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Detecting the quantum phase transition from the perspective of quantum information in the Aubry–André model

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We investigate the effectiveness of entropic uncertainty, entanglement and steering in discerning quantum phase transitions (QPTs). Specifically, we observe significant fluctuations in entropic uncertainty as the driving parameter traverses the phase transition point. It is observed that the entropic uncertainty, entanglement and quantum steering, based on the electron distribution probability, can serve as indicators for detecting QPTs. Notably, we reveal an intriguing anticorrelation relationship between entropic uncertainty and entanglement in the Aubry–André model. Moreover, we explore the feasibility of detecting a QPT when the period parameter is a rational number. These observations open up new and efficient avenues for probing QPTs.

Keywords: quantum phase transition, entropic uncertainty, quantum entanglement, quantum steering

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1. Introduction

In his pioneering work, American physicist P. W. Anderson[1] introduced the concept of electron localization in disordered systems, where the disruption of lattice periodicity leads to an exponential decay of the electron's wave function, representing a localized state. Subsequently, Aubry and André[2] introduced the one-dimensional quasi-periodic Aubry–André (AA) model, which exhibits a distinct lack of translational symmetry and a noteworthy transition from extension to locality. Within the framework of the AA model, a crucial duality point marks a sharp transition from all eigenstates being extended to all becoming localized. Since then, researchers have extensively investigated the AA model and its generalized versions, shedding light on the intricate nature of transitions in these complex systems.[3–6]

In recent decades, quantum phase transitions (QPTs) have garnered significant attention in condensed matter physics.[7] These transitions occur in a system due to variations in its parameters, such as the incommensurate potential strength (Δ), near absolute zero temperature. Altering these parameters can bring the Hamiltonian to a critical point that leads to a transformation in the system's ground state primarily induced by quantum fluctuations. The study of QPTs provides insights into a range of quantum phenomena.[8–13] Meanwhile, QPTs far from equilibrium systems have extended our understanding of phase transitions and universality greatly. By a sudden change in the Hamiltonian, a quantum quench process can push the initial quantum system out of equilibrium, which permits us to study the quench dynamics of the nonequilibrium system. More recently, many researchers have concentrated on the critical phenomena present in quench dynamics, which are termed dynamical QPTs.[14–28]

Given that quantum fluctuations play a pivotal role in QPTs and considering the close connection between the uncertainty principle and these fluctuations, a natural question arises: can we detect QPTs through entropic uncertainty? In 1927, Heisenberg introduced the uncertainty principle, a cornerstone of quantum mechanics,[29] which imposes a lower limit on the predictability of measurements for any two incompatible observables in a quantum system. Subsequent efforts have been dedicated to refining and generalizing this uncertainty relation for multi-observable cases.[30–42] Currently, uncertainty relations find widespread applications, such as in quantum teleportation,[43] induction inequalities,[44] random number generation,[45] wave–particle duality,[46,47] quantum key distribution,[48,49] both sides of cryptography[50] and entanglement witnessing.[51] Notably, the emergence of quantum cryptography[52] is closely tied to the uncertainty relation, although its reliability is compromised when eavesdroppers become entangled with the tested system, biasing the results.[53] Addressing this concern, Renes and Berta introduced and generalized the quantum-memory-assisted entropic uncertainty relation,[54–61] filling a crucial gap in this context.

In the pursuit of understanding QPTs, our research unfolds on multiple fronts. Initially, we explore the use of entropic uncertainty and quantum entanglement[62,63] as pivotal tools for detecting QPTs. Our investigation goes a step further by scrutinizing the intricate relationship between entropic uncertainty and quantum entanglement within the unique context of the AA model. Broadening our scope, we extend our inquiry to encompass the detection of QPTs through the phe-

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nomenon of quantum steering,[64,65] unraveling the fascinating dynamics of how localized measurements can influence the state of a distant counterpart. As a final frontier in our exploration, we delve into the nuanced variations of entropic uncertainty and quantum entanglement under the influence of rational number periods. This comprehensive analysis not only contributes to our understanding of QPTs, but also sheds light on the potential roles of entropic uncertainty, quantum entanglement and quantum steering in diverse scenarios. The collective findings presented in this research article pave the way for a deeper comprehension of the intricate interplay between these quantum phenomena and their implications for discerning QPTs.

