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Extensive numerical simulations on competitive growth between the Edwards–Wilkinson and Kardar–Parisi–Zhang universality classes

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Extensive numerical simulations and scaling analysis are performed to investigate competitive growth between the linear and nonlinear stochastic dynamic growth systems, which belong to the Edwards–Wilkinson (EW) and Kardar–Parisi–Zhang (KPZ) universality classes, respectively. The linear growth systems include the EW equation and the model of random deposition with surface relaxation (RDSR), the nonlinear growth systems involve the KPZ equation and typical discrete models including ballistic deposition (BD), etching, and restricted solid on solid (RSOS). The scaling exponents are obtained in both the $(1+1)$- and $(2+1)$-dimensional competitive growth with the nonlinear growth probability p and the linear proportion $1-p$. Our results show that, when p changes from 0 to 1, there exist non-trivial crossover effects from EW to KPZ universality classes based on different competitive growth rules. Furthermore, the growth rate and the porosity are also estimated within various linear and nonlinear growths of cooperation and competition.

Keywords: competitive growth, scaling behavior, discrete growth model, Kardar–Parisi–Zhang universality class

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1. Introduction

In the past few decades, studying and understanding dynamic properties of interface growth has developed tremendously, as surface and interface growth can be related to many actual physical processes, such as chemical vapor deposition, molecular beam epitaxial (MBE) growth, bacterial propagation, and forest fire spread.[1–5] In the process of related research, a series of discrete growth models have been proposed, involving random deposition (RD), random deposition with surface relaxation (RDSR),[2] ballistic deposition (BD),[6,7] restricted solid-on-solid (RSOS),[8,9] Eden models,[10] etc. The proposal of these discrete models has also greatly helped to understand the physical mechanism of surface and interface growth.

Most of the existing theoretical research focuses on the dynamic scaling properties of a single growth model, but most growth processes in nature are extremely complicated, and it is difficult to accurately describe the complex growth process. In the detailed experimental investigation, distinct growth phases were observed in surface or interface roughening with different competitive factors.[11–13] Compared with a simple growth model, introducing competitive growth may better describe the growth phenomena of interface and surface in nature and materials science.

Usually, the competitive growth models are divided into two types, one is the competition among particles of different shapes and sizes,[14–19] and the other is the competitive growth of a single kind of particle according to different growth rules.[20–25] This work will focus on the latter, and the competitive growth within the early and saturated growth regions will lead to some non-trivial dynamic properties according to different growth rules. In the study of the competitive growth system, it is an interesting issue to explore the crossover process between Edwards–Wilkinson (EW) and Kardar–Parisi–Zhang (KPZ) growths, which belong to two important universality classes.[26,27]

Through investigating the competitive growth models, the related continuum competitive growth equations and new scaling may be proposed.[28–32] Oliveira proposed a competition between the RSOS and BD models, where particles are deposited with growth probabilities p and $1-p$ following the RSOS and BD rules. Near the critical threshold $p_c \approx 0.83$, the competition corresponds to $\lambda = 0$, which reduces to the EW universality class.[21] Muraca *et al.* studied the RD-RDSR, BD-RDSR models, and found that the coefficients of continuum growth equations have a quadratic dependence on p.[28] Horowitz and Albano obtained three new exponents with a scaling relation by studying the RD-BD model.[29] Braunstein and Lam derived exact scaling in the competitive growth models of RD competing with BD and a constrained EW, respectively.[30] Kolakowska and Novotny derived the continuum stochastic growth equations and the resulting scaling based on RD competing with other correlated-growth models.[31,32]

As a typical Langevin-type growth equation, the EW equation is constructed to account for the local interaction among deposition particles of the nearest neighbors, which can

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be described as

$$\frac{\partial h(x,t)}{\partial t} = \nu\nabla^2 h(x,t) + \eta(x,t), \tag{1}$$

where $h(x,t)$ is the surface height of position x and time t, the first term on the right is the Laplacian term with the diffusion coefficient ν, $\eta(x,t)$ is the noise with zero mean, and the second moment satisfies

$$\langle \eta(x,t)\eta(x',t')\rangle = 2D\delta^d(x-x')\delta(t-t'). \tag{2}$$

Here D is the amplitude of the noise, and d is the substrate dimension of the system. When considering lateral growth, the linear EW equation is generalized to the nonlinear KPZ growth system, which is expressed as

$$\frac{\partial h(x,t)}{\partial t} = \nu\nabla^2 h(x,t) + \frac{1}{2}\lambda(\nabla h)^2 + \eta(x,t), \tag{3}$$

where λ is the nonlinear coefficient.

Both the EW and KPZ equations represent two typical universality classes, related to plenty of kinetic roughening and nonequilibrium physical phenomena. Thus, the study on competitive growth between the EW and KPZ universality classes is an important research direction.[22–25,33,34] Chame *et al.*[22] and Silveira *et al.*[23] investigated the crossover effects between EW and KPZ. In the continuum limit, for any $p > 0$, the growth model belongs to the KPZ universality class for the $(1+1)$-dimensional case. Interestingly, when p takes a small value, it presents a slow crossover from the EW to KPZ growth. Pellegrini and Jullien[24,25] discussed a simple growth model considering the competition between "sticky" and "sliding" particles within different growth dimensions, and performed the calculation of some scaling exponents of the model. Due to the limitation of computer calculation, the simulation sizes mentioned above are relatively small and may be affected by finite-size effects. Furthermore, one important issue needed to be clarified is, although one of these typical discrete growth models has long been regarded as the same universality classes as the EW or KPZ equation, whether or not it still belongs to the EW-KPZ universality class when the competitive growth is introduced to these discrete growth models. Thus, we perform extensive numerical simulations and scaling analysis on competitive growth between RDSR model and BD, etching, RSOS models. In order to further clarify the universality classes of the above discrete competitive growth systems, we also explore directly competitive growth of the continuum growth equation, that is, EW-KPZ with different growth probabilities. Briefly, this work focuses on dynamic scaling of these competitive growth systems mentioned above, estimating the scaling exponents and revealing the nontrivial crossover effects of these competitive growth dependent on both substrate dimension and specific growth rules when the growth probability p varies within the range $[0,1]$.

