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Influence of network structure on spreading dynamics via tie range

Min Li(李敏)[1], Yurong Song(宋玉蓉)[1], Bo Song(宋波)[2], Ruqi Li(李汝琦)[3], Guo-Ping Jiang(蒋国平)[1,†], and Zhang Hui(张晖)[1,4]

[1] *College of Automation and College of Artificial Intelligence, Nanjing University of Posts and Telecommunications, Nanjing* 210023, *China*
[2] *School of Modern Posts, Nanjing University of Posts and Telecommunications, Nanjing* 210023, *China*
[3] *School of Computer Science, Nanjing University of Posts and Telecommunications, Nanjing* 210023, *China*
[4] *Full Truck Alliance Co. Ltd., Nanjing* 210012, *China*

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There are various phenomena of malicious information spreading in the real society, which cause many negative impacts on the society. In order to better control the spreading, it is crucial to reveal the influence of network structure on network spreading. Motifs, as fundamental structures within a network, play a significant role in spreading. Therefore, it is of interest to investigate the influence of the structural characteristics of basic network motifs on spreading dynamics. Considering the edges of the basic network motifs in an undirected network correspond to different tie ranges, two edge removal strategies are proposed, short ties priority removal strategy and long ties priority removal strategy. The tie range represents the second shortest path length between two connected nodes. The study focuses on analyzing how the proposed strategies impact network spreading and network structure, as well as examining the influence of network structure on network spreading. Our findings indicate that the long ties priority removal strategy is most effective in controlling network spreading, especially in terms of spread range and spread velocity. In terms of network structure, the clustering coefficient and the diameter of network also have an effect on the network spreading, and the triangular structure as an important motif structure effectively inhibits the spreading.

Keywords: network spreading, network motifs, tie range, edge removal strategy

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1. Introduction

The relationship between complex network structure and network function is a hot issue in the field of complex network research. Complex network structures include nodes,[1,2] edges,[3,4] motifs,[5] cycles,[6–8] and other high-order structures.[9,10] Network functions include information spreading,[11–13] robustness,[14,15] stability,[16,17] and so on. In particular, the spreading of malicious information such as cyber fraud, false information and epidemic spreading in recent years has had a great negative impact on human beings. When large-scale malicious information spreading occurs in the system, the security and stability of the system will be seriously damaged. Therefore, many scholars have devoted themselves to analyzing the relationship between network structure and network spreading, and studying methods to control the spreading of malicious information in networks.[18–21]

Earlier research by Watts and Strogatzt proposed a small-world network model and revealed the characteristics of small-world networks, that is, there are highly clustered nodes and short average paths in the network. The network structure enables information to spread quickly in the network.[22] Later, Barabási proposed the scale-free network model and showed that in the scale-free networks, only a few nodes have very large degrees, while most nodes have small degrees. The nodes with large degree are often called central nodes or key nodes, which play an important role in information spreading and network resilience.[23] Compared to small-world networks, scale-free networks have a low spread threshold, which makes it easier for information to spread in scale-free networks. With the proposal of small-world networks and scale-free networks, more and more people are beginning to pay attention to the relationship between the structure and function of complex networks.

Newman explained three common clustering characteristics of most real networks from the perspective of model development: short average path length, skewness distribution and clustering. At the same time, the structure and function of complex networks are systematically introduced, and many methods and indicators of network analysis are proposed, which play an important role in the development of network science.[24] Boccaletti *et al*. introduced the structure and dynamic characteristics of complex networks in detail, and discussed the influence of network structure on network functions such as synchronization, spreading and stability, which provided an important reference for further understanding the structure and function of complex networks.[25] With the deep-

†Corresponding author. E-mail: jianggp@njupt.edu.cn

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ening of research, the community structure in social networks has attracted people's attention. Girvan and Newman firstly proposed the community structure in *Proceedings of the National Academy of Sciences*, and studied the community structure in social networks and biological networks.[26] After that, many researchers have studied the influence of community structure on network function. For example, the network with community structure is more conducive to the spreading of information in the network.[27,28] The closer the internal connection of community structure in the network, the stronger the stability and robustness of the network.[29]

