfrac{rc\,(M_C+M_{SP})}{M} - A - \omega M_{SP}, \\ \Pi_{SP} = \dfrac{rc\,(M_C+M_{SP}+1)}{M} - c - A - H_{SP} + \dfrac{AM}{M_{SP}+1}, \end{cases} \tag{1}$$

where M_C, M_D and M_{SP} denote the number of individuals in the population who adopt strategies C, D and SP, respectively. ωM_{SP} and εM_{SP} are the number of penalties that defectors and cooperators receive from each strong altruistic punisher in the population, respectively. In particular, when $A=0$, it reverts to the case of a traditional strong altruistic punishment.

2.3. Tax-based strong altruistic reward

Let us examine another strategy next: tax-based strong altruistic reward. Like tax-based strong altruistic punishers, tax-based strong altruistic rewarders need to contribute c to the common pool, and they also have to contribute to the punishment pool a certain amount of H_{SR} $(H_{SR}>0)$. Accordingly, for each additional reward, the cooperators' payoffs increase by γ $(\gamma>0)$, and they also, conditional on their strong altruism, cause the payoffs of the strong altruistic rewarders to increase by η $(\eta>0)$. Here, we assume that $\gamma>\eta$. In order to increase the intensity of cooperation, we introduce a tax factor into the model to improve the rewarder's reward. Specifically, all participants in the population must pay a certain amount of tax A into the tax pool. The total tax in the tax pool is divided equally among all rewarders. Based on this description, the desired payoffs of the C, D, and SR strategies in the population are

$$\begin{cases} \Pi_C = \dfrac{rc\,(M_C+M_{SR}+1)}{M} - c - A + \gamma M_{SR}, \\ \Pi_D = \dfrac{rc\,(M_C+M_{SR})}{M} - A, \\ \Pi_{SR} = \dfrac{rc\,(M_C+M_{SR}+1)}{M} - c - A - H_{SR} \\ \qquad + \dfrac{AM}{M_{SR}+1} + \eta M_{SR}, \end{cases} \tag{2}$$

where M_C, M_D and M_{SR} denote the number of individuals in the population who employ strategies C, D, and SR, respectively. γM_{SR} and ηM_{SR} are the total number of rewards received by cooperators and strongly altruistic cooperators from each strongly altruistic rewarder in the population, respectively. In particular, when $A=0$, it reverts to the case of a traditional strong altruistic reward.

2.4. Replication equations

We then examined the evolutionary dynamics of each of the three strategies C, D, and J ($J=SP$ or SR) using the replication equation,[50] their frequencies being x_1, x_2, and x_3, respectively, where the conditions are $x_1,x_2,x_3>0$ and $x_1+x_2+x_3=1$. Thus, the replication equation is

$$\begin{cases} \dot{x}_1 = x_1\,(F_C - \bar{F}), \\ \dot{x}_2 = x_2\,(F_D - \bar{F}), \\ \dot{x}_3 = x_3\,(F_J - \bar{F}), \end{cases} \tag{3}$$

where F_C, F_D, and F_J are the expected payoffs to C, D, and J, respectively, and $\bar{F} = x_1F_C + x_2F_D + x_3F_J$ denotes the expected payoffs to the population. Therefore, the expected payoffs for C, D, and J, respectively, are as follows:

$$F_L = \sum_{M_C=0}^{M-1}\sum_{M_D=0}^{M-M_C-1} \binom{M-1}{M_C}\binom{M-M_C-1}{M_D} \times x_1^{M_C}x_2^{M_D}x_3^{M-M_C-M_D-1}\prod_L, \quad L=C,D,J, \tag{4}$$

where $\binom{M-1}{M_C}\binom{M-M_C-1}{M_D}x_1^{M_C}x_2^{M_D}x_3^{M-M_C-M_D-1}$ describes the probability of having M_C cooperators, M_D defectors, and M_J tax-based strong altruistic punishers or rewarders among $M-1$ participants.

3. Results

We have used numerical calculations to investigate the evolution dynamics of the three strategies C, D, and J ($J = SP$ or SR) separately and further analyzed them according to the resulting conditions. Here, we first begin by considering tax-based strong altruistic punishment. The computational procedure in Appendix A demonstrates the theoretical analysis of the system. Figure 1 shows the flowchart within simplex S_3 at the tax value. Figure 1(a) shows the case without a tax, i.e., $A = 0$, and Fig. 1(b) shows the case with a tax, i.e., $A = 0.2$. The comparative analysis is then performed with and without a tax to see how including a tax promotes and sustains cooperation. Consider first the case of no tax, i.e., $A = 0$ (Fig. 1(a)). When $A = 0$, by analyzing the whole dynamical system, we get the unstable points as G_1 and G_2 at the DP-edge and CP-edge, respectively, and also the three vertices $C(1,0,0)$, $D(0,1,0)$, and $SP(0,0,1)$ respectively are analyzed for stability. We can get that only $D(0,1,0)$ is stable, so all individuals of the whole system move towards the state of the defector (vertex D). This suggests that in the absence of taxes, the strong altruistic punishers are not punishing enough to sustain the emergence of cooperation. The strongly altruistic punisher contributes c to the common pool and contributes H_{SP} to the punishment pool. The higher costs cause the strong altruistic punisher not to survive, which causes the model to change to a classical two-strategy game. In the end, only the defector survives. However, when the tax is large enough to offset the cost of punishment, the strong altruistic punisher receives a sizeable compensation that makes it superior to other strategies, thus allowing the strong altruistic punisher to exist and to be able to coexist with the cooperator. With the introduction of taxes (Fig. 1(b)), the system will move towards the coexistence of cooperators and strong altruistic punishers so that cooperation can survive. Here, it can be seen that the introduced tax factor can remove defectors (first-order free-riders) from the population. Still, there is no way to eliminate cooperators (second-order free-riders). Thus, taxes not only enable strong altruistic punishers to survive but also effectively deter defectors and enable cooperators to survive. In particular, we denote by G_3 the boundary equilibrium point on the side of CP, i.e., $(1-v_1,0,v_1)$, and $0 < v_1 < 1$. It is determined by the real root v_1 of the function $H_{SP} = \frac{A\left[1-(1-x_3)^M\right]}{x_3} + \varepsilon(M-1)x_3$ and is stable when $v_1 > \frac{1}{(\omega-\varepsilon)(M-1)}\frac{(M-r)c}{M}$. Thus, our numerical results are consistent with theoretical results suggesting that tax-based strong altruistic punishment has an evolutionary advantage over strong altruistic punishment in maintaining cooperation.