The structure of this paper is as follows. In Section 2, the model Hamiltonian, dynamical QPTs, entropic uncertainty, quantum entanglement and quantum steering are briefly reviewed. In Section 3, we first explore the use of entropic uncertainty and quantum entanglement to detect QPTs. We then investigate the relationship between entropic uncertainty and quantum entanglement in the AA model. Furthermore, we explore the detection of QPTs by quantum steering. Finally, we explore the variation of the entropic uncertainty and quantum entanglement of the system in the case of rational number periods. A summary is provided in Section 4.

2. Model and method

2.1. Model

We consider the AA model with the Hamiltonian

$$H = -J\sum_{j=1}^{L-1}(c_j^\dagger c_{j+1}+\text{H.c.})+\Delta\sum_{j=1}^{L}\cos(2\pi\alpha j)c_j^\dagger c_j, \quad (1)$$

where $c_j^\dagger$ (c_j) denotes the creation (annihilation) operator of fermions at site j ($j=1,\ldots,L$, with L being the total number of lattice sites), J is the hopping amplitude and we fix $J=1$, and Δ is the strength of the incommensurate potential. Here the period parameter α is an irrational number and we fix $\alpha=(\sqrt{5}-1)/2$ for convenience. The incommensurate potential strength Δ drives the system undergoing a delocalization–localization transition at a critical point $\Delta/J=2$. When $\Delta/J<2$, all the eigenstates are extended but localized as $\Delta/J>2$.[25,66]

A general Hamiltonian $H(\lambda)$ in which a QPT appears, where λ is a driving parameter that drives the QPT. Supposing the system is initially prepared as the ground state of the Hamiltonian λ_i, we investigate the dynamic evolution by changing the driving parameter to λ_f; that is, the sudden quench process is realized by changing the driving parameter. Here we consider the quench process described by the sudden change in the incommensurate potential strength Δ. We use the ground state of the Hamiltonian $H(\Delta_\text{i})$ as the initial state of the system and obtain an initial quantum state $|\Psi(0)\rangle$, then suddenly quench to Hamiltonian $H(\Delta_\text{f})$ and consider the evolution of the initial state under the action of this Hamiltonian. Thus, the final quantum state

$$|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H(\Delta_\text{f})t}|\Psi(0)\rangle. \quad (2)$$

By denoting the eigenstate of H as $|\Psi\rangle=\sum_j\psi(j)c_j^\dagger|0\rangle$, we obtain the following eigenvalue equation:

$$-[\psi(j+1)+\psi(j-1)]+\Delta\cos(2\pi\alpha j)\psi(j)=E\psi(j), \quad (3)$$

where E is the energy eigenvalue and $\psi(j)$ represents the amplitude of the corresponding normalized wave function $|\Psi\rangle$ at site j. Consider the probability of the electron appearance at each lattice site and divide the system into two parts: A and B. We can obtain the density matrix of the system at this time

$$\rho_{AB}=p|01\rangle+\sqrt{1-p^2}|10\rangle, \quad (4)$$

where $p^2=\sum_{j=1}^{L/2}|\psi(j)|^2$ represents the sum of the probabilities of the electron appearing on part A.