The rest of the paper is organized as follows. In Section 2, we introduce these discrete and continuum competitive growth systems, and describe the general scaling properties of characteristic physical quantities. In Section 3 we present the detailed simulation results of competitive growth in both $(1+1)$ and $(2+1)$-dimensions. Finally, a brief conclusion is given.

2. Competitive growth models and concepts

2.1. Discrete growth models

The growth rule of the RDSR model belonging to the EW universality class is: A particle chooses randomly a position to deposit to the substrate. When there is a neighbor whose height is lower than this location, the particle relaxes to the lowest neighbor, otherwise the particles are directly deposited at this location. If there are more than one lowest neighbor, the lowest neighbor is randomly chosen to deposit. The interface fluctuations of the RDSR model can be described by Eq. (1).

Discrete growth models belonging to the KPZ universality class include BD,[6,7] RSOS,[8] etching,[35] etc. Among them, the rule of the BD model reads: A particle randomly selects a location to deposit. If the height of the nearest neighbors are not higher than that of this position, the particle is deposited directly at the location; otherwise, it will paste at the highest point of the nearest neighbor. The RSOS rule is as follows: Randomly select a location, if the height difference between the location and its nearest neighbors are both less than k, a given positive integer, the particle is allowed to deposit here; otherwise, the deposition is forbidden. The rule of the etching model is as follows: When a position on the surface is randomly selected, the height of this position is increased by 1. Its nearest neighbors are increased to the height of the position before deposition if they are lower than the position before deposition, and if the nearest neighbor column is not lower than it, it has no effect on the nearest neighbor column. The fluctuations of the growing interface of these discrete growth models can be mapped by Eq. (3).

2.2. Competitive growth rules

During the numerical simulations of discrete competitive growth models, the selected probability of the nonlinear growth is set as p, and then the linear growth probability is $1-p$. The steps of the growth process can be described as follows:

Step 1: Randomly select a location on the substrate.

Step 2: Select the deposition obedience according to a given growth probability p.

Step 3: Deposit particles according to the selected growth rules.

Step 4: Repeat the above steps.

For the continuum competitive growth, the rules are as follows: at each growth step, a binary choice is allowed from the EW and KPZ equations as the basis for the next step of growth, where the selected probability of KPZ is p and that of EW is $1-p$. Similar to the competitive growth between two discrete models, the competitive intensity of these two continuum growth equations is mainly affected by the value of p.

2.3. Basic concepts

Surface width is often used to indicate dynamic scaling properties of the surface growth process, which is defined as

$$W(L,t)=\left\langle \frac{1}{L}\sum[h(x,t)-\langle h(x,t)\rangle]^2\right\rangle^{1/2}, \tag{4}$$

where $\langle\cdots\rangle$ represents ensemble averaging. This physical quantity follows the power-law scaling with growth time t and system size L, respectively,[36]

$$W(L,t)\sim\begin{cases} t^{\beta}, & t\ll L^{z},\\ L^{\alpha}, & t\gg L^{z}, \end{cases} \tag{5}$$

where β is the growth exponent, α is called the roughness exponent, and z is the dynamic exponent, which satisfies scaling relation $\beta=\alpha/z$.

In addition to the surface width, the height difference correlation function is another important physical quantity to characterize the scaling behavior, and it reads

$$G(l,t)=\langle[h(x+l,t)-h(x,t)]^2\rangle_x, \tag{6}$$

where l is the local window size. When the growth time $t\gg l^z$, the correlation function satisfies the scaling relationship

$$G(l,t)\sim l^{2\alpha_{\rm loc}}. \tag{7}$$

Furthermore, the calculation of growth rate and porosity is also highly meaningful for studying dynamic properties of surface growth. The average growth rate is written as

$$\bar{v}=\frac{1}{L}\int_{x=0}^{L}\left(\frac{\partial h}{\partial t}\right)\mathrm{d}x=\frac{\partial\bar{h}}{\partial t}. \tag{8}$$

The definition of porosity is

$$\sigma=\frac{N_{\rm v}}{N_{\rm v}+N_{\rm p}}, \tag{9}$$

where $N_{\rm v}$ and $N_{\rm p}$ are the numbers of voids and deposited particles, respectively.

3. Numerical simulations and results

3.1. (1 + 1)-dimensions

In this subsection, we simulate the competitive growth between the EW and KPZ equations in the $(1+1)$-dimensions, and the corresponding discrete competitive growth models, including RDSR-BD, RDSR-etching, and RDSR-RSOS. Firstly, we calculate the surface width and height difference correlation function to estimate the scaling exponents β and $\alpha_{\rm loc}$, respectively. Then, we compute the porosity and growth rate of the discrete competitive growth systems. During these simulations, we notice that there exist very similar results with different L, which indicates that finite-size effects are rather weak for simulating these competitive growth models.

Figure 1 exhibits the variation of $\log_2 W$ with $\log_2 t$ for these four competitive growth systems, and the variations of β with p are shown in the corresponding insets of Fig. 1. When p varies from 0 to 1, the estimated values of β are obtained by adjusting the value of p. Based on data fitting, we obtain the variation curve of β with p. More specifically, figure 1(a) shows the change of $W(L,t)$ with t for different growth probabilities for a given system size for the EW-KPZ competitive growth. Here, without loss of generality, $L=4096$ is used. When $p\to 0$, β is close to 1/4, which reduces to that of the EW equation, and when $p\to 1$, β is approximately $1/3$, which is consistent with the KPZ equation. Thus, with the increase of p, the estimated values of β gradually crossover from $1/4$ to $1/3$. By fitting data, we find that β and p exhibit good linear relationship. Figures 1(b)–1(d) show the changes of $\log_2 W$ with $\log_2 t$ based on the RDSR-BD, RDSR-etching and RDSR-RSOS competitive growth models. It can be seen from these results that, when $p\to 0$, β displays a estimated value close to $1/4$, this just is the scaling exponent of the pure RDSR model belonging to EW universality class. When $p\to 1$, β is roughly $1/3$, which is the scaling value of KPZ class. When p varies from 0 to 1, the RDSR-BD and RDSR-etching behaves similarly to that of the EW competing with KPZ, and the scaling exponents crossover monotonically from EW to KPZ universality class. However, RDSR-RSOS exhibits non-trivial behavior, the estimate values of β increase first and then decrease with increasing p.

