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Effects of quantum noise on teleportation of arbitrary two-qubit state via five-particle Brown state

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We propose a new protocol for quantum teleportation (QT) which adopts the Brown state as the quantum channel. This work focuses on the teleportation of a single unknown two-qubit state via a Brown state channel in an ideal environment. To validate the effectiveness of our proposed scheme, we conduct experiments by using the quantum circuit simulator Quirk. Furthermore, we investigate the effects of four noisy channels, namely, the phase damping noise, the bit-flip noise, the amplitude damping noise, and the phase-flip noise. Notably, we employ Monte Carlo simulation to elucidate the fidelity density under various noise parameters. Our analysis demonstrates that the fidelity of the protocol in a noisy environment is influenced significantly by the amplitude of the initial state and the noise factor.

Keywords: quantum communication, Brown state, fidelity

PACS: 03.67.Hk, 03.67.Dd, 42.50.Lc, 03.67.–a **DOI:** 10.1088/1674-1056/ad3dc8

1. Introduction

Quantum teleportation (QT) is pivotal part in quantum communication, and can transfer unknown quantum states from one place to another through the classical operations and quantum entanglement.[1] Quantum entanglement is widely recognized as one of the fundamental phenomena in the field of quantum information processing, with significant applications in diverse areas, including quantum computation,[2] dense coding,[3] quantum key distribution,[4] and entanglement-based cryptography.[5] Since Bennett *et al.*[6] proposed the original QT scheme in 1993, it has received considerable attention in recent years. Numerous teleportation protocols have been developed based on different entangled states, including the Bell state,[7] W state,[8,9] Brown state,[10] and others.[11,12] In these protocols, the sender and the receiver share a pair of pre-established Einstein–Podolski–Rosen (EPR[13]) states, which allows the receiver to faithfully recover the original state by employing classical channels and local operations.

In 1997, Bouwmeester *et al.*[14] successfully implemented the first experimental QT, marking a significant milestone in the field of quantum mechanics. In 2005, Brown *et al.*[15] utilized the numerical optimization program to successfully discover a five-qubit quantum state which is recognized as the Brown state. Through negative partial transpose and von Neumann entropy measurements, this state exhibited genuine multipartite entanglement. In 2007, Muralidharan and Panigrahi[16] introduced one generalized multi-qubit Brown state which presents a high degree of quantum entanglement. Hence, Brown state attracted widespread attention due to its significant potential in QT and remote state preparation protocols. In 2012, Chen *et al.*[17] proposed a new protocol for remote state preparation by using the Brown state as a quantum channel. Before this, the coefficients of the prepared state were mostly real and relatively simple. This new protocol reduced the restrictions on the coefficients of the prepared state and provided a detailed security analysis. In 2017, Fang and Jiang[18] introduced a new protocol for deterministic controlled bidirectional quantum information transmission based on a five-bit Brown state which enables the mutual transmission of state information, whether known or unknown, under the control of a third party through a 5-bit Brown state. In 2018, Dong and Ma[19] studied a scheme for preparing arbitrary two-qubit states under noisy conditions based on the Brown state. This was the first time that the noise had been introduced into protocols for the state preparation by using a Brown state channel, and the results indicate that the fidelity of the prepared state depends on the coefficients of the prepared state and the noise parameters. In 2020, Wang and Ma[20] proposed a hierarchical controlled quantum state preparation scheme based on the Brown state channel. Compared with the previous protocols for hierarchical controlled remote preparation of arbitrary single-qubit states, this scheme utilized the Brown state as the entanglement resource and required the consent of at least two other receivers for the higher-level receiver, which enhances the security.

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In the context of QT, noise inevitably arises as a crucial factor that must be carefully and thoroughly considered. When distributing the entangled channel among multiple participants, the qubits comprising it become susceptible to quantum noise. This influence significantly undermines the fidelity[21] of the output state in QT protocols. Hence, it becomes imperative to conduct a comprehensive analysis of the influence of noise in order to optimize the performance of the QT scheme. In 2001, Banaszek[22] investigated the fidelity of QT with non-maximally entangled states. Oh *et al.*[23] investigated QT through noisy quantum channels by solving analytically and numerically a master equation in the Lindblad form in 2018. Raphael and Rigolin[24] investigated how the noise or decoherence on individual qubits affects the effectiveness of QT protocol in the whole the transmission process. In 2020, He *et al.*[25] investigated the teleportation of an arbitrary single-qubit state in noisy environments. In 2021, Seida *et al.*[26] studied the bidirectional QT of single-qubit states, which explores a scenario where qubits from the quantum channel of teleportation are distributed through commonly encountered correlated noisy channels. Although there are some QT schemes of single-qubit states with Brown state. However, there has been no proposal for QT of two-qubit state based on Brown state under the noisy environment.