The above is to analyze the impact on network function from the perspective of the low-order structure of the network, and with the deepening of research, more and more attention is paid to the interrelationship between the higher-order structure of the network and network function, such as the network motif structure. Network scientist Milo *et al*. proposed the network motif structure.[5] The common motifs include triangle (3-node motif) structure, quadrilateral (4-node motif) structure, and other motif structures. The emergence of motif structure has attracted the interest of researchers in the neighborhood of complex networks. The research on the influence of network motif on network function includes the analysis of network motif structure to reveal the characteristics and functions of the network, the change of motif in the process of network evolution and the influence of motif on network stability and robustness.[30–32]

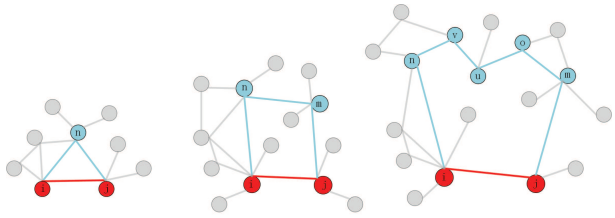
In view of some problems of malicious information spreading in real life, it is meaningful to study the influence of basic network motifs on network spreading and network structure. Inspired by Park and Lyu's research on long ties,[33,34] we define the weights of edges between two connected nodes in terms of tie range. The second shortest path length between two connected nodes is defined as tie range, and different tie ranges correspond to different motif structures. To study the effect of basic network motifs on information spreading, we perform different edge removal strategies based on the edge weights, which are short ties priority removal strategy and long ties priority removal strategy. The effects of different edge removal strategies on network spreading are analyzed by simulation in the susceptible–infected–recovered (SIR) model. And the effects of different structural features on spreading are analyzed from the perspective of network structure. It is found that the LTPR strategy has the best control effect on spreading dynamics. Moreover, the network structure characteristics such as Fiedler value, clustering coefficient and network diameter have the most prominent influence on spreading dynamics. Finally, we conclude that the high-order structure in the network has the effect of promoting spreading dynamics, and the triangular (3-node motif) structure has the effect of inhibiting spreading dynamics.

The structure of this article is as follow. In Section 2, tie range and proposed edge removal strategies are introduced. In Section 3, a large number of simulation results are shown to reflect the effect of different edge removal strategies on controlling spreading and changing network structure. The analysis also explores the influence of network structure on spreading. The research results are summarized in Section 4.

2. Tie range and edge removal strategy

2.1. Define the tie range

Tie range is defined as the second shortest path length between two connected nodes.[31,32] As shown in Fig. 1(a), nodes i and j and their common neighbor n form a triangular structure, then the tie range between nodes i and j is 2, called short ties; as shown in Fig. 1(b), node n, a neighbor of node i, and node m, a neighbor of node j, are neighbors of each other, the tie range between nodes i and j is 3, and nodes i, n, m, j form a quadrilateral structure; as shown in Fig. 1(c), the tie range between node i and node j is 6, which corresponds to the largest tie range in the network, known as long ties.



(a) Tie range = 2 (b) Tie range = 3 (c) Tie range = 6

Fig. 1. The definition of the tie range. The tie range (red line) is represented by the length of the second shortest path between two connected nodes (blue line).

2.2. The proposed edge removal strategy

The tie range is used to measure the weight of the edge e_{ij} between two connected nodes i and j in the network. Different tie ranges correspond to distinct basic network motif structures. If the weight of edge e_{ij} is 2, it means that the edge e_{ij} is part of a triangular (3-node motif) structure. If the weight of edge e_{ij} is 3, it means that the edge e_{ij} is part of a quadrilateral (4-node motif) structure. And so on. To investigate the impact of various motif structures on network spreading, two edge removal strategies are introduced: the short ties priority removal (STPR) strategy, which sorts in ascending order based on the weights of the edges, and the long ties priority removal (LTPR) strategy, which sorts in descending order based on the weights of the edges.

The specific process of the edge removal strategy is as follows.

Step 1 Networking

An initial network $G(V,E)$, where V and E denote the set of nodes and the set of edges in the network, respectively.

$N = |V|$ and $M = |E|$ are the number of nodes and edges of the network, respectively.

Step 2 Calculating

The tie ranges between two connected nodes i and j are calculated, and tie range is defined as the weight of the edge e_{ij} between nodes i and j.

