

Preparation of entangled W states based on the cavity QED system

Ke Li(李可) and Jun-Long Zhao(赵军龙)

Citation: Chin. Phys. B, 2024, 33 (**9**): 090306. DOI: 10.1088/1674-1056/ad50bf
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Algorithm for evaluating distance-based entanglement measures

Yixuan Hu(胡奕轩), Ye-Chao Liu(刘烨超), and Jiangwei Shang(尚江伟)
Chin. Phys. B, 2023, 32 (**8**): 080307. DOI: 10.1088/1674-1056/acd5c5

Generation of microwave photon perfect *W* states of three coupled superconducting resonators

Xin-Ke Li(李新克), Yuan Zhou(周原), Guang-Hui Wang(王光辉), Dong-Yan Lv(吕东燕),Fazal Badshah, and Hai-Ming Huang(黄海铭)
Chin. Phys. B, 2023, 32 (**4**): 040306. DOI: 10.1088/1674-1056/aca394

Entanglement properties of GHZ and W superposition state and its decayed states

Xin-Feng Jin(金鑫锋), Li-Zhen Jiang(蒋丽珍), and Xiao-Yu Chen(陈小余)
Chin. Phys. B, 2021, 30 (**6**): 060301. DOI: 10.1088/1674-1056/abd74e

Fast generation of W state via superadiabatic-based shortcut in circuit quantum electrodynamics

Xue-Mei Wang(王雪梅), An-Qi Zhang(张安琪), Peng Xu(许鹏, and Sheng-Mei Zhao(赵生妹)
Chin. Phys. B, 2021, 30 (**3**): 030307. DOI: 10.1088/1674-1056/abd75b

Fault tolerant controlled quantum dialogue against collective noise

Li-Wei Chang(常利伟), Yu-Qing Zhang(张宇青), Xiao-Xiong Tian(田晓雄), Yu-Hua Qian(钱宇华), Shi-Hui Zheng(郑世慧)
Chin. Phys. B, 2020, 29 (**1**): 010304. DOI: 10.1088/1674-1056/ab5786

Preparation of entangled W states based on the cavity QED system

Ke Li(李可)[1,†] and Jun-Long Zhao(赵军龙)[2]

[1] *Department of Physics, Shanghai Second Polytechnic University, Shanghai* 201209, *China*
[2] *Quantum Information Research Center, Shangrao Normal University, Shangrao* 334001, *China*

(Received 4 March 2024; revised manuscript received 22 May 2024; accepted manuscript online 28 May 2024)

We present a qubit-loss-free (QLF) fusion scheme for generating large-scale atom W states in cavity quantum electrodynamics (QED) system. Compared to the most current fusion schemes which are conditioned on the case where one particle can be extracted from each initial W state to the fusion process, our scheme will access one or two particles from each W state. Based on the atom–cavity-field detuned interaction, three $|W\rangle_{n+m+t}$ states can be generated from the $|W\rangle_n$, $|W\rangle_m$, and $|W\rangle_t$ states with the help of two auxiliary atoms, and three $|W\rangle_{n+m+t+q}$ states can be generated from $|W\rangle_n$, $|W\rangle_m$, $|W\rangle_t$, and a $|W\rangle_q$ state with the help of three auxiliary atoms. Comparing the numerical simulations of the resource cost of fusing three small-size W states based on the previous schemes, our fusion scheme seems to be more efficient. This QLF fusion scheme can be generalized to the case of fusing k different or identical particle W states. Furthermore, with no qubit loss, it greatly reduces the number of fusion steps and prepares W states with larger particle numbers.

Keywords: W state, detuned interaction, state fusion, cavity quantum electrodynamics

PACS: 03.67.Ac, 03.67.Hk, 03.65.Ud, 03.67.Bg **DOI:** 10.1088/1674-1056/ad50bf

1. Introduction

Quantum entanglement has attracted a great deal of attention due to its various striking features. As one of the most important physical resources, it has been widely applied in quantum information processing, such as quantum computation,[1] quantum teleportation,[2–4] quantum key distribution (QKD),[5] and biological organisms,[6] etc. Compared with bipartite entangled states, the multiparticle entangled state has more complex and diverse entangled structures. As we know, there are many different kinds of states when the number of particles forming an entangled state exceeds two, such as W states,[7] GHZ states,[8] and cluster states,[9] etc. These inequivalent classes of states cannot be converted into each other with stochastic local operations and classical communication (SLOCC).[10] In these entangled states, the W state is more special because the entanglement is robust against particle loss.[7] For a maximally entangled W state, the properties of the remaining particle are still entangled even when any particle is lost, which is attracting more attention in the field of quantum information.[11–24]

As the most commonly used preparation schemes, quantum state expansion and fusion technology are seen as effective ways to generate large-scale maximally entangled W states.[25–38] Although the number of particles of the initial entangled state is expanded by one or two each time with expansion technology, the fusion technology can obviously increase the number of particles in preparing target W state by fusing two or three initial small-size W states, simultaneously. Hence, the fusion process has attracted considerable attention in recent years. In 2011, Ozdemir *et al.* first put forward the fusion scheme of fusing two initial small-size W states (W_n, W_m, $n,m \geq 3$) into a W_{n+m-2} state by accessing one particle from each W state to the fusion mechanism. Then, several new preparation schemes of W state fusion with the help of controlled quantum gates and ancilla qubits have been proposed in optical systems.[32–34] The other preparation schemes are similarly conditioned on the case where one particle from each initial W state is permitted to enter the different fusion systems. However, these fusion schemes inevitably suffer from the qubit loss issue, which will decrease the fusion efficiency and increase the number of fusion steps.

Based on a two-outcome positive-operator valued measurement (POVM) on two qubits extracted from two small-size W states, Yang Ming research group first proposed a universal approach of qubit-loss-free (QLF) fusion scheme to generate a large-scale W state in 2016.[39] The number of the output target entangled particles is just equal to the sum of the input entangled particles. Motivated by this idea, Wang *et al.* proposed a QLF fusion scheme for fusing three polarization entangled W states, simultaneously, where the elements of a two-outcome POVM and the appropriate joint unitary operation for realizing a POVM are given.[40] Then, Li and Ding *et al.* also realized the QLF fusion schemes in a cavity QED system based on the resonant interactions between the atoms and the cavity modes.[41,42] The main advantage of this QLF fusion scheme is that it clearly increases the fusion efficiency and greatly reduces the number of fusion steps.