In this study, we investigate the QT protocol by utilizing the Brown state channel, considering both ideal environmental scenario and noisy environmental scenario. The rest of this paper is organized as follows. In Section 2, we present the QT method that employs the Brown state channel under ideal environmental conditions. To assess the feasibility and the effectiveness of our proposed QT method, we conduct simulations by using the Quirk platform. In Section 3, we explore the QT method based on the Brown state channel in the presence of noise. We analyze the influence of noise on the performance of the QT protocol. Finally, in Section 4, we draw a concise conclusion summarizing the key outcomes of our study.

2. QT scheme in ideal environment

2.1. Fundamental principle of QT in ideal environment

In this subsection, we present a QT protocol for transmitting an arbitrary two-qubit state, which employs the Brown state as the quantum channel in the ideal environment.

$$|\chi\rangle_{jk} = (k_1|00\rangle + k_2|01\rangle + k_3|10\rangle + k_4|11\rangle)_{jk}, \tag{1}$$

where k_1, k_2, k_3, and k_4 are complex numbers, and satisfy the relationship $|k_1^2| + |k_2^2| + |k_3^2| + |k_4^2| = 1$. The Brown state channel can be written as

$$\begin{aligned}|\mathrm{Br}\rangle_{12345} = \frac{1}{2}\big(&|001\rangle|\psi^-\rangle + |010\rangle|\phi^-\rangle \\ &+ |100\rangle|\psi^+\rangle + |111\rangle|\phi^+\rangle\big),\end{aligned} \tag{2}$$

where $|\phi^\pm\rangle$ and $|\psi^\pm\rangle$ are Bell states given by

$$\begin{aligned}|\phi^\pm\rangle &= \frac{1}{\sqrt{2}}\big(|00\rangle \pm |11\rangle\big), \\ |\psi^\pm\rangle &= \frac{1}{\sqrt{2}}\big(|01\rangle \pm |10\rangle\big).\end{aligned} \tag{3}$$

Three participants share the Brown state. Particles 2 and 4 are in the sender Alice's possession. Particles 3 and 5 belong to the receiver Bob. Particle 1 is owned by the controller Charlie who plays the role of the controller in this scheme, acting as a supervisor. If he agrees, Alice and Bob can proceed with the operation. Otherwise, they cannot perform the operations. Additionally, he must act as a trusted third party. If the measurement result he provided is incorrect, the operation cannot be carried out. By requiring the consent of a third party, the scheme provides an additional layer of security. The role of the controller ensures that the teleportation process is conducted correctly and securely.

The state of the whole system can be expressed as follows:

$$|\mathrm{sys}\rangle_{jk12345} = |\chi\rangle_{jk} \otimes |\mathrm{Br}\rangle_{12345}, \tag{4}$$

$$|\mathrm{sys}\rangle_{j2k4135} = \frac{1}{\sqrt{2}}\begin{pmatrix} k_1\begin{pmatrix}|0000011\rangle - |0001010\rangle + |0100000\rangle - |0101001\rangle \\ + |0000101\rangle + |0001100\rangle + |0100110\rangle + |0101111\rangle\end{pmatrix} \\ +k_2\begin{pmatrix}|0010011\rangle - |0011010\rangle + |0110000\rangle - |0111001\rangle \\ + |0010101\rangle + |0011100\rangle + |0110110\rangle + |0111111\rangle\end{pmatrix} \\ +k_3\begin{pmatrix}|1000011\rangle - |1001010\rangle + |1100000\rangle - |1101001\rangle \\ + |1000101\rangle + |1001100\rangle + |1100110\rangle + |1101111\rangle\end{pmatrix} \\ +k_4\begin{pmatrix}|1010011\rangle - |1011010\rangle + |1110000\rangle - |1111001\rangle \\ + |1010101\rangle + |1011100\rangle + |1110110\rangle + |1111111\rangle\end{pmatrix}\end{pmatrix}_{j2k4135}. \tag{5}$$

The protocol for QT of a two-qubit state using the Brown state under ideal environmental conditions is depicted in Fig. 1. First, Alice performs a Bell state measurement onto the two-qubit pairs (j, 2) and (k, 4), respectively. At the same time, Bob executes a CNOT gate operation onto the particle 3 (marked as the control qubit) and particle 5 (marked as the target qubit), where $\mathrm{CNOT} = |0\rangle_3\langle 0| \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|)_5 + |1\rangle_3\langle 1| \otimes (|0\rangle\langle 1| + |1\rangle\langle 0|)_5$. Then Charlie performs a single-

qubit measurement with the basis of $|0\rangle$ and $|1\rangle$ on particle 1 and tells the measurement outcome to the receiver Bob. After the Bell state measurements are accomplished by Alice, the state of the remaining qubits collapses into the following state