Step 3 Removing

Firstly, the $m(m \leq M)$ edges with tie range in the network are numbered according to the sequence of natural numbers from 1 to m. Under the STPR (LTPR) strategy, the m edges are sorted in ascending (descending) order according to the weights of the edges. The numbers of the sorted edges are stored in the array L as the result of edge sorting. Based on the edge sorting results, the m_{r} edges are selected for removal, where it is important to note that network connectivity is ensured during edge removal. That is, if the network connectivity is destroyed when removing an edge, the removal of the edge is canceled, and the next edge is selected for removal in turn, until m_{r} edges are removed.

The m_{r} edges are removed to change the network structure. There are M edges in the network. The f_{r} represents the ratio of edge removing m_{r} to the M edges, and its expression is as follows:

$$f_{\mathrm{r}} = \frac{m_{\mathrm{r}}}{M}. \tag{1}$$

Take the network in Fig. 2 as an example. Figure 2 shows a network consisting of 10 nodes and 14 edges, where edges with tie range 2 are represented in red, edges with tie range 3 are represented in yellow, and edges with tie range 4 are represented in green. The black edge e_{13} in the figure does not have a second shortest path, and therefore does not participate in the edge ordering. The network edges in Fig. 2 are sorted according to the STPR strategy as follows: $\{e_{01}, e_{02}, e_{12}, e_{14}, e_{24}, e_{16}, e_{45}, e_{56}, e_{67}, e_{69}, e_{78}, e_{89}, e_{48}\}$; the result of sorting according to the LTPR strategy is $\{e_{48}, e_{16}, e_{45}, e_{56}, e_{67}, e_{69}, e_{78}, e_{89}, e_{01}, e_{02}, e_{12}, e_{14}, e_{24}\}$.

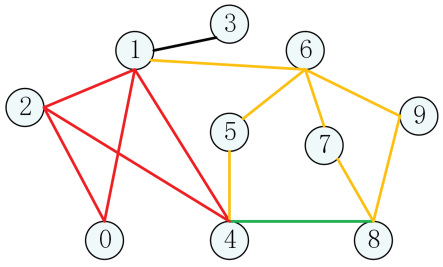


Fig. 2. A simple network with 10 nodes and 14 edges.

3. Data sets and simulation analysis

3.1. Data sets

In order to test the effectiveness of the proposed strategies, we apply it to synthetic networks and real world networks. The synthetic networks are WS small-world network[22] and BA scale-free network,[23] respectively. The WS network is constructed where the number of nodes $|V| = 1000$, edges $|E| = 3000$, and the average degree of the network is $\langle k \rangle = 6$. The BA network is taken as $|V| = 1000$, $m = 2$, $m_0 = 2$, edges $|E| = 3984$, and the average degree of the network is $\langle k \rangle \approx 8$. Real networks include Power networks and Email networks.[35] Table 1 shows some of the statistical structure characteristics of these four networks, including the number of nodes $|V|$, the number of edges $|E|$, the average degree of the network $\langle k \rangle$, average path length L, clustering coefficient C and network diameter D.

Table 1. Some of the statistical structure characteristics of the four networks.

Network	$\|V\|$	$\|E\|$	$\langle k \rangle$	L	C	D
WS	1000	3000	6	5.11	0.32	9
BA	1000	3984	8	3.20	0.04	5
Power	4941	6594	2.66	18.99	0.08	46
Email	1133	5451	9.62	3.61	0.22	8

3.2. Simulation analysis

In experiments, we have investigated the role of the proposed edge removal strategies on the spreading of information in the four networks as well as the structure of the network, especially in the process of removing edges to ensure the connectivity of the network. Firstly, the effects of the different strategies on the spreading capability of the four networks, including spread range, spread velocity and spread threshold, are analyzed under the SIR model.[36] Then, the reasons for the different effects of the two proposed strategies on spreading are analyzed from the perspective of network structure. Some properties of the network structure under different strategies, including Fiedler, average path length L, clustering coefficient C and network diameter D. Finally, the influence of network structure on the spread of information is analyzed.