In this paper, we also propose a QLF fusion scheme for generating large-scale W states in a cavity quantum electrodynamics (QED) system, which is based on the atom–cavity-

[†]Corresponding author. E-mail: like@sspu.edu.cn

© 2024 Chinese Physical Society and IOP Publishing Ltd

field detuned interaction. In our scheme, by fusing three small-size W states ($|W\rangle_n$, $|W\rangle_m$, $|W\rangle_t$) or four small-size W states ($|W\rangle_n$ $|W\rangle_m$, $|W\rangle_t$, $|W\rangle_q$), respectively, three $|W\rangle_{n+m+t}$ states or three $|W\rangle_{n+m+t+q}$ states can be generated. In comparison with the resource cost of fusing three small-size W states using our QLF fusion scheme and the previous schemes, the numerical simulation results show that this QLF fusion scheme is more efficient.

2. Detuned interaction between three atoms and the vacuum cavity mode

The key component of our fusion mechanism is based on the atom–cavity-field detuned interaction. The detuned interaction between n identical two-level atoms and the cavity mode in the interaction picture can be described by the following interaction Hamiltonian:[43]

$$H_i = g\sum_{j=1}^{n}\left(\mathrm{e}^{-\mathrm{i}\delta t}a^{\dagger}s_j^{-}+\mathrm{e}^{\mathrm{i}\delta t}as_j^{+}\right), \tag{1}$$

where g is the atom–cavity coupling strength, δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω, $s_j^{+}=|e\rangle_{jj}\langle g|$, $s_j^{-}=|g\rangle_{jj}\langle e|$, $|e\rangle_j$ and $|g\rangle_j$ are the excited and ground states of the jth ($j=1,2,\ldots,n$) atom, $a^{\dagger}$ and a the creation and annihilation operator for the cavity model. Conditioned on the case of $\delta \gg g$, the interaction is far-off-resonant and there is no energy exchange between the atomic system and the cavity. The detailed derivation process can be developed by referring to the introduction in Ref. [44]. The effective Hamiltonian can be written as

$$H = \lambda\sum_{i,j=1}^{n}\left(s_j^{+}s_i^{-}aa^{\dagger}-s_j^{-}s_i^{+}a^{\dagger}a\right), \tag{2}$$

where $\lambda = g^2/\delta$. Assume that the cavity mode is initially prepared in the vacuum state, for the case of $n=3$, the effective Hamiltonian becomes

$$H = \lambda\left(\sum_{j=1}^{3}|e\rangle_{jj}\langle e|+\sum_{i,j=1,i\neq j}^{3}s_i^{+}s_i^{-}\right). \tag{3}$$

Therefore, the time evolution operator induced by the expression of this Hamiltonian can be expressed as follows:

$$\begin{aligned}
|e_1\rangle|e_2\rangle|g_3\rangle &\rightarrow \mathrm{e}^{-\mathrm{i}\lambda t}(k_1|g_1\rangle|e_2\rangle|e_3\rangle+k_1|e_1\rangle|g_2\rangle|e_3\rangle\\
&\quad +k_2|e_1\rangle|e_2\rangle|g_3\rangle),\\
|e_1\rangle|g_2\rangle|e_3\rangle &\rightarrow \mathrm{e}^{-\mathrm{i}\lambda t}(k_2|g_1\rangle|e_2\rangle|e_3\rangle+k_1|e_1\rangle|g_2\rangle|e_3\rangle\\
&\quad +k_1|e_1\rangle|e_2\rangle|g_3\rangle),\\
|g_1\rangle|e_2\rangle|e_3\rangle &\rightarrow \mathrm{e}^{-\mathrm{i}\lambda t}(k_2|g_1\rangle|e_2\rangle|e_3\rangle+k_1|e_1\rangle|g_2\rangle|e_3\rangle\\
&\quad +k_1|e_1\rangle|e_2\rangle|g_3\rangle),\\
|g_1\rangle|g_2\rangle|e_3\rangle &\rightarrow k_2|g_1\rangle|g_2\rangle|e_3\rangle+k_1|g_1\rangle|e_2\rangle|g_3\rangle\\
&\quad +k_1|e_1\rangle|g_2\rangle|g_3\rangle),\\
|e_1\rangle|g_2\rangle|g_3\rangle &\rightarrow k_1|g_1\rangle|g_2\rangle|e_3\rangle+k_1|g_1\rangle|e_2\rangle|g_3\rangle\\
&\quad +k_2|e_1\rangle|g_2\rangle|g_3\rangle,\\
|g_1\rangle|e_2\rangle|g_3\rangle &\rightarrow k_1|g_1\rangle|g_2\rangle|e_3\rangle+k_2|g_1\rangle|e_2\rangle|g_3\rangle\\
&\quad +k_1|e_1\rangle|g_2\rangle|g_3\rangle,\\
|g_1\rangle|g_2\rangle|g_3\rangle &\rightarrow |g_1\rangle|g_2\rangle|g_3\rangle,\\
|e_1\rangle|e_2\rangle|e_3\rangle &\rightarrow \mathrm{e}^{-\mathrm{i}3\lambda t}|e_1\rangle|e_2\rangle|e_3\rangle,
\end{aligned} \tag{4}$$

where $k_1=\frac{1}{3}(\mathrm{e}^{-\mathrm{i}3\lambda t}-1)$ and $k_2=\frac{1}{3}(\mathrm{e}^{-\mathrm{i}3\lambda t}+2)$.

3. Qubit-loss-free fusion scheme of three atom W states

In this section, the QLF fusion schemes for fusing three small-size maximally entangled W states ($|W\rangle_n, |W\rangle_m, |W\rangle_t$) into a large-scale maximally entangled ($|W\rangle_{n+m+t}$) state with the help of two auxiliary atoms ($|g_3\rangle, |g_6\rangle$) will be introduced. In the fusion process, only one or two atoms from each W state have been accessed and are sent to the cavity mode for completing the fusion with one auxiliary atom. The remaining atoms are kept intact at their sites. The three initial W states can be written as[25]

$$|W_n\rangle = \frac{1}{\sqrt{n}}[|(n-1)_g\rangle|e\rangle_1+\sqrt{n-1}|W_{n-1}\rangle|g\rangle_1], \tag{5a}$$

$$\begin{aligned}
|W_m\rangle &= \frac{1}{\sqrt{m}}[|(m-2)_g\rangle(|g\rangle_2|e\rangle_4+|e\rangle_2|g\rangle_4)\\
&\quad +\sqrt{m-2}|W_{m-2}\rangle|g\rangle_2|g\rangle_4],
\end{aligned} \tag{5b}$$

$$|W_t\rangle = \frac{1}{\sqrt{t}}[|(t-1)_g\rangle|e\rangle_5+\sqrt{t-1}|W_{t-1}\rangle|g\rangle_5]. \tag{5c}$$