$$\begin{aligned}
&\mathrm{CNOT}_{35\,j2}\left\langle\Phi^{+}\right|_{k4}\left\langle\Phi^{+}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|10\rangle-k_2|11\rangle+k_3|00\rangle-k_4|01\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|01\rangle+k_2|00\rangle+k_3|11\rangle+k_4|10\rangle)_{35}\big), \quad (6)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Phi^{+}\right|_{k4}\left\langle\Phi^{-}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|10\rangle+k_2|11\rangle+k_3|00\rangle+k_4|01\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|01\rangle-k_2|00\rangle+k_3|11\rangle-k_4|10\rangle)_{35}\big), \quad (7)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Phi^{+}\right|_{k4}\left\langle\Psi^{+}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(-k_1|11\rangle+k_2|10\rangle-k_3|01\rangle+k_4|00\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|00\rangle+k_2|01\rangle+k_3|10\rangle+k_4|11\rangle)_{35}\big), \quad (8)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Phi^{+}\right|_{k4}\left\langle\Psi^{-}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(-k_1|11\rangle-k_2|10\rangle-k_3|01\rangle-k_4|00\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|00\rangle-k_2|01\rangle+k_3|10\rangle-k_4|11\rangle)_{35}\big), \quad (9)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Phi^{-}\right|_{k4}\left\langle\Phi^{+}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|10\rangle-k_2|11\rangle-k_3|00\rangle+k_4|01\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|01\rangle+k_2|00\rangle-k_3|11\rangle-k_4|10\rangle)_{35}\big), \quad (10)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Phi^{-}\right|_{k4}\left\langle\Phi^{-}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|10\rangle+k_2|11\rangle-k_3|00\rangle-k_3|01\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|01\rangle-k_2|00\rangle-k_3|11\rangle+k_4|10\rangle)_{35}\big), \quad (11)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Phi^{-}\right|_{k4}\left\langle\Psi^{+}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(-k_1|11\rangle+k_2|10\rangle+k_3|01\rangle-k_4|00\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|00\rangle+k_2|01\rangle-k_3|10\rangle-k_4|11\rangle)_{35}\big), \quad (12)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Phi^{-}\right|_{k4}\left\langle\Psi^{-}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(-k_1|11\rangle-k_2|10\rangle+k_3|01\rangle+k_4|00\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|00\rangle-k_2|01\rangle-k_3|10\rangle+k_4|11\rangle)_{35}\big), \quad (13)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Psi^{+}\right|_{k4}\left\langle\Phi^{+}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|00\rangle-k_2|01\rangle+k_3|10\rangle-k_4|11\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|11\rangle+k_2|10\rangle+k_3|01\rangle+k_4|00\rangle)_{35}\big), \quad (14)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Psi^{+}\right|_{k4}\left\langle\Phi^{-}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|00\rangle+k_2|01\rangle+k_3|10\rangle+k_4|11\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|11\rangle-k_2|10\rangle+k_3|01\rangle-k_4|00\rangle)_{35}\big), \quad (15)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Psi^{+}\right|_{k4}\left\langle\Psi^{+}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(-k_1|01\rangle+k_2|00\rangle-k_3|11\rangle+k_4|10\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|10\rangle+k_2|11\rangle+k_3|00\rangle+k_4|01\rangle)_{35}\big), \quad (16)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Psi^{+}\right|_{k4}\left\langle\Psi^{-}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(-k_1|01\rangle-k_2|00\rangle-k_3|11\rangle-k_4|10\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|10\rangle-k_2|11\rangle+k_3|00\rangle-k_4|01\rangle)_{35}\big), \quad (17)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Psi^{-}\right|_{k4}\left\langle\Phi^{+}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|00\rangle-k_2|01\rangle-k_3|10\rangle+k_4|11\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|11\rangle+k_2|10\rangle-k_3|01\rangle-k_4|00\rangle)_{35}\big), \quad (18)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Psi^{-}\right|_{k4}\left\langle\Phi^{-}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|00\rangle+k_2|01\rangle-k_3|10\rangle-k_4|11\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|11\rangle-k_2|10\rangle-k_3|01\rangle+k_4|00\rangle)_{35}\big), \quad (19)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Psi^{-}\right|_{k4}\left\langle\Psi^{+}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(-k_1|01\rangle+k_2|00\rangle+k_3|11\rangle-k_4|10\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|10\rangle+k_2|11\rangle-k_3|00\rangle-k_4|01\rangle)_{35}\big), \quad (20)\\
&\mathrm{CNOT}_{35\,j2}\left\langle\Psi^{-}\right|_{k4}\left\langle\Psi^{-}\right|\mathrm{sys}\rangle\\
&=\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(-k_1|01\rangle-k_2|00\rangle+k_3|11\rangle+k_4|10\rangle)_{35}\\
&\quad+|1\rangle_1\otimes(k_1|10\rangle-k_2|11\rangle-k_3|00\rangle+k_4|01\rangle)_{35}\big). \quad (21)
\end{aligned}$$

The 16 outcomes are the Bell state measurements performed by Alice, which are necessary for Bob to reconstruct the original two-qubit state. Then Charlie performs a single-qubit measurement with the basis of $|0\rangle$ and $|1\rangle$ on particle 1 and tells the measurement results to the receiver Bob. It should be noted that Alice must perform two Bell measurements, while Bob conducts a single-qubit measurement. Additionally, they need to publish their measurement results. As a result, the quantum measurement resource consumes 5 bits.