2.2. Entropic uncertainty

In order to explore the performance of entropic uncertainty relation in probing QPTs, we recall the quantum-memory-assisted entropic uncertainty relation as[55]

$$S(X|B)+S(Z|B)\geq S(A|B)-\log_2 c(X,Z), \quad (5)$$

where $S(X|B)=S(\rho_{XB})-S(\rho_B)$ denotes the conditional von Neumann entropy[67] of post-measurement state with

$$\rho_{XB}=\sum_i\left(|\psi_i^X\rangle_A\langle\psi_i^X|\otimes\mathbb{1}_B\right)\rho_{AB}\left(|\psi_i^X\rangle_A\langle\psi_i^X|\otimes\mathbb{1}_B\right), \quad (6)$$

where $\mathbb{1}_B$ is an identical operator in the Hilbert space of B, likewise for ρ_{ZB}. In addition, $S(A|B)=S(\rho_{AB})-S(\rho_B)$ represents the conditional von Neumann entropy of systemic density operator ρ_{AB} with $S(\rho_{AB})=-\text{Tr}(\rho_{AB}\log_2\rho_{AB})$ and $\rho_B=\text{Tr}_A(\rho_{AB})$. Here, X and Z resort to $\hat{\sigma}_x$ and $\hat{\sigma}_z$, respectively. As a result, Eq. (5) can be reduced to the form of

$$S(X|B)+S(Z|B)\geq S(A|B)+1, \quad (7)$$

entropic uncertainty of the system $U=S(X|B)+S(Z|B)$.

2.3. Quantum entanglement

It is necessary to find a universal and reasonable way to quantify the quantum entanglement of the system. Here, we investigate the entanglement of formation of the AA model. For a pure state, its entanglement of formation[68] is defined as the von Neumann entropy of the subsystem A or B of the pure state

$$E(|\phi\rangle)=S(\text{Tr}_B(|\phi\rangle\langle\phi|))=S(\text{Tr}_A(|\phi\rangle\langle\phi|)), \quad (8)$$

where $S(\text{Tr}_B(|\phi\rangle\langle\phi|))$ and $S(\text{Tr}_A(|\phi\rangle\langle\phi|))$ are the reduction density matrices of the A and B edges of the pure state $|\phi\rangle$,

respectively. $S(\mathrm{Tr}_B(|\phi\rangle\langle\phi|))$ and $S(\mathrm{Tr}_A(|\phi\rangle\langle\phi|))$ are the corresponding von Neumann entropies.

Concurrence[69] as a quantum resource also has the ability to measure entanglement. Considering that there is a functional relationship between concurrence and entanglement of formation

$$E(\rho) = h\left(\frac{1+\sqrt{1-C^2(\rho)}}{2}\right), \tag{9}$$

where a binary entropy is denoted by $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$. For a two-qubit pure state, its concurrence is defined as

$$C(|\psi\rangle) = |\langle\psi|\widetilde{\psi}\rangle| = \left|\langle\psi|\sigma_y\otimes\sigma_y|\psi^*\rangle\right|. \tag{10}$$

Generally, the concurrence for two-qubit X-type states can be reduced to

$$\begin{aligned} C(\rho^{\mathrm{X}}) = 2\max\{0, &\sqrt{\rho_{14}\rho_{41}} - \sqrt{\rho_{22}\rho_{33}}, \\ &\sqrt{\rho_{23}\rho_{32}} - \sqrt{\rho_{11}\rho_{44}}\}. \end{aligned} \tag{11}$$

2.4. Quantum steering

The three-setting linear steering inequality[70] is used to detect the steerability of a state and is considered as a very useful criterion of the quantification of steering. For a two-qubit system, it can be expressed as

$$S_{AB} = \mathrm{Tr}(T_{AB}^{\mathrm{T}} T_{AB}) > 1, \tag{12}$$

where the superscript T represents the transpose of the correlation matrix T_{AB},

$$T_{AB} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix}. \tag{13}$$

Here, $T_{AB} = [t_{ij}]$, $t_{ij} = \mathrm{Tr}[\rho_{AB}(\sigma_i\otimes\sigma_j)]$ and $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ with σ_i $(i = 1, 2, 3)$ denoting the Pauli matrices. When the inequality holds, it shows that the quantum state is steerable. Furthermore, the upper limit of the inequality is 3.