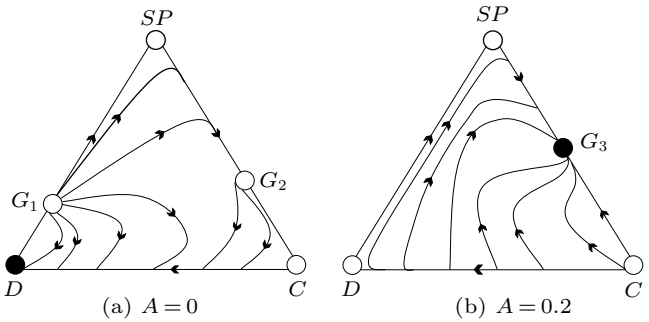


Fig. 1. The triangles represent the state space $S_3 = \{(x_1,x_2,x_3) : x_1,x_2,x_3 \geq 0$, and $x_1+x_2+x_3 = 1\}$, where x_1, x_2, x_3 are the frequencies of C, D, and SP, respectively. Filled circles represent stable fixed points, and open circles represent unstable fixed points. The tax value of the left-hand graph is (a) $A = 0$ and the tax value of the right-hand graph is (b) $A = 0.2$. The other parameters in the model are $M = 15$, $r = 3$, $c = 1$, $H_{SP} = 2.5$, $\omega = 0.7$ and $\varepsilon = 0.5$.

Next, we consider tax-based strong altruistic reward. The computational procedure in Appendix B demonstrates the theoretical analysis of the system. Similarly, Fig. 3 shows the flowchart within simplex S_3 at the tax value. Figure 3(a) shows the case without a tax, i.e., $A = 0$, and Fig. 3(b) shows the case with a tax, i.e., $A = 0.2$. When $A = 0$, by analyzing the whole dynamical system, we get the unstable points as G_4 at the DR-edge, and the three vertices $C(1,0,0)$, $D(0,1,0)$, and $SR(0,0,1)$ are also analyzed for stability. We can get that only $D(0,1,0)$ is stable, so all individuals of the whole system move towards the state of the defector (vertex D). This suggests that cooperation can be inhibited and that the rewards provided by strong altruistic rewarders are not sufficient to sustain the emergence of cooperation. The strongly altruistic rewarder contributes c to the common pool and H_{SR} to the reward pool. The higher costs make the strong altruistic rewarder unviable, leading to the model's degeneration into a classic two-strategy game. In the end, only the defector (D) survives. With the introduction of the tax (Fig. 3(b)), the strong altruistic rewarder receives a substantial compensation that makes it superior to other strategies when the tax is large enough to offset the cost of the reward, thus allowing the strong altruistic rewarder to exist and to be able to coexist with the cooperator. The system will evolve to a state where cooperators and tax-based strong altruistic rewarders coexist, which can show that cooperation in the group can be maintained. In particular, we have a bounded equilibrium $(1-v_2,0,v_2)$ and $0 < v_2 < 1$ denoted by G_5. It is determined by the real root v_2 of the function

$$H_{SR} = \frac{A\left[1-(1-x_3)^M\right]}{x_3} + \eta(M-1)x_3 - \gamma(M-1)x_3,$$

and is stable when $v_2 > \frac{1}{\gamma(M-1)}\frac{(M-r)c}{M}$. Thus, our numerical results are consistent with theoretical results suggesting that

strong tax-based altruistic rewards have an evolutionary advantage over strong altruistic rewards in maintaining cooperation.

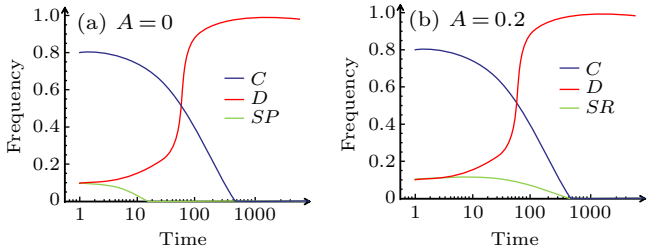


Fig. 2. Time evolution of the three strategies C, D, and SP for different tax values A. Other parameters: $M=15$, $r=3$, $c=1$, $H_{SP}=2.5$, $\omega=0.7$, and $\varepsilon=0.5$. Initial conditions: $(x_1,x_2,x_3)=(0.8,0.1,0.1)$. The tax value of the left-hand graph is (a) $A=0$ and the tax value of the right-hand graph is (b) $A=0.2$.

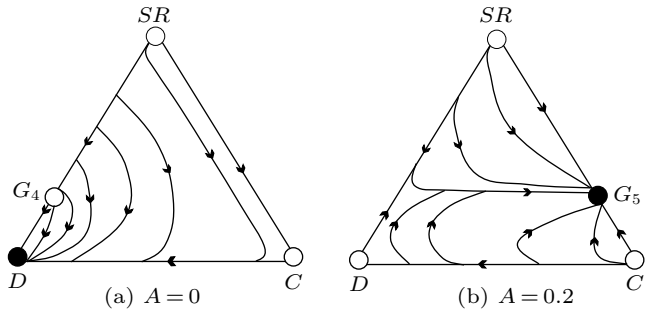


Fig. 3. The triangles represent the state space $S_3=\{(x_1,x_2,x_3): x_1,x_2,x_3\geq 0, \text{ and } x_1+x_2+x_3=1\}$, where x_1, x_2, x_3 are the frequencies of C, D, and SR, respectively. The tax value of the left-hand graph is (a) $A=0$ and the tax value of the right-hand graph is (b) $A=0.2$. The other parameters in the model are $M=15$, $r=3$, $c=1$, $H_{SR}=2.5$, $\gamma=0.7$, and $\eta=0.5$.

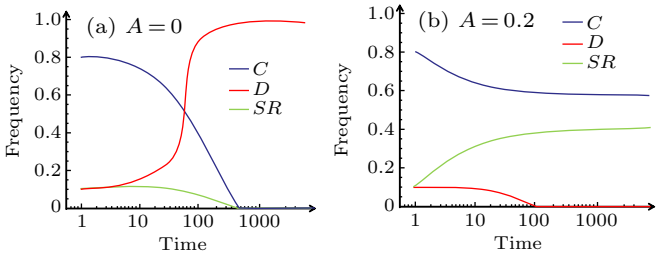


Fig. 4. Time evolution of the three strategies C, D and SR for different tax values A. Other parameters: $M=15$, $r=3$, $c=1$, $H_{SR}=2.5$, $\gamma=0.7$, and $\eta=0.5$. Initial conditions: $(x_1,x_2,x_3)=(0.8,0.1,0.1)$. The tax value of the left-hand graph is (a) $A=0$ and the tax value of the right-hand graph is (b) $A=0.2$.

Finally, this paper compares tax-based strong altruistic punishment with tax-based strong altruistic reward. The theoretical results show that equilibrium point $(1-v_I,0,v_I)$ on side CI is stable if $v_I>\frac{1}{\theta_I(M-1)}\frac{(M-r)c}{M}$, where the values of $I=1$ or 2, $\theta_1=\omega-\varepsilon$ and $\theta_2=\gamma$. v_1 and v_2 are respectively determined by functions

$$H_{SP}(x_3)=\frac{A\left[1-(1-x_3)^M\right]}{x_3}+\varepsilon(M-1)x_3,$$

$$H_{SR}(x_3)=\frac{A\left[1-(1-x_3)^M\right]}{x_3}+\eta(M-1)x_3-\gamma(M-1)x_3.$$

Based on these two functions, it can be obtained that the value of v_1 at the boundary equilibrium of CP-edge is greater than the value of v_2 at the boundary equilibrium of CR-edges. From Figs. 2(b) and 4(b), it can be seen that the frequency of cooperators in Fig. 4(b) is greater than the frequency of cooperators in Fig. 2(b). Therefore, this result can also be derived from the numerical analysis. It is confirmed that tax-based strong altruistic reward can lead to higher cooperation levels than tax-based strong altruistic punishment.