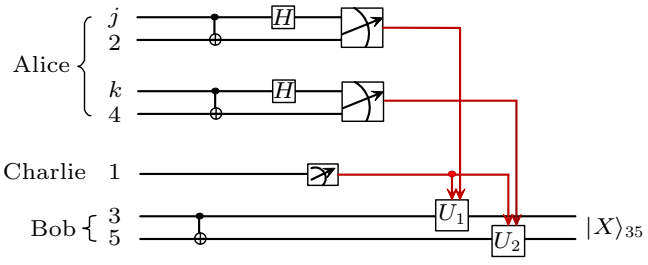


Fig. 1. QT based on the Brown state channel.

Table 1. Relationships among measurement results of particles j, 2, and 1 and the corresponding operations of particle 3.

Bell measurement results of particles j and 2	Measurement results of particles 1	Bob's operation
$\|\Phi^+\rangle$	$\|0\rangle$	X
	$\|1\rangle$	I
$\|\Phi^-\rangle$	$\|0\rangle$	$X\otimes Z$
	$\|1\rangle$	Z
$\|\Psi^+\rangle$	$\|0\rangle$	I
	$\|1\rangle$	X
$\|\Psi^-\rangle$	$\|0\rangle$	Z
	$\|1\rangle$	$X\otimes Z$

Table 2. Relationships among measurement results of particles k, 4, and 1 and the corresponding operations of particle 5.

Bell measurement results of particles k and 4	Measurement results of particles 1	Bob's operation
$\|\Phi^+\rangle$	$\|0\rangle$	Z
	$\|1\rangle$	X
$\|\Phi^-\rangle$	$\|0\rangle$	I
	$\|1\rangle$	$X\otimes Z$
$\|\Psi^+\rangle$	$\|0\rangle$	$Z\otimes X$
	$\|1\rangle$	I
$\|\Psi^-\rangle$	$\|0\rangle$	$Z\otimes X\otimes Z$
	$\|1\rangle$	Z

To recover the two-qubit state, depending on the measurement results from Alice and Charlie, the receiver Bob will perform two unitary operations onto particles 3 and 5 respectively. The relationship between the measurement results of the Bell state for particle pairs (j, 2) and the measurement result for particle 1, along with the corresponding operations on particle 3, is illustrated in Table 1. Similarly, Table 2 demonstrates the relationship between the measurement outcomes of the Bell state for particle pairs (k, 4) and the measurement result for particle 1, along with the corresponding operations on particle 5. For instance, if the collapsed state is

$$\frac{1}{4\sqrt{2}}\big(|0\rangle_1\otimes(k_1|10\rangle-k_2|11\rangle+k_3|00\rangle-k_4|01\rangle)_{35}$$
$$+|1\rangle_1\otimes(k_1|01\rangle+k_2|00\rangle+k_3|11\rangle+k_4|10\rangle)_{35}\big)$$

and the measurement result of particle 1 is $|0\rangle$, Bob needs to perform X operation onto particles 3, and then performs Z operation onto particles 5.

2.2. Simulation experiment on QT in ideal environment

Quirk[27] is a quantum circuit simulator that allows users to visualize and simulate quantum circuits. It uses a state vector approach to accurately simulate circuits of up to 16 qubits. We use Quirk to verify our scheme without considering the influence of noise. To prepare a Brown state in the simulator, we use three H gates and seven CNOT gates as shown in Fig. 2

To illustrate the experimental process, we generate a Brown state. Figure 3 shows our experimental circuit using this Brown state to transmit an arbitrary two-qubit state, which is used for specific verification.

The circuit we simulate is shown in Fig. 3 by using Quirk, in which the particle pairs (j, 2) and (k, 4) are $|\Phi^+\rangle$ and $|\Phi^+\rangle$, respectively. The measurement outcome of the particle 1 is $|0\rangle$. Through specific X and Z operating on particles 3 and 5, we can then recover the arbitrary two-qubit state $|\chi\rangle_{jk}$ on particles 3 and 5. This simulation demonstrates the feasibility of the proposed scheme.

This part describes the quantum teleportation of an unknown two-qubit state by using the Brown state channel in the ideal environment, where the Brown state is shared among Alice, Charlie, and Bob. However, in practical applications, quantum noise is the unavoidable factor that can affect the fidelity of QT. In the next section, we proceed to examine how the fidelity of the output state is affected in the teleportation process under various types of quantum noise when employing the Brown state channel.