3. QPTs in the AA model

3.1. Detection of a QPT via entropic uncertainty

In Fig. 1(a), we show the time evolution of the entropic uncertainty for systems with the same initial incommensurate potential strength $\Delta_i = 1.1$ under the influence of different Hamiltonian $H(\Delta_\mathrm{f})$. We can find that the fluctuation intensity of the entropic uncertainty reaches the maximum at final incommensurate potential strength $\Delta_\mathrm{f} = 2.0$. By changing the final incommensurate potential strength Δ_f multiple times, it is found that the fluctuation strength of the entropic uncertainty increases at $\Delta_\mathrm{f} < 2.0$ and decreases at $\Delta_\mathrm{f} > 2.0$. To show this change more clearly, we represent the fluctuation intensity by the expected value of the entropic uncertainty $\overline{U} = \frac{1}{\tau}\int_0^\tau U\,\mathrm{d}t$ (τ shows the time of the system evolution, here $\tau = 1.0\times10^4$) in Figs. 1(b) and 1(c). In Fig. 1(b), we can find that the expected value of the system entropic uncertainty at $\Delta_\mathrm{i} < 2.0$ always reaches the maximum at $\Delta_\mathrm{f} = 2.0$. At final incommensurate potential strength $\Delta_\mathrm{f} < 2.0$, the expected value of the entropic uncertainty tends upward, but as long as Δ_f exceeds the critical value, the expected value of the entropic uncertainty then decreases gradually. We also explore the changes in the system when initial incommensurate potential strength $\Delta_\mathrm{i} > 2$ in Fig. 1(c). Comparing Figs. 1(b) and 1(c), we can see that as long as Δ_i exceeds the critical value, the system property will also change. When $\Delta_\mathrm{i} > 2$, we find that the expected value of the entropic uncertainty increases sharply around the critical value ($\Delta_\mathrm{f} = 2$) and the gradient in areas away from the critical value becomes smaller. Our results indicate that when initial incommensurate potential strength Δ_i or final incommensurate potential strength Δ_f above the critical value $\Delta = 2$ will cause mutation in the system nature. This means that $\Delta = 2$ is the phase transition point, which is virtually consistent with the result of the previous work.[25,66]

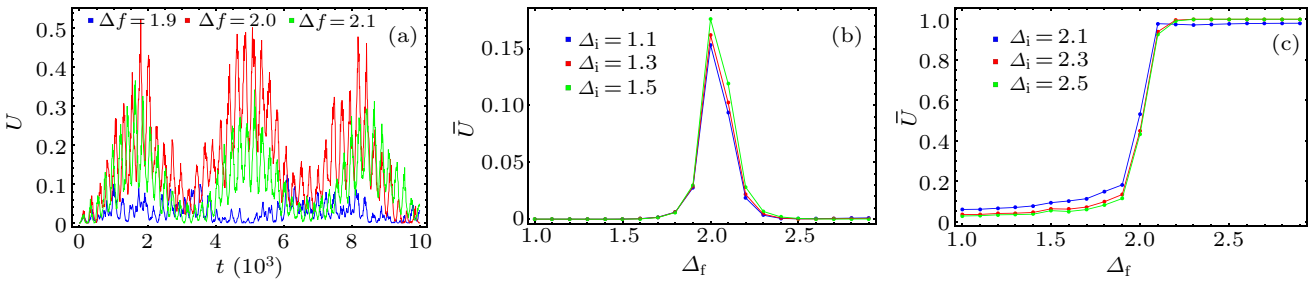


Fig. 1. The behavior of the entropic uncertainty of the AA model with a total number of lattice sites $L = 100$. (a) The entroropic uncertainty after time evolution of a system with initial incommensurate potential strength $\Delta_\mathrm{i} = 1.1$ and final incommensurate potential strengths $\Delta_\mathrm{f} = 1.9, 2.0$, and 2.1. (b) and (c) The expected value of the entropic uncertainty of the AA model at different final incommensurate potential strengths and initial incommensurate potential strengths $\Delta_\mathrm{i} = 1.1, 1.3, 1.5, 2.1, 2.3$, and 2.5.