4. Conclusion

In this paper, we consider separately strong altruistic punishment and strong altruistic reward, and introduce tax factors into the model to analyze them. In the real world, the government gives a fixed share of the tax value to specialized punishing and rewarding institutions, causing these institutions to punish offenders or reward people of good character. In this article, we consider the same taxes in the field of evolutionary games as they would in real life. Our study finds that strong altruistic punishment (reward) is sensitive to taxes, which can lead to a situation where cooperators and punishers (rewarders) coexist, as shown in Figs. 1(b) and 3(b). Cooperative behavior is incentivized by including taxes and the equal distribution of tax revenues to the strong altruistic punishers (rewarders).

This study's results suggest that tax-based punishment (reward) has an evolutionary advantage over traditional tax-based punishment (reward). Tax-based punishment (reward) can effectively promote the emergence of cooperative behaviors. When there is no tax, individuals in the population all turn to the defection strategy, resulting in the cooperation strategy not surviving and the defection strategy becoming dominant. This result suggests that sufficient tax compensation can promote the emergence of cooperation. Most studies on PGG indicate that punishment is a powerful tool to prevent free-rider behavior and that punishment is stronger than rewards. However, our study shows that tax-based strong altruistic reward is more vital to promoting cooperation than tax-based strong altruistic punishment when the tax conditions are all the same. This illustrates the strong altruistic reward of taxes that better promotes cooperation. Overall, the tax-based model of strong altruistic punishment (reward) considered in this paper promotes cooperation and contributes to a better future study and understanding of the issue.

Appendix A: Tax-based strong altruistic punisher

By calculating Eq. (4), we can obtain the expected returns for C, D, and SP as

$$\begin{cases} F_C = \dfrac{rc\left[(M-1)x_1+(M-1)x_3\right]}{M} - \dfrac{(M-r)c}{M} - A - \varepsilon(M-1)x_3, \\ F_D = \dfrac{rc\left[(M-1)x_1+(M-1)x_3\right]}{M} - A - \omega(M-1)x_3, \\ F_{SP} = \dfrac{rc\left[(M-1)x_1+(M-1)x_3\right]}{M} - \dfrac{(M-r)c}{M} - A - H_{SP} + \dfrac{A\left[1-(1-x_3)^M\right]}{x_3}, \end{cases} \tag{A1}$$

where $(M-1)x_1$ and $(M-1)x_3$ denote the expected number of cooperators and strong altruistic punishers, respectively, for individuals in the $M-1$ population, $\varepsilon(M-1)x_3$, and $\omega(M-1)x_3$ denote the expected fines for cooperators and defectors, respectively, and $(1-x_3)^M$ denotes the probability that there are no punishers in the population.

Because of $x_1 = 1-x_2-x_3$, we rewrite the replicated equation of Eq. (3) as

$$\begin{cases} \dot{x}_2 = \psi_1(x_2,x_3) = x_2\left[x_3(F_C-F_{SP}) - (1-x_2)(F_C-F_D)\right], \\ \dot{x}_3 = \varphi_1(x_2,x_3) = x_3\left[x_2(F_C-F_D) - (1-x_3)(F_C-F_{SP})\right], \end{cases} \tag{A2}$$

where

$$\begin{cases} F_C - F_D = \omega(M-1)x_3 - \varepsilon(M-1)x_3 - \dfrac{(M-r)c}{M}, \\ F_C - F_{SP} = H_{SP} - \varepsilon(M-1)x_3 - \dfrac{A\left[1-(1-x_3)^M\right]}{x_3}. \end{cases} \tag{A3}$$

Based on the above equations, we analyze the existence and stability of the equilibrium in further detail in Appendices A1 and A2, respectively.

A1. The existence of equilibria

Equilibrium of the system is determined by the roots of $\psi_1(x_2,x_3)=0$ and $\varphi_1(x_2,x_3)=0$. The three vertex immobilizers of $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$ are obtained by computation. Next, we study the simplex S_3.

(1) The CD-edge $(x_3=0, 0<x_1,x_2<1, x_1+x_2=1)$

Because of $x_3=0$ and $x_1+x_2=1$, the system of equations described by Eq. (A2) reduces to $\dot{x}_2 = x_2(1-x_2)(F_D-F_C) = x_2(1-x_2)\frac{(M-r)c}{M} > 0$. Thus, there is no fixed point on the CD-edge, and the dynamic direction runs from C to D.

(2) The DP-edge $(x_1=0, 0<x_2,x_3<1, x_2+x_3=1)$

Because of $x_1=0$ and $x_2+x_3=1$, the system of equations described by Eq. (A2) reduces to $\dot{x}_3 = x_3(F_{SP}-\bar{F}) = x_3(1-x_3)(F_{SP}-F_D)$, solving $F_{SP}=F_D$ has $H_{SP} = \frac{A\left[1-(1-x_3)^M\right]}{x_3} + \omega(M-1)x_3 - \frac{(M-r)c}{M}$. Therefore, there exists a boundary equilibrium $(0,1-\mu_1,\mu_1)$, where $0<\mu_1<1$ and $H_{SP} = \frac{A\left[1-(1-\mu_1)^M\right]}{\mu_1} + \omega(M-1)\mu_1 - \frac{(M-r)c}{M}$. Otherwise, there is no fixed point.

(3) The CP-edge $(x_2=0, 0<x_1,x_3<1, x_1+x_3=1)$

Because of $x_2=0$ and $x_1+x_3=1$, the system of equations described by Eq. (A2) reduces to $\dot{x}_2 = x_2(F_D-\bar{F}) = x_2(1-x_2)(F_{SP}-F_C)$, solving $F_{SP}=F_C$ has $H_{SP} = \frac{A\left[1-(1-x_3)^M\right]}{x_3} + \varepsilon(M-1)x_3$. Therefore, there exists a boundary equilibrium $(0,1-\nu_1,\nu_1)$, where $0<\nu_1<1$ and ν_1 is uniquely determined by equation $H_{SP} = \frac{A\left[1-(1-\nu_1)^M\right]}{\nu_1} + \varepsilon(M-1)\nu_1$. Otherwise, there is no fixed point.