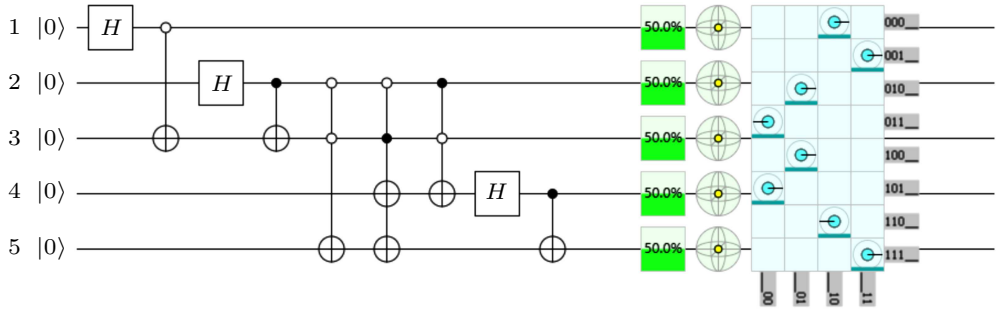


Fig. 2. Quantum circuit of generating one five-qubit Brown state.

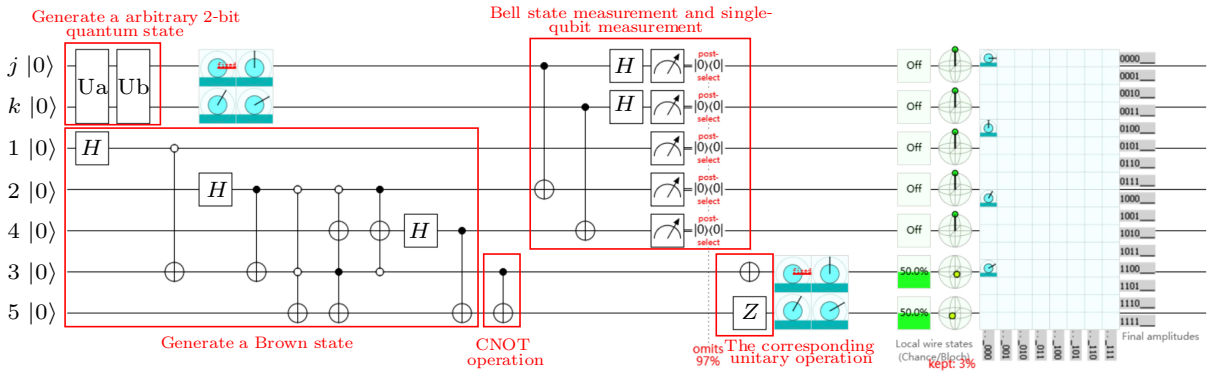


Fig. 3. Verification experimental circuit for transmitting arbitrary two-qubit state using one Brown state.

3. QT scheme in noisy environment

In this part we investigate the effects of quantum noise on the QT scheme. In the presence of noise, an initially pure input state undergoes a transformation and becomes a mixed state. When Alice transmits particles 3 and 5 to Bob with the Brown state channel, it is important to consider the presence of quantum noise that may affect the integrity of the channel. In our scheme, we will examine the effects of four types of noise: the amplitude damping, the phase damping, the bit-flip, and the phase-flip. We will then proceed to calculate and analyze the fidelities of the output states.

3.1. Amplitude damping noise

The amplitude-damping noise can be characterized as the process of dissipating energy from a quantum system, resulting in energy loss effects. The Kraus operators representing the amplitude damping noise can be formulated as follows:

$$E_0^{\mathrm{AD}}=\begin{bmatrix}1 & 0\\ 0 & \sqrt{1-\lambda_{\mathrm{AD}}}\end{bmatrix},\quad E_1^{\mathrm{AD}}=\begin{bmatrix}0 & \sqrt{\lambda_{\mathrm{AD}}}\\ 0 & 0\end{bmatrix}, \tag{22}$$

where $(0\leq\lambda_{\mathrm{AD}}\leq 1)$ indicates the noise factor.

3.2. Phase damping noise

The phase-damping noise causes the loss of quantum information without energy dissipation. It specifically affects the phase of quantum states. The Kraus operators representing the phase-damping noise can be written as follows:

$$\begin{aligned}E_0^{\mathrm{PD}}&=\sqrt{1-\lambda_{\mathrm{PD}}}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\\ E_1^{\mathrm{PD}}&=\sqrt{\lambda_{\mathrm{PD}}}\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\\ E_2^{\mathrm{PD}}&=\sqrt{\lambda_{\mathrm{PD}}}\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix},\end{aligned} \tag{23}$$

where $(0\leq\lambda_{\mathrm{PD}}\leq 1)$ indicates the noise factor.

3.3. Pauli (bit-flip, phase-flip) noise

The bit-flip noise makes state $|1\rangle$ turn into $|0\rangle$ or $|0\rangle$ turn into $|1\rangle$ with probability λ_{BF}. The phase-flip noise makes state $|1\rangle$ turn into $-|1\rangle$ with the probability λ_{PF}. Their Kraus operators are respectively given as

$$E_0^{\mathrm{BF}}=\sqrt{1-\lambda_{\mathrm{BF}}}I,\quad E_1^{\mathrm{BF}}=\sqrt{\lambda_{\mathrm{BF}}}\sigma_X, \tag{24}$$

$$E_0^{\mathrm{PF}}=\sqrt{1-\lambda_{\mathrm{PF}}}I,\quad E_1^{\mathrm{PF}}=\sqrt{\lambda_{\mathrm{PF}}}\sigma_Z, \tag{25}$$

where λ_{BF} and λ_{PF} represent the noise factors, and their values are in the ranges: $(0\leq\lambda_{\mathrm{BF}}\leq 1)$ and $(0\leq\lambda_{\mathrm{PF}}\leq 1)$, σ_X and σ_Z denote Pauli operations.

