

Event-based nonfragile state estimation for memristive recurrent neural networks with stochastic cyber-attacks and sensor saturations

Xiao-Guang Shao(邵晓光), Jie Zhang(张捷), and Yan-Juan Lu(鲁延娟)

Citation: Chin. Phys. B, 2024, 33 (**7**): 070203. DOI: 10.1088/1674-1056/ad3dcb
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Target layer state estimation in multi-layer complex dynamical networks considering nonlinear node dynamics

Yayong Wu(吴亚勇), Xinwei Wang(王欣伟), and Guo-Ping Jiang(蒋国平)
Chin. Phys. B, 2024, 33 (**4**): 040205. DOI: 10.1088/1674-1056/ad20d7

Finite-time H_∞ filtering for Markov jump systems with uniform quantization

Jingjing Dong(董敬敬), Xiaofeng Ma(马晓峰), Xiaoqing Zhang(张晓庆), Jianping Zhou(周建平), and Zhen Wang(王震)
Chin. Phys. B, 2023, 32 (**11**): 110202. DOI: 10.1088/1674-1056/acedf5

Comparison of differential evolution, particle swarm optimization, quantum-behaved particle swarm optimization, and quantum evolutionary algorithm for preparation of quantum states

Xin Cheng(程鑫), Xiu-Juan Lu(鲁秀娟), Ya-Nan Liu(刘亚楠), and Sen Kuang(匡森)
Chin. Phys. B, 2023, 32 (**2**): 020202. DOI: 10.1088/1674-1056/ac65ee

Robustness measurement of scale-free networks based on motif entropy

Yun-Yun Yang(杨云云), Biao Feng(冯彪), Liao Zhang(张辽), Shu-Hong Xue(薛舒红), Xin-Lin Xie(谢新林), and Jian-Rong Wang(王建荣)
Chin. Phys. B, 2022, 31 (**8**): 080201. DOI: 10.1088/1674-1056/ac6942

Memory-augmented adaptive flocking control for multi-agent systems subject to uncertain external disturbances

Ximing Wang(王希铭), Jinsheng Sun(孙金生), Zhitao Li(李志韬), and Zixing Wu(吴梓杏)
Chin. Phys. B, 2022, 31 (**2**): 020203. DOI: 10.1088/1674-1056/ac21c1

Event-based nonfragile state estimation for memristive recurrent neural networks with stochastic cyber-attacks and sensor saturations

Xiao-Guang Shao(邵晓光)[1], Jie Zhang(张捷)[1,†], and Yan-Juan Lu(鲁延娟)[2]

[1] *School of Automation, Nanjing University of Science and Technology, Nanjing* 210094, *China*
[2] *School of Electronics and Information Engineering, Changchun University of Science and Technology, Changchun* 130022, *China*

(Received 23 January 2024; revised manuscript received 28 February 2024; accepted manuscript online 12 April 2024)

This paper addresses the issue of nonfragile state estimation for memristive recurrent neural networks with proportional delay and sensor saturations. In practical engineering, numerous unnecessary signals are transmitted to the estimator through the networks, which increases the burden of communication bandwidth. A dynamic event-triggered mechanism, instead of a static event-triggered mechanism, is employed to select useful data. By constructing a meaningful Lyapunov–Krasovskii functional, a delay-dependent criterion is derived in terms of linear matrix inequalities for ensuring the global asymptotic stability of the augmented system. In the end, two numerical simulations are employed to illustrate the feasibility and validity of the proposed theoretical results.

Keywords: memristor-based neural networks, proportional delays, dynamic event-triggered mechanism, sensor saturations

PACS: 02.30.Yy, 02.30.Ks, 07.05.Dz **DOI:** 10.1088/1674-1056/ad3dcb

1. Introduction

For decades, recurrent neural networks (RNNs) have received considerable attention due to their extraordinary applications in a variety of areas such as control engineering, dynamic optimization, and computer vision.[1–3] Dynamical analyses (e.g., dissipativity, stability, and synchronization), as theoretical foundations of RNNs, are key steps for designs of RNNs in practical engineering, which have attracted considerable interests, see Refs. [4–6]. As the fourth basic circuit element, memristor[7] was proposed in 1971. The memristance depends on the amount of charge that has passed through the memristor in a specific direction which satisfies $M = \mathrm{d}\varphi/\mathrm{d}q$. The physical entity of the first memristor was not developed until 2008 by HP Labs, as shown in Fig. 1. Due to these features (e.g., nanometer size, memory characteristics) of memristors, a host of systems based on memristor have become a popular research area, particularly in the field of RNNs, see Refs. [8–11]. When a memristor is integrated into a computer system, it has the capability to retrieve information that was appeared in the past time. It is worth noting that even in the event of a power failure, the information remains accessible once the connection is restored.

It is often proved to be difficult to fully obtain neuron states due to the large size of neural networks and the utility of the measuring equipment.[12] Therefore, state estimation (SE) issue has developed into a research hotspot, e.g., a great deal of significant results were reported in Refs. [13,14] and the references therein. The SE issue for delayed RNNs has been introduced for the first time in Ref. [14]. The extended dissipative SE problem of delayed RNNs has been discussed in Ref. [6], in which the developed results have been verified by a quadruple tank process system. As is known, time delays are frequently inevitable during information transmission and limited processing speeds which may cause oscillation and instability. Discrete time delay and distributed time delay are both types of common time delays. In addition, the concept of proportional delay has recently been the subject of extensive research. It has been shown that proportional time delay also has a crucial impact on control systems and web quality of service (QoS) routing.[15,16] In addition, the authors studied the polynomial stability of complex-valued NNs. By means of an exponential function e^t, the proportional delay was converted into the discrete delay in Ref. [17]. However, the condition based on such a method is delay-independent and conservative. In Ref. [18], a delay-dependent criterion has been inferred by a novel Lyapunov–Krasovskii functional (LKF), but it cannot be directly applied in research of SE issue.

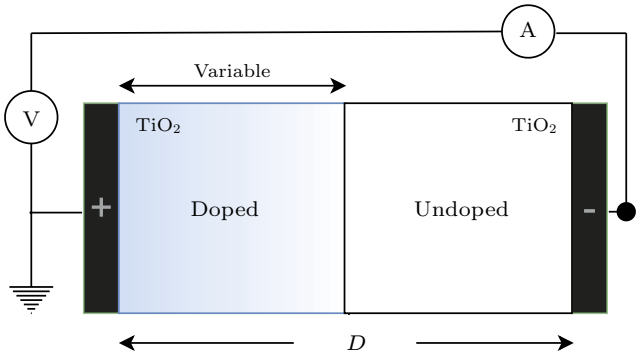


Fig. 1. Physical model of memristor element.[11]

In practical systems, many unnecessary signals are trans-

†Corresponding author. E-mail: jie_zhang_njust@163.com

© 2024 Chinese Physical Society and IOP Publishing Ltd

ferred to estimators through networks. In order to avoid the waste of network resources caused by data transmission, several event-triggered schemes have been proposed and have attracted considerable attention, see Refs. [19–23]. In Ref. [20], the problem of nonfragile SE for RNNs with sensor nonlinearity was investigated. A simplified event-triggered mechanism (ETM) that relies on incomplete measurements was introduced. For fractional-order systems, an event-triggered algorithm was applied to multi-agent systems in Ref. [21]. Recently, there has been a growing amount of attention focused on the dynamic event-triggered mechanism (DETM) which is increasingly preferred in practical applications due to the time-varying event-triggered threshold. For instance, the event-based SE issue for RNNs with unknown time delays has been studied in Ref. [22]. The H_∞ SE issue of complex networks under DETM subject to cyber attacks has been addressed in Ref. [23]. In regards of memristive recurrent neural networks (MRNNs), the nonfragile estimation problem subject to proportional delay under DETM has not been thoroughly investigated.

On the other hand, the output of the sensor is limited by the physical characteristics of its components and technology. Not only network delays but also sensor saturation could impact the system performance. Taking into consideration of the sensor saturation, time delay for a class of MRNNs is practical. Furthermore, most studies on the SE issues of NNs fail to consider the potential impact of perturbations in real-world scenarios. These perturbations can arise from the complex and dynamic environments, leading to the fragility of the designed estimator up to date. As such, nonfragile techniques have drawn significant research attention for various types of dynamic systems in order to tolerate the effects of estimator gain perturbations.[20] In Ref. [24], the desired filter is supposed to have gain variations composed of linear fractional parametric uncertainties. Due to the insertion of network, a few challenges have happened during the data transmission such as packet dropouts and communication delay. Moreover, cyber attacks jeopardize the stability of systems, which may cause physical damage.[7,23] For instance, the authors of Ref. [25] investigated the SE issue of NNs with cyber-attacks under hybrid triggered scheme. Recently, a novel DETM was applied to the control problem of MRNNs with multiple cyber-attacks.[26] In spite of this, there are few results about designs of a nonfragile estimator for MRNNs with proportional delay and sensor saturation under DETM, which in a way motivates our investigation.

Inspired by the aforementioned reasons, our objective is to address the nonfragile SE issue for delayed MRNNs with ROSS under the DETM. The main merits of our investigation are summarized as follows: (i) This paper discusses the SE issue of delayed MRNNs subject to cyber-attacks, sensor saturation and gain perturbation. (ii) To decrease the unnecessary signals transmissions, a DETM is introduced into the SE issue of MRNNs in this paper and communication delay is considered in the signals transmissions. (iii) Based on the Lyapunov theory and the reciprocal convexity lemma, a less conservative delay-dependent criterion has been derived to ensure the asymptotic stability of the augmented system. (iv) To illustrate the usefulness and advantages of our presented estimation algorithm for MRNNs, two numerical simulations are presented.

Notation In this paper, $\mathbb{R}^n$ denotes the set of n-dimensional vector. $\mathcal{M}$, $\mathcal{M}^{\mathrm{T}}$ represents the transpose, the symbol $*$ stands for the symmetric term in a symmetric matrix, and $\mathrm{col}\{\cdot\}$ stands for an column vector. I and 0 denote the identity matrix and the zero matrix of compatible dimensions, respectively. $\mathcal{E}\{\cdot\}$ means the mathematical expectation operator.