To explain this kind of expression of W state, we give the explicit expressions for $n=2,3$ cases, i.e., a tripartite W state is written as $|W_3\rangle=\frac{1}{\sqrt{3}}(|gge\rangle+|geg\rangle+|egg\rangle)=\frac{1}{\sqrt{3}}(|2_g\rangle|1_e\rangle_1+\sqrt{2}|W_2\rangle|1_g\rangle_1)$ with W_2 being $|W_2\rangle=\frac{1}{\sqrt{2}}(|ge\rangle+|eg\rangle)$. As shown in Fig. 1, the atom 1 from $|W_n\rangle$, atom 2 from $|W_m\rangle$ and a auxiliary atom $|g_3\rangle$ cross the cavity A, while atom 4 from $|W_m\rangle$, atom 5 from $|W_t\rangle$ and a auxiliary atom $|g_6\rangle$ cross the cavity B, simultaneously. The state of the total system can be described as

$$\begin{aligned}
|W_n\rangle\otimes|W_m\rangle\otimes|W_t\rangle\otimes|g_3\rangle\otimes|g_6\rangle &= \frac{\sqrt{n-1}\sqrt{m-2}\sqrt{t-1}}{\sqrt{nmt}}|W_{n-1}\rangle_g|W_{m-2}\rangle|W_{t-1}\rangle|g_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle|g_5\rangle|g_6\rangle\\
&\quad +\frac{1}{\sqrt{nmt}}|(n-1)\rangle_g|(m-2)\rangle_g|(t-1)\rangle_g(|e_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|e_5\rangle|g_6\rangle+|e_1\rangle|e_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle)\\
&\quad +\frac{\sqrt{t-1}}{\sqrt{nmt}}|(n-1)\rangle_g|(m-2)\rangle_g|W_{t-1}\rangle(|e_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|g_5\rangle|g_6\rangle+|e_1\rangle|e_2\rangle|g_3\rangle|g_4\rangle|g_5\rangle|g_6\rangle)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{\sqrt{n-1}}{\sqrt{nmt}}|W_{n-1}\rangle|(m-2)\rangle_g|(t-1)\rangle_g(|g_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|e_5\rangle|g_6\rangle+|g_1\rangle|e_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle)\\
&+\frac{\sqrt{n-1}\sqrt{t-1}}{\sqrt{nmt}}|W_{n-1}\rangle|(m-2)\rangle_g|W_{t-1}\rangle(|g_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|g_5\rangle|g_6\rangle+|g_1\rangle|e_2\rangle|g_3\rangle|g_4\rangle|g_5\rangle|g_6\rangle)\\
&+\frac{\sqrt{m-2}}{\sqrt{nmt}}|(n-1)\rangle_g|W_{m-2}\rangle|(t-1)\rangle_g|e_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle\\
&+\frac{\sqrt{m-2}\sqrt{t-1}}{\sqrt{nmt}}|(n-1)\rangle_g|W_{m-2}\rangle|W_{t-1}\rangle|e_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle|g_5\rangle|g_6\rangle\\
&+\frac{\sqrt{n-1}\sqrt{m-2}}{\sqrt{nmt}}|W_{n-1}\rangle|W_{m-2}\rangle|(t-1)\rangle_g|g_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle. \qquad (6)
\end{aligned}
$$

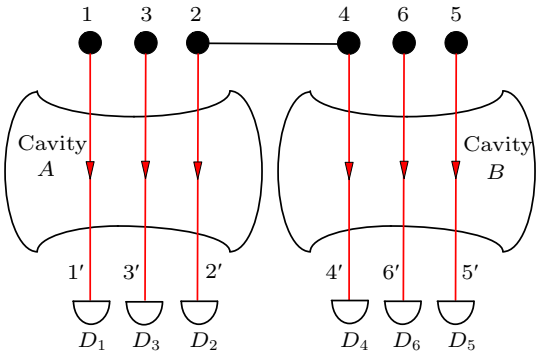


Fig. 1. The QLF fusion scheme for fusing three W states: atom 1 from $|W_n\rangle$, atom 2 from $|W_m\rangle$ and an auxiliary atom $|g_3\rangle$ cross the cavity A, atom 4 from $|W_m\rangle$, atom 5 from $|W_t\rangle$ and an auxiliary atom $|g_6\rangle$ cross the cavity B. The entanglement of atoms will be formed after the atom–cavity-field detuned interaction. D_1, D_2, D_3, D_4, D_5, D_6 denote the atomic detectors. The output atoms are labeled as $1'$, $2'$, $3'$, $4'$, $5'$, $6'$.

After the detuned interaction between the three atoms and the cavity mode depicted in Eq. (4), a lot of entangled states will be generated. Since there are more input states in Eq. (6) and more corresponding output states after fusion mechanism, we take the $|W_4\rangle$ state as an example to briefly explain which input states may produce the target W state. The maximally entangled $|W_4\rangle$ state can be written as

$$|W_4\rangle=\frac{1}{2}(|ggge\rangle+|ggeg\rangle+|gegg\rangle+|eggg\rangle). \qquad (7)$$

Obviously, each term in $|W_4\rangle$ state is made up of three g atoms and one e atom. The n-atom maximally entangled W state is similar. There should be three e atoms in each of its terms in the expression of the final large-scale W state with the QLF fusion scheme for fusing three $|W_4\rangle$ states. Hence, at least two e atoms have to be detected in order to generate a maximally entangled W state. In Eq. (4), if three g atoms ($|g_1\rangle|g_2\rangle|g_3\rangle$ or $|g_4\rangle|g_5\rangle|g_6\rangle$) are input into the cavity mode, then it is impossible to detect an e atom in the output atoms. But in the case where every three atoms in cavity A and cavity B are all g, the product state of $|W_{n-1}\rangle$, $|W_{m-2}\rangle$ and $|W_{t-1}\rangle$ can be generated if the six g atoms in the output mode are all detected. This product state can be re-fused in the next fusion. Therefore, we only need to consider the fusion with the following input states:

$$
\begin{aligned}
|\psi\rangle=&\frac{1}{\sqrt{nmt}}|(n-1)\rangle_g|(m-2)\rangle_g|(t-1)\rangle_g|e_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|e_5\rangle|g_6\rangle+\frac{1}{\sqrt{nmt}}|(n-1)\rangle_g|(m-2)\rangle_g|(t-1)\rangle_g|e_1\rangle|e_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle\\
&+\frac{\sqrt{t-1}}{\sqrt{nmt}}|(n-1)\rangle_g|(m-2)\rangle_g|W_{t-1}\rangle|e_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|g_5\rangle|g_6\rangle+\frac{\sqrt{m-2}}{\sqrt{nmt}}|(n-1)\rangle_g|W_{m-2}\rangle|(t-1)\rangle_g|e_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle\\
&+\frac{\sqrt{n-1}}{\sqrt{nmt}}|W_{n-1}\rangle|(m-2)\rangle_g|(t-1)\rangle_g|g_1\rangle|e_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle\\
&+\frac{\sqrt{n-1}\sqrt{m-2}\sqrt{t-1}}{\sqrt{nmt}}|W_{n-1}\rangle_g|W_{m-2}\rangle|W_{t-1}\rangle|g_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle|g_5\rangle|g_6\rangle. \qquad (8)
\end{aligned}
$$