3.4. Effect of noise on QT

In the domain of quantum information and quantum communication theory the fidelity serves as a metric for evaluating the similarity or closeness between the two quantum states. In our study, we will employ the fidelity as a measure to assess the difference between the output state and the target state.

The operator form of the state to be teleported can be expressed as

$$\rho_{\mathrm{target}}=|\chi\rangle\langle\chi|. \tag{26}$$

Suppose that the Brown state channel is affected by the noise when distributing qubits to Bob. This effect can be written as

$$\rho\rightarrow\xi(\rho)=\sum_i\sum_j E_i^3E_j^5\rho E_i^{3+}E_j^{5+}, \tag{27}$$

where $(i,j)\in\{0,1\}$.

When a series of operations is completed by the participants, the fidelity of the resulting output state can be described as

$$F=\left\langle\chi|\rho_{\mathrm{out}}^{3,5}|\chi\right\rangle. \tag{28}$$

We can calculate the fidelity in the AD noise according to Eq. (28) from

$$\begin{aligned}F_{\mathrm{AD}}=&\frac{1}{2}\left(\lambda_{\mathrm{AD}}^2\left(a^2c^2+d^2b^2\right)\right)\\ &+\left(ad\sqrt{\lambda_{\mathrm{AD}}(1-\lambda_{\mathrm{AD}})}+bc\sqrt{\lambda_{\mathrm{AD}}}\right)^2\\ &+\frac{1}{2}\left(ab\sqrt{\lambda_{\mathrm{AD}}(1-\lambda_{\mathrm{AD}})}+cd\sqrt{\lambda_{\mathrm{AD}}}\right)^2\\ &+\frac{1}{2}\left(cd\sqrt{\lambda_{\mathrm{AD}}(1-\lambda_{\mathrm{AD}})}+ab\sqrt{\lambda_{\mathrm{AD}}}\right)^2\\ &+\frac{1}{2}\left(a^2(1-\lambda_{\mathrm{AD}})+b^2\sqrt{1-\lambda_{\mathrm{AD}}}+c^2+d^2\sqrt{1-\lambda_{\mathrm{AD}}}\right)^2\\ &+\frac{1}{2}\left(a^2\sqrt{1-\lambda_{\mathrm{AD}}}+b^2+c^2\sqrt{1-\lambda_{\mathrm{AD}}}+d^2(1-\lambda_{\mathrm{AD}})\right)^2.\end{aligned} \tag{29}$$

The fidelity in the PD noise is calculated from

$$\begin{aligned}F_{\mathrm{PD}}=&(1-\lambda_{\mathrm{PD}})^2\\ &+\lambda_{\mathrm{PD}}(1-\lambda_{\mathrm{PD}})\left(\left(a^2+b^2\right)^2+\left(b^2+c^2\right)^2\right.\\ &\left.+\left(c^2+d^2\right)^2+\left(d^2+a^2\right)^2\right)^2\\ &+\lambda_{\mathrm{PD}}^2\left(a^4+b^4+c^4+d^4\right).\end{aligned} \tag{30}$$

The fidelity in the BF noise is calculated from

$$\begin{aligned}F_{\mathrm{BF}}=&(1-\lambda_{\mathrm{BF}})^2+4\lambda_{\mathrm{BF}}(1-\lambda_{\mathrm{BF}})\\ &\times\left((ad+bc)^2+(ab+cd)^2\right)\\ &+4\lambda_{\mathrm{BF}}^2(ac+bd)^2.\end{aligned} \tag{31}$$

The fidelity in the PF noise is calculated from

$$\begin{aligned}F_{\mathrm{PF}}=&(1-\lambda_{\mathrm{PF}})^2+\lambda_{\mathrm{PF}}(1-\lambda_{\mathrm{PF}})\\ &\times\left(\left(a^2+b^2-c^2-d^2\right)^2+\left(a^2-b^2-c^2+d^2\right)^2\right)\\ &+\lambda_{\mathrm{PF}}^2\left(a^2-b^2+c^2-d^2\right)^2.\end{aligned} \tag{32}$$