(4) The interior of the simplex S_3 $(0<x_1,\ x_2,\ x_3<1,\ x_1+x_2+x_3=1)$

Solving $F_D=F_C$ has $x_3 = \frac{1}{(\omega-\varepsilon)(M-1)}\frac{(M-r)c}{M}$. Similarly, solving $F_{SP}=F_C$ has $H_{SP} = \frac{A\left[1-(1-x_3)^M\right]}{x_3} + \varepsilon(M-1)x_3$. Thus, there exists an interior immovable point $(1-\rho_1-\sigma_1,\rho_1,\sigma_1)$, where $0<\rho_1,\sigma_1<1$ and, $0<\rho_1+\sigma_1<1$, $\sigma_1 = \frac{1}{(\omega-\varepsilon)(M-1)}\frac{(M-r)c}{M}$ and $H_{SP} = \frac{A\left[1-(1-\sigma_1)^M\right]}{\sigma_1} + \varepsilon(M-1)\sigma_1$. Otherwise, there is no fixed point.

A2. The stabilities of equilibria

By Eq. (A2), we get the Jacobian matrix as

$$J = \begin{pmatrix} \dfrac{\partial\psi_1(x_2,x_3)}{\partial x_2} & \dfrac{\partial\psi_1(x_2,x_3)}{\partial x_3} \\ \dfrac{\partial\varphi_1(x_2,x_3)}{\partial x_2} & \dfrac{\partial\varphi_1(x_2,x_3)}{\partial x_3} \end{pmatrix}, \tag{A4}$$

where

$$\begin{cases} \dfrac{\partial\psi_1(x_2,x_3)}{\partial x_2} = (2x_2-1)(F_C-F_D) + x_3(F_C-F_{SP}) + x_2\left[(x_2-1)\dfrac{\partial(F_C-F_D)}{\partial x_2} + x_3\dfrac{\partial(F_C-F_{SP})}{\partial x_2}\right], \\ \dfrac{\partial\psi_1(x_2,x_3)}{\partial x_3} = x_2\left[(F_C-F_{SP}) + (x_2-1)\dfrac{\partial(F_C-F_D)}{\partial x_3} + z\dfrac{\partial(F_C-F_{SP})}{\partial x_3}\right], \\ \dfrac{\partial\varphi_1(x_2,x_3)}{\partial x_2} = x_3\left[(F_C-F_D) + (x_3-1)\dfrac{\partial(F_C-F_{SP})}{\partial x_2} + x_2\dfrac{\partial(F_C-F_D)}{\partial x_2}\right], \\ \dfrac{\partial\varphi_1(x_2,x_3)}{\partial x_3} = (2x_3-1)(F_C-F_{SP}) + x_2(F_C-F_D) + x_3\left[(x_3-1)\dfrac{\partial(F_C-F_{SP})}{\partial x_3} + x_2\dfrac{\partial(F_C-F_D)}{\partial x_3}\right]. \end{cases} \tag{A5}$$

We analyze the stability of the immovable points of the vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$, as well as the boundary immovable points and the interior immovable points on the CP-edge and DP-edge. This analysis will be carried out by studying the Jacobian matrix. Here, we denote the fixed points $\left(x_1^0,x_2^0,x_3^0\right)$, where $0\le x_1^0,x_2^0,x_3^0\le 1$ and $x_1^0+x_2^0+x_3^0=1$.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1,0,0)$:

The Jacobian matrix is

$$J=\begin{pmatrix}\dfrac{(M-r)c}{M} & 0\\ 0 & AM-H_{SP}\end{pmatrix}, \tag{A6}$$

and thus $(1,0,0)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,1,0)$:

The Jacobian matrix is

$$J=\begin{pmatrix}-\dfrac{(M-r)c}{M} & H_{SP}-AM\\ 0 & AM-H_{SP}-\dfrac{(M-r)c}{M}\end{pmatrix}, \tag{A7}$$

and thus $(0,1,0)$ is stable if $H_{SP}>AM-\frac{(M-r)c}{M}$. While if $H_{SP}<AM-\frac{(M-r)c}{M}$, $(0,1,0)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,0,1)$:

The Jacobian matrix is

$$J=\begin{pmatrix}H_{SP}+\dfrac{(M-r)c}{M}-\omega(M-1)-A & 0\\ (\omega-\varepsilon)(M-1)-\dfrac{(M-r)c}{M} & H_{SP}-\varepsilon(M-1)-A\end{pmatrix}, \tag{A8}$$

and thus $(0,0,1)$ is stable if

$$H_{SP}<A+\min\left\{\omega(M-1)-\frac{(M-r)c}{M},\varepsilon(M-1)\right\}.$$

While if

$$H_{SP}>A+\min\left\{\omega(M-1)-\frac{(M-r)c}{M},\varepsilon(M-1)\right\},$$

$(0,0,1)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,1-\mu_1,\mu_1)$ on the *DP*-edge, where $H_{SP}=\frac{A\left[1-(1-x_3)^M\right]}{x_3}+\omega(M-1)x_3-\frac{(M-r)c}{M}$ and $0<\eta_1<1$:

The Jacobian matrix is

$$J=\begin{pmatrix}(1-\mu_1)\left[(\omega-\varepsilon)(M-1)\mu_1-\dfrac{(M-r)c}{M}\right] & a_{12}\\ \mu_1\left[(\omega-\varepsilon)(M-1)\mu_1-\dfrac{(M-r)c}{M}\right] & a_{22}\end{pmatrix}, \tag{A9}$$

where

$$\begin{aligned}a_{12} &= -(1-\mu_1)\left\{\frac{(M-r)c}{M}+\varepsilon(M-1)\mu_1\right.\\ &\quad\left.+\frac{A\left[N\mu_1(1-\mu_1)^M-1+(1-\mu_1)^M\right]}{\mu_1}\right\},\\ a_{22} &= \mu_1\left\{\omega(M-1)-\varepsilon(M-1)\mu_1-\frac{(M-r)c}{M}\right.\\ &\quad\left.+\frac{A(1-\mu_1)\left[M\mu_1(1-\mu_1)^M-1+(1-\mu_1)^M\right]}{{\mu_1}^2}\right\}.\end{aligned}$$

Here, based on the first order Taylor series expansions for a_{12} and a_{22}, we can obtain

$$\begin{aligned}a_{12} &= -(1-\mu_1)\left[\frac{(M-r)c}{M}+\varepsilon(M-1)\mu_1\right],\\ a_{22} &= \mu_1\left[\omega(M-1)-\varepsilon(M-1)\mu_1-\frac{(M-r)c}{M}\right],\end{aligned}$$

and the Jacobian matrix then simplifies to

$$J=\begin{pmatrix}(1-\mu_1)\left[(\omega-\varepsilon)(M-1)\mu_1-\dfrac{(M-r)c}{M}\right] & -(1-\mu_1)\left[\dfrac{(M-r)c}{M}+\varepsilon(M-1)\mu_1\right]\\ \mu_1\left[(\omega-\varepsilon)(M-1)\mu_1-\dfrac{(M-r)c}{M}\right] & \mu_1\left[\omega(M-1)-\varepsilon(M-1)\mu_1-\dfrac{(M-r)c}{M}\right]\end{pmatrix}, \tag{A10}$$

and the eigenvalues of the matrix are $H_{SP}-AM-\varepsilon(M-1)\mu_1$ and

$$(1-\mu_1)\left[H_{SP}-AM+\frac{(M-r)c}{M}-\varepsilon(M-1)\mu_1\right].$$

Since

$$(1-\mu_1)\left[H_{SP}-AM+\frac{(M-r)c}{M}-\varepsilon(M-1)\mu_1\right]>0,$$

so $(0,1-\mu_1,\mu_1)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1-\nu_1,0,\nu_1)$ on the *CP*-edge, where $H_{SP}=\frac{A\left[1-(1-\nu_1)^M\right]}{\nu_1}+\varepsilon(M-1)\nu_1$ and $0<\nu_1<1$:

The Jacobian matrix is

$$J=\begin{pmatrix}\dfrac{(M-r)c}{M}-(\omega-\varepsilon)(M-1)\nu_1 & 0\\ \nu_1\left[(\omega-\varepsilon)(M-1)v_P-\dfrac{(M-r)c}{M}\right] & a_{22}\end{pmatrix}, \tag{A11}$$

where

$$\begin{aligned}a_{22} &= \nu_1\left\{\varepsilon(M-1)(1-\nu_1)\right.\\ &\quad\left.+\frac{A(1-\nu_1)\left[M\nu_1(1-\nu_1)^{M-1}-1+(1-\nu_1)^M\right]}{\nu_1^2}\right\}.\end{aligned}$$

and thus $(1-\nu_1,0,\nu_1)$ is stable if $\nu_1>\frac{1}{(\omega-\varepsilon)(M-1)}\frac{(M-r)c}{M}$. While if $\nu_1<\frac{1}{(\omega-\varepsilon)(M-1)}\frac{(M-r)c}{M}$, $(1-\nu_1,0,\nu_1)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1-\rho_1-\sigma_1,\rho_1,\sigma_1)$ in the interior of simplex S_3, where $\sigma_1=\frac{1}{(\omega-\varepsilon)(M-1)}\frac{(M-r)c}{M}$, $H_{SP}=\frac{A\left[1-(1-\sigma_1)^M\right]}{\sigma_1}+\varepsilon(M-1)\sigma_1$ and $0<\rho_1,\sigma_1<1$:

The Jacobian matrix is

$$J=\begin{pmatrix}0 & a_{12}\\ 0 & a_{22}\end{pmatrix}, \tag{A12}$$

where

$$\begin{aligned} a_{12} =& -\rho_1\Bigg\{(\omega-\varepsilon)(M-1)(1-\rho_1)+\varepsilon(M-1)\sigma_1 \\ &+\frac{A\left[M\sigma_1(1-\sigma_1)^{M-1}-1+(1-\sigma_1)^M\right]}{\sigma_1}\Bigg\}, \\ a_{22} =& \sigma_1\Bigg\{(\omega-\varepsilon)(M-1)\rho_1+\varepsilon(M-1)(1-\sigma_1) \\ &+\frac{A(1-\sigma_1)\left[M\sigma_1(1-\sigma_1)^{M-1}-1+(1-\sigma_1)^M\right]}{\sigma_1^2}\Bigg\}. \end{aligned}$$

Here, based on the first order Taylor series expansions for a_{12} and a_{22}, and we can obtain

$$\begin{aligned} a_{12} &= -\rho_1\left[(\omega-\varepsilon)(M-1)(1-\rho_1)+\varepsilon(M-1)\sigma_1\right], \\ a_{22} &= \sigma_1\left[(\omega-\varepsilon)(M-1)\rho_1+\varepsilon(M-1)(1-\sigma_1)\right], \end{aligned}$$

and the Jacobian matrix then simplifies to

$$J=\begin{pmatrix} 0 & -\rho_1\left[(\omega-\varepsilon)(M-1)(1-\rho_1)+\varepsilon(M-1)\sigma_1\right] \\ 0 & \sigma_1\left[(\omega-\varepsilon)(M-1)\rho_1+\varepsilon(M-1)(1-\sigma_1)\right] \end{pmatrix}, \tag{A13}$$

and thus $(1-\rho_1-\sigma_1,\rho_1,\sigma_1)$ is unstable.

In particular, the traditional strong altruistic punishment equilibrium is stabilized as follows.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1,0,0)$:

The Jacobian matrix is

$$J=\begin{pmatrix} \frac{(M-r)c}{M} & 0 \\ 0 & -H_{SP} \end{pmatrix}, \tag{A14}$$

and thus $(1,0,0)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,1,0)$:

The Jacobian matrix is

$$J=\begin{pmatrix} -\frac{(M-r)c}{M} & H_{SP} \\ 0 & -H_{SP}-\frac{(M-r)c}{M} \end{pmatrix}, \tag{A15}$$

and thus $(0,1,0)$ is stable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,0,1)$:

The Jacobian matrix is

$$J=\begin{pmatrix} H_{SP}+\frac{(M-r)c}{M}-\omega(M-1) & 0 \\ (\omega-\varepsilon)(M-1)-\frac{(M-r)c}{M} & H_{SP}-\varepsilon(M-1) \end{pmatrix}, \tag{A16}$$

and thus $(0,0,1)$ is stable if

$$H_{SP}<\min\left\{\omega(M-1)-\frac{(M-r)c}{M},\varepsilon(M-1)\right\}.$$

While if

$$H_{SP}>\min\left\{\omega(M-1)-\frac{(M-r)c}{M},\varepsilon(M-1)\right\},$$

$(0,0,1)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,1-\mu_1,\mu_1)$ on the *DP*-edge, where $0<\mu_1<1$:

The Jacobian matrix is

$$J=\begin{pmatrix} (1-\mu_1)\left[(\omega-\varepsilon)(M-1)\mu_1-\frac{(M-r)c}{M}\right] & -(1-\mu_1)\left[\frac{(M-r)c}{M}+\varepsilon(M-1)\mu_1\right] \\ \mu_1\left[(\omega-\varepsilon)(M-1)\mu_1-\frac{(M-r)c}{M}\right] & \mu_1\left[\omega(M-1)-\varepsilon(M-1)\mu_1-\frac{(M-r)c}{M}\right] \end{pmatrix}, \tag{A17}$$

and the eigenvalues of the above matrix are $H_{SP}-\varepsilon(M-1)\mu_1$ and $(1-\mu_1)\left[H_{SP}+\frac{(M-r)c}{M}-\varepsilon(M-1)\mu_1\right]$, so $(0,1-\mu_1,\mu_1)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1-\nu_1,0,\nu_1)$ on the *CP*-edge, where $0<\nu_1<1$:

The Jacobian matrix is

$$J=\begin{pmatrix} \frac{(M-r)c}{M}-(\omega-\varepsilon)(M-1)\nu_1 & 0 \\ \nu_1\left[(\omega-\varepsilon)(M-1)\nu_1-\frac{(M-r)c}{M}\right] & \varepsilon(M-1)\nu_1(1-\nu_1) \end{pmatrix}, \tag{A18}$$

so $(1-\nu_1,0,\nu_1)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1-\rho_1-\sigma_1,\rho_1,\sigma_1)$ in the interior of simplex S_3:

The Jacobian matrix is

$$J=\begin{pmatrix} 0 & -\rho_1\left[(\omega-\varepsilon)(M-1)(1-\rho_1)+\varepsilon(M-1)\sigma_1\right] \\ 0 & \sigma_1\left[(\omega-\varepsilon)(M-1)\rho_1+\varepsilon(M-1)(1-\sigma_1)\right] \end{pmatrix}, \tag{A19}$$

and by the central prevalence theorem, we can judge $\left(x_1^0,x_2^0,x_3^0\right)=(1-\rho_1-\sigma_1,\rho_1,\sigma_1)$ to be unstable.