After the atom–cavity-field detuned interaction, the evolution of the total system, and the success and failure properties of the output states are listed in Table 1. Three maximally entangled target $|W_{n+m+t}\rangle$ states will be generated by different detections in the output atoms. For example, after flying out of the cavity, when the results $|e\rangle_{3'}|e\rangle_{6'}$ have been detected on the D_3 and D_6, a $(m+n+t)$-qubit W-like state can be obtained as the following situation:

$$
\begin{aligned}
|\widetilde{W}_{n+m+t}\rangle=&\frac{1}{\sqrt{nmt}}\mathrm{e}^{-\mathrm{i}\lambda t}|(n-1)\rangle_g|(m-2)\rangle_g|(t-1)\rangle_g(k_1^2|ggeg\rangle_{1'2'4'5'}+k_1^2|ggge\rangle_{1'2'4'5'})\\
&+\frac{1}{\sqrt{nmt}}\mathrm{e}^{-\mathrm{i}\lambda t}|(n-1)\rangle_g|(m-2)\rangle_g|(t-1)\rangle_g(k_1^2|gegg\rangle_{1'2'4'5'}+k_1^2|eggg\rangle_{1'2'4'5'})\\
&+\frac{\sqrt{t-1}}{\sqrt{nmt}}|(n-1)\rangle_g|(m-2)\rangle_g|W_{t-1}\rangle k_1^2|gggg\rangle_{1'2'4'5'}+\frac{\sqrt{m-2}}{\sqrt{nmt}}|(n-1)\rangle_g|W_{m-2}\rangle|(t-1)\rangle_g k_1^2|gggg\rangle_{1'2'4'5'}
\end{aligned}
$$

$$+\frac{\sqrt{n-1}}{\sqrt{nmt}}|W_{n-1}\rangle|(m-2)\rangle_g|(t-1)\rangle_g k_1^2|gggg\rangle_{1'2'4'5'}. \tag{9}$$

By controlling the velocity of the atoms appropriately, the interaction time satisfies the relation $\lambda t = 2\pi/9$, and the state in Eq. (9) will become

$$\begin{aligned}|\widetilde{W}_{n+m+t}\rangle \;=\; & \frac{1}{3\sqrt{nmt}}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{9}}|(n-1)\rangle_g|(m-2)\rangle_g|(t-1)\rangle_g(|ggeg\rangle_{1'2'4'5'}+|ggge\rangle_{1'2'4'5'}\\ & +|(n-1)\rangle_g|(m-2)\rangle_g|(t-1)\rangle_g|gegg\rangle_{1'2'4'5'}+|eggg\rangle_{1'2'4'5'})+\frac{\sqrt{t-1}}{3\sqrt{nmt}}|(n-1)\rangle_g|(m-2)\rangle_g|W_{t-1}\rangle|gggg\rangle_{1'2'4'5'}\\ & +\frac{\sqrt{m-2}}{3\sqrt{nmt}}|(n-1)\rangle_g|W_{m-2}\rangle|(t-1)\rangle_g|gggg\rangle_{1'2'4'5'}+\frac{\sqrt{n-1}}{3\sqrt{nmt}}|W_{n-1}\rangle|(m-2)\rangle_g|(t-1)\rangle_g|gggg\rangle_{1'2'4'5'},\end{aligned} \tag{10}$$

where the global phase factor $\mathrm{e}^{-\mathrm{i}\frac{5}{3}\pi}$ has been discarded. Although the above state is not the maximally entangled $|W_{n+m+t}\rangle$ state, there is only one relative phase difference between the $|\widetilde{W}_{n+m+t}\rangle$ state and the standard $|W_{n+m+t}\rangle$ state, which can be removed by a classical pulse on any one of the $(n+m+t)$ atoms. Similarly, as shown in Table 1, the other two maximally entangled $|W_{n+m+t}\rangle$ states can be generated by detecting $|e\rangle_{1'}|e\rangle_{4'}$ or $|e\rangle_{2'}|e\rangle_{5'}$, respectively. It should be stated that different entanglement states can be generated probabilistically. However, in each implementation of the scheme, only one of the entangled states could be generated. The success probability of each target $|W_{n+m+t}\rangle$ state is

$$P_{\mathrm{s}}(W_n, W_m, W_t) = \frac{n+m+t}{9nmt}. \tag{11}$$

Table 1. Truth table of the QLF fusion scheme for fusing three W states. "Input " indicates the atoms at the input of the cavity A and cavity B, and "Output" indicates the atoms after the cavity modes. "Result " indicates the success (S), failure (F) and recyclable (R) properties of the output states of the fusion scheme. "S_1 ", "S_2 ", and "S_3 " indicate one target W state by different detections in the output atoms.