3.2.1. Spreading capacity analysis

In the SIR model, there are three types of nodes, susceptible (S), infected (I), and recovery (R). The susceptible node is changed into an infected node by the infected node under a certain probability. The infected node will also become a recovery node under a certain probability. During the experiment, we set the infection probability to 0.3 and the recovery probability to 0.3. The impact of the proposed strategies on the spreading capacity is analyzed in terms of spread range I, spread velocity $\bar{v}$ and spread threshold λ_{c}. The spread range I is described as the final infection rate at the steady state. The effects of different strategies on the spread range of the four networks are studied under the SIR model. The results are shown in Fig. 3, from which it can be seen that the spread ranges I of the STPR and LTPR strategies corresponding to

the four networks show different decreasing trends with the increase of the removed edge ratio f_r. From Fig. 3(a), it can be seen that in the WS network, the spread range I of the network corresponding to the STPR strategy is the smallest when the removed edge ratio $f_r \leq 0.14$; as the removed edge ratio f_r increases, the spread range I of the LTPR strategy corresponding network decreases continuously. Especially when the edge removed ratio $f_r > 0.14$, the spread range I decreases rapidly, and when $f_r = 0.24$, the best effect of controlling the network spreading is basically achieved. From Fig. 3(b), it can be seen that the spread range I of the LTPR strategy corresponding network is the smallest in the BA network as removed edge ratio f_r increases. From Fig. 3(c), it can be seen that in the Power network, the LTPR strategy corresponding network has the relatively smallest spread range I as the removed edge ratio f_r increases. From Fig. 3(d), it can be seen that in the Email network, the spread range I of the LTPR strategy corresponding network is always the smallest as the removed edge ratio f_r increases.

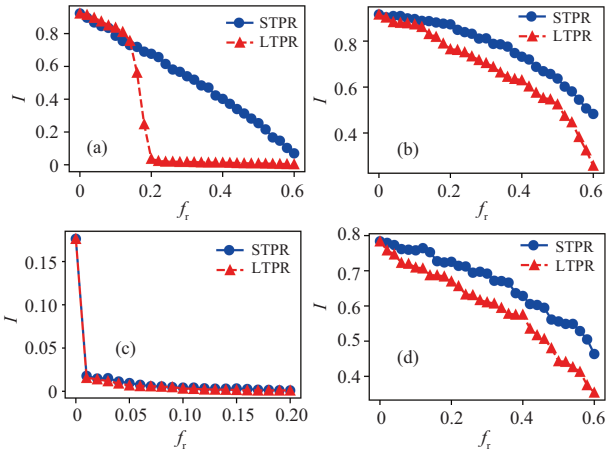


Fig. 3. The effect of the STPR and LTPR strategies on the spread ranges of different networks in the SIR model: (a) WS network, (b) BA network, (c) Power network, and (d) Email network. The f_r denotes the removed edge ratio, and I denotes the spread range of the network when the spreading reaches the steady state in terms of the total infected rate of the network.

It can be seen that the corresponding spread range I under different networks have different trends with the increase of the removed edge ratio f_r, which is related to the structural characteristics of the network, the number of edges with different tie ranges and the properties of edges. In the WS network with small-world property, the proportion of edges with tie range greater than 2 in the network is about 0.24, and these edges play an important role in shortening the distance between nodes, so the LTPR strategy has the best control effect on the WS network spread range I. However, for networks with scale-free property, such as BA and Email networks, due to the close distance between nodes in the network, the spread phenomenon is more likely to occur. Therefore, the spread range I of the scale-free network corresponding to the STPR and LTPR strategies decreases slowly with the increase of the removed edge ratio f_r. In the Power network, because the network is relatively sparse, the removal of a small number of edges will have a great impact on the network spreading, and the edges with long tie play a more important role in narrowing the distance between nodes and promoting spread. Therefore, for sparse networks, the spread range I will shrink rapidly when the removed edge ratio is very small, and then slowly change with the increase of the removed edge ratio. From the point of view of controlling the spread range of the networks, we find that the LTPR strategy has the best effect of controlling the spreading of the four networks, especially for the WS network and the Power network, which basically achieves the effect of completely suppressing the spreading.