2. Preliminaries and problem statement

In this section, we consider the following MRNNs subject to proportional delay:

$$\begin{aligned}\dot{\xi}_i(t) =& -h_i(\xi_i(t))\xi_i(t) + \sum_{j=1}^{n} a_{ij}(\xi_j(t))g_j(\xi_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}(\xi_j(t))g_j(\xi_j(\rho_j t)), \\ \xi_i(t_0) =& \tilde{\varphi}_i(t_0), \ \ t_0 \in [\rho, 1], \end{aligned} \tag{1}$$

where $\xi_i(t)$ represents the state of the ith neuron at time t; $h_i > 0$, $g_j(\cdot)$ stands for the activation function; ρ denotes the proportional delay factor satisfying $0 < \rho \leq 1$ and $\rho t = t - (1-\rho t)$, in which $(1-\rho)t$ presents the transmission function. The initial condition $\tilde{\varphi}_i(\cdot) \in C([\rho, 1], \mathbb{R})$. Here $a_{ij}(\cdot)$ and $b_{ij}(\cdot)$ stand for the memristive connection weights and can be expressed as

$$\begin{aligned} h_i(\xi_i(t)) &= \frac{1}{\tilde{C}_i}\Big[\sum_{j=1}^{n}(\tilde{W}_{ij}^a + \tilde{W}_{ij}^b) \times \mathrm{sgn}_{ij} + \frac{1}{R_i}\Big], \\ a_{ij}(\xi_j(t)) &= \frac{\tilde{W}_{ij}^a}{\tilde{C}_i} \times \mathrm{sgn}_{ij}, \ \ b_{ij}(\xi_j(t)) = \frac{\tilde{W}_{ij}^b}{\tilde{C}_i} \times \mathrm{sgn}_{ij}, \end{aligned}$$

where $\tilde{W}_{ij}^a$ and $\tilde{W}_{ij}^b$ are the memductances of memristors F_{aij} with F_{aij} satisfying $\tilde{W}_{ij}^a = 1/F_{aij}$; $\tilde{W}_{ij}^b = 1/F_{bij}$; $\tilde{C}_i$ is the capacitor; $\mathrm{sgn}_{ij} = 1$ if $i \neq j$, otherwise -1. Combine the memristor structure and circuit analysis, the model of MRNNs is shown in Fig. 2. Here ξ_i is the neural state of the ith subsystem; F_{ij}, R_i, and C_i stand for the memristors, resistor, and capacitor, respectively. For convenience, we make z represent h, a and b throughout this paper. Due to the characteristics of the memristor, the connection weight $z_{ij}(\xi_i(t))$ in Eq. (1) can be defined as

$$h_i(\xi_i(t)) = \begin{cases} h_i', & |\xi_i(t)| \leq \mathcal{T}, \\ h_i'', & |\xi_i(t)| > \mathcal{T}, \end{cases}$$

$$z_{ij}(\xi_j(t)) = \begin{cases} z'_{ij}, & |\xi_j(t)| \le \mathcal{T}, \\ z''_{ij}, & |\xi_j(t)| > \mathcal{T}, \end{cases} \tag{2}$$

in which $\mathcal{T} > 0$ stands for the switching jump, z'_{ij} and z''_{ij} are constants. Notice that $z_{ij}(\xi_j(t))$ is switching by the states. It is obvious that system (1) is a discontinuous switched system. For this reason, the solutions of MRNNs should be handled in Filippov's sense.[27] According to the differential inclusions and set-valued maps,[28] we obtain

$$\begin{aligned} \dot{\xi}_i(t) \in & -\mathrm{co}\{h'_i, h''_i\}\xi_i(t) + \sum_{j=1}^{n} \mathrm{co}\{a'_{ij}, a''_{ij}\} g_j(\xi_j(t)) \\ & + \sum_{j=1}^{n} \mathrm{co}\{b'_{ij}, b''_{ij}\} g_j(\xi_j(\rho_j t)), \end{aligned} \tag{3}$$

where $\mathrm{co}\{h'_i, h''_i\} = [\underline{h}_i, \overline{h}_i]$, $\mathrm{co}\{z'_{ij}, z''_{ij}\} = [\underline{z}_{ij}, \overline{z}_{ij}]$, with $\underline{h}_i = \min\{h'_i, h''_i\}$, $\overline{h}_i = \max\{h'_i, h''_i\}$, $\underline{z}_{ij} = \min\{z'_{ij}, z''_{ij}\}$, $\overline{z}_{ij} = \max\{z'_{ij}, z''_{ij}\}$.

For presentation simplicity, we take $[\underline{\mathcal{H}}, \overline{\mathcal{H}}] = [\underline{h}_i, \overline{h}_i]_{n\times n}$, $[\underline{\mathcal{A}}, \overline{\mathcal{A}}] = [\underline{a}_{ij}, \overline{a}_{ij}]_{n\times n}$, $[\underline{\mathcal{B}}, \overline{\mathcal{B}}] = [\underline{b}_{ij}, \overline{b}_{ij}]_{n\times n}$, $\xi(t) = \mathrm{col}_n\{\xi_i(t)\}$, $g(\xi(t)) = \mathrm{col}_n\{g_j(\xi_j(t))\}$, $g(\xi(\rho t)) = \mathrm{col}_n\{g_j(\xi_j(\rho t))\}$. Then, systems (3) can be expressed in the matrix form as

$$\dot{\xi}(t) = -\mathcal{H}(t)\xi(t) + \mathcal{A}(t)g(\xi(t)) + \mathcal{B}(t)g(\xi(\rho t)), \tag{4}$$

where $\mathcal{H}(t) \in [\underline{\mathcal{H}}, \overline{\mathcal{H}}]$, $\mathcal{A}(t) \in [\underline{\mathcal{A}}, \overline{\mathcal{A}}]$, and $\mathcal{B}(t) \in [\underline{\mathcal{B}}, \overline{\mathcal{B}}]$ are measurable functions.

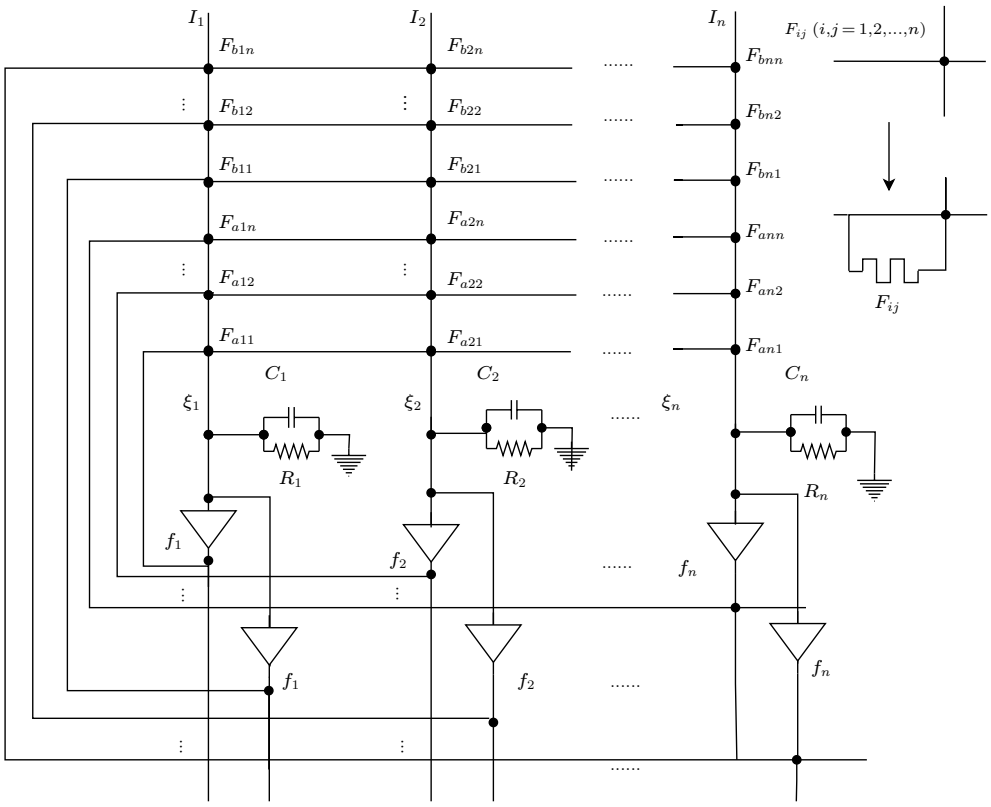


Fig. 2. Circuit of MRNNs.

Assumption 1 The activation function $\hat{g}(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ satisfies the following sector-bounded condition:

$$\begin{aligned} & [\hat{g}(\kappa_1) - \hat{g}(\kappa_2) - \check{G}_1(\kappa_1 - \kappa_2)]^{\mathrm{T}}[\hat{g}(\kappa_1) - \hat{g}(\kappa_2) \\ & - \check{G}_2(\kappa_1 - \kappa_2)] \le 0, \ \ \forall \kappa_1, \kappa_2 \in \mathbb{R}^n, \end{aligned} \tag{5}$$

where $\hat{g}(0) = 0$, the real matrices $\check{G}_1$ and $\check{G}_2$ are known with compatible dimensions.

Based on Lemma 1 of Ref. [29], system model (4) is further rewritten as

$$\begin{aligned} \dot{\xi}(t) = & -(\mathcal{H} + \Delta\mathcal{H})\xi(t) + (\mathcal{A} + \Delta\mathcal{A})g(\xi(t)) \\ & + (\mathcal{B} + \Delta\mathcal{B})g(\xi(\rho t)), \\ \pi(t) = & \ \mathrm{Sat}(E\xi(t))), \end{aligned} \tag{6}$$

where $\mathcal{H} = \frac{\underline{\mathcal{H}} + \overline{\mathcal{H}}}{2}$, $\mathcal{Z} = \frac{\underline{\mathcal{Z}} + \overline{\mathcal{Z}}}{2}$, $\Delta\mathcal{Z} = \mathcal{H}_z\mathcal{F}_z(t)\mathcal{N}_z$, with $\mathcal{F}_h^{\mathrm{T}}(t)\mathcal{F}_h(t) \le I_{n^2}$, $\mathcal{F}_z^{\mathrm{T}}(t)\mathcal{F}_z(t) \le I_{n^2}$, $\mathcal{H}_h = [\mathcal{H}_{h1}, \mathcal{H}_{h2}, \ldots, \mathcal{H}_{hn}]$, $\mathcal{H}_z = [\mathcal{H}_{z1}, \mathcal{H}_{z2}, \cdots, \mathcal{H}_{zn}]$, $\mathcal{N}_h = \mathcal{N}_z = \underbrace{[I_n, I_n, \ldots, I_n]}_{n}^{\mathrm{T}}$, $\mathcal{F}_h(t) = \mathrm{diag}\{\underbrace{0, \cdots, 0}_{\ell-1}, \ \mathcal{F}_{h\ell}(t), \ \underbrace{0, \cdots, 0}_{n-\ell}\}$, $\mathcal{F}_z(t) = \mathrm{diag}\{\mathcal{F}_{z11}(t), \cdots, \mathcal{F}_{z1n}(t), \ldots, \mathcal{F}_{zn1}(t), \ldots, \mathcal{F}_{znn}(t)\}$, $\mathcal{H}_{z\ell} = \begin{bmatrix} 0_{\ell-1,n} \\ z_{\ell 1}, z_{\ell 2}, \ldots, z_{\ell n} \\ 0_{n-\ell,n} \end{bmatrix}$ with $z_{\ell i} = \frac{\overline{z}_{\ell i} - \underline{z}_{\ell i}}{2}$, $\pi(t) \in \mathbb{R}^m$ is the actual measurement output. E is a known constant matrix with compatible dimensions. $\mathrm{Sat}(\cdot) \in \mathbb{R}^m \to \mathbb{R}^m$ is a saturation function defined as

$$\mathrm{Sat}(v) = \mathrm{col}\{\mathrm{Sat}(v_1)\ \mathrm{Sat}(v_2)\ \cdots\ \mathrm{Sat}(v_m)\}, \tag{7}$$

where $\mathrm{Sat}(v_j) = \mathrm{sign}(v_j)\min\{v_{j,\max}, |v_j|\}$ $(j = 1,2,\ldots,m)$, $v_{j,\max}$ is the saturation level.

