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Non-Gaussian quantum states generated via quantum catalysis and their statistical properties

Xiao-Yan Zhang(张晓燕)[1], Chun-Yan Yang(杨春燕)[2], Ji-Suo Wang(王继锁)[1,†], and Xiang-Guo Meng(孟祥国)[3,‡]

[1] *Shandong Provincial Key Laboratory of Laser Polarization and Information Technology, School of Physics and Physical Engineering, Qufu Normal University, Qufu* 273165, *China*

[2] *Shandong National Secondary Vocational School, Qingzhou* 262500, *China*

[3] *School of Basic Medical Sciences, Weifang Medical University, Weifang* 261000, *China*

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A new kind of non-Gaussian quantum catalyzed state is proposed via multiphoton measurements and two-mode squeezing as an input of thermal state. The characteristics of the generated multiphoton catalysis output state depends on the thermal parameter, catalyzed photon number and squeezing parameter. We then analyze the nonclassical properties by examining the photon number distribution, photocount distribution and partial negativity of the Wigner function. Our findings indicate that nonclassicality can be achieved through the implementation of multiphoton catalysis operations and modulated by the thermal parameter, catalyzed photon number and squeezing parameter.

Keywords: two-mode squeezing, multiphoton catalysis, nonclassicality, Wigner function

PACS: 03.65.–w, 05.30.–d, 42.50.–p **DOI:** 10.1088/1674-1056/ad2a74

1. Introduction

Non-Gaussian quantum states with nonclassical properties play an important role in the fields of quantum optics and quantum information.[1,2] It is well known that different optical fields can exhibit various nonclassical properties of light.[3,4] Thus, the generation and modulation of these states has emerged as an important subject with immense potential for applications in quantum metrology,[5,6] quantum information processing[7,8] and quantum communication.[9,10] In recent times, there has been an increasing focus on the quantum theory and its applications in the pursuit of non-Gaussian quantum states with nonclassical characteristics.[11–14] For instance, Agarwal and Tara first proposed the concept of a multiphoton excited coherent state by implementing photon subtraction operations in coherent states.[15] Additionally, an approach to experimentally generate single-photon-added thermal states has been suggested.[16] The non-Gaussian quantum states of a multimode light field were demonstrated by removing a single photon in a mode-selective manner from a Gaussian state. The interplay between non-Gaussianity and quantum entanglement has also been investigated.[17] Moreover, by virtue of the nonlinear optical process of parametric down-conversion, the entanglement of continuous-variable states has been achieved through a superposition of number-conserving operations. The application of a coherent superposition operation serves to enhance both entanglement and the fidelity of quantum teleportation.[18,19]

On the other hand, multiphoton catalysis has become another prominent non-Gaussian operation, distinct from the process of photon subtraction, photon addition and their coherent superposition. The quantum optical catalysis scheme was first proposed, and the preparation and characterization of coherent superposition states via conditional measurements on a beam splitter were then reported.[20] Bartley *et al.* successfully generated nonclassical multiphoton states by using a beam splitter to perform single-photon quantum catalysis.[21] Interestingly, similar to a beam splitter, nonlinear manipulation of the two-mode squeezing process can enhance nonclassicality and entanglement. By combining quantum catalysis and a parametric-down amplifier operated on the two-mode squeezing vacuum state, Laguerre-polynomial-weighted squeezed vacuum states have been generated and their entanglement properties were investigated.[22] Notably, these types of multiphoton catalysis operations have also been employed to enhance self-referenced continuous-variable quantum key distribution[23] and have been useful for phase estimation and phase sensitivity in quantum optical interferometry.[24]

Inspired by previous research, this article introduces a theoretical scheme for generating non-Gaussian quantum states by utilizing multiphoton catalysis and two-mode squeezing, with the thermal state as the input. We demonstrate that the performance of the multiphoton catalysis operation can be significantly improved by increasing the number of catalyzed photons and the squeezing parameter, both of which rely on the improvement of the input states. The organization of this paper is as follows: Section 2 presents the proposed scheme for generating non-Gaussian quantum states and evaluates the success probabilities of detection using multiphoton

†Corresponding author. E-mail: jswang@qfnu.edu.cn

‡Corresponding author. E-mail: mengxiangguo1978@sina.com

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catalysis and two-mode squeezing. Sections 3–5 provide a detailed analysis of the nonclassical properties of the output states produced by multiphoton catalysis, examining photon number distribution (PND), photocount distribution and the negativity of the Wigner function (WF). The main results of this study are summarized in Section 6.