Input	Output	Result	Input	Output	Result
	$\mathrm{e}^{-\mathrm{i}\lambda t}k_1^2\|ggegee\rangle_{1'2'3'4'5'6'}$	S_3		$\mathrm{e}^{-\mathrm{i}\lambda t}k_1k_2\|geegeg\rangle_{1'2'3'4'5'6'}$	S_2
	$\mathrm{e}^{-\mathrm{i}\lambda t}k_1^2\|ggeege\rangle_{1'2'3'4'5'6'}$	S_3		$\mathrm{e}^{-\mathrm{i}\lambda t}k_1^2\|geegge\rangle_{1'2'3'4'5'6'}$	S_3
	$\mathrm{e}^{-\mathrm{i}\lambda t}k_1k_2\|ggeeeg\rangle_{1'2'3'4'5'6'}$	F		$\mathrm{e}^{-\mathrm{i}\lambda t}k_1^2\|geeegg\rangle_{1'2'3'4'5'6'}$	F
$\|e_1\rangle\|g_2\rangle\|g_3\rangle$	$\mathrm{e}^{-\mathrm{i}\lambda t}k_1^2\|geggee\rangle_{1'2'3'4'5'6'}$	S_2	$\|e_1\rangle\|e_2\rangle\|g_3\rangle$	$\mathrm{e}^{-\mathrm{i}\lambda t}k_1^2\|egeegg\rangle_{1'2'3'4'5'6'}$	S_1
$\|e_4\rangle\|e_5\rangle\|g_6\rangle$	$\mathrm{e}^{-\mathrm{i}\lambda t}k_1^2\|gegege\rangle_{1'2'3'4'5'6'}$	F	$\|g_4\rangle\|e_5\rangle\|g_6\rangle$	$\mathrm{e}^{-\mathrm{i}\lambda t}k_1^2\|egegge\rangle_{1'2'3'4'5'6'}$	S_3
	$\mathrm{e}^{-\mathrm{i}\lambda t}k_1k_2\|gegeeg\rangle_{1'2'3'4'5'6'}$	S_2		$\mathrm{e}^{-\mathrm{i}\lambda t}k_1k_2\|egegeg\rangle_{1'2'3'4'5'6'}$	F
	$\mathrm{e}^{-\mathrm{i}\lambda t}k_2k_1\|egggee\rangle_{1'2'3'4'5'6'}$	F		$\mathrm{e}^{-\mathrm{i}\lambda t}k_2k_1\|eeggge\rangle_{1'2'3'4'5'6'}$	F
	$\mathrm{e}^{-\mathrm{i}\lambda t}k_2k_1\|eggege\rangle_{1'2'3'4'5'6'}$	S_1		$\mathrm{e}^{-\mathrm{i}\lambda t}k_2^2\|eeggeg\rangle_{1'2'3'4'5'6'}$	S_2
	$\mathrm{e}^{-\mathrm{i}\lambda t}k_2^2\|eggeeg\rangle_{1'2'3'4'5'6'}$	S_1		$\mathrm{e}^{-\mathrm{i}\lambda t}k_2k_1\|eegegg\rangle_{1'2'3'4'5'6'}$	S_1
	$k_1k_2\|ggeegg\rangle_{1'2'3'4'5'6'}$	F		$k_1k_2\|ggegeg\rangle_{1'2'3'4'5'6'}$	F
	$k_1^2\|ggegeg\rangle_{1'2'3'4'5'6'}$	F		$k_1^2\|ggegge\rangle_{1'2'3'4'5'6'}$	S_3
	$k_1^2\|ggegge\rangle_{1'2'3'4'5'6'}$	S_3		$k_1^2\|ggeegg\rangle_{1'2'3'4'5'6'}$	F
$\|e_1\rangle\|g_2\rangle\|g_3\rangle$	$k_1k_2\|gegegg\rangle_{1'2'3'4'5'6'}$	F	$\|e_1\rangle\|g_2\rangle\|g_3\rangle$	$k_1k_2\|geggeg\rangle_{1'2'3'4'5'6'}$	S_2
$\|e_4\rangle\|g_5\rangle\|g_6\rangle$	$k_1^2\|geggeg\rangle_{1'2'3'4'5'6'}$	S_2	$\|g_4\rangle\|e_5\rangle\|g_6\rangle$	$k_1^2\|geggge\rangle_{1'2'3'4'5'6'}$	F
	$k_1^2\|geggge\rangle_{1'2'3'4'5'6'}$	F		$k_1^2\|gegegg\rangle_{1'2'3'4'5'6'}$	F
	$k_2^2\|eggegg\rangle_{1'2'3'4'5'6'}$	S_1		$k_2^2\|egggeg\rangle_{1'2'3'4'5'6'}$	F
	$k_2k_1\|egggeg\rangle_{1'2'3'4'5'6'}$	F		$k_2k_1\|egggge\rangle_{1'2'3'4'5'6'}$	F
	$k_2k_1\|eggggg\rangle_{1'2'3'4'5'6'}$	F		$k_2k_1\|eggegg\rangle_{1'2'3'4'5'6'}$	S_1
	$k_1k_2\|ggegeg\rangle_{1'2'3'4'5'6'}$	F			
	$k_1^2\|ggegge\rangle_{1'2'3'4'5'6'}$	S_3			
	$k_1^2\|ggeegg\rangle_{1'2'3'4'5'6'}$	F			
$\|g_1\rangle\|e_2\rangle\|g_3\rangle$	$k_2^2\|geggeg\rangle_{1'2'3'4'5'6'}$	S_2	$\|g_1\rangle\|g_2\rangle\|g_3\rangle$		
$\|g_4\rangle\|e_5\rangle\|g_6\rangle$	$k_2k_1\|geggge\rangle_{1'2'3'4'5'6'}$	F	$\|g_4\rangle\|g_5\rangle\|g_6\rangle$	$\|gggggg\rangle_{1'2'3'4'5'6'}$	R
	$k_2k_1\|gegegg\rangle_{1'2'3'4'5'6'}$	F			
	$k_1k_2\|egggeg\rangle_{1'2'3'4'5'6'}$	F			
	$k_1^2\|egggge\rangle_{1'2'3'4'5'6'}$	F			
	$k_1^2\|eggegg\rangle_{1'2'3'4'5'6'}$	S_1			

4. Qubit-loss-free fusion scheme of four atom W states

In this section, we will discuss the detailed process of generating a large-scale maximally entangled $|W_{n+m+t+q}\rangle$ state by fusing state $|W_n\rangle$, $|W_m\rangle$ and $|W_t\rangle$ with the help of three auxiliary atoms ($|g_3\rangle, |g_6\rangle, |g_9\rangle$). The fusion process is depicted in Fig. 2. The four W initial states can be written as

$$|W_n\rangle = \frac{1}{\sqrt{n}}[|(n-1)_g\rangle|e\rangle_1 + \sqrt{n-1}|W_{n-1}\rangle|g\rangle_1], \quad (12a)$$

$$|W_m\rangle = \frac{1}{\sqrt{m}}[|(m-2)_g\rangle(|g\rangle_2|e\rangle_4 + |e\rangle_2|g\rangle_4) + \sqrt{m-2}|W_{m-2}\rangle|g\rangle_2|g\rangle_4], \quad (12b)$$

$$|W_t\rangle = \frac{1}{\sqrt{t}}[|(t-2)_g\rangle(|g\rangle_5|e\rangle_7 + |e\rangle_5|g\rangle_7) + \sqrt{t-2}|W_{t-2}\rangle|g\rangle_5|g\rangle_7], \quad (12c)$$

$$|W_q\rangle = \frac{1}{\sqrt{q}}[|(q-1)_g\rangle|e\rangle_8 + \sqrt{q-1}|W_{q-1}\rangle|g\rangle_8]. \quad (12d)$$