Remark 1 It is challenging to analyze the dynamic behaviors of MRNNs because of the state-dependent nature of the memristive connection weights. What is more, traditional methods cannot be directly applied to investigate the stability and SE issues for MRNNs. Distinct from the existing works (e.g., Refs. [4,30]), MRNNs (1) have been transformed into a class of uncertain systems by incorporating the switching functions in this paper.

Considering the real environment, a nonfragile estimator to mitigate the impact of possible gain perturbations and the following estimator is constructed:

$$\begin{aligned}\dot{\hat{\xi}}(t) =& -\mathcal{H}\hat{\xi}(t) + \mathcal{A}g(\hat{\xi}(t)) + \mathcal{B}g(\hat{\xi}(\rho t)) \\ &+ (\mathcal{K} + \Delta\mathcal{K})(\pi_s(t) - E\hat{\xi}(t)), \\ \hat{\pi}(t) =& E\hat{\xi}(t),\end{aligned} \tag{8}$$

where $\hat{\xi}(t)$, $\hat{\pi}(t)$, and $\mathcal{K}$ denote the estimation of $\xi(t)$, $\pi(t)$, and the estimator gain matrix to be designed, respectively. In Eq. (8), $\pi_s(t) - E\hat{\xi}(t)$ is usually called an innovation. $\Delta\mathcal{K}$ signifies the fluctuations with $\mathcal{H}_k\mathcal{F}_k(t)\mathcal{N}_k$, $\mathcal{F}_k^{\mathrm{T}}(t)\mathcal{F}_k(t) \le I$.

In reality, numerous extraneous signals are transmitted into the estimator through the networks. An ETM which relies on the available output measurements is often employed to select valuable data so as to decrease the strain on communication bandwidth.

Assumption 2 Time delay always exists when signals travel through a communication channel in a practical system. Assuming that the communication delay is $\nu_k \in (0, \bar{\nu})$ with $\bar{\nu} = \max\{\nu_k\}$; the sampling sequence is $\{0, \hbar, 2\hbar, \ldots, n\hbar\}$, in which $\hbar$ is the sampling period; the set $\{t_k\hbar | t_k \in \mathbb{N}\}$ presents the successfully transmitted sequence; and the current sampling sequence is denoted by the set $\{t_k\hbar + \ell\hbar\}$ $(\ell = 1,2,\ldots,t_{k+1} - t_k - 1)$. Considering the effect of network delay, the released data at the instant $t_k\hbar$ will arrive at the designed estimator at the release instant $t_k\hbar + \nu_k$.

The last sampled data $\pi(t_k\hbar + \ell\hbar)$ will be transmitted by the DETM when the following condition is violated:

$$\begin{aligned}&[\pi(t_k\hbar + \ell\hbar) - \pi(t_k\hbar)]^{\mathrm{T}}\mathcal{U}[\pi(t_k\hbar + \ell\hbar) - \pi(t_k\hbar)] \\ &\le \epsilon(t)\pi^{\mathrm{T}}(t_k\hbar + \ell\hbar)\mathcal{U}\pi(t_k\hbar + \ell\hbar),\end{aligned} \tag{9}$$

where $\mathcal{U} \in \mathbb{R}^m > 0$ is a symmetric matrix. The function $\epsilon(t)$ represents the DETM threshold[31] satisfying

$$\begin{aligned}\dot{\epsilon}(t) =& \frac{1}{\epsilon(t)}\left[\frac{1}{\epsilon(t)} - \rho\right][\pi(t_k\hbar + \ell\hbar) - \pi(t_k\hbar)]^{\mathrm{T}} \\ &\times \mathcal{U}[\pi(t_k\hbar + \ell\hbar) - \pi(t_k\hbar)],\end{aligned} \tag{10}$$

where $\rho > 0$ is a given constant.

Remark 2 From Eq. (10), it is not difficult to see that $\epsilon(t)$ is a time-varying function. Compared to the existing static ETMs in Ref. [19,32,33], the DETM employed in this paper is more reasonable. Furthermore, if the error approaches zero, $\dot{\epsilon}(t) \to 0$. Namely, the dynamic threshold $\epsilon(t)$ will be fixed. It is worth noting that the proposed DETM (9) will be degenerated into the traditional one with a fixed threshold when we select $\epsilon(0) = 1/\rho$.

Similar to the method proposed in Ref. [34], we end up with the following representations of functions $\nu(t)$ and $e_s(t)$:

$$\nu(t) = \begin{cases} t - t_k\hbar, & t \in \Theta_0, \\ t - t_k\hbar - \ell\hbar, & t \in \Theta_\ell, \\ t - t_k\hbar - d_M\hbar, & t \in \Theta_{d_M}, \end{cases} \tag{11}$$

$$e_s(t) = \begin{cases} 0, & t \in \Theta_0, \\ \pi(t_k\hbar) - \pi(t_k\hbar + \ell\hbar), & t \in \Theta_\ell, \\ \pi(t_k\hbar) - \pi(t_k\hbar + d_M\hbar), & t \in \Theta_{d_M}, \end{cases} \tag{12}$$

where $\Theta_0 = [t_k\hbar + \nu_k,\ t_k\hbar + h + \bar{\nu})$, $\Theta_\ell = [t_k\hbar + \ell\hbar + \bar{\nu},\ t_k\hbar + \ell\hbar + h + \bar{\nu})$, $\ell = 1,2,\ldots,d_M - 1$, $\Theta_{d_M} = [t_k\hbar + d_M h + \bar{\nu},\ t_{k+1}h + \nu_{k+1})$. Then one has $0 < \underline{\nu} \le \nu(t) \le \ell + \bar{\nu} \triangleq \nu$ from Eq. (11), in which ν denotes the upper bound of $\nu(t)$.

Based on the above analysis, the available input of the designed estimator (8) is

$$\pi_s(t) = \pi(t_k\hbar) = e_s(t) + \mathrm{Sat}(E\xi(t - \nu(t))). \tag{13}$$

Combining with Eqs. (11) and (12), we have

$$\dot{\epsilon}(t) = \frac{1}{\epsilon(t)}\left[\frac{1}{\epsilon(t)} - \rho\right]e_s^{\mathrm{T}}(t)\mathcal{U}e_s(t). \tag{14}$$

This paper considers the impact of cyber-attacks when sampling data is transmitted over the network. When the cyber-attacks occur, the sampling data and the aggressive signals are transmitted to the designed estimator. The actual input of the designed estimator can be expressed by

$$\begin{aligned}\tilde{\pi}(t) &= (1 - \varepsilon(t))\pi_s(t) + \varepsilon(t)\hbar(\xi(t - \delta(t))) \\ &= (1 - \varepsilon(t))[e_s(t) + \mathrm{Sat}(E\xi(t - \nu(t)))] \\ &\quad + \varepsilon(t)\hbar(\xi(t - \delta(t))),\end{aligned} \tag{15}$$

where $0 \le \delta(t) \le d$, $\hbar(\xi(t))$ denotes the cyber-attacks; $\varepsilon(t)$ is a stochastic variable describing the random nature of cyber-attacks and obeys the Bernoulli distributed satisfying $\mathrm{Prob}\{\varepsilon(t) = 1\} = \mathcal{E}\{\varepsilon(t)\} = \bar{\varepsilon}$.

Next, by defining $e(t) = \xi(t) - \hat{\xi}(t)$ and $\tilde{g}(e(t)) = g(\xi(t)) - g(\hat{\xi}(t))$, the error dynamical can be obtained by

$$\begin{aligned}\dot{e}(t) =& -\mathcal{H}e(t) + \mathcal{A}\tilde{g}(e(t)) + \Delta\mathcal{A}g(\xi(t)) + \mathcal{B}\tilde{g}(\xi(\rho t)) \\ &+ \Delta\mathcal{B}g(\xi(\rho t)) - E\hat{\xi}(t) - [(1 - \varepsilon(t))e_s(t) \\ &+ \mathrm{Sat}(E\xi(t - \nu(t)))] - \varepsilon(t)(\mathcal{K} + \Delta\mathcal{K})\hbar(\xi(t - \delta(t))),\end{aligned} \tag{16}$$

where $\upsilon_t \triangleq [\xi^{\mathrm{T}}(t)e^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\varphi_g(t) \triangleq [g^{\mathrm{T}}(\xi(t))\tilde{g}^{\mathrm{T}}(e(t))]^{\mathrm{T}}$, $\upsilon(t - \nu_t) \triangleq \upsilon(t - \nu(t))$, $\upsilon(t - \delta_t) \triangleq \upsilon(t - \delta(t))$. From Eqs. (6) and (16), one has

$$\dot{\upsilon}_t = -(\tilde{\mathcal{H}} + \mathcal{I}_1\bar{E})\upsilon(t) + \tilde{\mathcal{A}}\varphi_g(t) + \tilde{\mathcal{B}}\varphi_g(\rho t) - \tilde{\varepsilon}\tilde{\mathcal{K}}e_s(t)$$

$$+\tilde{\varepsilon}\tilde{\mathcal{K}}\mathrm{Sat}(E\bar{\mathcal{I}}\upsilon(t-\nu_t))+\psi_t-\bar{\varepsilon}\tilde{\mathcal{K}}\hbar(\upsilon(t-\delta_t)), \quad (17)$$

where $\tilde{\varepsilon}=1-\bar{\varepsilon}$, $\varepsilon_0=\bar{\varepsilon}-\varepsilon(t)$, $\psi_t=\varepsilon_0\tilde{\mathcal{K}}\hbar(\xi(t-\delta_t))-\varepsilon_0\tilde{\mathcal{K}}e_s(t)-\varepsilon_0\tilde{\mathcal{K}}\mathrm{Sat}(E\xi(t-\nu_t))$, $\bar{E}=[E\ -E]$, $\bar{\mathcal{I}}=[\mathcal{I}\ 0]$,

$$\tilde{\mathcal{H}}=\begin{bmatrix}\mathcal{H}+\Delta\mathcal{H} & 0\\ \Delta\mathcal{H} & \mathcal{H}\end{bmatrix},\ \tilde{\mathcal{A}}=\begin{bmatrix}\mathcal{A}+\Delta\mathcal{A} & 0\\ \Delta\mathcal{A} & \mathcal{A}\end{bmatrix},\ \tilde{\mathcal{B}}=\begin{bmatrix}\mathcal{B}+\Delta\mathcal{B} & 0\\ \Delta\mathcal{B} & \mathcal{B}\end{bmatrix},$$
$$\mathcal{I}_1=\begin{bmatrix}0\\ \mathcal{I}\end{bmatrix},\ \tilde{\mathcal{K}}=\begin{bmatrix}0\\ \mathcal{K}+\Delta\mathcal{K}\end{bmatrix}.$$

Assumption 3[33] Cyber-attacks $\hbar(\cdot)$ satisfy the following inequality:

$$\|\hbar(\xi(t))\|_2\le\|\mathcal{G}\xi(t)\|_2, \quad (18)$$

where $\mathcal{G}$ is the constant matrix upper bound of the nonlinear function $\hbar(\cdot)$.