Appendix B: Tax-based strong altruistic rewarder

By calculating Eq. (4), we can obtain the expected returns for C, D and SR as

$$\begin{cases} F_C = \dfrac{rc\left[(M-1)x_1+(M-1)x_3\right]}{M} - \dfrac{(M-r)c}{M} - A + \gamma(M-1)x_3, \\ F_D = \dfrac{rc\left[(M-1)x_1+(M-1)x_3\right]}{M} - A, \\ F_{SR} = \dfrac{rc\left[(M-1)x_1+(M-1)x_3\right]}{M} - \dfrac{(M-r)c}{M} - A - H_{SR} + \dfrac{A\left[1-(1-x_3)^M\right]}{x_3} + \eta(M-1)x_3, \end{cases} \tag{B1}$$

where $(M-1)x_1$ and $(M-1)x_3$ denote the expected number of cooperators and strong altruistic rewarders, respectively, for individuals in the $M-1$ population, $\gamma(N-1)x_3$ and $\eta(N-1)x_3$ denote the expected rewards for cooperators and strong altruistic rewarders, respectively, and $(1-x_3)^M$ denotes the probability that there are no strong altruistic rewarders in the population.

Because of $x_1 = 1-x_2-x_3$, we rewrite the replicated equation of Eq. (3) as

$$\begin{cases} \dot{x}_2 = \psi_2(x_2,x_3) = x_2\left[x_3(F_C-F_{SR}) - (1-x_2)(F_C-F_D)\right], \\ \dot{x}_3 = \varphi_2(x_2,x_3) = x_3\left[x_2(F_C-F_D) - (1-x_3)(F_C-F_{SR})\right], \end{cases} \tag{B2}$$

where

$$\begin{cases} F_C - F_D = \gamma(M-1)x_3 - \dfrac{(M-r)c}{M}, \\ F_C - F_{SR} = H_{SR} - \dfrac{A\left[1-(1-x_3)^M\right]}{x_3} \\ \qquad\qquad - \eta(M-1)x_3 + \gamma(M-1)x_3. \end{cases} \tag{B3}$$

Based on these equations, we analyze the existence and stability of the equilibrium in further detail in Appendices B1 and B2, respectively.

B1. The existence of equilibria

Equilibrium of the system is determined by the roots of $\psi_2(x_2,x_3)=0$ and $\varphi_2(x_2,x_3)=0$. The three vertex immobilizers of $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ are obtained by computation. Next, we study the simplex S_3.

(1) The CD-edge $(x_3=0, 0<x_1,x_2<1, x_1+x_2=1)$

Because $x_3=0$ and $x_1+x_2=1$, the system of equations described by Eq. (B2) reduces to $\dot{x}_2 = x_2(1-x_2)(F_D-F_C) = x_2(1-x_2)\frac{(M-r)c}{M} > 0$. Thus, there is no fixed point on the CD-edge, and the dynamic direction runs from C to D.

(2) The DR-edge $(x_1=0, 0<x_2,x_3<1, x_2+x_3=1)$

Because $x_1=0$ and $x_2+x_3=1$, the system of equations described by Eq. (B2) reduces to $\dot{x}_3 = x_3(F_{SR}-\bar{F}) = x_3(1-x_3)(F_{SR}-F_D)$, solving $F_{SR}=F_D$ has $H_{SR} = \frac{A\left[1-(1-x_3)^M\right]}{x_3} + \eta(M-1)x_3 - \frac{(M-r)c}{M}$. Therefore, there exists a boundary equilibrium $(0,1-\mu_2,\mu_2)$, where $0<\mu_2<1$ and $H_{SR} = \frac{A\left[1-(1-\mu_2)^M\right]}{\mu_2} + \eta(M-1)\mu_2 - \frac{(M-r)c}{M}$. Otherwise, there is no fixed point.

(3) The CR-edge $(x_2=0, 0<x_1,x_3<1, x_1+x_3=1)$

Because $x_2=0$ and $x_1+x_3=1$, the system of equations described by Eq. (B2) reduces to $\dot{x}_2 = x_2(F_D-\bar{F}) = x_2(1-x_2)(F_{SR}-F_C)$, solving $F_{SR}=F_C$ has $H_{SR} = \frac{A\left[1-(1-x_3)^M\right]}{x_3} + \eta(M-1)x_3 - \gamma(M-1)x_3$. Therefore, there exists a boundary equilibrium $(1-\nu_2,0,\nu_2)$, where $0<\nu_2<1$ and ν_2 is uniquely determined by equation $H_{SR} = \frac{A\left[1-(1-\nu_2)^M\right]}{\nu_2} + \eta(M-1)\nu_2 - \gamma(M-1)\nu_2$. Otherwise, there is no fixed point.

(4) The interior of the simplex S_3 $(0<x_1,\ x_2,\ x_3<1,\ x_1+x_2+x_3=1)$

Solving $F_D=F_C$ has $x_3 = \frac{1}{\gamma(M-1)}\frac{(M-r)c}{N}$. Similarly, solving $F_{SR}=F_C$ has $H_{SR} = \frac{A\left[1-(1-x_3)^M\right]}{x_3} + \eta(M-1)x_3 - \gamma(M-1)x_3$. Thus, there exists an interior immovable point $(1-\rho_2-\sigma_2,\rho_2,\sigma_2)$, where $0<\rho_2,\sigma_2<1$, $0<\rho_2+\sigma_2<1$, $\sigma_2 = \frac{1}{\gamma(M-1)}\frac{(M-r)c}{M}$ and $H_{SR} = \frac{A\left[1-(1-\sigma_2)^M\right]}{\sigma_2} + \eta(M-1)\sigma_2 - \gamma(M-1)\sigma_2$. Otherwise, there is no fixed point.

