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Detecting the core of a network by the centralities of the nodes

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Many networks exhibit the core/periphery structure. Core/periphery structure is a type of meso-scale structure that consists of densely connected core nodes and sparsely connected peripheral nodes. Core nodes tend to be well-connected, both among themselves and to peripheral nodes, which tend not to be well-connected to other nodes. In this brief report, we propose a new method to detect the core of a network by the centrality of each node. It is discovered that such nodes with non-negative centralities often consist in the core of the networks. The simulation is carried out on different real networks. The results are checked by the objective function. The checked results may show the effectiveness of the simulation results by the centralities of the nodes on the real networks. Furthermore, we discuss the characters of networks with the single core/periphery structure and point out the scope of the application of our method at the end of this paper.

Keywords: complex network, core/periphery structure, the objective function

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1. Introduction

Many networks exhibit the core/periphery structure, which is important for scientists to understand the properties and the dynamics of the networks.[1] The constitution of the structure often refers to a simple two-class partition.[2–4] The definition of core/periphery has been formalized in Ref. [2]. The notion of the core in the network is often used to describe the connectivity between the nodes with large degree, and it has been applied to profile meso-scale properties in networks by examining the density of connections.[5–10] The core in the network influences the functionality of itself. It is demonstrated in the transmission of rumors or information in social networks,[11,12] in the organization of the human connection in neurodevelopment,[13,14] in the transportation networks of airline flights,[15] and in the characterizing data patterns.[16]

The core of the network is often regarded to be comprised of the densely inter-connected high-degree nodes, which impact adaptability, flexibility, and controllability.[17,18] Lots of profiling methods have been proposed based on optimizing a suitable fitness function using the coreness value to define the density of links inside the core,[2] referring to a quality index with respect to the size of the expected core and the fuzziness of the boundary,[19] or applying Markov chains to describe random walks to index the coreness of individual nodes.[20] These methods rely on subjective fine-tuning due to the presence of one or more free parameters, which could be obtained arbitrarily or by techniques. Other examples include maximizing the closeness centrality within the core by an ensemble of random networks to define a coefficient that characterizes the core.[3] Recently, some scientists have proposed to use an influence propagation process to detect the pairs of core-periphery nodes.[21] Generally, these methods always tend to be relatively complex in nature, and therefore scalability issues are likely to be encountered when applied to the huge networks.

Interestingly, the presence of a network with core/periphery structure could be divided into two parts. However, there is, at present, no general method to define the core nodes. Due to the importance of the core/periphery structure in the networks, we present a method to profile such structure in this brief report. First, we define the centrality of each node. It could be considered that the core of a network consists of the nodes with non-negative centralities. Second, the results can be checked by the objective function. It is discovered that the nodes with non-negative centralities may be included to the core by detecting the core/periphery structure. Third, we discuss the character of the network with single core/periphery structure on the synthetic network. Furthermore, we describe the scope of the application of our method.

2. Method

Consider an unweighted and undirected network. A complex network can be described as a graph set $G(V,E)$. The node set is V, and the connection set is E. In this paper, it is considered that these connections are undirected in the network. Each element in the adjacency matrix A of the network is 0 or 1, when node i is connected with node j, the element $a_{ij}=1$, otherwise $a_{ij}=0$. The degree of node i is $k_i=\sum_j a_{ij}$. Then, we can define the centrality $C(i)$ of the node i in the network. It is considered that the core node in the star network may have the highest centrality, while the periphery nodes have the lowest centrality. The centrality of node i is defined

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as

$$C(i) = \frac{1}{k_i} \sum_{j=1}^{N} \frac{a_{ij}(k_i - k_j)}{k_i + k_j - 2}. \quad (1)$$

It is discovered that the value of the node centrality $C(i)$ only depends on the degree of node i itself and its neighbors' degrees instead of all the nodes in the network.