Lemma 1[35] For any integrable vector function $f(\cdot):[t_0,t_1]\to\mathbb{R}^n$ and positive matrix $\mathcal{Q}$, we have

$$\Big(\int_{t_0}^{t_1}f(\tau)\,\mathrm{d}\tau\Big)^{\mathrm{T}}\mathcal{Q}\Big(\int_{t_0}^{t_1}f(\tau)\,\mathrm{d}\tau\Big)+3\varkappa^{\mathrm{T}}\mathcal{Q}\varkappa$$
$$\le(t_1-t_0)\int_{t_0}^{t_1}f^{\mathrm{T}}(\tau)\mathcal{Q}f(\tau)\,\mathrm{d}\tau,$$

where $\varkappa=\int_{t_0}^{t_1}f(s)ds-\frac{2}{t_1-t_0}\int_{t_0}^{t_1}\int_{s}^{t_0}f(u)\,\mathrm{d}u\,\mathrm{d}s$.

Lemma 2[36] For $\nu\in[0,1]$, matrices $Y>0,L$ and vectors χ_1,χ_2. The following inequality holds

$$-\frac{1}{\nu}\chi_1^{\mathrm{T}}Y\chi_1-\frac{1}{1-\nu}\chi_2^{\mathrm{T}}Y\chi_2\le-\begin{bmatrix}\chi_1\\ \chi_2\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}Y & L\\ * & Y\end{bmatrix}\begin{bmatrix}\chi_1\\ \chi_2\end{bmatrix},$$

in which

$$\begin{bmatrix}Y & L\\ * & Y\end{bmatrix}>0.$$

3. Main results

For the sake of description, denote

$$\tilde{\zeta}_t=\mathrm{col}\Big\{\upsilon_t\ \upsilon(t-\nu)\ \upsilon(t-\nu_t)\ \frac{1}{\nu(t)}\int_{t-\nu_t}^{t}\upsilon(s)\mathrm{d}s$$
$$\times\frac{1}{\varpi_\nu(t)}\int_{t-\nu}^{t-\nu_t}\upsilon(s)\mathrm{d}s\ \upsilon(t-d)\ \upsilon(t-\delta_t)\ \frac{1}{\delta_t}\int_{t-\delta_t}^{t}\upsilon(s)\mathrm{d}s$$
$$\times\frac{1}{\varpi_\delta(t)}\int_{t-\nu}^{t-\nu_t}\upsilon(s)\mathrm{d}s\ \varphi_g(t)\ \varphi_g(\rho t)\ \dot{\upsilon}_t\ e_s(t)$$
$$\times\mathrm{Sat}(E\bar{\mathcal{I}}\upsilon(t-\nu_t))\ \hbar(\upsilon(t-\delta_t))\Big\};$$
$$\tilde{\mu}=\mu(t)=\frac{\nu_t}{\nu}<1,\ \ \varpi_\nu(t)=\frac{1}{\nu-\nu_t}.$$

In this section, a nonfragile estimator for system (1) is designed. Then, a delay-dependent sufficient criterion will be derived for guaranteeing the asymptotic stability of the augmented system by constructing a sensible LKF.

Theorem 1 Suppose that ν, d, and gain matrix $\mathcal{K}$ are given. The augmented system (17) is asymptotically stable if there exist symmetric definite matrices $\mathcal{O}$, $\mathcal{Z}_1$, $\mathcal{Z}$, $\mathcal{Q}$, $\mathcal{Q}_1$, $\mathcal{P}$, matrices L_d, L_ν, $\mathcal{W}$ and positive scalar κ_1, κ_2 such that

$$\tilde{\Upsilon}<0, \quad (19)$$

$$\Omega_d=\begin{bmatrix}\mathrm{diag}\{\mathcal{Z},\mathcal{Z}\} & L_d\\ * & \mathrm{diag}\{\mathcal{Z},\mathcal{Z}\}\end{bmatrix}>0,$$
$$\Omega_\nu=\begin{bmatrix}\mathrm{diag}\{\mathcal{Q},\mathcal{Q}\} & L_\nu\\ * & \mathrm{diag}\{\mathcal{Q},\mathcal{Q}\}\end{bmatrix}>0, \quad (20)$$

where $\tilde{\Upsilon}=\sum_{k=1}^{5}\Upsilon_k+2l_1^{\mathrm{T}}\mathcal{W}\tilde{\chi}+2\mathfrak{a}l_{12}^{\mathrm{T}}\mathcal{W}\tilde{\chi}$, $\Upsilon_1=2l_1^{\mathrm{T}}\mathcal{P}l_{12}+l_{10}^{\mathrm{T}}\mathcal{O}l_{10}-l_{11}^{\mathrm{T}}\rho\mathcal{O}l_{11}+l_1^{\mathrm{T}}[\mathcal{Z}_1+\mathcal{Q}_1]l_1-l_6^{\mathrm{T}}\mathcal{Z}_1l_6-l_2^{\mathrm{T}}\mathcal{Q}_1l_2+l_{12}^{\mathrm{T}}[d^2\mathcal{Z}+\nu^2\mathcal{Q}]l_{12}$, $\Upsilon_2=-\Lambda_\nu^{\mathrm{T}}\Omega_\nu\Lambda_\nu-\Lambda_d^{\mathrm{T}}\Omega_d\Lambda_d$, $\Upsilon_3=l_{14}^{\mathrm{T}}\mathcal{U}l_{14}+l_{14}^{\mathrm{T}}\mathcal{U}l_{13}+l_{13}^{\mathrm{T}}\mathcal{U}l_{13}$, $\Upsilon_4=-\kappa_1l_1^{\mathrm{T}}F_1l_1+2\kappa_1l_1^{\mathrm{T}}F_2l_{10}-\kappa_1l_{10}^{\mathrm{T}}Il_{10}+\kappa_2l_7^{\mathrm{T}}\bar{\mathcal{I}}^{\mathrm{T}}\mathcal{G}^{\mathrm{T}}\mathcal{G}\bar{\mathcal{I}}l_7-\kappa_2l_{14}^{\mathrm{T}}Il_{14}$, $\Upsilon_5=l_{14}^{\mathrm{T}}E\bar{\mathcal{I}}l_3-l_{14}^{\mathrm{T}}Il_{14}$, $\tilde{\chi}=-(\tilde{\mathcal{H}}+\mathcal{I}_1\bar{E})l_1+\tilde{\mathcal{A}}l_{10}+\tilde{\mathcal{B}}l_{11}-\tilde{\varepsilon}\tilde{\mathcal{K}}l_{13}-\tilde{\varepsilon}\tilde{\mathcal{K}}l_{14}-\tilde{\varepsilon}\tilde{\mathcal{K}}l_{15}$, $\Lambda_\nu=\begin{bmatrix}\Lambda_1\ \Lambda_2\end{bmatrix}$, $\Lambda_1=[(l_1-l_3)^{\mathrm{T}}\ \sqrt{3}(l_1+l_3-2l_4)^{\mathrm{T}}]$, $\Lambda_2=[(l_3-l_2)^{\mathrm{T}}\ \sqrt{3}(l_3+l_2-2l_5)^{\mathrm{T}}]$, $\Lambda_d=\begin{bmatrix}\tilde{\Lambda}_1\ \tilde{\Lambda}_2\end{bmatrix}$, $\tilde{\Lambda}_1=[(l_1-l_3)^{\mathrm{T}}\ \sqrt{3}(l_1+l_3-2l_4)^{\mathrm{T}}]$, $\tilde{\Lambda}_2=[(l_3-l_2)^{\mathrm{T}}\ \sqrt{3}(l_3+l_2-2l_5)^{\mathrm{T}}]$, $F_1=I\otimes(\hat{G}_1^{\mathrm{T}}\hat{G}_2+\hat{G}_2^{\mathrm{T}}\hat{G}_1)/2$, $F_2=-I\otimes(\hat{G}_1^{\mathrm{T}}+\hat{G}_2^{\mathrm{T}})/2$, $l_\ell=[0_{2n\times(\ell-1)2n}\ I_{2n}\ 0_{2n\times(15-\ell)2n}]$ $(\ell=1,2,\ldots,15)$.

Proof Choose the following LKF for Eq. (17):

$$\mathrm{V}_t=\sum_{h=1}^{4}\mathrm{V}_{ht}, \quad (21)$$

where

$$\mathrm{V}_{1t}=\upsilon_t^{\mathrm{T}}\mathcal{P}\upsilon_t,$$
$$\mathrm{V}_{2t}=\int_{\rho t}^{t}\varphi_g^{\mathrm{T}}(\zeta)\mathcal{O}\varphi_g(\zeta)\mathrm{d}\zeta+\frac{1}{2}\epsilon^2(t),$$
$$V_{3t}=\int_{t-\nu}^{t}\upsilon(\zeta)^{\mathrm{T}}\mathcal{Q}_1\upsilon(\zeta)\mathrm{d}\zeta+\int_{t-d}^{t}\upsilon(\zeta)^{\mathrm{T}}\mathcal{Z}_1\upsilon(\zeta)\mathrm{d}\zeta,$$
$$V_{4t}=\nu\int_{-\nu}^{0}\int_{t+\zeta}^{t}\dot{\upsilon}(\zeta)^{\mathrm{T}}\mathcal{Q}\dot{\upsilon}(\zeta)\mathrm{d}\varsigma\mathrm{d}\zeta$$
$$+d\int_{-d}^{0}\int_{t+\zeta}^{t}\dot{\upsilon}(\zeta)^{\mathrm{T}}\mathcal{Z}\dot{\upsilon}(\zeta)\mathrm{d}\varsigma\mathrm{d}\zeta, \quad (22)$$

Defining the infinitesimal operator $\mathcal{L}$ of V_t as in Ref. [37]: $\mathcal{L}\mathrm{V}_t=\lim\limits_{\Delta\to0^+}\frac{1}{\Delta}\{\mathcal{E}\{\mathrm{V}_(\upsilon_{t+\Delta}|\upsilon_t)\}-\mathrm{V}(\upsilon_t)\}$, we obtain

$$\mathcal{E}\{\mathcal{L}\mathrm{V}_{1t}\}=\mathcal{E}\{2\upsilon_t^{\mathrm{T}}\mathcal{P}\dot{\upsilon}_t\}, \quad (23)$$
$$\mathcal{E}\{\mathcal{L}\mathrm{V}_{2t}\}=\mathcal{E}\{\varphi_g^{\mathrm{T}}(t)\mathcal{O}\varphi_g(t)-\varphi_g^{\mathrm{T}}(\rho t)\rho\mathcal{O}\varphi_g(\rho t)$$
$$+\epsilon_s(t)\dot{\epsilon}_s(t)\}, \quad (24)$$
$$\mathcal{E}\{\mathcal{L}\mathrm{V}_{3t}\}=\mathcal{E}\{\upsilon_t^{\mathrm{T}}[\mathcal{Z}_1+\mathcal{Q}_1]\upsilon_t-\upsilon^{\mathrm{T}}(t-d)\mathcal{Z}_1\upsilon(t-d)$$
$$-\upsilon^{\mathrm{T}}(t-\nu)\mathcal{Q}_1\upsilon(t-\nu)\}, \quad (25)$$
$$\mathcal{E}\{\mathcal{L}\mathrm{V}_{4t}\}=\mathcal{E}\{\dot{\upsilon}_t^{\mathrm{T}}[d^2\mathcal{Z}+\nu^2\mathcal{Q}]\dot{\upsilon}_t-\nu\int_{t-\nu}^{t}\dot{\upsilon}^{\mathrm{T}}(\zeta)\mathcal{Q}\dot{\upsilon}(\zeta)\mathrm{d}\zeta$$
$$-d\int_{t-d}^{t}\dot{\upsilon}^{\mathrm{T}}(\zeta)\mathcal{Z}\dot{\upsilon}(\zeta)\mathrm{d}\zeta\}. \quad (26)$$