Similarly, atom 1 from $|W_n\rangle$, atom 2 from $|W_m\rangle$, and an auxiliary atom $|g_3\rangle$ cross the cavity A, atom 4 from $|W_m\rangle$, atom 5 from $|W_t\rangle$ and a auxiliary atom $|g_6\rangle$ cross the cavity B, atom 7 from $|W_t\rangle$, atom 8 from $|W_q\rangle$ and an auxiliary atom $|g_9\rangle$ cross the cavity C, simultaneously. The state of the system can be described as

$$\begin{aligned}
&|W_n\rangle\otimes|W_m\rangle\otimes|W_t\rangle\otimes|W_q\rangle\otimes|g_3\rangle\otimes|g_6\rangle\otimes|g_9\rangle\\
=\ &+\frac{1}{\sqrt{nmtq}}|(n-1)\rangle_g|(m-2)\rangle_g|(t-2)\rangle_g|(q-1)\rangle_g(|e_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|g_5\rangle|g_6\rangle|e_7\rangle|e_8\rangle|g_9\rangle\\
&+|e_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|e_5\rangle|g_6\rangle|g_7\rangle|e_8\rangle|g_9\rangle+|e_1\rangle|e_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle|g_7\rangle|e_8\rangle|g_9\rangle)\\
&+\frac{\sqrt{t-2}}{\sqrt{nmtq}}|(n-1)\rangle_g|(m-2)\rangle_g|W_{t-2}\rangle|(q-1)\rangle_g|e_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|g_5\rangle|g_6\rangle|g_7\rangle|e_8\rangle|g_9\rangle\\
&+\frac{\sqrt{n-1}}{\sqrt{nmtq}}|W_{n-1}\rangle|(m-2)\rangle_g|(t-2)|(q-1)\rangle\rangle_g|g_1\rangle|e_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle|g_7\rangle|e_8\rangle|g_9\rangle\\
&+\frac{\sqrt{m-2}}{\sqrt{nmtq}}|(n-1)\rangle_g|W_{m-2}\rangle|(t-2)\rangle_g|(q-1)\rangle_g|e_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle|e_5\rangle|g_6\rangle|g_7\rangle|e_8\rangle|g_9\rangle\\
&+\frac{\sqrt{q-1}}{\sqrt{nmtq}}|(n-1)\rangle_g|m-2\rangle_g|t-2\rangle_g|W_{q-1}\rangle|e_1\rangle|g_2\rangle|g_3\rangle|e_4\rangle|g_5\rangle|g_6\rangle|e_7\rangle|g_8\rangle|g_9\rangle\\
&+\frac{\sqrt{n-1}\sqrt{m-2}\sqrt{t-2}\sqrt{q-1}}{\sqrt{nmtq}}|W_{n-1}\rangle_g|W_{m-2}\rangle|W_{t-2}\rangle|W_{q-1}\rangle|g_1\rangle|g_2\rangle|g_3\rangle|g_4\rangle|g_5\rangle|g_6\rangle|g_7\rangle|g_8\rangle|g_9\rangle. \quad (13)
\end{aligned}$$

We also consider the input states which can generate target W state and will be re-fused in the next fusion. Through the detuned interaction of atom–cavity-field, three maximally entangled $|W_{n+m+t+q}\rangle$ states will be obtained by detecting the output atoms. When the results $|e\rangle_{3'}|e\rangle_{6'}|e\rangle_{9'}$ on the D_3, D_6, D_9 are detected, one of three $|W_{n+m+t+q}\rangle$ can be described as follows:

$$\begin{aligned}
|\widetilde{W}_{n+m+t+q}\rangle = \ &\frac{1}{\sqrt{nmtq}}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{9}}|(n-1)_g\rangle|(m-2)_g\rangle|(t-2)\rangle_g|(q-1)\rangle_g(|ggggeg\rangle_{1'2'4'5'7'8'}+|gggggе\rangle_{1'2'4'5'7'8'}\\
&+|gggegg\rangle_{1'2'4'5'7'8'}+|ggeggg\rangle_{1'2'4'5'7'8'}+|gegggg\rangle_{1'2'4'5'7'8'}+|eggggg\rangle_{1'2'4'5'7'8'})\\
&+\frac{\sqrt{n-1}}{\sqrt{nmtq}}|W_{n-1}\rangle|(m-2)\rangle_g\rangle|(t-2)\rangle_g|(q-1)\rangle_g|gggggg\rangle_{1'2'4'5'7'8'}\rangle\\
&+\frac{\sqrt{m-2}}{\sqrt{nmtq}}|(n-1)\rangle_g|W_{m-2}\rangle\rangle|(t-2)\rangle_g|(q-1)\rangle_g|gggggg\rangle_{1'2'4'5'7'8'}\rangle\\
&+\frac{\sqrt{t-2}}{\sqrt{nmtq}}|(n-1)\rangle_g|(m-2)\rangle_g|W_{t-2}\rangle|(q-1)\rangle_g|gggggg\rangle_{1'2'4'5'7'8'}\rangle\\
&+\frac{\sqrt{q-1}}{\sqrt{nmtq}}|(n-1)\rangle_g|(m-2)\rangle_g|(t-2)\rangle|W_{q-1}\rangle|gggggg\rangle_{1'2'4'5'7'8'}\rangle, \quad (14)
\end{aligned}$$

where the global phase factor $\mathrm{e}^{-\mathrm{i}\frac{5}{2}\pi}$ has been discarded. The relative phase $\mathrm{e}^{-\mathrm{i}\frac{2\pi}{9}}$ can be removed by a classical pulse on any one of the $(n+m+t+q)$ atoms. If the output atoms $|e\rangle_{2'}|e\rangle_{5'}|e\rangle_{8'}$ or $|e\rangle_{1'}|e\rangle_{4'}|e\rangle_{7'}$ have been detected, then another two target W states of $W_{n+m+t+q}$ will be generated. The maximum successful probability of each target $|W_{n+m+t+q}\rangle$ state is

$$P_{\mathrm{s}}(W_n, W_m, W_t, W_q) = \frac{n+m+t+q}{27nmtq}. \quad (15)$$

In the case where nine g atoms have been detected, the product

state of $|W_{n-1}\rangle$, $|W_{m-2}\rangle$, $|W_{t-2}\rangle$ and $|W_{q-1}\rangle$ can be generated and will be re-fused in the next fusion. Compared with most of current schemes, our scheme not only realizes the QLF fusion of fusing four W states simultaneously but also prepares three target W states. This is a great progress in the successful preparation of the W state. In addition, this scheme can be generalized to the case of the QLF fusion to fuse k different or same particles W states. The $(k-1)$ cavity modes and $(k-1)$ auxiliary g atoms are indispensable. For example, three maximally entangled target W_{kn} will be obtained by detecting $(k-1)$ e atoms if fusing the same particle (n) W states. The maximum successful probability of each target $|W_{kn}\rangle$ state is $P_{\rm s} = kn/(3^{k-1}n^k)$.