For the RDSR-BD model, when p is small, due to weak coupling,[37] the scaling exponents are in agreement with that of the EW equation within the early growth regime, and with increasing p, the competitive growth model gradually falls into KPZ universality class. Therefore, it is manifested clearly as the gradual transition from EW to KPZ universality classes. For the RDSR-etching model, when $p<0.5$, the model shows that the linear growth gradually transitions to the nonlinear growth, and when $p\geq 0.5$, the component of the nonlinear growth is always dominant relative to the linear growth. For the RDSR-RSOS model, when $p<0.7$, the model increases slowly from EW to KPZ universality classes. However, when $p\geq 0.7$, $W(L,t)$ increases greatly with time evolution, and the estimated values of β far exceed $1/3$ of KPZ class. Interestingly, when $p\to 1$, $\beta\to 1/3$, which indicates that the competitive model reduces suddenly to KPZ universality class. Fig-

ure 2 shows the competitive morphologies of the RDSR-RSOS model as a typical representative. It can be seen that when $p > 0.7$, the ruggedness of the surface increases sharply, especially when $p \approx 0.9$, one can observe clearly the anomalous behavior. When $p \to 1$, the surface morphology is slightly smooth, which fits perfectly the KPZ case.

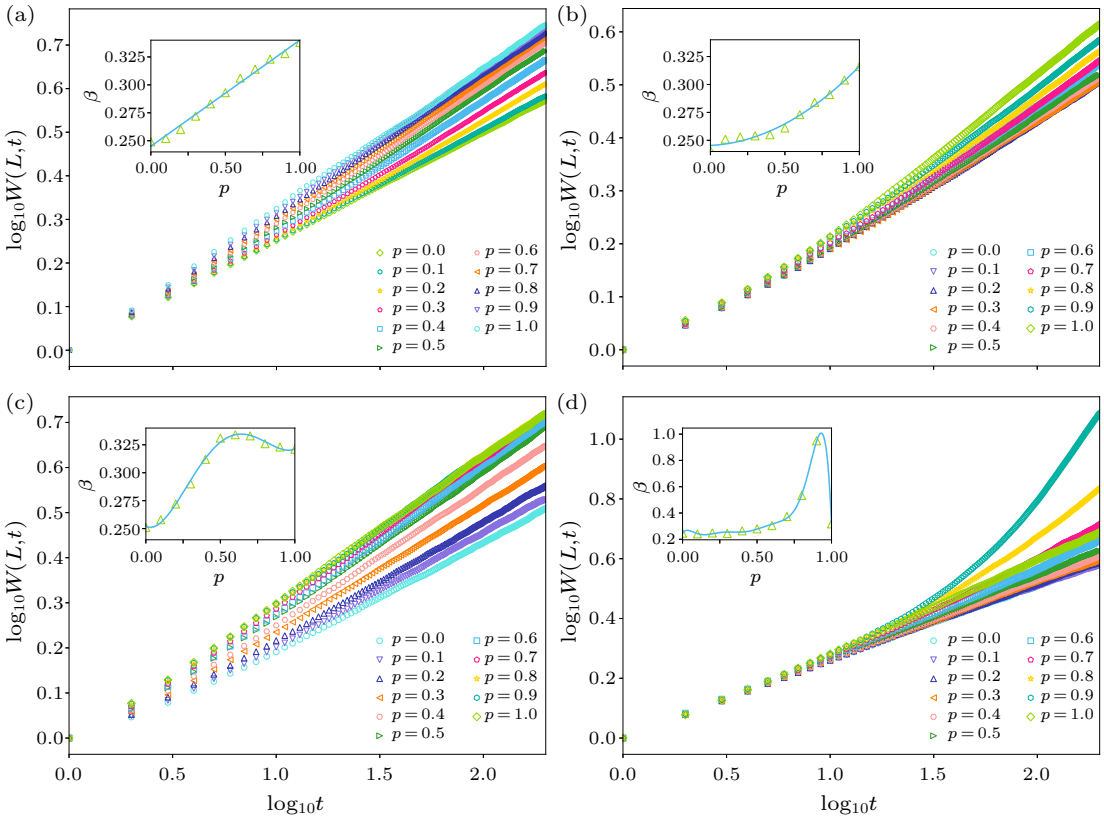


Fig. 1. Log–log plot of $W(L,t)$ as a function of growth time t of the $(1+1)$-dimensional competitive growth systems with different growth probabilities p: (a) the EW-KPZ equation, (b) the RDSR-BD model, (c) the RDSR-etching model, (d) the RDSR-RSOS model. For clear comparison, each curve is shifted moderately along the vertical coordinate without changing the scaling results. Insets exhibit the estimated values of β versus p correspondingly.