3.2. Detection of a QPT via the quantum entanglement

In order to more intuitively reflect the influence of different final incommensurate potential strengths on the evolution of the system, we mainly focus on the expected value of the entanglement. Figure 2(a) shows that entanglement with increasing final incommensurate potential strength exhibits two changing trends. When $\Delta_\mathrm{f} < 2$, the expected value of the en-

tanglement of formation $\overline{E} = \frac{1}{\tau}\int_0^\tau E\,\mathrm{d}t$ (τ represents the time of the system evolution; here, $\tau = 1.0\times10^4$ is hold) is constantly decreasing and the descent velocity increases rapidly around the critical value $\Delta_\mathrm{f} = 2$. The expected value of the entanglement of formation gradually increases and eventually remains stable at $\Delta_\mathrm{f} > 2$. This indicates that the quantum entanglement of the system is hardly affected by the environment when the final incommensurate potential strength is low or high. However, the final incommensurate potential strength near the critical value will have a significant effect on the quantum entanglement of the system. We also explored the effect of the final incommensurate potential strength on the different initial quantum states in Fig. 2(b). The expected value of the entanglement of formation first drops slowly, then decreases sharply around the critical value and finally tends to stabilize. This means that the quantum entanglement of the system vanishes quickly after the final incommensurate potential strength exceeds the critical value. By comparing Figs. 2(a) and 2(b), one can find that if the initial incommensurate potential strength used to prepare the initial state exceeds the critical value $\Delta_\mathrm{i} = 2$, the quantum entanglement of the system tends differently with the environment.

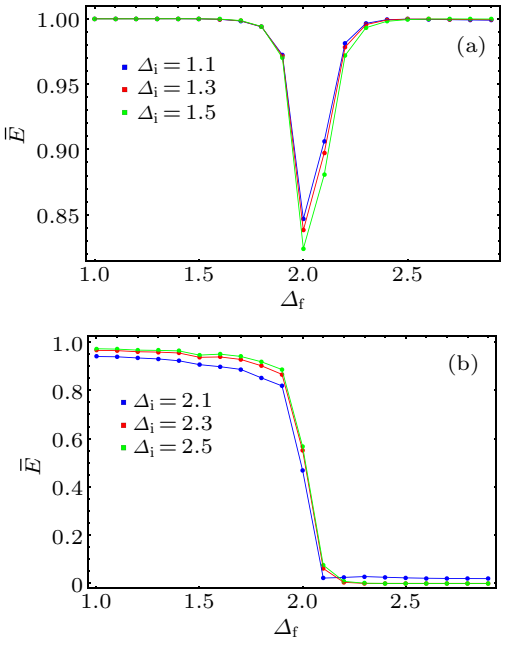


Fig. 2. The behavior of the entanglement of formation of the AA model with a total number of lattice sites $L = 100$. (a) The expected value of the entanglement of formation at different final incommensurate potential strengths and initial incommensurate potential strengths $\Delta_\mathrm{i} = 1.1, 1.3$ and 1.5. (b) The expected value of the entanglement of formation at different final incommensurate potential strengths and initial incommensurate potential strengths $\Delta_\mathrm{i} = 2.1, 2.3$ and 2.5.

3.3. The relationship between entroropic uncertainty and entanglement in determining QPT

For the quantum entanglement of the system, it is found that the dynamic of entanglement is essentially opposite to the entropic uncertainty in the AA model; in this sense, we are able to explore the relationship between the entanglement and uncertainty, as shown in Fig. 3. By comparing Figs. 1 and 2, we find that the changing trends of entropic uncertainty and entanglement of formation in the same case are exactly opposite and of the same order of magnitude. This means that they may have a numerical relationship. The sum of the expected values of the entanglement of formation and entropic uncertainty (S_EU) is shown in Fig. 3(a). Thus, it reveals that there is a completely inverse correlation between them