3.5. Average fidelity of QT scheme

More specifically, we can write the arbitrary two-qubit state as

$$\begin{aligned}|\chi\rangle_{jk} = &\cos\frac{\theta_0}{2}|00\rangle + \sin\frac{\theta_0}{2}\cos\frac{\theta_1}{2}\mathrm{e}^{\mathrm{i}\phi_1}|01\rangle \\ &+ \sin\frac{\theta_0}{2}\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\mathrm{e}^{\mathrm{i}\phi_2}|10\rangle \\ &+ \sin\frac{\theta_0}{2}\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\mathrm{e}^{\mathrm{i}\phi_3}|11\rangle \end{aligned} \tag{33}$$

with $0 \le \theta_i \le \pi, 0 \le \phi_j \le 2\pi$ $(i = j = 1,2,3)$. Equation (1) can be converted into Eq. (33) since they only differ by a global phase with no physical significance. Considering the fact that each state in Eq. (1) can occur with varying probabilities, we can define

$$\begin{aligned}\langle F\rangle = &\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} F(\theta_0,\theta_1,\theta_2,\phi_1,\phi_2,\phi_3) \\ &\times \sin^5\theta_0\cos\theta_0\sin^3\theta_1\cos\theta_1 \\ &\times \sin\theta_2\cos\theta_2\,\mathrm{d}\theta_0\,\mathrm{d}\theta_1\,\mathrm{d}\theta_2\,\mathrm{d}\phi_1\,\mathrm{d}\phi_2\,\mathrm{d}\phi_3 \end{aligned} \tag{34}$$

as average fidelity according to Ref. [28]. We choose the Monte Carlo method instead of calculating it directly, since the integral of Eq. (34) is trivial. Figure 4 clearly shows the curves of average fidelity F_{av} with the Monte Carlo method.

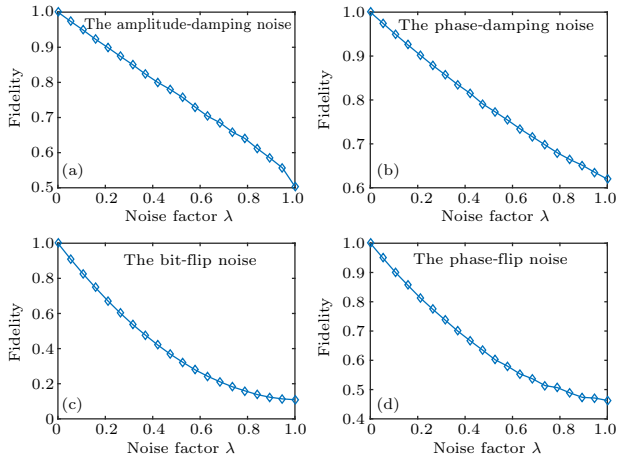


Fig. 4. Average fidelity curves in four-type noisy environment.

It can be seen that the average fidelity F constantly decreases with the noise parameter increasing in all kinds of noisy circumstances.

Here we employ Monte Carlo simulation to elucidate the fidelity density under various noise parameters. Specifically, we assume that the teleported states are independent of the noisy channels, which are randomly selected in variable space. The resulting fidelities are then calculated for specific values of the noise parameter, and the density of fidelity is contingent upon the resulting fidelity of the teleported states. Figure 5 visually represents this relationship, where the brightness of the color corresponds to higher values in the probabilistic distribution. This color intensity reflects the average fidelity depicted in Fig. 4. Notably, it is apparent that the performance is significantly affected by bit-flip noises.

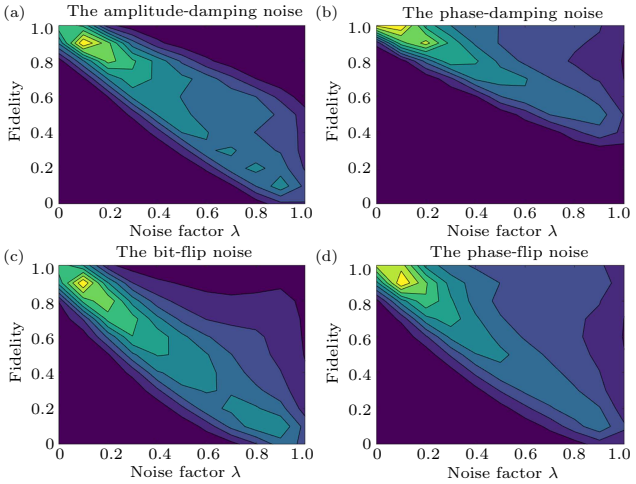


Fig. 5. Relationships between probability density of average fidelity (F) and noise factor magnitude $\lambda_{\mathrm{ad,pd,bf,pf}}$ under four different noise conditions, indicating that the brighter the color, the higher the probability density of the communication system's fidelity is.

3.6. Analysis

It can be found that the fidelities of QT are completely relevant to the noise parameter and the amplitude information according to Eqs. (29)–(32). Here we make a special example to compare the influences of four kinds of noise on fidelity. When the state to be transmitted is the maximally entangled state $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$, the fidelities in the four types of noisy environments are presented in Table 3. It can be noted that the fidelity of the teleportation process is entirely contingent upon the noise factor of the channel. Figure 6 clearly illustrates the effect of the noise factor of four-type noise on the fidelity F. It can be easily observed that the fidelity F always decreases with the noise factor increasing under the amplitude-damping, phase-damping, and phase-flip noises. It should be noted that regardless of how the noise parameter varies, the fidelity remains consistently at 1 under the bit-flip noise. According to the four fidelity curves obtained, it can be seen that the value of the noise factor has the most important effect on the fidelity under phase-flip noise. However, it is mentioned that in the special case when the quantum state that is transmitted is a maximally entangled state, it is most resistant to bit-flip noise.