For convenience, we define the network spread velocity $\bar{v}$ as the average value of the change of network infection rate over time T, where time T is taken to correspond to when the network spread reaches stability.

$$\bar{v} = \frac{\sum_{t=1}^{t=T} I(t) - I(t-1)}{T}, \tag{2}$$

where $I(t)$ denotes the infection rate of nodes in the network at time t, $I(t-1)$ denotes the infection rate of nodes in the network at time $t-1$. The time T is taken to be when the network spreading reaches stability. In the SIR model, the effect of the proposed strategies on different network spread velocities is investigated. The network spread velocity $\bar{v}$ is calculated, and the results are shown in Fig. 4, which shows that the network spread velocity $\bar{v}$ continues to slow down as the removed edge ratio f_r increases.

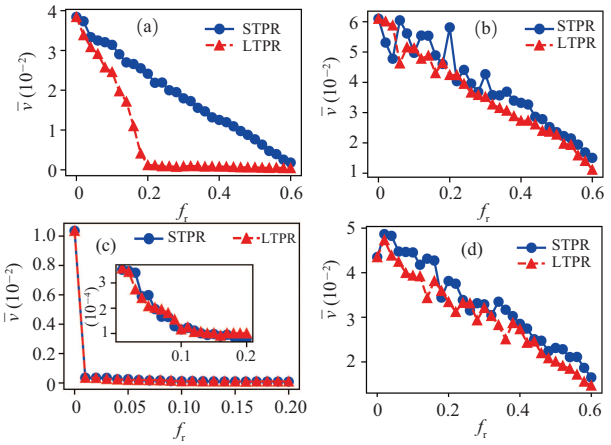


Fig. 4. In the SIR model, the effect of the STPR and LTPR strategies on different network spread velocities: (a) WS network, (b) BA network, (c) Power network, and (d) Email network. f_r denotes the removed edge ratio and $\bar{v}$ denotes the spread velocity.

Compared to the STPR strategy, the LTPR strategy has the greatest impact on the network spread velocity $\bar{v}$. Especially in the WS and Power networks, the network spreading is greatly reduced when the removed edge ratio f_r reaches a certain value. In both BA and Email networks, there are occasional increases in network spread velocity $\bar{v}$ with different

removed edge ratios f_r, but overall the LTPR strategy corresponds to the fastest decrease in network spread velocity $\bar{v}$. The network spread velocity $\bar{v}$ is related to the spread range I when the network spreading reaches a steady state in the network, so we can see that the change of the spread velocity $\bar{v}$ in Fig. 4 is roughly the same as the change of the spread range I in Fig. 3.

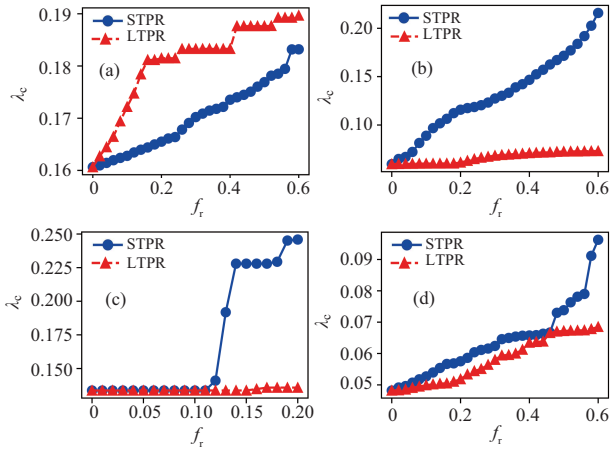


Fig. 5. The effect of the STPR and LTPR strategies on different network spread thresholds: (a) WS network, (b) BA network, (c) Power network, and (d) Email network. f_r denotes the removed edge ratio and λ_c denotes the spread threshold.

Spread threshold λ_c can similarly reflect the network spreading capacity. When the network contagion rate $\lambda > \lambda_c$, the information will spread rapidly in the network; when $\lambda \leq \lambda_c$, the information will not spread in the network. Therefore, the larger the value of λ_c, the more unfavourable the spread of the network will be.[37] The spread threshold of a complex network is the reciprocal of the largest eigenvalue of the network adjacency matrix A.[38]