Obviously, the star network possesses the most significant core/periphery structure. Thus, the centrality of the core node in the star network is equal to 1, while the centrality of the periphery node in the star network is equal to -1. Usually, the heterogeneity between a pair of nodes in the complex network is not as large as the one in the star network. Yet the centrality of the node in the regular network with the period boundary condition or in the globally connected network may be equal to 0 because of the same degree of each node. In this paper, it is defined that the node is a core node if its centrality is larger or equal to zero.[22] The node with negative centrality is regarded as a peripheral one.[23]

For instance, we can calculate the nodes of the centralities in the karate club network[24] by formula (1). The results are listed in order in Table 1. We may establish a core set of these core nodes. The core set is $\{34,1,33,2,3,6,7,4\}$. Thus, it may be considered that the core of the karate club network contains these eight nodes at least.

Table 1. The centrality of each node in the karate club network.

Node	Centrality	Node	Centrality	Node	Centrality	Node	Centrality
34	0.7232	24	–0.0895	31	–0.4633	10	–0.8412
1	0.6645	25	–0.2095	8	–0.4678	15	–0.8578
33	0.5444	30	–0.2246	17	–0.5000	16	–0.8578
2	0.3628	26	–0.2540	29	–0.6142	19	–0.8578
3	0.3200	28	–0.2818	27	–0.6912	21	–0.8578
6	0.0083	5	–0.3216	20	–0.7142	23	–0.8578
7	0.0083	11	–0.3216	13	–0.7708	12	–1.0000
4	0.0019	9	–0.3775	18	–0.8264		
32	–0.0188	14	–0.4016	22	–0.8264		

Then, we ought to check the correctness of the result whether only the core. The reason for this is that there is a possibility that the core of the network may contain the nodes with negative centralities. According to Ref. [25], the objective function may check the correctness of the result. The objective function is introduced as follows:

$$\Phi = \frac{2E_{cc}}{n_c(n_c-1)} + \frac{E_{cp}}{n_c \cdot n_p} - \frac{2E_{pp}}{n_p(n_p-1)}. \quad (2)$$

Here, n_c is the number of the core nodes, n_p is the number of the peripheral nodes, E_{cc} is the number of the connections in the core, E_{pp} is the number of the connections between each pair of the peripheral nodes, and E_{cp} is the number of the connections between the core nodes and the peripheral nodes. The first term $2E_{cc}/n_c/(n_c-1)$ is the connection density of the core nodes. The second term $E_{cp}/n_c/n_p$ is the connection density between the core and the peripheral nodes. The last term $2E_{pp}/n_p/(n_p-1)$ is the connection density of the peripheral nodes. If we choose the core nodes correctly, the objective function always maximizes the connection density among the core nodes, and between core and peripheral nodes, while minimizing the connection density among the peripheral nodes.

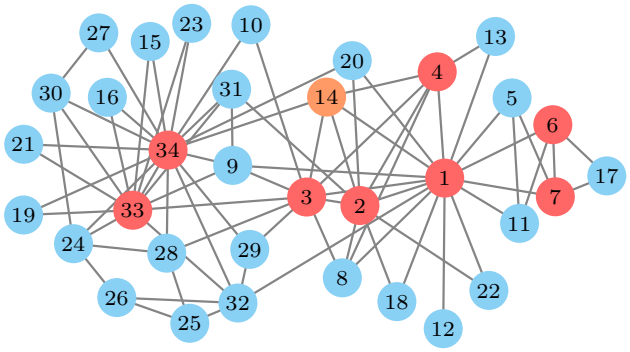


Fig. 1. The scheme of the karate club network. The red nodes are the central nodes with $C(i)>0$. These nodes consist of the core of the karate club network. The blue nodes are the periphery ones because of the negative centralities. Although node 14 has the negative centrality, it still belongs to the core of the karate network by the calculation with the objective function.