Table 3. Fidelity in noisy environment when state to be teleported is the maximum entangled state.

Noise	Fidelity F^*
Amplitude-damping	$\frac{1}{16}\lambda_{\mathrm{AD}}^2 + \frac{1}{8}\left(\sqrt{\lambda_{\mathrm{AD}}(1-\lambda_{\mathrm{AD}})} + \sqrt{\lambda_{\mathrm{AD}}}\right)^2$ $+\frac{1}{16}\left(1+2\sqrt{1-\lambda_{\mathrm{AD}}}+(1-\lambda_{\mathrm{AD}})\right)^2$
Phase-damping	$\left(\frac{1}{2}\lambda_{\mathrm{PD}}-1\right)^2$
Bit-flip	1
Phase-flip	$(\lambda_{\mathrm{PF}}-1)^2$

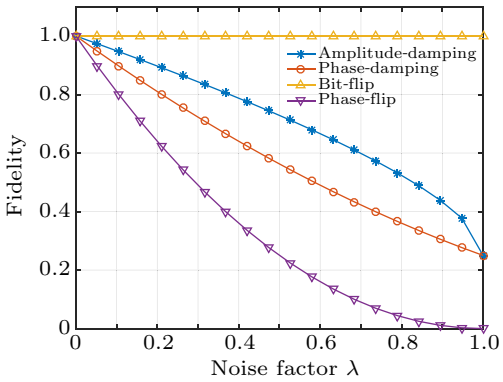


Fig. 6. Fidelity curves in four-type noisy environment when state to be teleported is the maximum entangled state.

4. Discussion and comparison

In recent years, several quantum communication schemes based on the Brown state have been reported. The differences between our scheme and these prior efforts are detailed in Table 4.

Our QT scheme contains a comprehensive noise analysis, covering phase damping, bit-flip, amplitude damping, and phase-flip noise. This enhances the robustness of the protocol for practical applications. The use of CNOT gates, Bell-state measurements, and single-qubit measurements achieves a balance between simplicity and the complexity needed for efficient quantum teleportation. Employing the Quirk simulator for feasibility and Monte Carlo simulations to evaluate fidelity across noise spectra showcases the protocol's resilience to noise and efficacy in maintaining fidelity, making it a substantial improvement over previous approaches. Of articular note is our detailed analysis of the protocol's performance for teleportation of maximum entangled states, which are essentially more resistant to bit-flip noise, further emphasizing the advantages of our QT scheme.

Table 4. Differences between previous schemes with our scheme.

	Quantum communication method	Noise types considered	Quantum channel	Implementation complexity	Number of qubits
Scheme 1[19]	controlled remote preparation	bit flip, phase flip, amplitude damping, phase damping	five-qubit Brown state	projective measurements, unitary operations	6
Scheme 2[18]	bidirectional QT	not mentioned	five-qubit Brown state	CNOT gate, Bell-state measurement, single-qubit measurements	7
Scheme 3[29]	QT	not mentioned	generalized seven-qubit Brown state	Von Neumann measurement	8
Our scheme	QT	bit flip, phase flip, amplitude damping, phase damping	five-qubit Brown state	CNOT gates, Bell-state measurements, single-qubit measurements	7

5. Conclusion

We present a QT scheme for transmitting an unknown two-qubit state which employs a five-qubit Brown state as the quantum channel. Firstly, we provide a detailed description for the specific implementation of our proposed scheme under the ideal environment condition. To validate the effectiveness of our proposed scheme, we conduct experiments using the quantum circuit simulator Quirk. Furthermore, we introduce four classic types of noises: the amplitude damping noise, the phase damping noise, the bit-flip noise, and the phase-flip noise. We then discuss and analyze the fidelity results under the influence of each noise. The comprehensive analysis of different noise types in the work opens up the prospect for further research aimed at improving the anti-noise robustness of the system, which may bring more practical and reliable QT protocols to the quantum communication systems in the real world.

We notice that the teleportation fidelity of the unknown arbitrary two-qubit state is greatly influenced by the existence of noise channels. As the noise parameter increases, the teleportation fidelity decreases accordingly. When the state to be transmitted is the maximum entangled state, the bit flip channel exhibits the most pronounced influence on the teleportation fidelity, whereas the phase damping channel exerts the weakest influence. These findings provide valuable insights into the practical implementation of QT in noisy environments.

In the future, we can focus on extending this scheme to teleport multi-qubit states, which will represent a significant advancement in quantum communication and quantum computing applications that require the manipulation and the transmission of more general quantum states.

Data availability statement

The data of this manuscript are included as electronic supplementary material, which is available to authorized users.

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