$$\lambda_c = \frac{1}{\lambda_{max}}, \tag{3}$$

where λ_{max} denotes the largest eigenvalue of the network adjacency matrix A. Figure 5 shows the variation of the corresponding network spread threshold λ_c with the increasing removed edge ratio f_r under different edge removal strategies. From Fig. 5, it can be seen that as the removed edge ratio f_r increases, the network spread threshold λ_c changes with different degrees of increase. In terms of spread threshold, the STPR and LTPR strategies have different effects on the four networks. Among them, the LTPR strategy increases the spread threshold λ_c of the WS network to a greater extent, and the STPR strategy increases the spread threshold λ_c of the BA, Power and Email networks to a greater extent. Spread threshold λ_c is related to the degree distribution of the network. The more uniform the network degree distribution is, the larger the spread threshold λ_c is. For the WS network with small-world property, its degree distribution obeys Poisson distribution, and the spread threshold of the network is large, and the degree values of the two connected nodes corresponding to the long tie may be larger. The LTPR strategy will make the degree distribution of the network more uniform, so the LTPR strategy increases the spread threshold of the WS network to a greater extent. For BA, power and Email networks with scale-free property, the degree distribution obeys power-law distribution, spread threshold of the network is small, and the degree value of the two connected nodes corresponding to the short tie may be larger. Therefore, the STPR strategy will make the degree distribution of the network more uniform and increase the propagation threshold of the network faster.

3.2.2. The effect of network structure on spreading

In order to further observe the changes in the structural characteristics of the network, as well as to analyse the impact of structural characteristics on spreading, the following work was carried out. The Laplace matrix is a powerful tool for studying network dynamics, especially the second smallest eigenvalue of the Laplace matrix, also known as the Fiedler value λ_2 or algebraic connectivity.[7] The Fiedler value λ_2 measures not only the algebraic connectivity of the network, but also the synchronizability and spread velocity of the network, a larger Fiedler value λ_2 indicates that the network is more synchronised and has a faster spread velocity.[39] Figure 6 shows the variation of the Fiedler values λ_2 of the corresponding networks of the STPR and LTPR strategies with the removed edge ratio f_r. From Fig. 6, it can be seen that the Fiedler value λ_2 of the network corresponding to the LTPR strategy is smaller than that of the STPR strategy, and therefore the corresponding network has the slowest spread velocity.

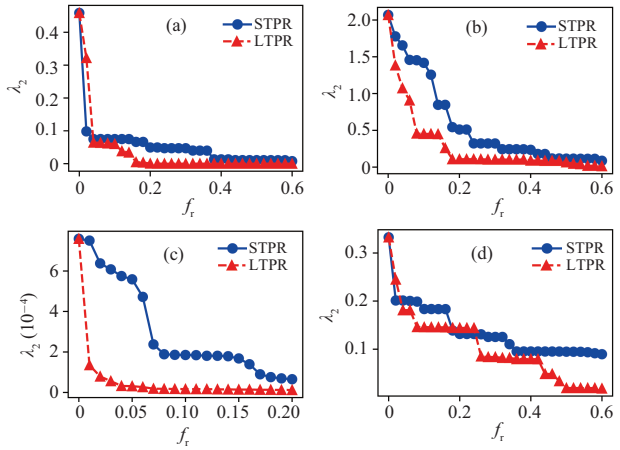


Fig. 6. The effect of the STPR and LTPR strategies on Fiedler values λ_2 for different networks with increasing removed edge ratio f_r. (a) WS network, (b) BA network, (c) Power network, and (d) Email network.

In addition to the Fiedler value λ_2, the average path length of the network L,[40] the clustering coefficient C,[41] and the network diameter D[42] are also basic characteristics that can reflect the structure of the network. The expression for the