2. Non-Gaussian quantum state

In Fig. 1, we present a model for generating the non-Gaussian quantum state via multiphoton catalysis and two-mode squeezing. Specifically, an arbitrary state $|\varphi\rangle_{\rm in}$ and a Fock state $|m\rangle_a$ are respectively sent to one of the input ports and a two-mode squeezed operation, then detected on the Fock state $\langle m|$ at another output port. We finally obtain the output state after performing the quantum catalysis operation on $|\varphi\rangle_{\rm in}$, which is of the form

$$|\psi\rangle_{\rm out} = D_m\, {}_a\langle m|\, S_2(r)\, |m\rangle_a\, |\varphi\rangle_{\rm in}\,, \tag{1}$$

where D_m is the normalization factor determined by $\mathrm{tr}\,\rho_{\rm out} = 1$ and $S_2(r)$ is a two-mode squeezed operation with the squeezing r. Using the unnormalized coherent state ($\|\alpha\rangle = \exp(\alpha a^\dagger)|0\rangle$) representation of $|m\rangle$ and the normal-ordering products of the operators $S_2(r)$ and $\mathrm{e}^{\lambda b^\dagger b}$, we thus obtain

$$\begin{aligned} &{}_a\langle m|\, S_2(r)\, |m\rangle_a \\ &= \frac{(-\operatorname{sech} r)^m}{m!} :\mathrm{H}_{m,m}\left(b^\dagger \sinh r, b\sinh r\right): \mathrm{e}^{\left(b^\dagger b+1\right)\ln\operatorname{sech} r} \\ &\equiv S_m(r), \end{aligned} \tag{2}$$

where : : means normal ordering and $\mathrm{H}_{m,m}(x,y)$ refers to a two-variable Hermite polynomial. Indeed, the process of quantum catalysis can be comprehended as the quantum catalysis operator $S_m(r)$ acting on an input state to produce the catalyzed state as output; that is $|\psi\rangle_{\rm out} \to S_m(r)\,|\varphi\rangle_{\rm in}$. Combining Eqs. (1) and (2), we give the output state $|\psi\rangle_{\rm out}$ as

$$|\psi\rangle_{\rm out} = D_m S_m(r)\,|\varphi\rangle_{\rm in}\,. \tag{3}$$

Considering a thermal state $\rho_{\rm th} = (1-\mathrm{e}^{\lambda})\,\mathrm{e}^{\lambda a^\dagger a}$ as an input state, where $\lambda = -\hbar\omega/(k_{\rm B}T)$, ω, $k_{\rm B}$ and T are respectively the oscillation frequency, Boltzmann constant and thermodynamic temperature of the thermal field. Using the operator identity $\mathrm{e}^{\lambda a^\dagger a} = \mathrm{e}^{-\lambda}{:}\exp[(1-\mathrm{e}^{-\lambda})a^\dagger a]{:}$, ⋮ ⋮ refers to antinormal ordering.[25] Further, using Eqs. (2) and (3), the multiphoton catalysis output state $\rho_{\rm out}$ is obtained as

$$\begin{aligned} \rho_{\rm out} &= {}_a\langle m|\, S_m(r)\, |m\rangle_a\, \rho_{\rm th}\, {}_a\langle m|\, S_m(r)\, |m\rangle_a \\ &= \frac{(1-\mathrm{e}^{\lambda})\,\mathrm{e}^{-\lambda}\,(\operatorname{sech} r)^{2m+1}}{(m!)^2} \\ &\quad \times :\mathrm{H}_{m,m}\left(b^\dagger\sinh r, b\sinh r\right): \\ &\quad \times \mathrm{e}^{b^\dagger b\ln\operatorname{sech} r}{:}\exp\left[\left(1-\mathrm{e}^{-\lambda}\right)b^\dagger b\right]{:} \\ &\quad \times \mathrm{e}^{b^\dagger b\ln\operatorname{sech} r} :\mathrm{H}_{m,m}\left(b^\dagger\sinh r, b\sinh r\right): . \end{aligned} \tag{4}$$