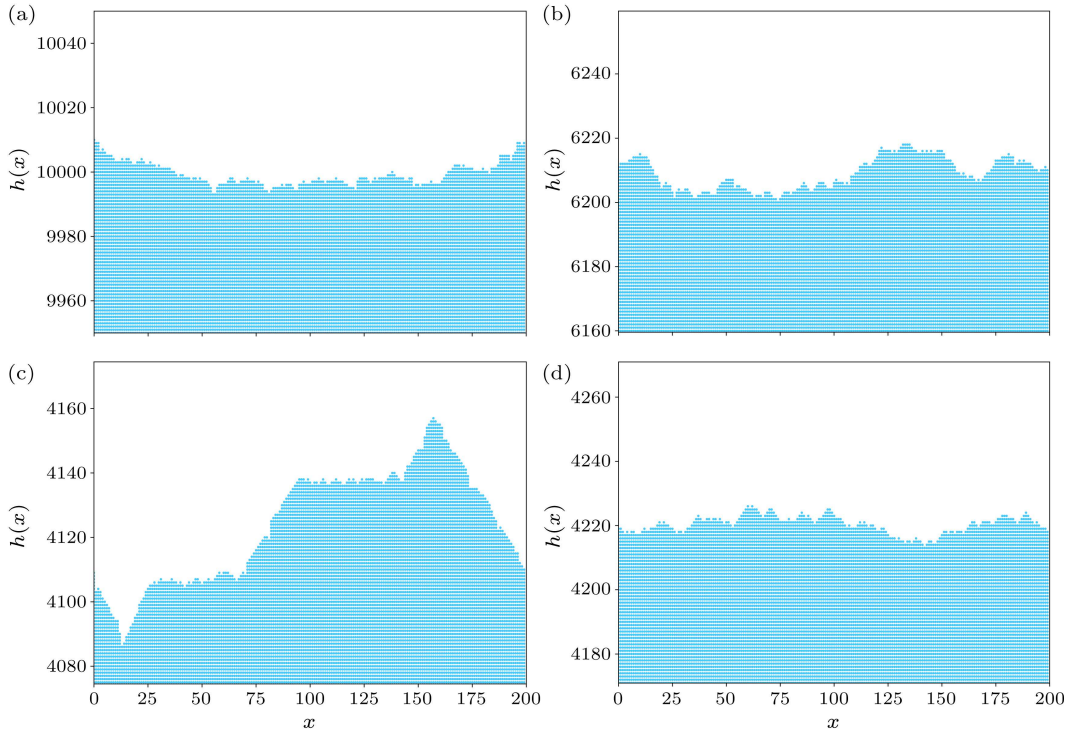


Fig. 2. Comparison of typical interfaces of RDSR-RSOS competitive growth at the saturation time ($t = 10000$) with different growth probabilities: (a) $p = 0$, (b) $p = 0.7$, (c) $p = 0.9$, (d) $p = 1$.

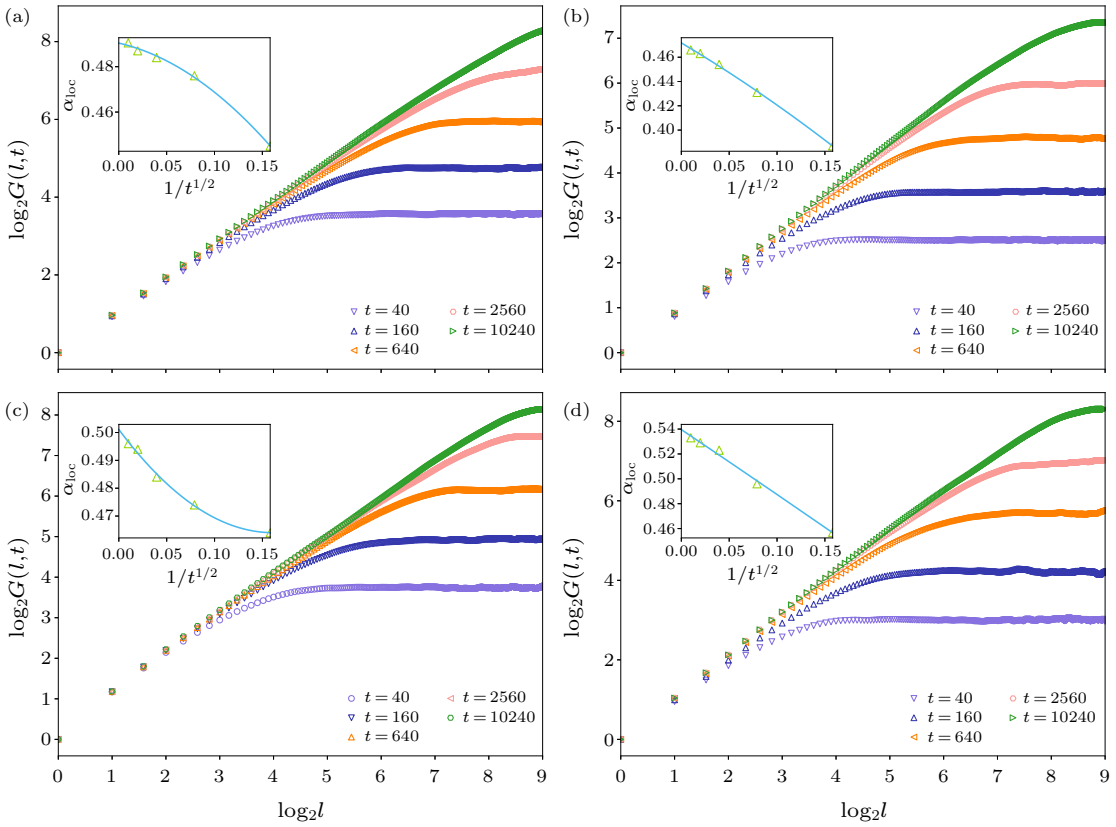


Fig. 3. Log–log plot of $G(l,t)$ as a function of l at different growth time with $p=0.6$ in the (1+1)-dimensional (a) EW-KPZ, (b) RDSR-BD, (c) RDSR-etching, and (d) RDSR-RSOS competitive growth systems. Insets show correspondingly the estimated values of $\alpha_{\rm loc}$ versus $1/t^{1/2}$ based on finite-time corrections.

Through calculating the height difference correlation function, we find that the scaling results are affected evidently by the competitive probability in the EW-KPZ, RDSR-BD, RDSR-etching, RDSR-RSOS competitive growth systems. As shown in Fig. 3, when $p=0.6$, the correlation function changes with the local window size l within different growth times.