average path length L of the network is defined as

$$L = \frac{1}{\frac{1}{2}N(N-1)} \sum_{i \geq j} d_{ij}, \tag{4}$$

where d_{ij} is denoted as the number of edges on the shortest path between node i and node j and N is the number of nodes in the network. Figure 7 shows the effect of the STPR and LTPR strategies on the average path length L of the network. From Fig. 7, it can be seen that the average path length L becomes longer as the removed edge ratio f_r increases. In particular, in the WS network, the average path length L of the LTPR strategy corresponding network grows rapidly, as shown in Fig. 7(a), which shows that the average path length L of the LTPR strategy corresponding network basically reaches the longest length at the removed edge ratio $f_r = 0.24$, and then there is very little change in L with the increase of f_r. Looking at the effect of the STPR and LTPR strategies on the average path length L and spreading of different networks, we find that the LTPR strategy corresponds to the longest average path lengths L of the WS, Power and Email networks, as well as the best effect on spreading control. However, in BA networks, the LTPR strategy corresponds to networks with shorter average path lengths L than the STPR strategy, but performs best in controlling spreading. This is due to the fact that the average path length L of the BA network itself is relatively short and the STPR strategy reduces the number of edges with a tie range of 3 and 4 in the BA network, resulting in many nodes needing longer distances to complete the connection between them; the LTPR strategy reduces the number of edges with long tie range in the BA network, resulting in small nodes needing longer distances to complete the connection between them. Compared with other networks, the impact of STPR and LTPR strategies on the average path length L of the BA networks is not very obvious. Therefore, maybe there are other factors that affect the spreading.

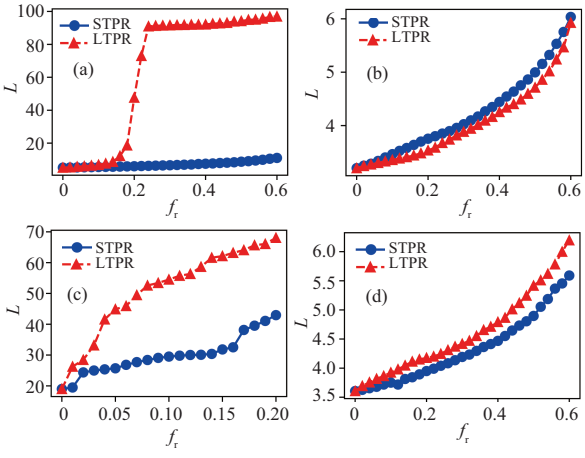


Fig. 7. Effect of the STPR and LTPR strategies on the average path length L for different networks with increasing removed edge ratio f_r. (a) WS network, (b) BA network, (c) Power network, and (d) Email network.

The network clustering coefficient C is a measure of the degree of clustering of nodes in a network. In general, there is a certain between clustering coefficient C and network spreading capability. The network clustering coefficient C is defined as follows:

$$C = \frac{1}{N} \sum_{i=1}^{N} C_i, \tag{5}$$

where C_i denotes the clustering coefficient of node i, C_i is proportional to the number of triangles n_1 connected to node i, and inversely proportional to the number of triples n_2 connected to node i, that is, $C_i = n_1/n_2$, and N denotes the number of nodes in the network.

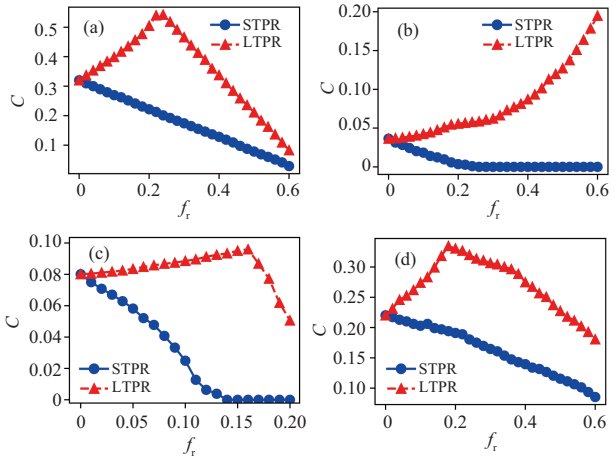


Fig. 8. The effect of the STPR and LTPR strategies on the clustering coefficient C for different networks with increasing removed edge ratio f_r. (a) WS network, (b) BA network, (c) Power network, and (d) Email network.

Figure 8 shows the effect of the STPR and LTPR strategies on the network clustering coefficient C. It can be seen from Fig. 8 that the clustering coefficient C of the STPR strategy corresponding to the four networks show a decreasing trend as the removed edge ratio f_r increases. This is because the STPR strategy preferentially removes the edges that constituent the triangular (3-node motif) structure, resulting in a decrease in the clustering coefficient C of the corresponding nodes. However, the clustering coefficient C of the LTPR strategy corresponding networks tend to increase and then decrease with the increase of the removed edge ratio f_r in the WS, Power, and Email networks. The clustering coefficient C of the LTPR strategy corresponding to the BA network show an increasing trend as the removed edge ratio f_r increases. In addition, the network clustering coefficient C corresponding to the LTPR strategy is larger than that of the STPR strategy. This is because the LTPR strategy preferentially removes the edges that constituent the high-order structure, resulting in a decrease in the number of triples corresponding to some nodes. Combining $C_i = n_1/n_2$, when the removed edge ratio f_r