By means of Lemma 1, one gets

$$-\nu\int_{t-\nu}^{t}\dot{\upsilon}^{\mathrm{T}}(\zeta)\mathcal{Q}\dot{\upsilon}(\zeta)\mathrm{d}\zeta$$
$$=-\nu\Big\{\int_{t-\nu_t}^{t}\dot{\upsilon}^{\mathrm{T}}(\zeta)\mathcal{Q}\dot{\upsilon}(\zeta)\mathrm{d}\zeta+\int_{t-\nu}^{t-\nu_t}\dot{\upsilon}^{\mathrm{T}}(\zeta)\mathcal{Q}\dot{\upsilon}(\zeta)\mathrm{d}\zeta\Big\}$$

$$
\begin{aligned}
&\leq -\nu\Big\{\frac{1}{\nu_t}[\upsilon_t-\upsilon(t-\nu_t)]^{\mathrm{T}}\mathcal{Q}[\upsilon_t-\upsilon(t-\nu_t)]\\
&\quad+\frac{3}{\nu_t}[\upsilon_t+\upsilon(t-\nu_t)-\frac{2}{\nu_t}\int_{t-\nu_t}^{t}\upsilon(\zeta)\mathrm{d}\zeta]^{\mathrm{T}}\\
&\quad\times\mathcal{Q}[\upsilon_t+\upsilon(t-\nu_t)-\frac{2}{\nu_t}\int_{t-\nu_t}^{t}\upsilon(\zeta)\mathrm{d}\zeta]\\
&\quad+\frac{1}{\varpi_\nu(t)}[\upsilon(t-\nu_t)-\upsilon(t-\nu)]^{\mathrm{T}}\mathcal{Q}[\upsilon(t-\nu_t)\\
&\quad-\upsilon(t-\nu)]+\frac{3}{\varpi_\nu(t)}[\upsilon(t-\nu_t)+\upsilon(t-\nu)\\
&\quad-\frac{2}{\varpi_\nu(t)}\int_{t-\nu_t}^{t-\nu}\upsilon(\zeta)\mathrm{d}\zeta]^{\mathrm{T}}\mathcal{Q}[\upsilon(t-\nu_t)\\
&\quad+\upsilon(t-\nu)-\frac{2}{\varpi_\nu(t)}\int_{t-\nu_t}^{t-\nu}\upsilon(\zeta)\mathrm{d}\zeta]\Big\}.
\end{aligned}
$$

Then, we have

$$
\begin{aligned}
&-\nu\int_{t-\nu}^{t}\dot{\upsilon}^{\mathrm{T}}(\zeta)\mathcal{Q}\dot{\upsilon}(\zeta)\mathrm{d}\zeta\\
&\leq-\frac{1}{\tilde{\mu}}\tilde{\zeta}_t[l_1-l_3]^{\mathrm{T}}\mathcal{Q}[l_1-l_3]\tilde{\zeta}_t-\frac{3}{\tilde{\mu}}\tilde{\zeta}_t^{\mathrm{T}}[l_1+l_3-2l_4]^{\mathrm{T}}\mathcal{Q}\big[l_1+l_3\\
&\quad-2l_4]\tilde{\zeta}_t-\frac{1}{1-\tilde{\mu}}\tilde{\zeta}_t[l_3-l_2]^{\mathrm{T}}\mathcal{Q}[l_3-l_2]\tilde{\zeta}_t\\
&\quad-\frac{3}{1-\tilde{\mu}}\tilde{\zeta}_t^{\mathrm{T}}[l_3+l_2-2l_5]^{\mathrm{T}}\mathcal{Q}[l_3+l_2-2l_5]\tilde{\zeta}_t. \qquad (27)
\end{aligned}
$$

According to the definition of $\mu(t)$ and Lemma 2, one has

$$
-\nu\int_{t-\nu}^{t}\dot{\upsilon}^{\mathrm{T}}(\zeta)\mathcal{Q}\dot{\upsilon}(\zeta)\mathrm{d}\zeta\leq-\tilde{\zeta}_t^{\mathrm{T}}\Lambda_\nu^{\mathrm{T}}\Omega_\nu\Lambda_\nu\tilde{\zeta}_t. \qquad (28)
$$

Similar to Eq. (28), we have

$$
-d\int_{t-d}^{t}\dot{\upsilon}^{\mathrm{T}}(\zeta)\mathcal{Z}\dot{\upsilon}(\zeta)\mathrm{d}\zeta\leq-\tilde{\zeta}_t^{\mathrm{T}}\Lambda_d^{\mathrm{T}}\Omega_d\Lambda_d\tilde{\zeta}_t. \qquad (29)
$$

Recalling the triggering protocol (9) and the adaptive law (13), one obtains

$$
\begin{aligned}
\epsilon(t)\dot{\epsilon}(t)&\leq y^{\mathrm{T}}(t_k\hbar)\mathcal{U}\pi(t_k\hbar)-\rho e_s^{\mathrm{T}}(t)\mathcal{U}e_s(t)\\
&=\mathrm{Sat}^{\mathrm{T}}(E\bar{\mathcal{I}}\xi(t-\nu_t))\mathcal{U}\mathrm{Sat}(E\bar{\mathcal{I}}\xi(t-\nu_t))\\
&\quad+\mathrm{Sat}^{\mathrm{T}}(E\bar{\mathcal{I}}\xi(t-\nu_t))\mathcal{U}e_s(t)+(1-\rho)e_s^{\mathrm{T}}(t)\mathcal{U}e_s(t).
\end{aligned} \qquad (30)
$$

In view of Assumptions 1 and 3, we can reach

$$
\begin{aligned}
&\kappa_1[-\upsilon_t^{\mathrm{T}}F_1\upsilon_t+2\upsilon_t^{\mathrm{T}}F_2\varphi_g(t)-\varphi_g^{\mathrm{T}}(t)\varphi_g(t)]>0,\\
&\kappa_2[\upsilon^{\mathrm{T}}(t-\delta_t)\bar{\mathcal{I}}^{\mathrm{T}}\mathcal{G}^{\mathrm{T}}\mathcal{G}\bar{\mathcal{I}}\upsilon(t-\delta_t)\\
&-\hbar^{\mathrm{T}}(\xi(t-\delta_t))\hbar(\xi(t-\delta_t))]\geq0. \qquad (31)
\end{aligned}
$$

According to the analysis of saturation function in Ref. [38], we have

$$
\mathrm{Sat}^{\mathrm{T}}(E\bar{\mathcal{I}}\upsilon(t-\nu_t))\big[E\bar{\mathcal{I}}\upsilon(t-\nu_t)-\mathrm{Sat}(E\bar{\mathcal{I}}\upsilon(t-\nu_t))\big]>0. \qquad (32)
$$

Then, for any matrix $\mathcal{W}$, it can be refereed from Eq. (17) that

$$
\begin{aligned}
&\mathcal{E}\{2\tilde{\zeta}_t^{\mathrm{T}}[l_1^{\mathrm{T}}\mathcal{W}+\mathfrak{a}l_{12}^{\mathrm{T}}\mathcal{W}][-(\tilde{\mathcal{H}}+\mathcal{I}_1\bar{E})l_1+\tilde{\mathcal{A}}l_{10}+\tilde{\mathcal{B}}l_{11}\\
&\quad-\tilde{\varepsilon}\tilde{\mathcal{K}}l_{13}-\tilde{\varepsilon}\tilde{\mathcal{K}}l_{14}-\bar{\varepsilon}\tilde{\mathcal{K}}l_{15}-Il_{12}+\psi_t]\tilde{\zeta}_t\}=0. \qquad (33)
\end{aligned}
$$

Taking Eqs. (23)–(33) into account, it can be concluded that $\mathcal{E}\{\mathcal{L}\mathrm{V}_t\}\leq\tilde{\zeta}_t^{\mathrm{T}}\tilde{\Upsilon}\tilde{\zeta}_t$. Then, combining the inequalities (19) and (20), it is realized that the augmented system (17) is stochastically asymptotically stable. The proof is complete.

Theorem 1 shows the performance analysis under the assumption of a known matrix gain $\mathcal{K}$. Next, we calculate the estimator gain $\mathcal{K}$ in the following subsection based on the above condition.

Theorem 2 For given scalars $d>0$ and $\nu>0$, the augmented system (17) is asymptotically stable under DETM if there exist scalars κ_1, $\kappa_2>0$, $u>0$, symmetric positive matrices $\mathcal{P}$, $\mathcal{O}$, $\mathcal{Z}_1$, $\mathcal{Z}$, $\mathcal{Q}$, $\mathcal{Q}_1$, and any matrices $\mathcal{W}$, L_d, L_ν such that

$$
\begin{bmatrix}\hat{\Upsilon}&\tilde{\mathcal{H}}&u\tilde{\mathcal{N}}^{\mathrm{T}}*&-uI&0*&*&-uI\end{bmatrix}<0, \qquad (34)
$$

$$
\begin{aligned}
\Omega_d&=\begin{bmatrix}\mathrm{diag}\{\mathcal{Z},\mathcal{Z}\}&L_d*&\mathrm{diag}\{\mathcal{Z},\mathcal{Z}\}\end{bmatrix}>0,\\
\Omega_\nu&=\begin{bmatrix}\mathrm{diag}\{\mathcal{Q},\mathcal{Q}\}&L_\nu*&\mathrm{diag}\{\mathcal{Q},\mathcal{Q}\}\end{bmatrix}>0,
\end{aligned} \qquad (35)
$$

where $\hat{\Upsilon}=\bar{\Upsilon}'+\bar{\Psi}$, $\bar{\Upsilon}'=\sum_{k=1}^{5}\Upsilon_k$, $\bar{\Psi}=\mathrm{sym}\{(l_1^{\mathrm{T}}+\mathfrak{a}l_{12}^{\mathrm{T}})\bar{\Pi}\}$, $\bar{\Pi}=-\mathcal{W}_hl_1+\hat{\mathcal{A}}l_{10}+\hat{\mathcal{B}}l_{11}-\tilde{\varepsilon}\mathcal{W}_xl_{13}-\tilde{\varepsilon}\mathcal{W}_xl_{14}--\bar{\varepsilon}\mathcal{W}_xl_{15}$ with $\mathcal{W}_x=[0\ \mathcal{X}^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathcal{W}=\{\mathcal{W}_1,\mathcal{W}_2\}$; Υ_k has been defined in Theorem 1. If Eqs. (34) and (35) are feasible, the desired estimator gain matrix $\mathcal{K}$ is determined by $\mathcal{K}=\mathcal{W}_2^{-1}\mathcal{X}$.