To eliminate finite-time effects, we adopt time-dependence corrections to estimate the effective values of local roughen exponent $\alpha_{\rm loc}$. By fitting the simulation data, the predicted values of $\alpha_{\rm loc}$ are obtained as the growth time tends to infinity. Accordingly, the estimated values of $\alpha_{\rm loc}$ with growth time t are shown in the subplots of Fig. 3. To compare clearly with the results obtained from these four competitive growth systems, the estimated values of $\alpha_{\rm loc}$ with different p are shown in Table 1. It can be seen that when simulating EW-KPZ, RDSR-BD, and RDSR-etching systems are performed, the obtained values of $\alpha_{\rm loc}$ are almost 0.5 with changing p. Considering that the roughness exponents for both EW and KPZ classes are $1/2$, the estimated $\alpha_{\rm loc}$ values have not obvious dependence on p. For the RDSR-RSOS growth, the estimated values of $\alpha_{\rm loc}$ are also around 0.5 when $p \leq 0.7$. However, when $0.7 < p < 1$, the values gradually increase to deviate by 0.5, which is in line with the changing trend of β. Moreover, the surface morphologies also exhibits non-trivial properties correspondingly, especially within the large p region, which form gradually a pyramid-like structure at the earlier growth time, as shown in Fig. 2(c). Accordingly, the evident changes of surface morphology also cause directly sharp increase of $W(L,t)$ and $G(l,t)$ with time evolution.

Table 1. The estimated values of $\alpha_{\rm loc}$ for the four competitive growth systems at different p in the (1+1)-dimensions.

p	RDSR-BD	RDSR-etching	RDSR-RSOS	EW-KPZ
0	0.481	0.488	0.485	0.487
0.2	0.473	0.492	0.473	0.488
0.4	0.474	0.520	0.474	0.485
0.6	0.472	0.501	0.540	0.490
0.8	0.471	0.511	0.791	0.487
1	0.492	0.495	0.475	0.488

Furthermore, we also calculate the growth rate and the porosity of these models between linear and nonlinear competitive growth. Since there are differences in porosity and growth rates among these competitive growth models, we investigate further how these two characteristic quantities change as competitive growth occurs. As shown in Fig. 4, considering that the RDSR model has no voids and the BD and etching models have voids, the porosity of the competitive models gradually increases from 0 to the corresponding values of the BD and etching models. Figure 5 shows the growth rate of the competitive models as a function of p. Due to the existence of voids, the growth rates of RDSR-etching and RDSR-BD models are larger with the increasing parameter p.

In the RDSR-RSOS model, the probability of deposition failure gradually increases with increasing p, which is attributed to the deposition rules of the RSOS model. Consequently, the growth rate of the RDSR-RSOS model gradually decreases.

3.2. (2 + 1)-dimensions

In this subsection, we simulate the competitive growth between the EW and KPZ equations, and their corresponding discrete growth models in the $(2+1)$-dimensions. The simulating methods used here are similar with those in the $(1+1)$-dimensional case. Without loss of generality, the system sizes $L \times L = 256 \times 256$ are used, and 300 noise realizations are averaged during the following numerical simulations.

Figure 6 exhibits the variations of $\log_2 W$ with $\log_2 t$ in these four competitive growth systems, and the changing trends of β with p are shown in the insets correspondingly. When p varies from 0 to 1, the estimated values of β are obtained at every 0.1 interval. By fitting data, we obtain roughly the variation curves of β with p. Figure 6(a) shows $\log_2 W$ versus $\log_2 t$ for different probabilities for simulating EW-KPZ growth. One can easily find that, when $p \to 0$, $\beta \approx 0.05$, which is consistent with that of the EW equation $\beta = 0$. When $p \to 1$, β is about $1/4$, which is in agreement well with that of the KPZ case, and when p changes from 0 to 1, it gradually crossovers from EW to KPZ universality classes. Figures 6(b)–6(d) display the changes of $\log_2 W$ with $\log_2 t$ in the RDSR-BD, RDSR-etching and RDSR-RSOS models. It can

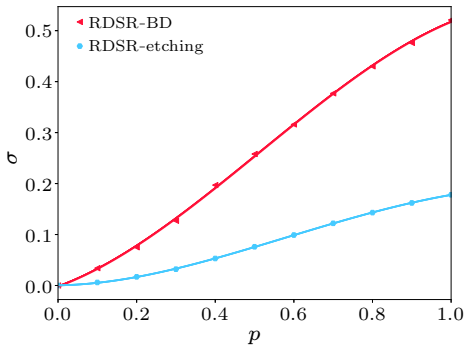


Fig. 4. The porosity σ versus p of the (1+1)-dimensional RDSR-BD and RDSR-etching competitive growths.

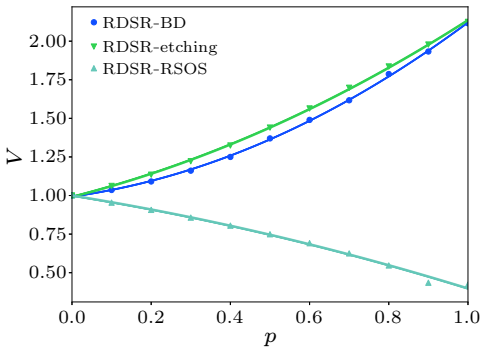


Fig. 5. The growth rate V versus p of the (1+1)-dimensional RDSR-BD, RDSR-RSOS and RDSR-etching competitive growths.