$$S_\mathrm{EU} = \overline{E} + \overline{U} = 1. \tag{14}$$

Combining Eqs. (9) and (14), we can get the relationship between concurrence and entropic uncertainty

$$S_\mathrm{CU} = \overline{C} + \overline{U} \geq 1, \tag{15}$$

where $\overline{C} = \frac{1}{\tau}\int_0^\tau C\,\mathrm{d}t$ ($\tau = 1.0\times10^4$ is chosen). We verify this relationship for entropic uncertainty and concurrence in Fig. 3(b). Moreover, it can be seen that S_CU has a singular point when $\Delta_\mathrm{f} = 2.0$, which directly indicates the QPT. In this regard, we say the concurrence and entropic uncertainty are available for detecting QPTs.

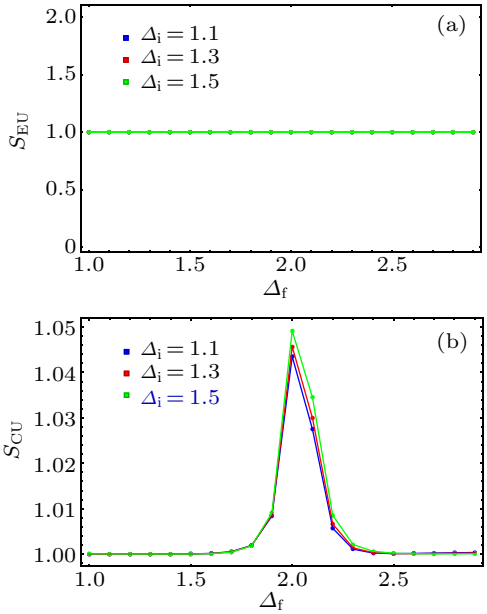


Fig. 3. (a) The sum of the expected values of the entanglement of formation and entropic uncertainty (S_EU) at different final incommensurate potential strengths and initial incommensurate potential strengths $\Delta_\mathrm{i} = 1.1, 1.3$ and 1.5. (b) The sum of the expected values of the concurrence and entropic uncertainty (S_CU) at different final incommensurate potential strengths and initial incommensurate potential strengths $\Delta_\mathrm{i} = 1.1, 1.3$ and 1.5.

3.4. Detection of a QPT via quantum steering

Quantum steering describes the ability to influence a nonlocalized state of another particle by making localized measurements of one particle in a two-body system far enough away. As long as the inequality Eq. (13) is met, we consider that it is steerable between parts A and B. In order to more intuitively reflect the influence of different final incommensurate potential strengths on the evolution of the system, we focus on

the expected value of the quantum steering $\overline{S_{AB}} = \frac{1}{\tau}\int_0^\tau S_{AB}\mathrm{d}t$ ($\tau = 1.0\times10^4$). Following Fig. 4, we find that the initial states prepared with different incommensurate potential strengths satisfy the inequality in different environments. This means that the steerability of the system is insusceptible to the incommensurate potential strength because it is always in a steerable state. The steerability gradually decreases as the final incommensurate potential strength approaches the critical value and the steerability of the system gradually returns to the maximum value after exceeding the critical value. Furthermore, we note the drastic changes in the steerability of the system near the critical value $\Delta_\mathrm{f} = 2.0$ and the steerability of the system in regions far from the critical value is near the upper limit of the inequality. Therefore, it is found that quantum steering is an alternative method for detecting QPTs.

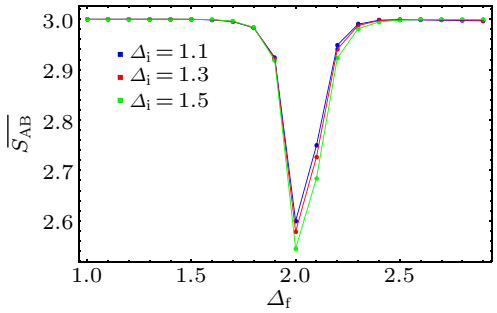


Fig. 4. The expected value of the quantum steering $\overline{S_{AB}}$ at different final incommensurate potential strengths and initial incommensurate potential strengths $\Delta_\mathrm{i} = 1.1, 1.3$ and 1.5.