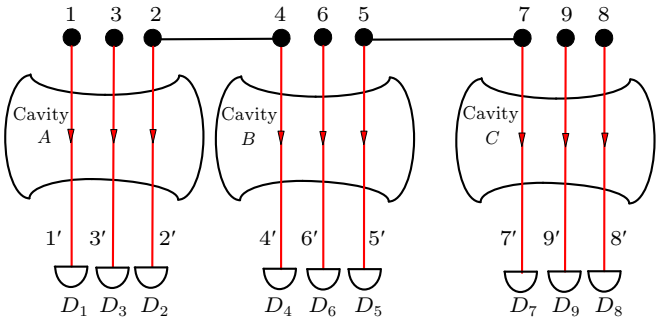


Fig. 2. The QLF fusion scheme for fusing four W states: atom 1 from $|W_n\rangle$, atom 2 from $|W_m\rangle$ and an auxiliary atom $|g_3\rangle$ cross the cavity A, atom 4 from $|W_m\rangle$, atom 5 from $|W_t\rangle$ and an auxiliary atom $|g_6\rangle$ cross the cavity B, atom 7 from $|W_t\rangle$, atom 8 from $|W_q\rangle$ and an auxiliary atom $|g_9\rangle$ cross the cavity C. D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9 denotes the atomic detectors. The output atoms are labeled as $1'$, $2'$, $3'$, $4'$, $5'$, $6'$, $7'$, $8'$, $9'$.

5. Cost analysis and discussion

In this section, we will estimate the performance of the QLF fusion scheme by presenting the results of numerical simulations and compare the resource cost of generating a larger-scale W state using our fusion scheme with the previous schemes. Because there are few schemes for fusing four W small-size states, we only compare the schemes that fuse three W states. The resource cost of preparing a $|W_{n+m+t}\rangle$ state by fusing $|W_n\rangle$, $|W_m\rangle$ and $|W_t\rangle$ is defined as follows:[25]

$$R[W_{n+m+t}] = \frac{R[W_n]+R[W_m]+R[W_t]}{P_{\rm s}(W_n,W_m,W_t)}, \tag{16}$$

where $P_{\rm s}(W_n,W_m,W_t)$ is the success probability of fusing three small-size W states. Since the three target W states can be obtained by our QLF fusion scheme, the success probability of fusing three W states is

$$P_{\rm s}(W_m,W_n,W_t) = \frac{m+n+t}{3mnt}. \tag{17}$$

Although our QLF fusion scheme can work for $|W_2\rangle$ states, most of the previous schemes can only start with $|W_3\rangle$ states; therefore, for the purpose of comparison, we are going to define the basic resource for preparing $|W_3\rangle$ state as unit cost, i.e., $R[W_3] = 1$. According to the discussion in Ref. [25], the closer the sizes of the two resource states are, the lower the cost is. Based on this principle, we use the formula to calculate the optimal cost of our fusion schemes for W states and those of previous ones in Fig. 3. The figure shows that our QLF fusion scheme is more efficient than the previous fusion schemes except for Ref. [32]. Although the resource cost of our scheme seems higher than that in Ref. [32] at $N > 30$, a single Fredkin gate was used in Ref. [32], which clearly adds complexity to the physical implementation. Another very important issue in a repeated fusion operation is the availability of quantum memories to store the throughput W states obtained at each fusion step. This requirement clearly increases the complexity of physical realization considerably. Therefore, the number of fusion steps to achieve a W state of a target size will be greatly reduced by applying the QLF fusion scheme. Furthermore, because three target W states can be prepared, our scheme has another distinct advantage: the number of particles of the final W state prepared by our scheme is much larger than that of the previous schemes. For instance, if the impact of requirement of the intermediate quantum memories in repeat fusion operation is ignored, we define the three $|W_3\rangle$ states as the initial states, three $|W_9\rangle$ states will be generated by using our QLF fusion scheme, and three $|W_{27}\rangle$ states will be obtained by fusing these three $|W_9\rangle$ states, and then we continue fusing these $|W_{27}\rangle$ states into three $|W_{81}\rangle$, etc. Of course, even if it is not in the ideal state, the ability to generate three target W states still makes our scheme a great progress in the successful preparation of the target W state.

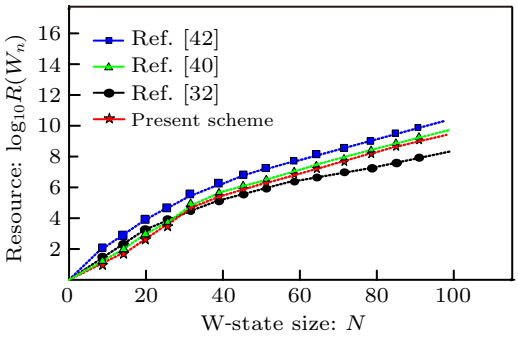


Fig. 3. The resource cost comparison: the fusion scheme in Ref. [42] (blue square), the fusion scheme in Ref. [40] (green triangle), our fusion scheme for fusing three W states (red asterisk), and the fusion scheme in Ref. [32] (black circle).

It is also necessary to give a brief discussion on the experimental feasibility of our scheme. The radiative time of the Rydberg atoms with principal quantum numbers 49, 50, and 51 is $T_{\rm r} = 3\times10^{-2}$ s, and the atom–cavity coupling strength is $g = 2\pi\times24$ kHz.[45–47] The required atom–cavity interaction time for our fusion scheme is about $t = 10^{-4}$ s when $\delta = 10g$. In Ref. [47], the efficient decay time of the cavity (50 GHz) is about $t = 3\times10^{-2}$ s if we control the quality factor of order $Q = 10^8$. In this case, the time required for accomplishing the

whole process in our QLF fusion scheme is much shorter than the time of cavity decay and atomic radiation. So, our fusion schemes can be realized in the current cavity QED technology.