Proof The matrix $\tilde{\Upsilon}$ can be decomposed as follows:

$$
\tilde{\Upsilon}=\bar{\Upsilon}+\Delta\bar{\Upsilon}, \qquad (36)
$$

where $\bar{\Upsilon}=\bar{\Upsilon}'+\Psi$, $\bar{\Upsilon}'=\sum_{k=1}^{5}\Upsilon_k$, $\Psi=\mathrm{sym}\{(l_1^{\mathrm{T}}+\mathfrak{a}l_{12}^{\mathrm{T}})\Pi\}$, $\Pi=\mathcal{W}\hat{\mathcal{H}}l_1+\mathcal{W}\hat{\mathcal{A}}_{10}+\mathcal{W}\hat{\mathcal{B}}_{11}-\tilde{\varepsilon}\mathcal{W}\hat{\mathcal{K}}l_{13}-\tilde{\varepsilon}\mathcal{W}\hat{\mathcal{K}}_{14}-\tilde{\varepsilon}\mathcal{W}\hat{\mathcal{K}}l_{15}$, $\hat{\mathcal{H}}=\begin{bmatrix}\mathcal{H}&0\\E&\mathcal{H}-E\end{bmatrix}$, $\hat{\mathcal{Z}}=\begin{bmatrix}\mathcal{Z}&0\\0&\mathcal{Z}\end{bmatrix}$, $\hat{\mathcal{K}}=\begin{bmatrix}0\\\mathcal{K}\end{bmatrix}$, $\Delta\bar{\Upsilon}=\tilde{\mathcal{H}}\tilde{\mathcal{F}}(t)\tilde{\mathcal{N}}+\tilde{\mathcal{N}}^{\mathrm{T}}\tilde{\mathcal{F}}^{\mathrm{T}}(t)\tilde{\mathcal{H}}^{\mathrm{T}}$ with $\tilde{\mathcal{H}}=[\widehat{\mathcal{W}}^{\mathrm{T}}\ 0\ \mathfrak{a}\widehat{\mathcal{W}}^{\mathrm{T}}\ 0]^{\mathrm{T}}$, $\widehat{\mathcal{W}}=[\mathcal{W}\tilde{\mathcal{H}}_h\ \mathcal{W}\tilde{\mathcal{H}}_a\ \mathcal{W}\tilde{\mathcal{H}}_b\ \mathcal{W}\tilde{\mathcal{H}}_k]$, $\tilde{\mathcal{F}}(t)=\mathrm{diag}\{\tilde{\mathcal{F}}_h(t),\ \tilde{\mathcal{F}}_a(t),\ \tilde{\mathcal{F}}_b(t),\ \mathcal{F}_k(t)\}$, $\tilde{\mathcal{H}}_h=\mathrm{diag}\{\mathcal{H}_h,\mathcal{H}_h\}$, $\tilde{\mathcal{H}}_z=\mathrm{diag}\{\mathcal{H}_z,\mathcal{H}_z\}$, $\tilde{\mathcal{F}}_h(t)=\mathrm{diag}\{\mathcal{F}_h(t),\mathcal{F}_h(t)\}$, $\tilde{\mathcal{F}}_z(t)=\mathrm{diag}\{\mathcal{F}_z(t),\mathcal{F}_z(t)\}$, $\tilde{\mathcal{N}}_h=[\mathcal{N}_h\ \ 0;\ \ \mathcal{N}_h\ \ 0]$, $\tilde{\mathcal{N}}_z=\begin{bmatrix}\mathcal{N}_z&0\\\mathcal{N}_z&0\end{bmatrix}$, $\tilde{\mathcal{H}}_k=\begin{bmatrix}0\\\mathcal{H}_k\end{bmatrix}$, $\tilde{\mathcal{N}}=\begin{bmatrix}-\tilde{\mathcal{N}}_h&0&0&0&0&0&0&0\\0&0&\tilde{\mathcal{N}}_a&0&0&0&0&0\\0&0&0&\tilde{\mathcal{N}}_b&0&0&0&0\\\tilde{\mathcal{N}}_k\bar{E}&0&0&0&0&-\tilde{\varepsilon}\tilde{\mathcal{N}}_k&-\tilde{\varepsilon}\tilde{\mathcal{N}}_k&-\bar{\varepsilon}\mathcal{N}_k\end{bmatrix}$.

Through the above transformation and Eq. (19), we can rewrite Eq. (36) as

$$
\tilde{\Upsilon}=\bar{\Upsilon}+\tilde{\mathcal{H}}\tilde{\mathcal{F}}(t)\tilde{\mathcal{N}}+\tilde{\mathcal{N}}^{\mathrm{T}}\tilde{\mathcal{F}}^{\mathrm{T}}(t)\tilde{\mathcal{H}}^{\mathrm{T}}\leq0. \qquad (37)
$$

Based on the S-procedure that Eq. (37) is true if there exist a positive scalar u such that

$$
\tilde{\Upsilon}<\bar{\Upsilon}+u\tilde{\mathcal{H}}\tilde{\mathcal{H}}^{\mathrm{T}}+u^{-1}\tilde{\mathcal{N}}^{\mathrm{T}}\tilde{\mathcal{N}}<0. \qquad (38)
$$

By using the Schur complement and denoting $\mathcal{X} = \mathcal{W}_2\mathcal{K}$, it is easy to prove that the inequality (38) is satisfied if Eqs. (34) and (35) hold. This completes the proof.

When the gain perturbations $\Delta\mathcal{K}$ and cyber-attacks are not considered, the desired estimator (8) will be degenerated into a traditional one with an ideal estimator gain under DETMs. In this case, the following corollary is concluded easily with $\Delta\mathcal{K} = 0$, $\varepsilon(t) = 0$.

Corollary 1 For given scalars $\nu > 0$, the augmented system is asymptotically stable if there exist scalars $\kappa_1 > 0$, $\mu > 0$, positive matrices $\mathcal{P}$, $\mathcal{O}$, $\mathcal{Q}$, $\mathcal{Q}_1$, and matrices L_ν, $\mathcal{W} = \{\mathcal{W}_1, \mathcal{W}_2\}$ satisfying

$$\begin{bmatrix} \bar{\Upsilon} & \tilde{\mathcal{H}}' & \mu\tilde{\mathcal{N}}'^T \\ * & -\mu I & 0 \\ * & * & -\mu I \end{bmatrix} < 0, \tag{39}$$

$$\begin{bmatrix} \mathrm{diag}\{\mathcal{Q}, \mathcal{Q}\} & L_\nu \\ * & \mathrm{diag}\{\mathcal{Q}, \mathcal{Q}\} \end{bmatrix} > 0, \tag{40}$$

where

$$\begin{aligned}
&\bar{\Upsilon} = \sum_{k=1}^{5} \Xi_k, \\
&\Xi_1 = \Upsilon_1 + \Upsilon_2 + l_1^{\mathrm{T}}\mathcal{Q}_1 l_1 - l_2^{\mathrm{T}}\mathcal{Q}_1 l_2 - \Lambda_\nu^{\mathrm{T}}\Omega_\nu\Lambda_\nu, \\
&\Xi_2 = \mathrm{sym}\{-l_1^{\mathrm{T}}\mathcal{W}_h l_1 + l_1^{\mathrm{T}}\mathcal{W}\hat{\mathcal{A}} l_6 + l_1^{\mathrm{T}}\mathcal{W}\hat{\mathcal{B}} l_7 - l_1^{\mathrm{T}}\mathcal{W} l_8 \\
&\qquad - l_1^{\mathrm{T}}\mathcal{W}_s l_9 - l_1^{\mathrm{T}}\mathcal{W}_{sat} l_{10}\}, \\
&\Xi_5 = \mathfrak{a}\mathrm{sym}\{-l_8^{\mathrm{T}}\mathcal{W}_h l_1 + l_8^{\mathrm{T}}\mathcal{W}\hat{\mathcal{A}} l_6 + l_8^{\mathrm{T}}\mathcal{W}\hat{\mathcal{B}} l_7 - l_8^{\mathrm{T}}\mathcal{W} l_8 \\
&\qquad - l_8^{\mathrm{T}}\mathcal{W}_s l_9 - l_8^{\mathrm{T}}\mathcal{W}_{sat} l_{10}\}, \\
&\Xi_3 = 2l_{10}^{\mathrm{T}}\mathcal{U} l_9 + l_{10}^{\mathrm{T}}\mathcal{U} l_{10} + l_{10}^{\mathrm{T}}(1-\rho) l_9, \\
&\Xi_4 = 2l_{10}^{\mathrm{T}} E\bar{\mathcal{I}}_2 l_3 - 2l_{10}^{\mathrm{T}} I l_{10}, \text{ with } \mathcal{W}_s = \mathcal{W}_{sat} = \begin{bmatrix} 0 & \mathcal{X}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\
&\tilde{\mathcal{H}} = \begin{bmatrix} \Omega^{\mathrm{T}} 0 & 0 & 0 & 0 & 0 & 0 & \mathfrak{a}\Omega^{\mathrm{T}} & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \\
&\Omega = \begin{bmatrix} \tilde{\mathcal{H}}_h & 0 & 0 & 0 & 0 & \tilde{\mathcal{H}}_a & \tilde{\mathcal{H}}_b & 0 & 0 & 0 \end{bmatrix}, \\
&l_\ell = [0_{2n\times(\ell-1)2n}\ I_{2n}\ 0_{2n\times(10-\ell)2n}], \quad (\ell = 1, 2, \ldots, 10), \\
&\mathcal{W}_h = \begin{bmatrix} \mathcal{W}_1\mathcal{H} & 0 \\ -\mathcal{X}E & \mathcal{W}_2\mathcal{H} + \mathcal{X}E \end{bmatrix}, \\
&\tilde{\mathcal{N}} = \begin{bmatrix} \tilde{\mathcal{N}}_h & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{N}}_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{N}}_b & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

$\hat{\mathcal{A}}$, $\hat{\mathcal{B}}$, Υ_1, and Υ_2 have been defined in Theorem 2. Then the desired estimator gain matrix $\mathcal{K}$ is determined by $\mathcal{K} = \mathcal{W}_2^{-1}\mathcal{X}$.

Remark 3 In this section, we have tackled the nonfragile state estimation issue of MRNNs with proportional delays under DETMs and sensor saturations. In Theorem 1, a delay-dependent criterion has been established to guarantee the scholastic asymptotic stability. Then, the gain matrix is obtained by solving the solutions of LMI (34) in Theorem 2.

Remark 4 So far, only a few methods have been utilized to study the dynamic analysis of MRNNs. The methods introduced in Refs. [4,26] utilize few variables but overlook the excitatory and inhibitory influence of $z_{ij}(\xi_j(t))$, leading to conservatism. In addition, the authors in Refs. [30,39] consider MRNNs as switched systems with finite subsystems, in which the number of subsystems can reach up to 2^{2n^2}. Considering the switched property of memristive weights $a_{ij}(\xi(t))$ and $b_{ij}(\xi(t))$, the switching function is employed to reconstruct the MRNNs (1) as an uncertain system. In contrast to the methods mentioned above, the robust analysis approach achieves a balance between conservatism and complexity.