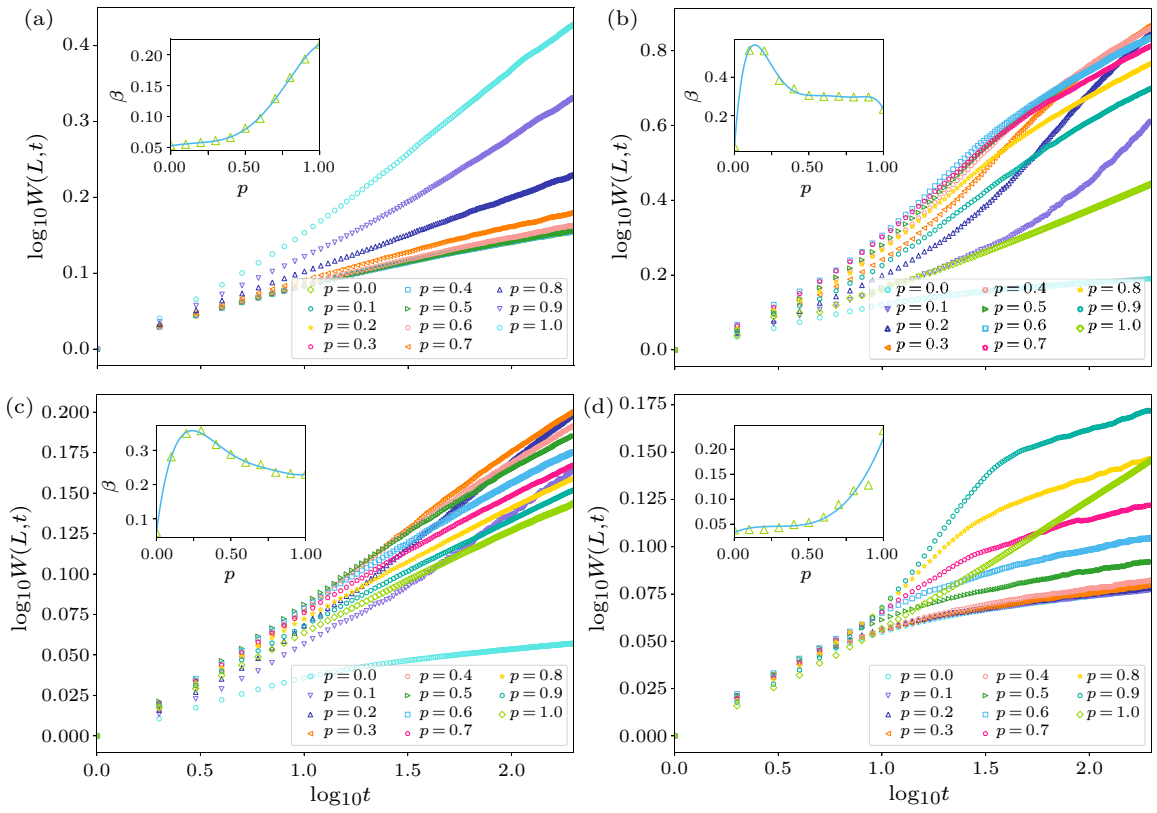


Fig. 6. Log–log plot of $W(L,t)$ as a function of t in the $(2+1)$-dimensional competitive growth systems with different p: (a) the EW-KPZ equation, (b) the RDSR-BD model, (c) the RDSR-etching model, (d) the RDSR-RSOS model. For the sake of clear comparison, each curve is shifted successively along the vertical coordinate without affecting the scaling results. Insets exhibit the estimated values of β versus p correspondingly.

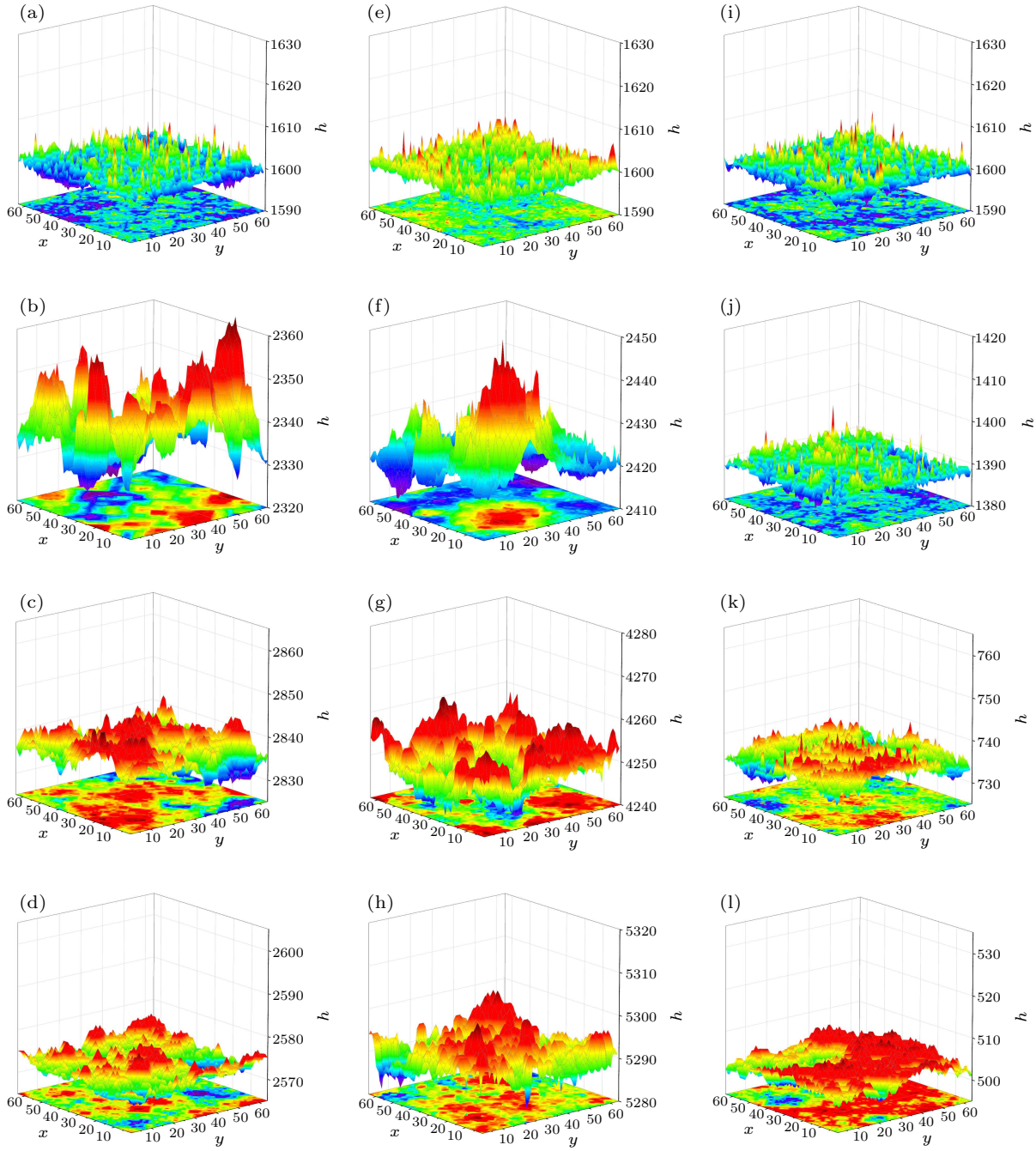


Fig. 7. Typical surface morphologies in the $(2+1)$-dimensional competitive growth systems: (a)–(d) RDSR-BD, (e)–(h) RDSR-etching, (i)–(l) RDSR-RSOS, with different growth probabilities $p = 0$, 0.2, 0.7, 1 from top to bottom.