6. Conclusion

In conclusion, the QLF fusion scheme has been presented for generating large-scale W states based on the atom–cavity-field detuned interaction. In this fusion scheme, by accessing one or two particles from each W state to the cavity quantum system, three $|W_n+m+t\rangle$ states can be prepared by fusing W_m, W_n and W_t states with the help of two auxiliary atoms. Three $|W_n+m+t+q\rangle$ states can also be obtained from fusing four small-size W states with the help of three auxiliary atoms. When more cavity fields and auxiliary atoms are added, the QLF fusion scheme can be realized in fusing more W states, simultaneously. The cost analysis shows that our fusion scheme is more efficient than most of the previous schemes. In addition, since three target W states are prepared, our scheme has a distinct advantage in that it generates more particles of the final W states. Moreover, the QLF fusion scheme will greatly reduce the number of fusion steps, as well as the requirement of quantum memories required to achieve a W state of a target size, which confirms that the present preparation scheme can be implemented with a simpler and easier method.

Acknowledgements

Project supported by the National Natural Science Foundation of China (Grant No. 12204311) and the Jiangxi Natural Science Foundation (Grant No. 20224BAB211025).

References

[1] Raussendorf R, Browne D E and Briegel H J 2003 *Phys. Rev. A* **68** 022312
[2] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[3] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575
[4] Özdemir Ş K, Bartkiewicz K and Liu Y X 2007 *Phys. Rev. A* **76** 042325
[5] Gisin N, Ribordy G, Tittel W and Zbinden H 2002 *Rev. Mod. Phys.* **74** 145
[6] Li T C and Yin Z Q 2016 *Sci. Bull.* **61** 163
[7] Dür W 2001 *Phys. Rev. A* **63** 020303
[8] Greenberger D M, Horne M A, Shimony A and Zeilinger A 1990 *Am. J. Phys.* **58** 1131
[9] Briegel H J and Raussendorf R 2001 *Phys. Rev. Lett.* **86** 910
[10] Dür W, Vidal G and Cirac J I 2000 *Phys. Rev. A* **62** 062314
[11] Gräfe M 2014 *Nat. Photonics* **8** 791
[12] Ng H T and Kim K 2014 *Opt. Commun.* **331** 353
[13] Ozaydin F 2014 *Phys. Lett. A* **378** 3161
[14] Yu N, Guo C and Duan R 2014 *Phys. Rev. Lett.* **112** 160401
[15] Murao M, Jonathan DM, Plenio B and Vedral V 1999 *Phys. Rev. A* **59** 156
[16] Shi B S and Tomita A 2002 *Phys. Lett. A* **296** 161
[17] Joo J, Park, Y J, Oh S and Kim J 2003 *New J. Phys.* **5** 136
[18] Anders K and Mohamed B 1998 *Phys. Rev. A* **58** 4394
[19] Bose S, Vedral V and Kninght 1998 *Phys. Rev. A* **57** 822
[20] Liu X S, Long G L, Tong D M and Li F 2002 *Phys. Rev. A* **65** 022304
[21] D'Hondt E and Panangaden P 2006 *Quantum Inf. Comput.* **6** 173
[22] Ozaydin F 2014 *Phys. Lett. A* **378** 3161
[23] Ozaydin F, Altintas A, Yesilyurt C, Bugu S and Erol V 2015 *Acta Phys. Pol. A* **127** 1233
[24] Dag C B and Mustecaplioglu O E 2015 *Entropy* **18** 244
[25] Özdemir Ş K, Matsunaga E, Tashima T, Yamamoto T, Koashi M and Imoto N 2011 *New J. Phys.* **13** 103003
[26] Tashima T, Özdemir Ş K, Yamamoto T, Koashi M and Imoto N 2008 *Phys. Rev. A* **77** 030302
[27] Tashima T, Özdemir Ş K, Yamamoto T, Koashi M and Imoto N 2009 *New J. Phys.* **11** 023024
[28] Zang X P, Yang M, Wu W F, Fang S D and Cao Z L 2014 *Indian J. Phys.* **88** 1141
[29] Zang X P, Yang M, Ozaydin F, Song W and Cao Z L 2016 *Opt. Express* **24** 12293
[30] Yesilyurt C, Bugu S, Ozaydin F, Altintas A, Tame M, Yang L and Özdemir Ş K 2016 *J. Opt. Soc. Am. B* **33** 2313
[31] Tashima T, Kitano T, Özdemir Ş K, Yamamoto T, Koashi M and Imoto N 2010 *Phys. Rev. Lett.* **105** 210503
[32] Ozaydin F, Bugu S, Yesilyurt C, Altintas A A, Tame M and Özdemir Ş K 2014 *Phys. Rev. A* **89** 042311
[33] Yesilyurt C, Bugu S and Ozaydin F 2013 *Quantum Inf. Process.* **12** 2965
[34] Bugu S, Yesilyurt C and Ozaydin F 2013 *Phys. Rev. A* **87** 032331
[35] Zang X P, Yang M, Ozaydin F, Song W and Cao Z L 2015 *Sci. Rep.* **5** 16245
[36] Dikera F, Ozaydinb F and Arika M 2015 *Acta Phys. Pol. A* **127** 1189
[37] Li N, Yang J and Ye L 2015 *Quantum Inf. Process.* **14** 1933
[38] Li K, Chen T T, Mao H B and Wang J Q 2018 *Quantum Inf. Process.* **17** 307
[39] Li K, Kong F Z, Yang M, Yang Q and Cao Z L 2016 *Phys. Rev. A* **94** 062315
[40] Wang M Y, Hao Q Z, Yan F L and Gao T 2018 *Laser Phys. Lett.* **15** 055201
[41] Li K, Chen T T, Mao H B and Wang J Q 2019 *Quantum Inf. Process.* **18** 273
[42] Ding C Y, Kong F Z, Yang M, Yang Q and Cao Z L 2018 *Quantum Inf. Process.* **17** 124
[43] Guo G P, Li C F, Li J and Guo G C 2002 *Phys. Rev. A* **65** 042102
[44] Alexandre B, Grimsmo A L, Girvin S M and Andreas W 2021 *Rev. Mod. Phys.* **93** 25005
[45] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
[46] Yang M and Cao Z L 2006 *Phys. A* **366** 243
[47] Brune M, Hagley E, Dreyer J, Maitre X, Maali A, Wunderlich C, Raimond J M and Haroche S 1996 *Phys. Rev. Lett.* **77** 4887