The checking steps could be introduced with the karate club network as follows. As shown in Table 1, we have obtained eight nodes with the positive centralities. Thus, it could be thought that the core of the network contains these eight nodes at least. First, we may calculate the value of the objective function, and its value is 0.6282. (Here, $n_c = 8$, $n_p = 26$, $E_{cc} = 11$, $E_{cp} = 56$, and $E_{pp} = 11$.) It is obvious that there are 24 neighbors connecting the core. Next, we may choose a node from the neighbors, and add it to the core. Thus, the core set is $\{34,1,33,2,3,6,7,4,X\}$. Here, X is the neighbor connecting with the core nodes. We may obtain a new value of the objective function. Here, n_c is 9 and n_p is 25 in the karate club network. For example, it is assumed that node 5 is chosen and the new core set is $\{34,1,33,2,3,6,7,4,5\}$. According to formula (2), the value of the objective function is 0.5722. We could repeat the process above, and obtain 24 different values of the objective function. In each calculation process, we keep n_c at 9 and n_p at 25, and change the neighbor node only. The calculation results show that the increment of the objective function is positive, if the new core set is $\{34,1,33,2,3,6,7,4,14\}$. The value of the objective function is 0.6344. (Here $n_c = 9$, $n_p = 25$, $E_{cc} = 16$, $E_{cp} = 51$, and $E_{pp} = 11$.) The other results of the increments of the objective function are all negative. As shown in Fig. 1, it is discovered that the neighbors of node 14 are all core nodes with positive centralities. The connection density of the core nodes would be more if node 14 is regarded as a core one. Now the core set is $\{34,1,33,2,3,6,7,4,14\}$. What will happen if we add one node to the core once again? As expected, the value of the objective function will decrease. The objective function gets

the maximum locally, if the core of the karate club network is the set $\{1,2,3,4,6,7,14,33,34\}$. We have obtained the core of the karate club network by the centrality, and checked the result by the objective function. The results of the two methods differ by only one node.

It is thought that generally the core of the karate club network contains five nodes at least (node 1, node 2, node 3, node 33 and node 34).[26,27] Yet, some scientists have concluded that the core of the karate club network contains 14 nodes.[20] Thus, it could be acceptable that the core of the network is made up of the nodes with non-negative centralities.

3. Simulation and analysis

In this section, we examine the core/periphery structure in some real-world networks, and analyze the effectiveness of the results on a synthetic benchmark network.

3.1. The dolphin network

We may apply our method to the dolphin network, which has an obvious core/periphery structure. The dolphin network describes the associations between 62 dolphins living in Doubtful Sound.[28] In this network, the dolphins represented as nodes have connections with each other if they are observed together more often than expected by chance during the years from 1994 to 2001. There are 159 connections in the dolphin network. First, we calculate the centrality of each node by formula (1), and the simulation results are listed in Table 2. It shows that there are 21 core nodes in the dolphin network. The core set is $\{52,18,58,15,46,30,38,34,2,21,39,14,37,16,10,55,44,43,41,48,28\}$. According to formula (2), the value of the objective function is 0.2667. (Here $n_c = 21$, $n_p = 41$, $E_{cc} = 38$, $E_{cp} = 98$, and $E_{pp} = 23$.) Then, we used the objective function to check the core of the dolphin network. Via the simulation, we obtain the core set, which is $\{2,10,14,15,16,18,21,28,30,34,37,38,39,41,43,44,46,48,51,52,55,58\}$. The additional node is node 51, whose centrality is negative but its neighbors are node 15, node 17, node 21, node 34, node 43, node 46 and node 52. Six of them are core nodes. Thus, it could enhance the connection density of the core nodes and let the objective function get the local maximum. The value of the objective function is 0.2680. (Here $n_c = 22$, $n_p = 40$, $E_{cc} = 44$, $E_{cp} = 93$, and $E_{pp} = 22$.)