be shown that, when $p \to 0$, the estimated values of β approach to 0, which just is the exact value of the standard RDSR model in the (2+1)-dimensions. When $p \to 1$, the values of β nearly equal 0.24, which crossovers to that of KPZ class as expected. It should be noted that, when p changes between 0 and 1, there exists a significant difference in comparison with the values of these three competetive growth models. The changing trend of RDSR-RSOS growth is similar to that of the EW-KPZ equation, and the estimated values of β increase monotonically with increasing p. However, both RDSR-BD and RDSR-etching models exhibit non-trivial scaling behavior, and the maximum values of β in these two models are reached when p is nearly 0.2, which differ evidently with EW-KPZ competitive class.

More specifically, for RDSR-BD and RDSR-etching models, the estimated values first increase sharply with p increasing. When $p > 0.2$, the obtained values of β gradually decrease with increasing p. When $p \to 1$, the β values are almost 1/4, which belongs to KPZ universality class. The surface topographies of these two competitive growth models are shown in Figs. 7(a)–7(h). Interestingly, when $p = 0.2$, the surface morphologies are most rugged, and then the surface morphologies are gradually smooth. For the RDSR-RSOS model, the dynamic properties are similar to those of the competition

between EW and KPZ equations. The estimated values of β gradually increase with increasing p. We find that there exist the similar changing trends of surface morphologies, which are gradually rugged with increasing p, as shown in Figs. 7(i)–7(l).

Next, we quantitatively analyze the height difference correlation function affected by the competitive probability. As shown in Fig. 8, when $p = 0.6$, this correlation function of EW competing with KPZ, and RDSR competing with BD, etching, RSOS models changes with the local window size within different growth times. The values of $\alpha_{\rm loc}$ are estimated with the same way as the $(1+1)$-dimensions. The variations of $\alpha_{\rm loc}$ with t are shown in the corresponding insets of Fig. 8. By fitting the obtained data, the $\alpha_{\rm loc}$ values are estimated when the growth time tends to infinity as the finite-time correction. Table 2 lists the obtained values of $\alpha_{\rm loc}$ when different values of p are chosen. It can be seen that when EW competes with the KPZ equation, the RDSR model competes with the RSOS model, the values of $\alpha_{\rm loc}$ change from 0 to 1/3, which is due to the gradual transition from EW to KPZ universality classes. However, for the RDSR-etching, the values of $\alpha_{\rm loc}$ gradually increase when $p < 0.2$, and then decrease to 1/3 when $0.2 < p < 1$, which reduces to KPZ universal class. For the RDSR-BD model, the estimated values of $\alpha_{\rm loc}$ are around 0.5 when p changes from 0 to 1. Thus, the competitive growth for both RDSR-BD and RDSR-etching exhibit non-trivial behavior in the $(2+1)$-dimensions.

Table 2. The estimated values of $\alpha_{\rm loc}$ for the four competitive growth systems at different p in the $(2+1)$-dimensions.

p	RDSR-BD	RDSR-etching	RDSR-RSOS	EW-KPZ
0	0.098	0.103	0.094	0.123
0.2	0.532	0.433	0.088	0.128
0.4	0.524	0.408	0.105	0.139
0.6	0.510	0.380	0.179	0.189
0.8	0.508	0.358	0.261	0.237
1	0.378	0.346	0.375	0.339

The porosity and growth rate of the above discrete competitive growth models in the $(2+1)$-dimensions are shown in Figs. 9 and 10, respectively.We find that the changing trends are similar to those in the $(1+1)$-dimensions. As shown in Fig. 9, with the increase of p, the estimated values of porosity gradually increase from 0 to 0.19 in the RDSR-BD model, and the corresponding values increase from 0 to 0.25 in the RDSR-etching model. Figure 10 shows the change of the growth rate of these competitive growth models with p. With the increase of p, the variations of RDSR-etching and RDSR-BD exhibit a nearly linear increase, yet the variation of RDSR-RSOS growth presents non-trivial behavior, which gradually decreases with increasing p.