4. Numerical example

To illustrate the effectiveness of the designed estimator, one numerical simulation is provided.

Example 1 Consider the following parameters of MRNNs (1) as

$$\begin{aligned}
&a_{11}(\xi_1) = \begin{cases} -0.4, & |\xi_1| \le 1, \\ -0.6, & |\xi_1| > 1, \end{cases} \quad a_{12}(\xi_1) = \begin{cases} 0.1, & |\xi_1| \le 1, \\ 0.2, & |\xi_1| > 1, \end{cases} \\
&a_{21}(\xi_2) = \begin{cases} 0.1, & |\xi_2| \le 1, \\ -0.2, & |\xi_2| > 1, \end{cases} \quad a_{22}(\xi_2) = \begin{cases} -0.3, & |\xi_2| \le 1, \\ -0.2, & |\xi_2| > 1, \end{cases} \\
&b_{11}(\xi_1) = \begin{cases} -0.3, & |\xi_1| \le 1, \\ -0.1, & |\xi_1| > 1, \end{cases} \quad b_{12}(\xi_1) = \begin{cases} -0.05, & |\xi_1| \le 1, \\ -0.15, & |\xi_1| > 1, \end{cases} \\
&b_{21}(\xi_2) = \begin{cases} -0.3, & |\xi_2| \le 1, \\ 0.1, & |\xi_2| > 1, \end{cases} \quad b_{22}(\xi_2) = \begin{cases} -0.4, & |\xi_2| \le 1, \\ -0.3, & |\xi_2| > 1, \end{cases} \\
&d_1(\xi_1) = \begin{cases} 0.8, & |\xi_1| \le 1, \\ 0.6, & |\xi_1| > 1, \end{cases} \quad d_2(\xi_2) = \begin{cases} 0.9, & |\xi_2| \le 1, \\ 0.7, & |\xi_2| > 1, \end{cases}
\end{aligned}$$

$$E = \begin{bmatrix} 0.5 & 0.4 \\ -0.3 & 0.1 \end{bmatrix}, \quad \mathcal{H}_k = [1;\ 1], \quad \mathcal{N}_k = [0.2\ 0.2].$$

Assume that the transmission delay $\nu = 0.1$, the PD factor $\rho_i = 0.4$ $(i = 1, 2)$. Moreover, we set the activation functions as

$$g(\xi(t)) = \begin{bmatrix} 0.4\xi_1(t) - \tanh(0.2\xi_1(\mathrm{t})) + 0.2\xi_2(\mathrm{t}) \\ 0.9\xi_2(t) + \tanh(0.7\xi_2(\mathrm{t})) \end{bmatrix},$$

which implies $\hat{G}_1 = \begin{bmatrix} 0.2 & 0.2 \\ 0 & 0.2 \end{bmatrix}$ and $\hat{G}_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0 & 0.9 \end{bmatrix}$.

Next, two cases are presented to illustrate the usefulness of the proposed method.

Case A Suppose that the cyber-attack is ignored, which means $\varepsilon(t) = 0$. Choose the sampling period $h = 0.1$. We have $\epsilon(0) = 0.36$, $\rho = 4$, $d = 0.15$. Solving the linear matrix inequalities (LMIs) (34) and (35), we can reach

$$\mathcal{W}_2 = 10^{-2} \times \begin{bmatrix} 2.46 & -1.37 \\ -1.37 & 3.38 \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} 1.2602 & -0.3911 \\ 1.3526 & 0.6403 \end{bmatrix},$$

and $\kappa_1 = 0.5201$, $\kappa_2 = 0.1940$, $u = 0.0122$.

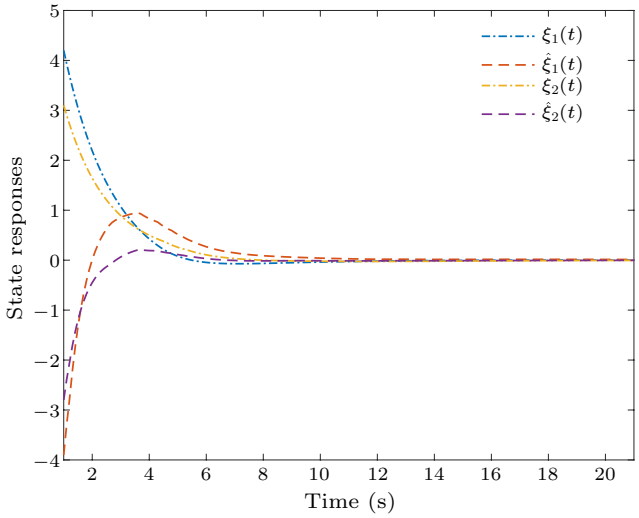


Fig. 3. Trajectories of $\xi_i(t)$ and $\hat{\xi}_i(t)$ with $\varepsilon(t) = 0$.

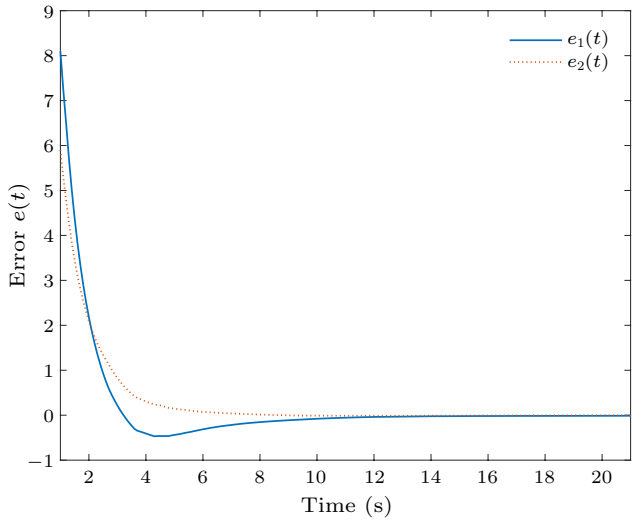


Fig. 4. Estimation errors $e_i(t)$ with $\varepsilon(t) = 0$.

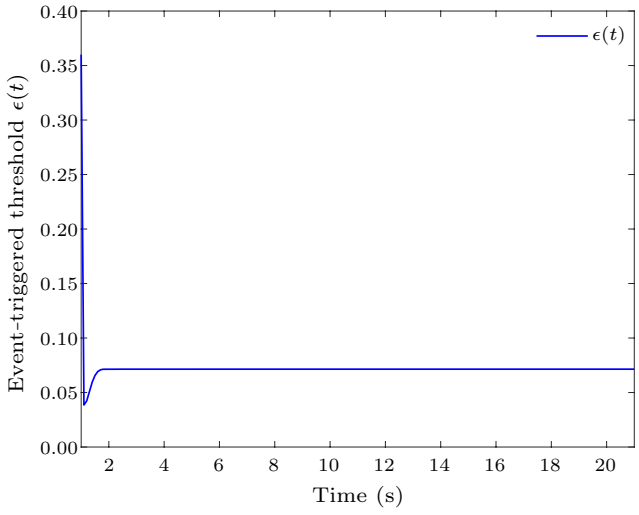


Fig. 5. Adaptive law $\epsilon(t)$ of the proposed DETM with $\varepsilon(t) = 0$.

In this case, the estimation system with a gain fluctuation under DETMs is taken into consideration. The initial parameters of system (1) are chosen as $\xi(0) = [4.2;\ 3.1]$, $\hat{\xi} = [-3.9; -2.8]$. Based on the above parameters, Figs. 3–6 plot all the simulation results. Figure 3 depicts the state of neurons and the corresponding estimates. Figure 4 plots the estimation errors between $\xi_i(t)$ and $\hat{\xi}_i(t)$. Figure 5 shows the trajectory of adaptive law $\epsilon(t)$ with $\epsilon(0) = 0.36$. As can be seen from Fig. 5, $\epsilon(t)$ converges to a constant gradually. In Fig. 6, the results of release instants and release interval under DETMs are described. In Fig. 4, only 33 of the $20/h = 200$ sampled data are transmitted to the nonfragile estimator. That is to say, 16.5% of the measured data is chosen under the DETMs. It is obvious that $v_1(t), v_2(t)$ converge to zeros finally which verify our theoretical analysis.

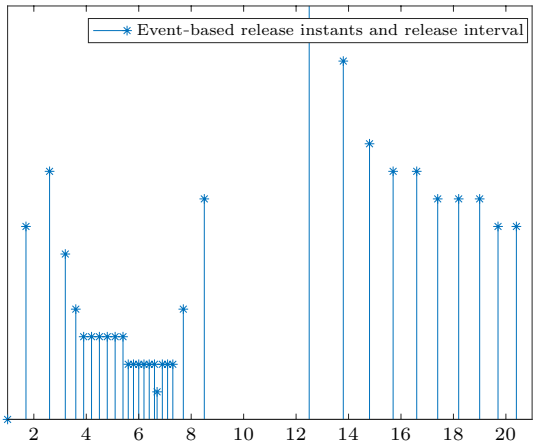


Fig. 6. Release instants with adaptive threshold $\epsilon(t)$ in case A.

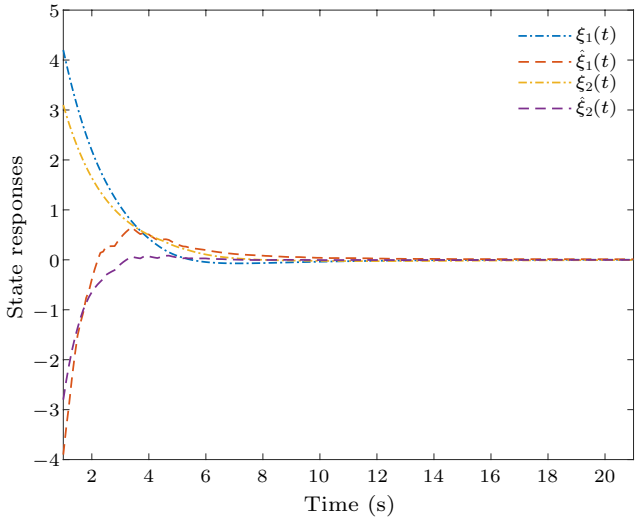


Fig. 7. Trajectories of $\xi_i(t)$ and $\hat{\xi}_i(t)$ with cyber-attacks.

Case B We take the influence of cyber attacks into account and suppose the function $\hbar(\xi(t)) = \begin{bmatrix} \tanh(0.02\xi_1(t)) \\ \tanh(0.1\xi_2(t)) \end{bmatrix}$. Under Assumption 3, we choose $\mathcal{G} = \text{diag}\{0.02,\ 0.1\}$. The sampling period $h = 0.1$ s. the initial condition of adaptive law $\epsilon(0) = 0.36$, $\rho = 4$, $d = 0.15$. Solving the LMIs (34) and (35), we have the following feasible solutions:

$$\mathcal{W}_2 = \begin{bmatrix} 1.3396 & -0.0814 \\ -0.0814 & 0.8253 \end{bmatrix},\quad \mathcal{K} = \begin{bmatrix} -0.0106 & -0.3293 \\ 0.2459 & -0.1053 \end{bmatrix},$$

and $\kappa_1 = 11.1938$, $\kappa_2 = 5.2842$, $u = 0.31111$.