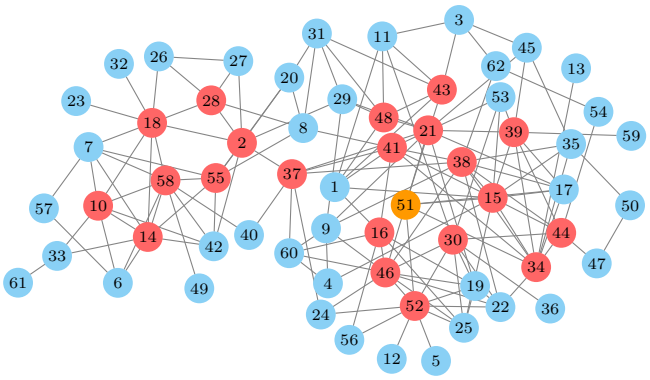


Fig. 2. The scheme of the dolphin network. The red nodes are the central ones with $C(i) > 0$. Although the orange node (node 51) has the negative centrality, it still belongs to the core of the dolphin network by the calculation with the objective function.

Table 2. The centrality of each node in the dolphin network.

Node	Centrality	Node	Centrality	Node	Centrality	Node	Centrality
52	0.4414	44	0.0664	51	–0.1293	54	–0.5238
18	0.3823	43	0.0636	29	–0.1657	57	–0.5833
58	0.3499	41	0.0587	6	–0.1720	24	–0.6010
15	0.3363	48	0.0171	62	–0.1778	40	–0.7460
46	0.2735	28	0.0121	25	–0.2344	56	–0.7571
30	0.2628	31	–0.0159	17	–0.2471	5	–1.0000
38	0.2524	33	–0.0185	45	–0.2494	12	–1.0000
34	0.2453	7	–0.0239	20	–0.2548	13	–1.0000
2	0.2326	9	–0.0411	42	–0.2558	23	–1.0000
21	0.2195	3	–0.0482	22	–0.2600	32	–1.0000
39	0.2037	8	–0.0660	27	–0.2963	36	–1.0000
14	0.1769	19	–0.0791	50	–0.3000	49	–1.0000
37	0.1239	1	–0.0869	26	–0.3111	59	–1.0000
16	0.1099	11	–0.1048	47	–0.3571	61	–1.0000
10	0.1088	35	–0.1074	53	–0.4565		
55	0.0754	60	–0.1213	4	–0.4847		

3.2. The scientific collaboration network

We also apply our core/periphery structure-finding method to the collaboration network of the scientists at Santa Fe Institute during any part of calendar year 1999 or 2000.[29] We carry our research to the largest component of the collaboration graph. This network contains 118 nodes, and each node presents a scientist. There exists a connection between a pair of nodes if they have coauthored one or more articles during the same time period. There are 200 connections in this collaboration network. From Fig. 3, it may be discovered that each scientist has coauthored articles with about four others. Via the simulation, we may find that there are 21 nodes with the positive centralities in the network by formula (1). The core set is $\{2,24,12,1,27,56,53,22,9,4,6,17,15,5,11,66,72,51,23,47,28\}$. According to formula (2), the value of the objective function is 0.1739. (Here, $n_c = 21$, $n_p = 97$, $E_{cc} = 25$, $E_{cp} = 131$, and $E_{pp} = 44$.) Then, we detect the core of the collaboration network from these core nodes by the objective function. Via the simulation, we obtain the core set of the collaboration network is $\{1,2,3,4,5,6,9,11,12,15,17,22,23,24,27,28,47,51,53,56,66,72\}$. The additional node is node 3, whose centrality is negative but its neighbors are node 4, node 5, node 6, node 24, node 73 and node 74. Four of them are core nodes. Thus, it could enhance the connection density of the core nodes and let the objective function get local maximum. The value of the objective function is 0.1774. (Here, $n_c = 22$, $n_p = 96$, $E_{cc} = 29$, $E_{cp} = 129$, and $E_{pp} = 42$.)