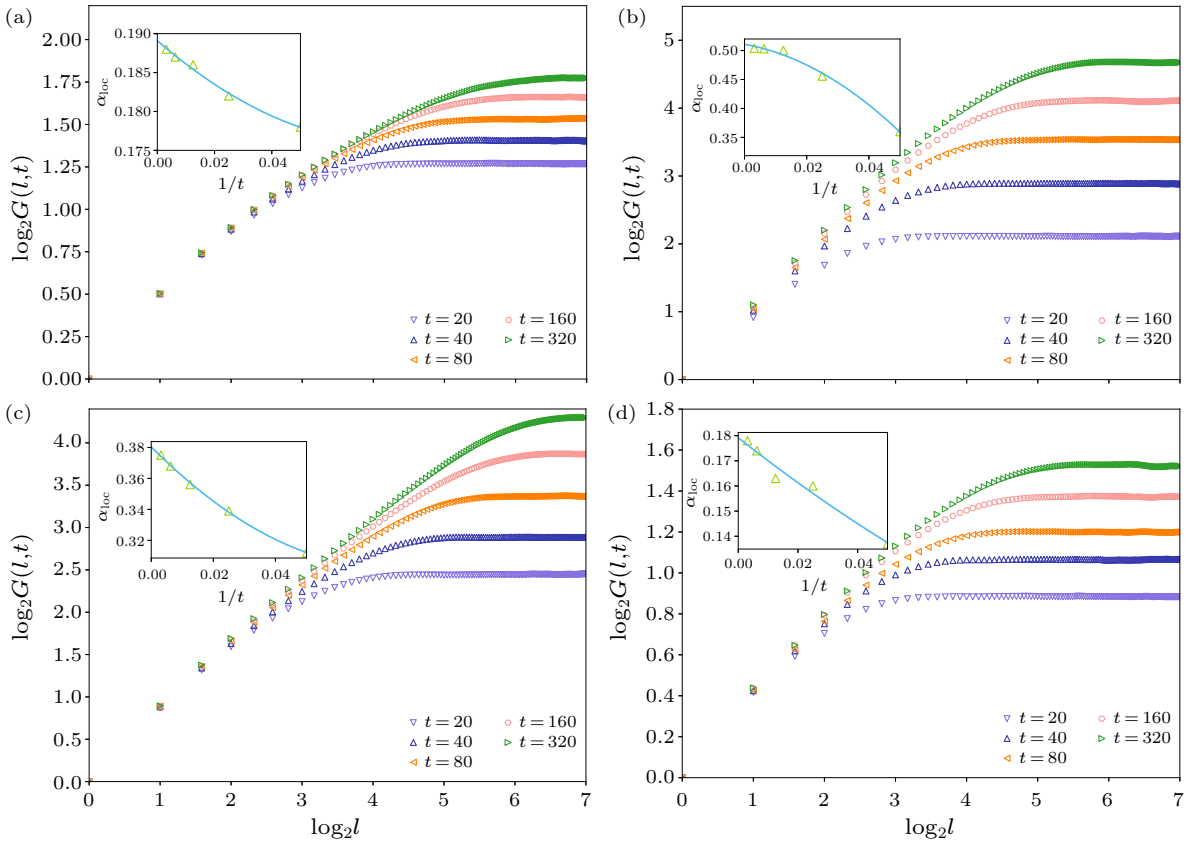


Fig. 8. Log–log plot of $G(l,t)$ as a function of l at different growth times with $p = 0.6$ in the $(2+1)$-dimensional (a) EW-KPZ, (b) RDSR-BD, (c) RDSR-etching, and (d) RDSR-RSOS competitive growth systems. Insets show the correspondingly estimated values of $\alpha_{\rm loc}$ with finite time corrections.

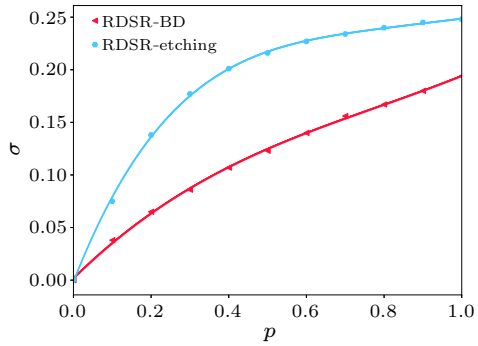


Fig. 9. Porosity σ versus p of the $(2+1)$-dimensional RDSR-BD and RDSR-etching competitive growths.

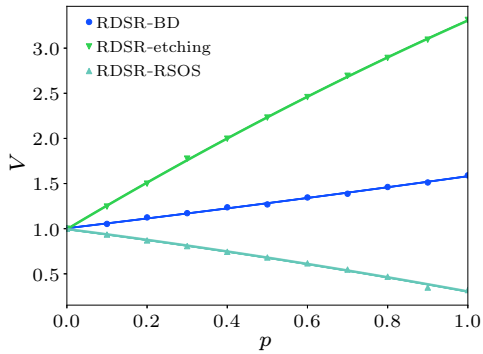


Fig. 10. Growth rate V versus p in the $(2+1)$-dimensional RDSR-BD, RDSR-RSOS and RDSR-etching competitive growth models.

4. Conclusion

In summary, we have performed a large number of numerical simulations on competitive growth between EW and KPZ universality classes in the $(1+1)$- and $(2+1)$-dimensions, the growth exponent β and local roughness exponent $\alpha_{\rm loc}$ are obtained by calculating the surface width and height difference correlation function, respectively. Our results show that, by adjusting the growth probability p from 0 to 1, there exist non-trivial scaling properties and crossover effects from EW to KPZ classes using different competitive growth rules.

In the $(1+1)$-dimensions, with the change of p, the estimated values of β from simulating RDSR-BD and RDSR-etching models show the crossover from EW to KPZ classes, yet specific changing trends in these two discrete models are obviously different. For the RDSR-RSOS model, it shows a gradual transition from EW to KPZ at $p \leq 0.7$, increases greatly at $0.7 < p < 1$, and abruptly returns to the KPZ universality class when $p \to 1$. Considering that the $\alpha_{\rm loc}$ values for both EW and KPZ universality classes are $1/2$ in the $(1+1)$-dimensions, the roughness exponents related to competitive growth for whether the continuum or the discrete growth systems have not obvious dependence on p. However, for the RDSR-RSOS model, the values of $\alpha_{\rm loc}$ depend evidently on p, first slowly changing, then increasing sharply, and finally returning to $1/2$ abruptly. Similarly, through the simulation of a discrete erosion model in $(1+1)$-dimensions, the non-trivial scaling results of the smooth transition from EW to KPZ universality classes are also found.[38]

For the $(2+1)$-dimensional competitive growth, the estimated values of β and $\alpha_{\rm loc}$ for simulating EW-KPZ and RDSR-RSOS systems gradually exhibit the crossover from EW to KPZ with increasing p. However, for the RDSR-BD and RDSR-etching models, the estimated values of β increase gradually, and reach the maximum value when p is nearly 0.2, and then decrease monotonically with increasing p. For the RDSR-etching, the value of $\alpha_{\rm loc}$ gradually increases at $p \leq 0.2$ and gradually decreases to $1/3$ at $0.2 < p < 1$, which just falls into the KPZ class. Another special situation happens in the RDSR-BD model, the estimated values of $\alpha_{\rm loc}$ remain at about 0.5 when $0 < p < 1$, but there exist sudden changes of $\alpha_{\rm loc}$ when p is chosen with two limit cases, 0 and 1. In addition, we calculate the porosity and growth rate for these competitive growth models with different competitive probabilities. The simulation results within limit conditions could be consistent with the theoretical predictions on the independent growth models without introducing competitive growth mechanism. Therefore, through our extensive numerical simulations between EW and KPZ universality classes, the scaling properties and crossover effects on various competitive growth rules have been obtained with different growth probabilities, which will provide reference for the in-depth study of the growth processes in nature and experimental research.

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