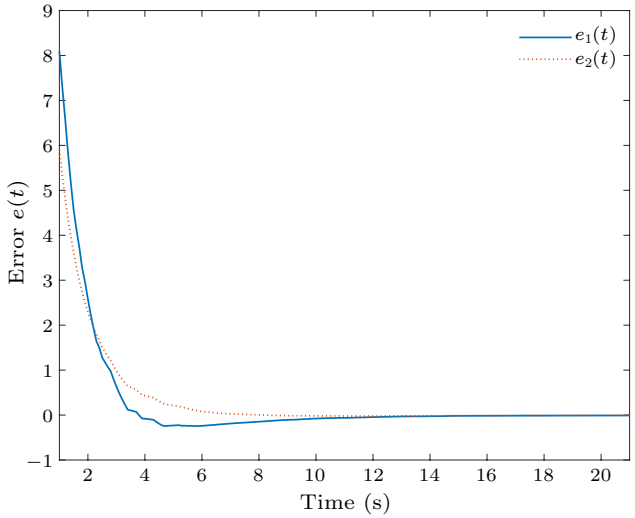


Fig. 8. Estimation errors $e_i(t)$ under DETM.

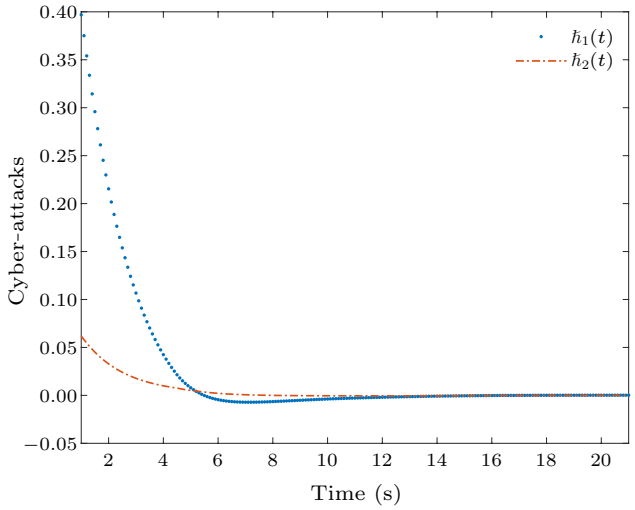


Fig. 9. Graph of $\hbar_i(t)$, $i = 1, 2$.

In this simulation, both the static ETMs and DETMs are taken into consideration. Choose the probability of cyber-attacks to be $\bar{\varepsilon} = 0.2$. Based on the above parameters, Figs. 7–11 plot all the simulation results in detail. Figure 7 depicts the state of neurons and corresponding estimates. Figure 8 plots the estimation errors between $\xi_i(t)$ and $\hat{\xi}_i(t)$ with cyber-attacks. It is obvious that the errors $\upsilon_i(t)$ converge to 0, implying that system (17) exhibits asymptotic stability ($i = 1, 2$). Figure 9 shows the graph of cyber-attacks. The adaptive law $\epsilon(t)$ is shown in Fig. 10. In Fig. 11, the results of release instants under static ETMs and DETMs are described. Under the static ETMs and DETMs, the measured data transmitted to the designed estimator is 16% and 11%, respectively. We could find that the amount of signal data transmitted to the designed estimator through the network is reduced obviously than the static ETMs in Ref. [20]. Therefore, the validity of the proposed theoretical results is verified in this numerical simulation.

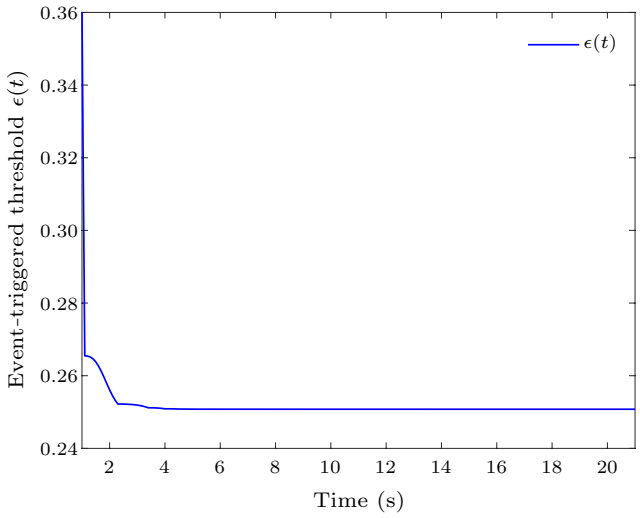


Fig. 10. Adaptive law $\epsilon(t)$ of the proposed DETM.

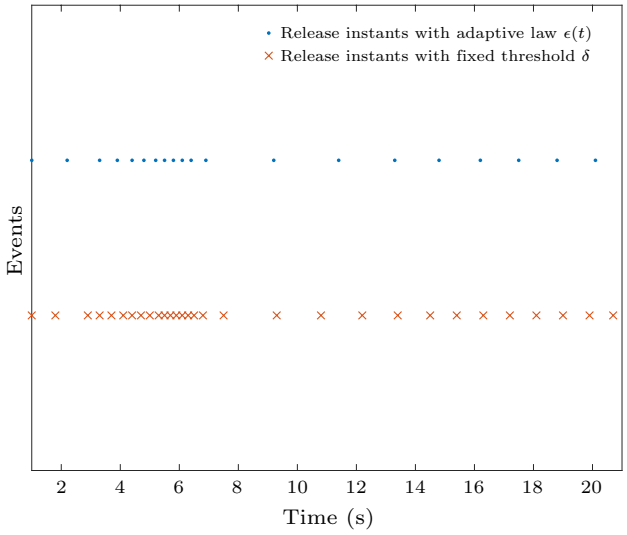


Fig. 11. Release instants with adaptive threshold $\epsilon(t)$ and fixed threshold in Ref. [20].

5. Conclusion

We have investigated the nonfragile SE for MRNNs with proportional delay subject to sensor saturations. In order to reduce the load of communication networks, a DETM is employed to select the meaningful signals. By constructing a novel LKF, a delay-dependent criterion is established under which the augmented system is scholastically asymptotic stability. The effectiveness of the approved method in this study is illustrated by a numerical example. In addition, it is possible to extend our method and scheme to uncertain systems or complex-valued NNs. Hybrid delays[40] and attacks[23] could be taken into account to reflect the complex environment.

References

[1] Yang C, Peng G, Li Y, *et al.* 2008 *IEEE Trans. Cybernet.* **49** 2568
[2] Vecsei P M, Choo K, Chang J, *et al.* 2019 *Phys. Rev. B* **99** 245120
[3] Qin C, Schlemper J, Caballero J, *et al.* 2019 *IEEE Trans. Med. Imaging* **18** 280

[4] Strukov D B, Snider G S, Stewart D R, *et al.* 2022 *Chin. Phys. B* **31** 100201
[5] Qiu S B, Liu X G, Wang F X, *et al.* 2018 *Neural Comput. Appl.* **30** 211
[6] Nagamani G, Adhira B and Soundararajan G 2021 *Nonlinear Dynam.* **104** 451
[7] Chua L 1971 *Memristor — The missing circuit element, circuit theory* (New York: IEEE) p. 507
[8] Guo T, Sun B, Ranjan S, *et al.* 2020 *ACS Appl. Mater. Interfaces* **12** 54243
[9] Tsafack N, Iliyasu A M, De Dieu N J, *et al.* 2021 *J. Inf. Secur. Appl.* **61** 2214
[10] Duan S, Hu X, Dong Z, *et al.* 2015 *IEEE Trans. Neural Netw. Learn. Syst.* **13** 68
[11] Strukov D B, Snider G S, Stewart D R, *et al.* 2008 *Nature* **453** 80
[12] Zhang X M, Han Q L and Ge X 2019 *Inform. Sci.* **478** 83
[13] Li J, Dong H, Wang Z, *et al.* 2020 *IEEE Trans. Neural Netw. Learning* **31** 3747
[14] Wang Z, Ho D W C and Liu X 2005 *IEEE Trans. Neural Netw. Learning* **16** 279
[15] Lee S C M, Lui J C S and Yau D K Y 2004 *IEEE Trans. Parall. Distr.* **15** 385
[16] Tang Z, Park J H and Wang Y 2019 *IEEE Trans. Cybernet.* **49** 3105
[17] Zhang Y and Zhou L 2022 *Neural Comput. Appl.* **34** 2193
[18] Zhou L 2018 *Neurocomputing* **308** 235
[19] Mu C, Liao K and Wang K 2021 *Nonlinear Dynam.* **103** 2645
[20] Zha L, Fang J, Liu J, *et al.* 2018 *Neurocomputing* **273** 1
[21] Wang F and Yang Y 2017 *Int. J. Syst. Sci.* **48** 571
[22] Huong D C and Trinh H 2022 *Nature* **32** 6267
[23] Deng Y, Mo Z and Lu H 2022 *Chin. Phys. B* **31** 020503
[24] Liu G, Park J H, Xu S, *et al.* 2019 *Nonlinear Anal-Hybrid Systems* **32** 65
[25] Liu J, Xia J, Cao J, *et al.* 2018 *Neurocomputing* **291** 35
[26] Wang X, Park J H, Liu Z, *et al.* 2022 *IEEE Trans. Neural Netw. Learning* **45** 1
[27] Filippov A F 1988 1960 *Matematicheskii Sbornik* **1** 99
[28] Aubin J P and Frankowska H 2009 *Set-valued analysis* (Berlin: Springer)
[29] Zhang R, Zeng D, Zhong S, *et al.* 2017 *Appl. Math. Comput.* **310** 57
[30] Wang Z, Ding S, Huang Z and Zhang H 2016 *IEEE Trans. Neural Netw. Learn. Syst.* **27** 2337
[31] Gu Z, Yue D, Liu J, *et al.* 2017 *J. Franklin Inst.* **354** 3540
[32] Tan Y, Xiong M, Zhang B, *et al.* 2022 *IEEE Trans. Syst. Man Cybernet. A* **52** 2121
[33] Chen X, Yin L Y, Liu Y T, *et al.* 2019 *Chin. Phys. B* **28** 090701
[34] Hu S and Yue D 2012 *ISA Trans.* **51** 153
[35] Boyd S, El Ghaoui L, Feron E, *et al.* 1994 *Linear matrix inequalities in system and control theory* (Philadelphia: SIAM) p. 106
[36] Park P G, Ko J W and Jeong C 2011 *Automatica* **47** 235
[37] Sakthivel R, Anbuvithya R, Mathiyalagan K, *et al.* 2015 *Neurocomputing* **168** 1111
[38] Yang F and Li Y 2009 *Automatica* **45** 1896
[39] Wen S, Huang T, Zeng Z, *et al.* 2015 *Neural Netw.* **63** 48
[40] Qian W, Liu H, Zhao Y, *et al.* 2022 *Appl. Math. Comput.* **424** 127016