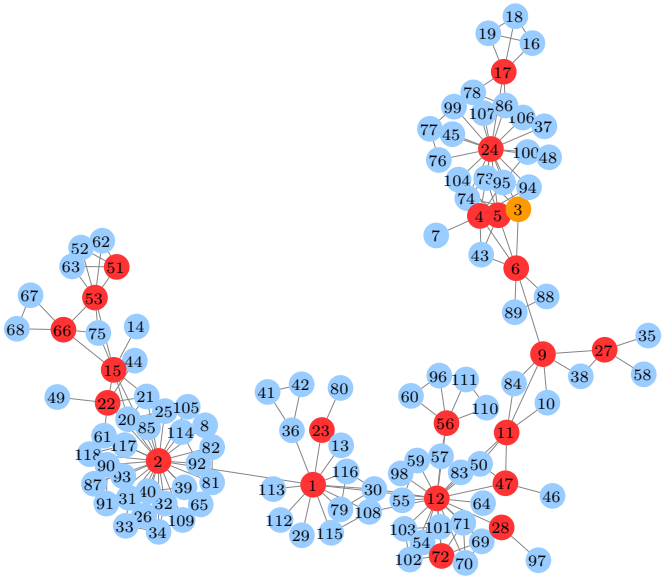


Fig. 3. The scheme of the scientific collaboration network. The red nodes are the central nodes with $C(i) > 0$. Although the orange node (node 3) has negative centrality, it still belongs to the core of the collaboration network by the calculation with the objective function.

Table 3. The centrality of each node in the collaboration network.

Node	Centrality	Node	Centrality	Node	Centrality	Node	Centrality
2	0.8828	42	–0.1667	106	–0.4500	7	–1.0000
24	0.7955	31	–0.2016	107	–0.4500	8	–1.0000
12	0.7885	32	–0.2016	55	–0.4546	13	–1.0000
1	0.6521	33	–0.2016	59	–0.4546	14	–1.0000
27	0.5625	34	–0.2016	75	–0.4630	26	–1.0000
56	0.4530	62	–0.2333	39	–0.4655	29	–1.0000
53	0.3815	63	–0.2333	40	–0.4655	35	–1.0000
22	0.3605	30	–0.2375	81	–0.4655	37	–1.0000
9	0.3598	69	–0.2708	82	–0.4655	44	–1.0000
4	0.2866	70	–0.2708	90	–0.4655	45	–1.0000
6	0.2795	71	–0.2708	91	–0.4655	46	–1.0000
17	0.2599	101	–0.2708	92	–0.4655	48	–1.0000
15	0.2569	102	–0.2708	114	–0.4655	49	–1.0000
5	0.2360	103	–0.2708	117	–0.4655	54	–1.0000
11	0.2328	67	–0.3000	118	–0.4655	58	–1.0000
66	0.2121	68	–0.3000	50	–0.4849	64	–1.0000
72	0.2063	79	–0.3048	94	–0.5217	65	–1.0000
51	0.1417	116	–0.3048	43	–0.5370	80	–1.0000
23	0.0769	60	–0.3333	38	–0.5833	83	–1.0000
47	0.0500	96	–0.3333	115	–0.5897	85	–1.0000
28	0.0455	110	–0.3333	52	–0.6071	87	–1.0000
36	–0.0159	111	–0.3333	76	–0.6167	93	–1.0000
77	–0.0476	73	–0.3443	99	–0.6167	97	–1.0000
3	–0.0846	74	–0.3443	10	–0.6167	98	–1.0000
20	–0.0960	88	–0.3571	84	–0.6167	105	–1.0000
95	–0.1068	89	–0.3571	100	–0.7000	109	–1.0000
16	–0.1429	108	–0.4024	25	–0.7655	112	–1.0000
18	–0.1429	78	–0.4127	57	–0.7879	113	–1.0000
19	–0.1429	86	–0.4127	61	–0.8227		
41	–0.1667	21	–0.4207	104	–0.8250		