3.5. QPT in the case of rational number periods

If the period parameter α is an irrational number, this means that the modulation period cannot match the lattice period. The system will lose translational symmetry and the eigenstate of the system will undergo a delocalization-localization transition at $\Delta = 2J$. Now we use the method of quantum information to study the change of the system in the case of rational number periods ($\alpha = 0.5$). In Fig. 5(a), the expected value of the entropic uncertainty increases slowly, implying an ordered link between the entropic uncertainty and final incommensurate potential strength. The mutation generated at $\Delta_\mathrm{f} = 2$ shows that a QPT still exists in the entropic uncertainty of the system even if the period parameter is a rational number. This may mean that there are still many more QPTs. In Fig. 5(b), we explore the relation between entropic uncertainty and entanglement of formation in the periodic case of rational numbers. The blue line indicates the sum of entropic uncertainty and entanglement of formation. The sum is constant at different final incommensurate potential strengths, which means entropic uncertainty and entanglement of formation still have full anti-correlations. The red line indicates the expected value of the entanglement of formation. The decreasing trend shows that the higher final incommensurate potential strength environment leads to lower quantum entanglement in the system. The mutation at $\Delta_\mathrm{f} = 2$ suggests that a QPT also exists for the quantum entanglement of the system.

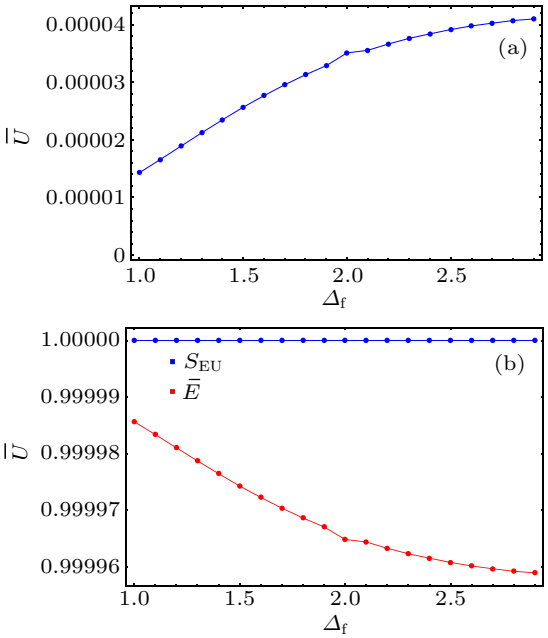


Fig. 5. (a) The expected value of the entropic uncertainty of the AA model with a total number of lattice sites $L = 200$ and $\Delta_\mathrm{i} = 1.1$. (b) The red line indicates the expected value of the entanglement of formation with a total number of lattice sites $L = 200$ and $\Delta_\mathrm{i} = 1.1$; the blue line indicates the sum of the expected values of the entanglement of formation and entropic uncertainty.

4. Conclusions

In summary, we have studied entropic uncertainty, quantum entanglement and quantum steering in the AA model, and shown that the properties of system change significantly when the driving parameter crosses the phase transition points. For the AA model, we find that the change gradient of entropic uncertainty, quantum entanglement and quantum steering increases dramatically as the driving parameters approach the phase transition point. These phenomena prove that we can detect QPTs by means of quantum information. Furthermore, we find that there is an anticorrelation relationship between entropic uncertainty and concurrence and entanglement of formation in the AA model, where the entropic uncertainty and entanglement of formation are fully anticorrelation relations. Finally, we find that when the period parameter is rational number, entropic uncertainty and quantum entanglement still have a weak mutation. Thereby, it is claimed that the current investigation provides new avenues to detect QPTs in terms of quantum information technologies.

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