Finally, using the completeness of coherent states and the identity $g^{b^\dagger b}|z\rangle = \exp[(g^2-1)\,|z|^2/2]\,|gz\rangle$, the state $\rho_{\rm out}$ reads

$$\begin{aligned} \rho_{\rm out} &= \mathcal{N}_{\rm c}{:}\sum_{n=0}^{m}\mathrm{H}_{m,m-n}\left(b^\dagger\varkappa, b\sigma\right)\mathrm{H}_{m,m-n}\left(b\varkappa, b^\dagger\sigma\right) \\ &\quad \times \exp\left[\left(\mathrm{e}^{\lambda}\operatorname{sech}^2 r-1\right)b^\dagger b\right]: , \end{aligned} \tag{5}$$

where

$$\mathcal{N}_{\rm c} = \frac{\left(1-\mathrm{e}^{\lambda}\right)\mathrm{e}^{n\lambda}\tanh^{2n} r\operatorname{sech}^{2m+1} r}{n!\,[(m-n)!]^2}, \tag{6}$$

$\varkappa = \sinh r$, $\sigma = \mathrm{e}^{\lambda}\operatorname{sech} r\tanh r$. Obviously, $\rho_{\rm out}$ is a typical non-Gaussian quantum state in terms of the original definition of two-variable Hermite polynomials.

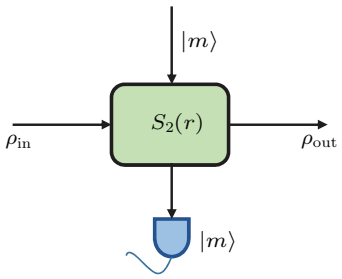


Fig. 1. Scheme for the quantum catalysis model of generating a non-Gaussian quantum state with a thermal state and an m-photon Fock state as inputs on two-mode squeezed device $S_2(r)$ with squeezing parameter r, the heralded output state at one output port can be achieved with conditional measurement at the other output port.

Now we give the normalization factor $\mathcal{N}_{\rm c}$ that can characterize the success probability of the state $\rho_{\rm out}$. Using the normalization condition $\mathrm{tr}\,\rho = 1$, the differential form of the six-variable special polynomial $\mathcal{F}_{n,m,l,k}(x,y,x',y';\nu,\upsilon)$ and its generating functions,[26] thus the success probability of detecting m photons in the optical field is

$$P_m = \mathfrak{N}_{\rm c}\sum_{n=0}^{m}\mathcal{O}_{m,m-n,m,m-n}\left(0,0,0,0;\frac{\varkappa^2}{1-\mathrm{e}^{\lambda}},\frac{\sigma^2}{1-\mathrm{e}^{\lambda}}\right), \tag{7}$$

$\mathfrak{N}_{\rm c} = \mathcal{N}_{\rm c}\left(\frac{1-\mathrm{e}^{\lambda}}{1-\mathrm{e}^{\lambda}\operatorname{sech}^2 r}\right)^{2m-n}$. Obviously, the detection probability is only related to six-variable special polynomials, which is significant for further illustrating the nonclassical properties of the multiphoton catalysis output state $\rho_{\rm out}$.

We plot the success probability of photon detection as a function of both the average thermal photon number $\overline{n}$ and the squeezing parameter r for various catalyzed photon number ($m = 0,1,2,3$) in Fig. 2. As depicted in Fig. 2, the success probability decreases with an increase in both the catalyzed photon number m and squeezing parameter r. For the large average thermal photon number $\overline{n}$, the success probability of detection changes very slightly and is more pronounced in regions with smaller values of squeezing r.

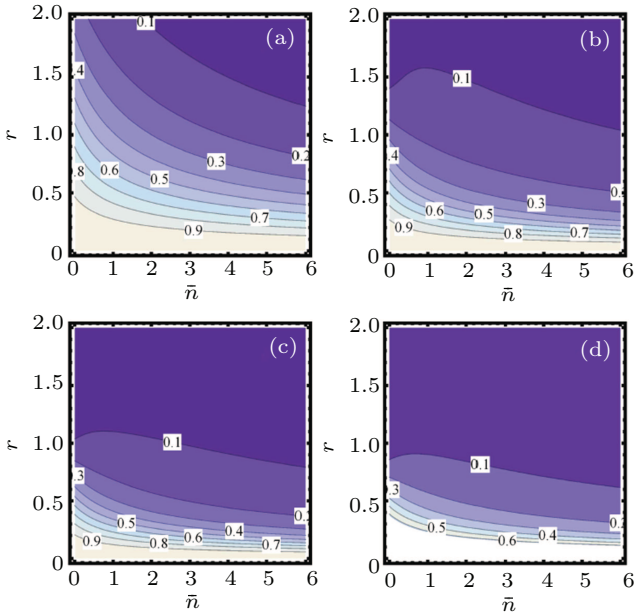


Fig. 2. The success probability P_m of detecting photons as functions of the average thermal photon number $\bar{n}$ and the squeezing parameter r (a)–(d) for catalyzed photon number $m = 0, 1, 2, 3$, respectively.