3.3. The politician blogs network

As a further test of our algorithm, we turn to the politician blogs network. The politician blogs network is a directed network of hyperlinks between weblogs on US politics.[30] This network contains 1224 nodes, and each node presents a politician. The connections were extracted from a crawl of the front page of the blog around the time of the 2004 presidential election. We convert it to an undirected network by treating all the directed connections as undirected connections, and reserve the largest component. The largest component contains 1222 nodes and 16714 connections. We calculate the centrality of each node. The simulation results show that there are 184 nodes with positive centralities in the politician blogs network, but it is found that these nodes are not connected as a whole. There are two nodes (node 643 and node 1022) isolated with positive centralities. It may be considered that these two nodes are peripheral. The calculation results are given in Appendix A. Thus, the number of the core nodes is 182. According to formula (2), the value of the objective function is 0.2642. (Here, $n_c = 182$, $n_p = 1040$, $E_{cc} = 3527$, $E_{cp} = 10440$, and $E_{pp} = 2747$.) Then, we could use the objective function to check the core of the politician blogs network. Via the simulation, it is discovered that there are 193 core nodes in the politician blogs network. The value of the objective function is 0.2677. (Here, $n_c = 193$, $n_p = 1029$, $E_{cc} = 4110$, $E_{cp} = 10066$, and $E_{pp} = 2538$.) Scientists often think that the politician blogs network may be divided into two communities according to the different political leanings, but our results reveal that this network also has a core/periphery structure. These politicians as the core members connect with each other densely, regardless of the political leanings.

3.4. Synthetic networks with a single core/periphery structure

Since the ground truths about the core/periphery structures in real networks are unknown, we generate a network to measure the correctness of our algorithms.[31] For the synthetic network with single core/periphery structure, the size of whole network, the size of core, and the size of periphery are set as $N = 500$, $n_c = 100$ and $n_p = 400$, respectively. (It is regarded that the nodes with the top 100 numbers are the core nodes, and the others are the periphery nodes in the network.) For a network with typical core/periphery structure, the connection probability among core nodes (P_{cc}) is larger than or equal to the connection probability between core nodes and peripheral nodes (P_{cp}), and is also larger than the connection probability among peripheral nodes (P_{pp}). Therefore, we may set $P_{pp} = 0.05$ and $P_{cc} = 4(P_{cp} - P_{pp}) + P_{pp}$. The sets of synthetic networks are generated by varying the value of P_{cp} whose variety range is from 0.05 to 0.2875, for P_{cc} must be less than or equal to 1. If P_{cp} is 0.05, the connection probability of the whole network is 0.05, and thus the network would not express the core/periphery structure. Yet, the synthetic network would show the core/periphery structure clearly with the increase of the value of P_{cp}. In order to describe the extent of the

core/periphery structure, we can introduce the global centrality C of the network. The global centrality C may be defined as

$$C = \frac{1}{N}\sum_i |C(i)|. \tag{3}$$

This is the mean value of the absolute value of each node's centrality. This shows that the more the global centrality is, the more obvious the core/periphery structure is. Figure 4 shows the relationship between the global centrality C and the connection probability $P_{\rm cp}$. The calculation results are in line with our expectations.

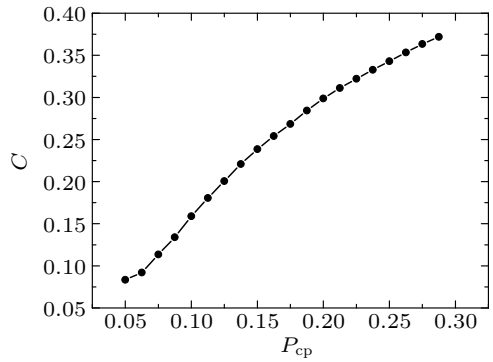


Fig. 4. The relationship between the global centrality C and the connection probability $P_{\rm cp}$.

After getting the centrality of each node by formula (1), we verify the correctness of our algorithms. First, we may define TP, TN, FP and FN, these four variables, to classify the simulation results. TP is defined as the number of the identified core nodes in the core nodes. TN is defined as the number of the identified periphery nodes in the periphery nodes, FP is defined as the number of the identified core nodes in the periphery nodes, and FN is defined as the number of the identified periphery nodes in the core nodes. Next, we calculate the accuracy, recall, specificity and precision with the defined variables above. Here, accuracy is (TP+TN)/N, recall is TP/$n_{\rm c}$, specificity is TN/$n_{\rm p}$, and precision is TP/(TP+FP). Finally, the results are shown in Fig. 5.