B2. The stabilities of equilibria

By Eq. (B2), we get the Jacobian matrix as

$$J = \begin{pmatrix} \dfrac{\partial\psi_2(x_2,x_3)}{\partial x_2} & \dfrac{\partial\psi_2(x_2,x_3)}{\partial x_3} \\ \dfrac{\partial\varphi_2(x_2,x_3)}{\partial x_2} & \dfrac{\partial\varphi_2(x_2,x_3)}{\partial x_3} \end{pmatrix}, \tag{B4}$$

where

$$\begin{cases} \dfrac{\partial\psi_2(x_2,x_3)}{\partial x_2} = (2x_2-1)(F_C-F_D) + x_3(F_C-F_{SP}) + x_2\left[(x_2-1)\dfrac{\partial(F_C-F_D)}{\partial x_2} + x_3\dfrac{\partial(F_C-F_{SP})}{\partial x_2}\right], \\ \dfrac{\partial\psi_2(x_2,x_3)}{\partial x_3} = x_2\left[(F_C-F_{SP}) + (x_2-1)\dfrac{\partial(F_C-F_D)}{\partial x_3} + z\dfrac{\partial(F_C-F_{SP})}{\partial x_3}\right], \\ \dfrac{\partial\varphi_2(x_2,x_3)}{\partial x_2} = x_3\left[(F_C-F_D) + (x_3-1)\dfrac{\partial(F_C-F_{SP})}{\partial x_2} + x_2\dfrac{\partial(F_C-F_D)}{\partial x_2}\right], \\ \dfrac{\partial\varphi_2(x_2,x_3)}{\partial x_3} = (2x_3-1)(F_C-F_{SP}) + x_2(F_C-F_D) + x_3\left[(x_3-1)\dfrac{\partial(F_C-F_{SP})}{\partial x_3} + x_2\dfrac{\partial(F_C-F_D)}{\partial x_3}\right]. \end{cases} \tag{B5}$$

We analyze the stability of the immovable points of the vertices $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$, as well as the boundary immovable points and the interior immovable points on the *CR*-edge and *DR*-edge. This analysis will be carried out by studying the Jacobian matrix. Here, we denote the fixed points $\left(x_1^0,x_2^0,x_3^0\right)$, where $0 \le x_1^0,x_2^0,x_3^0 \le 1$ and $x_1^0+x_2^0+x_3^0=1$.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1,0,0)$:

The Jacobian matrix is

$$J=\begin{pmatrix} \dfrac{(M-r)c}{M} & 0 \\ 0 & AM-H_{SR} \end{pmatrix}, \tag{B6}$$

and thus $(1,0,0)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,1,0)$:

The Jacobian matrix is

$$J=\begin{pmatrix} -\dfrac{(M-r)c}{M} & H_{SR}-AM \\ 0 & AM-H_{SR}-\dfrac{(M-r)c}{M} \end{pmatrix}, \tag{B7}$$

and thus $(0,1,0)$ is stable if $H_{SR} > AM-\frac{(M-r)c}{M}$. While if $H_{SR} < AM-\frac{(M-r)c}{M}$, $(0,1,0)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,0,1)$:

The Jacobian matrix is

$$J=\begin{pmatrix} H_{SR}+\dfrac{(M-r)c}{M}-A-\eta\,(M-1) & 0 \\ \gamma(M-1)-\dfrac{(M-r)c}{M} & H_{SR}-A+(\gamma-\eta)\,(M-1) \end{pmatrix}, \tag{B8}$$

and thus $(0,0,1)$ is stable if

$$H_{SR} < A+\min\left\{\eta\,(M-1)-\frac{(M-r)c}{M},\eta\,(M-1)-\gamma(M-1)\right\}.$$

While if

$$H_{SR} > A+\min\left\{\eta\,(M-1)-\frac{(M-r)c}{M},\eta\,(M-1)-\gamma(M-1)\right\},$$

$(0,0,1)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,1-\mu_2,\mu_2)$ on the *DR*-edge, where $H_{SR}=\frac{A\left[1-(1-\mu_2)^M\right]}{\mu_2}+\eta\,(M-1)\,\mu_2-\frac{(M-r)c}{M}$ and $0<\mu_2<1$:

The Jacobian matrix is

$$J=\begin{pmatrix} (1-\mu_2)\left[\gamma(M-1)\,\mu_2-\dfrac{(M-r)c}{M}\right] & a_{12} \\ \mu_2\left[\gamma(M-1)\,\mu_2-\dfrac{(M-r)c}{M}\right] & a_{22} \end{pmatrix}, \tag{B9}$$

where

$$a_{12}=-(1-\mu_2)\left\{\frac{(M-r)c}{M}-(\gamma-\eta)\,(M-1)\,\mu_2+\frac{A\left[M\mu_2(1-\mu_2)^{M-1}-1+(1-\mu_2)^M\right]}{\mu_2}\right\},$$

$$a_{22}=\mu_2\left\{(\gamma-\eta)\,(M-1)\,\mu_2+\eta\,(M-1)-\frac{(M-r)c}{M}+\frac{A\,(1-\mu_2)\left[M\mu_2(1-\mu_2)^{M-1}-1+(1-\mu_2)^M\right]}{\mu_2^2}\right\}.$$

Here, based on the first order Taylor series expansions for

$$a_{12}=-(1-\mu_2)\left[\frac{(M-r)c}{M}-(\gamma-\eta)\,(M-1)\,\mu_2\right] \text{ and } a_{22}=\mu_2\left[(\gamma-\eta)\,(M-1)\,\mu_2+\eta\,(M-1)-\frac{(M-r)c}{M}\right],$$

and the Jacobian matrix then simplifies to

$$J=\begin{pmatrix} (1-\mu_2)\left[\gamma(M-1)\,\mu_2-\dfrac{(M-r)c}{M}\right] & -(1-\mu_2)\left[\dfrac{(M-r)c}{M}-(\gamma-\eta)\,(M-1)\,\mu_2\right] \\ \mu_2\left[\gamma(M-1)\,\mu_2-\dfrac{(M-r)c}{M}\right] & \mu_2\left[(\gamma-\eta)\,(M-1)\,\mu_2+\eta\,(M-1)-\dfrac{(M-r)c}{M}\right] \end{pmatrix}, \tag{B10}$$

and the eigenvalues of the matrix are $\gamma(M-1)\,\mu_2-\frac{(M-r)c}{M}$ and $\mu_2\,(1-\mu_2)\,\eta\,(N-1)$. Since $\mu_2\,(1-\mu_2)\,\eta\,(N-1)>0$, so $(0,1-\eta_R,\eta_R)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1-\nu_2,0,\nu_2)$ on the *CR*-edge, where $H_{SR}=\frac{A\left[1-(1-\nu_2)^M\right]}{\nu_2}+\eta\,(M-1)\,\nu_2-\gamma(M-1)\,\nu_2$ and $0<\nu_2<1$:

The Jacobian matrix is

$$J=\begin{pmatrix} \dfrac{(M-r)c}{M}-\gamma(M-1)\nu_2 & 0 \\ \nu_2\left[\gamma(M-1)\nu_2-\dfrac{(M-r)c}{M}\right] & a_{22}\end{pmatrix}, \tag{B11}$$

where

$$\begin{aligned} a_{22} =& -\nu_2(1-\nu_2)\Bigg\{(\gamma-\eta)(M-1) \\ &-\frac{A\left[M\nu_2(1-\nu_2)^{M-1}-1+(1-\nu_2)^M\right]}{\nu_2}\Bigg\},\end{aligned}$$

and thus $(1-\nu_2,0,\nu_2)$ is stable if $\nu_2>\frac{1}{\gamma(M-1)}\frac{(M-r)c}{M}$. While if $[\nu_2<\frac{1}{\gamma(M-1)}\frac{(M-r)c}{M}$, $(1-\nu_2,0,\nu_2)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1-\rho_2-\sigma_2,\rho_2,\sigma_2)$ in the interior of simplex S_3, where $\sigma_2=\frac{1}{\gamma(M-1)}\frac{(M-r)c}{M}$, $H_{SR}=\frac{A\left[1-(1-\sigma_2)^M\right]}{\sigma_2}+\eta(M-1)\sigma_2-\gamma(M-1)\sigma_2$ and $0<\rho_2,\sigma_2<1$:

The Jacobian matrix is

$$J=\begin{pmatrix} 0 & a_{12} \\ 0 & a_{22}\end{pmatrix}, \tag{B12}$$

where

$$\begin{aligned} a_{12} =& \rho_2\Bigg\{(\rho_2+\sigma_2-1)\gamma(M-1)-\sigma_2\eta(M-1) \\ &-\frac{A\left[M\sigma_2(1-\sigma_2)^{M-1}-1+(1-\sigma_2)^M\right]}{\sigma_2}\Bigg\}, \\ a_{22} =& \sigma_2\Bigg\{(\rho_2+\sigma_2-1)\gamma(M-1)-(\sigma_2-1)\eta(M-1) \\ &+\frac{A(1-\sigma_2)\left[M\sigma_2(1-\sigma_2)^{M-1}-1+(1-\sigma_2)^M\right]}{\sigma_2^2}\Bigg\}.\end{aligned}$$

Here, based on the first order Taylor series expansions for a_{12} and a_{22}, we can obtain

$$\begin{aligned} a_{12} &= \rho_2\left[(\rho_2+\sigma_2-1)\gamma(M-1)-\sigma_2\eta(M-1)\right], \\ a_{22} &= \sigma_2\left[(\rho_2+\sigma_2-1)\gamma(M-1)-(\sigma_2-1)\eta(M-1)\right],\end{aligned}$$

and the Jacobian matrix then simplifies to

$$J=\begin{pmatrix} 0 & \rho_2\left[(\rho_2+\sigma_2-1)\gamma(M-1)-\sigma_2\eta(M-1)\right] \\ 0 & \sigma_2\left[(\rho_2+\sigma_2-1)\gamma(M-1)-(\sigma_2-1)\eta(M-1)\right]\end{pmatrix}, \tag{B13}$$

and thus $(1-\rho_2-\sigma_2,\rho_2,\sigma_2)$ is unstable.

In particular, the traditional strong altruistic reward equilibrium is stabilized as follows.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1,0,0)$:

The Jacobian matrix is

$$J=\begin{pmatrix} \dfrac{(M-r)c}{M} & 0 \\ 0 & -H_{SR}\end{pmatrix}, \tag{B14}$$

and thus $(1,0,0)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,1,0)$:

The Jacobian matrix is

$$J=\begin{pmatrix} -\dfrac{(M-r)c}{M} & H_{SR} \\ 0 & -H_{SR}-\dfrac{(M-r)c}{M}\end{pmatrix}, \tag{B15}$$

and thus $(0,1,0)$ is stable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,0,1)$:

The Jacobian matrix is

$$J=\begin{pmatrix} H_{SR}+\dfrac{(M-r)c}{M}-\eta(M-1) & 0 \\ \gamma(M-1)-\dfrac{(M-r)c}{M} & H_{SR}+(\gamma-\eta)(M-1)\end{pmatrix}, \tag{B16}$$

and thus $(0,0,1)$ is stable if

$$H_{SR}<\min\left\{\eta(M-1)-\frac{(M-r)c}{M},\eta(M-1)-\gamma(M-1)\right\}.$$

While if

$$H_{SR}>\min\left\{\eta(M-1)-\frac{(M-r)c}{M},\eta(M-1)-\gamma(M-1)\right\},$$

$(0,0,1)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(0,1-\mu_2,\mu_2)$ on the DR-edge, where $0<\mu_2<1$:

The Jacobian matrix is

$$J=\begin{pmatrix} (1-\mu_2)\left[\gamma(M-1)\mu_2-\dfrac{(M-r)c}{M}\right] & -(1-\mu_2)\left[\dfrac{(M-r)c}{M}-(\gamma-\eta)(M-1)\mu_2\right] \\ \mu_2\left[\gamma(M-1)\mu_2-\dfrac{(M-r)c}{M}\right] & \mu_2\left[(\gamma-\eta)(M-1)\mu_2+\eta(M-1)-\dfrac{(M-r)c}{M}\right]\end{pmatrix}, \tag{B17}$$

and the eigenvalues of the matrix are $\gamma(M-1)\mu_2-\frac{(M-r)c}{M}$ and $\mu_2(1-\mu_2)\eta(M-1)$, so $(0,1-\mu_2,\mu_2)$ is unstable.

• $\left(x_1^0,x_2^0,x_3^0\right)=(1-\nu_2,0,\nu_2)$ on the CP-edge, where $0<\nu_2<1$:

The Jacobian matrix is

$$J=\begin{pmatrix} \dfrac{(M-r)c}{M}-\gamma(M-1)\nu_2 & 0 \\ \nu_2\left[\gamma(M-1)\nu_2-\dfrac{(M-r)c}{M}\right] & -\nu_2(1-\nu_2)\left[(\gamma-\eta)(M-1)\right]\end{pmatrix}, \tag{B18}$$

and thus $(1-\nu_2, 0, \nu_2)$ is stable if $\nu_2 > \frac{1}{\gamma(M-1)}\frac{(M-r)c}{M}$. While if $\nu_2 < \frac{1}{\gamma(M-1)}\frac{(M-r)c}{M}$, $(1-\nu_2, 0, \nu_2)$ is unstable.

• $\left(x_1^0, x_2^0, x_3^0\right) = (1-\rho_2-\sigma_2, \rho_2, \sigma_2)$ in the interior of simplex S_3:

The Jacobian matrix is

$$J = \begin{pmatrix} 0 & \rho_2\left[(\rho_2+\sigma_2-1)\gamma(M-1)-\sigma_2\eta(M-1)\right] \\ 0 & \sigma_2\left[(\rho_2+\sigma_2-1)\gamma(M-1)-(\sigma_2-1)\eta(M-1)\right] \end{pmatrix}, \tag{B19}$$

and by the central prevalence theorem, we can judge $\left(x_1^0, x_2^0, x_3^0\right) = (1-\rho_2-\sigma_2, \rho_2, \sigma_2)$ to be unstable.

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