is small, the edges that constituent the triangular (3-node motif) structure in the network will not be removed, meaning n_1 unchanged, and the number of triples n_2 decreases, resulting in an increase in the clustering coefficient of the network. Then, when the removed edge ratio increases to a certain extent, the edges that constituent the triangular (3-node motif) structure in the network begin to be removed, resulting in a decrease in the clustering coefficient of some nodes. Therefore, under the LTPR strategy, with the increase of the proportion of removed edges, the clustering coefficient of the network has an inflection point. Combined with Fig. 3, it is not difficult to find that the larger the clustering coefficient, the smaller the propagation range of the corresponding network. It further proves that the larger the network clustering coefficient C is, the more effective it is in suppressing network spreading, which is evidenced by the references.[41]

The network diameter D is the maximum of the shortest path lengths between any two nodes in the network. The expression for the network diameter D is defined as

$$D = \max_{i,j} d_{ij}, \tag{6}$$

where i and j denote any two nodes in the network and d_{ij} denotes the shortest path length between node i and node j. Figure 9 shows the changes in the network diameter D of the four networks for the STPR and LTPR strategies with the increase of the removed edge ratio f_r. It can be seen from Fig. 9 that the network diameter D corresponding to the STPR and LTPR strategies continue to increase as the removed edge ratio f_r increases, and the network diameter D corresponding to the LTPR strategy is larger than or equal to the network diameter D corresponding to the STPR strategy. Corresponding to the network spreading capacity, it is easy to see that a large network diameter can inhibit network spreading.

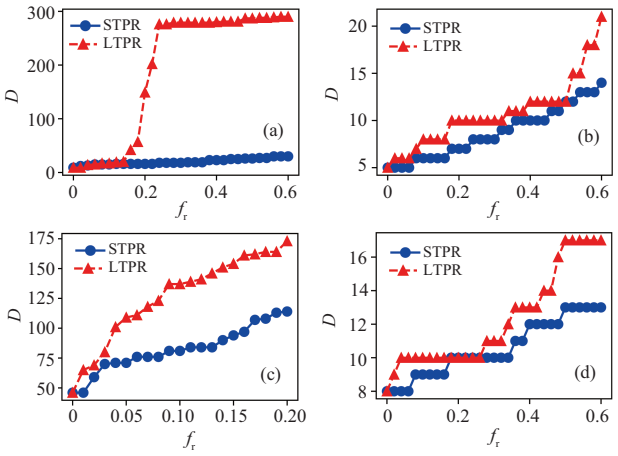


Fig. 9. The effect of the STPR and LTPR strategies on the network diameter D for different networks with increasing removed edge ratio f_r. (a) WS network, (b) BA network, (c) Power network, and (d) Email network.

4. Conclusion

In this paper, we propose two edge removal strategies, STPR strategy and LTPR strategy. These two edge removal strategies consider the second shortest path length between two connected nodes (tie range) and investigate the effect of the basic network motifs corresponding to different sizes of tie range on network spreading. In the SIR model, the effects of the proposed strategies on the network spreading including spread range, spread velocity and spread threshold are analysed and it is found that the LTPR strategy is the most effective in controlling the spread range and spread velocity of the network, and in terms of spread threshold, the LTPR strategy is the most effective in increasing the spread threshold of the WS network and the LTPR strategy is the most effective in increasing the spread thresholds of the BA, Power, and Email networks. At the same time, we statistically and analyse the effect of the STPR and LTPR strategies on the structural properties of the network part. Fiedler, clustering coefficients and network diameters were found to be factors affecting network spreading, and the LTPR strategy increased network clustering coefficients and network diameters and enabled better control of network spreading.

In conclusion, edges between two connected nodes in the network corresponding to a long tie range have a promoting effect on network spreading, while edges between two connected nodes corresponding to a short tie range have an inhibiting effect on network spreading. Analyzed from the structural point of view, higher-order structures promote spreading, whereas triangular (3-node motif) structures inhibit spreading.

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