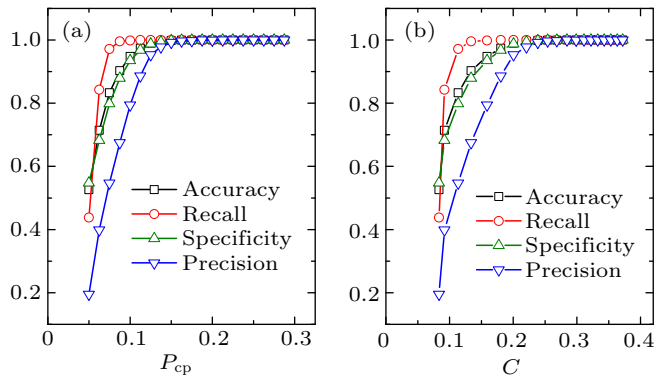


Fig. 5. Panel (a) shows the accuracy, recall, specificity, and precision as a function of $P_{\rm cp}$ on the sets of synthetic networks with single core/periphery structure. Panel (b) shows the accuracy, recall, specificity, and precision as a function of the global centrality on the sets of synthetic networks with single core/periphery structure.

It is discovered that the accuracy grows with the increase of the global centrality. If the global centrality of the network is larger than 0.18, the accuracy could be approximately equal to 1. What should be noticed is that the recall grows most sharply. This means that the core of the network can be identified accurately by the centrality of each node when the network represents a little bit of the core/periphery structure. Yet the specificity and the precision do not grow as sharply as the recall. The reason is that some periphery nodes are identified as core nodes if the core/periphery structure of the network is not very obvious. With the increase of $P_{\rm cp}$, the core/periphery structure of the network may get more and more clear. Meanwhile the accuracy, recall, specificity, and precision of these four variables tend towards 1. This means the core nodes and the peripheral nodes can be distinguished accurately in synthetic networks with a large global centrality.

4. Conclusion

In this brief report, we have defined the centrality of each node. It could be regarded that nodes with the positive centralities consist of the core of the complex network. We have carried out a simulation on real networks, i.e., the karate club network, the dolphin network, the scientific collaboration network, and the politician blogs network. Meanwhile, we have used the objective function to check the core of the network in order to ensure the reliability of the results. The checked results have shown the effectiveness of the simulation results by the centralities of the nodes on the real networks mentioned above. Furthermore, we have taken the calculation on the synthetic networks with single core/periphery structure, and have obtained consistent conclusions.

Generally, the centrality of each node represents the importance in the network. The core nodes with large degree may be regarded as the global central nodes, such as node 34 and node 1 in the karate club network. Yet, the core nodes with little degrees may be regarded as the local central nodes, such as node 6 and node 7 in the karate club network. Such local central nodes often connect two communities to each other. Perhaps, another question may be raised — why do the core nodes consist of the core of the network? The reason for this is that the core nodes and the peripheral nodes can be distinguished accurately when the global centrality of the network is larger than 0.2 from the simulation results in Fig. 5(b). Via formula (3), we may get the global centrality of the networks discussed. The global centrality of the karate club network is 0.504. The global centrality of the dolphin network is 0.343. The global centrality of the scientific collaboration network is 0.526. The global centrality of the politician blogs network is 0.593. It is discovered that the global centrality of these networks is all larger than 0.2. Thus, the viewpoint that the core

nodes consist of the core of the network is correct if the network represents the core/periphery structure clearly. Yet, our method may fail, if the global centrality of network is small enough, i.e., the network does not exhibit the core/periphery structure, such as the American college football network,[32] whose global centrality is 0.0317.

It may be universal to detect the core of complex network by the centrality of each node. This method could be applied to huge networks because of the low order complexity of the simulation. Huge networks often exhibit a multiple core/periphery structure. We hope that our method will also be useful for scientists who wish to detect the core/periphery structure of huge networks.

Appendix A

Table A1. The positive centralities of the nodes in the politician blogs network.

Node	Centrality	Node	Centrality	Node	Centrality	Node	Centrality	Node	Centrality
674	0.8413	808	0.4002	750	0.2690	141	0.1595	366	0.0745
127	0.8029	147	0.3927	21	0.2634	423	0.1561	853	0.0742
770	0.8027	731	0.3915	171	0.2628	625	0.1556	479	0.0731
840	0.7345	1201	0.3843	72	0.2597	1195	0.1555	49	0.0720
498	0.7176	302	0.3788	829	0.2581	472	0.1550	946	0.0686
48	0.7097	586	0.3785	101	0.2558	1013	0.1532	218	0.0683
1008	0.6804	403	0.3751	764	0.2511	633	0.1530	753	0.0672
1180	0.6666	377	0.3749	1022	0.2500	1079	0.1529	899	0.0630
924	0.6653	443	0.3747	501	0.2467	164	0.1492	1077	0.0627
801	0.6426	405	0.3676	935	0.2415	760	0.1460	1170	0.0611
892	0.6194	591	0.3633	62	0.2382	522	0.1447	778	0.0609
799	0.6184	909	0.3533	838	0.2355	701	0.1376	721	0.0602
1215	0.6068	649	0.3502	744	0.2296	991	0.1363	593	0.0574
568	0.5975	1202	0.3481	890	0.2219	318	0.1350	451	0.0551
832	0.5701	1130	0.3465	852	0.2104	855	0.1338	1009	0.0541
254	0.5641	155	0.3462	1152	0.2099	450	0.1312	1012	0.0520
884	0.5636	119	0.3447	617	0.2097	447	0.1300	818	0.0500
621	0.5398	514	0.3397	1214	0.2089	100	0.1282	923	0.0489
784	0.5317	844	0.3391	1070	0.2073	596	0.1242	442	0.0440
284	0.5202	84	0.3356	657	0.2028	566	0.1237	487	0.0438
359	0.5178	602	0.3346	384	0.1997	328	0.1234	226	0.0423
1032	0.5045	419	0.3295	912	0.1936	786	0.1230	65	0.0408
139	0.4946	977	0.3285	983	0.1928	157	0.1225	183	0.0349
793	0.4882	537	0.3250	1038	0.1879	81	0.1222	526	0.0320
341	0.4842	959	0.3221	198	0.1864	482	0.1148	507	0.0272
948	0.4597	998	0.3167	641	0.1859	930	0.1116	768	0.0242
673	0.4560	708	0.3063	584	0.1840	188	0.1053	174	0.0217
578	0.4514	234	0.3047	238	0.1784	382	0.0970	866	0.0210
694	0.4480	429	0.3017	126	0.1738	463	0.0962	13	0.0115
1081	0.4468	518	0.2923	37	0.1718	87	0.0906	379	0.0115
643	0.4444	835	0.2902	671	0.1717	922	0.0889	304	0.0096
785	0.4359	997	0.2895	387	0.1688	474	0.0861	640	0.0078
696	0.4339	675	0.2801	953	0.1664	600	0.0860	494	0.0069
499	0.4322	413	0.2745	400	0.1627	1172	0.0826	1155	0.0058
385	0.4212	60	0.2740	339	0.1626	97	0.0805	777	0.0038
446	0.4112	123	0.2721	943	0.1622	745	0.0781	293	0.0005
1061	0.4087	608	0.2692	716	0.1621	850	0.0773		

Here we only list the 184 nodes with positive centralities in the politician blogs network. Node 643 and node 1022 are isolated, so these two nodes could be considered as periperal nodes. According to formula (2), the core of politician blogs network contains not only 182 nodes with positive centralities but also contains an additional 11 nodes. These additional nodes are node 360, node 594, node 527, node 12, node 1080, node 444, node 761, node 647, node 365, node 270, and node 309.

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