3. Photon number distribution

We consider the PND of multiphoton catalysis output state via thermal state inputting, which is the probability of finding k photons in the optical field. Using the coherent state representation of the Fock state, the PND can be given by

$$\begin{aligned} &P(k) \\ &= \sum_{n=0}^{m} \frac{\mathfrak{N}_{\rm c}\mathcal{N}_{\rm c}}{k!} \frac{\partial^{2k}}{\partial\alpha'^{*k}\partial\alpha^{k}} \frac{\partial^{2m-n}}{\partial s^{m}\partial t^{m-n}} \frac{\partial^{2m-n}}{\partial s'^{m}\partial t'^{m-n}} \\ &\times \exp\left(-st - s't' + {\rm e}^{\lambda}\,{\rm sech}^2 r\alpha'^{*}\alpha - \left|\alpha'\right|^2/2 + |\alpha|^2/2\right. \\ &\left. + s\alpha'^{*}\varkappa + t\alpha\sigma + s'\alpha\varkappa + t'\alpha'^{*}\sigma\right)\big|_{s=t=s'=t'=\alpha=\alpha'^{*}=0}. \end{aligned} \tag{8}$$

In Fig. 3, we display the probability $P(k)$ as a function of photons k for catalyzed photon number $m = 1, 2$ with several different average thermal photon number $\bar{n}$ and squeezing parameter r. From Fig. 3 we can find that in the range of several photon numbers the oscillation of $P(k)$ can appear and for others the range is almost zero. As the catalyzed photon number m and the average thermal photon number $\bar{n}$ decrease, the probability $P(k)$ increases, reaching its maximum value when the number of photons k is small. As the squeezing parameter r increases, the probability $P(k)$ decreases and the peak value moves to a larger number of photons k. The non-Gaussian probability distribution is a consequence of the relationship between the output state $\rho_{\rm out}$ of multiphoton catalysis and the two-variable Hermite polynomials.

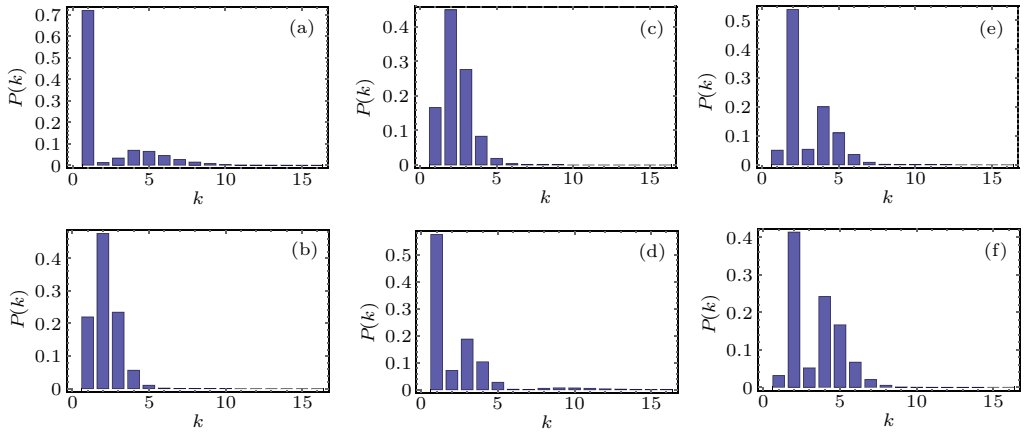


Fig. 3. The PND $P(k)$ as a function of k with several different values of $(m, r, \bar{n})$: (a) $(1, 0.8, 2)$, (b) $(1, 1.6, 2)$, (c) $(1, 1.6, 5)$, (d) $(2, 0.8, 2)$, (e) $(2, 1.6, 2)$, (f) $(2, 1.6, 5)$.

4. Photocount distribution

The concept of quantum photocount distribution was first introduced by Kelley and Kleiner,[27] describing the probability distribution of registering n photoelectrons within an interval. We employ multi-variable special polynomials to analytically investigate the photocount distribution of the multiphoton catalysis output state.

In terms of the reduced density operator ρ of the optical field, the new formula of calculating the photocount distribution can be defined as[28]

$$\begin{aligned} \mathcal{P}(\varpi) = &\left(\frac{\xi}{\xi-1}\right)^{\varpi} \int \frac{{\rm d}^2 z}{\pi}\, {\rm e}^{-\xi|z|^2} {\rm L}_{\varpi}\left(|z|^2\right) \\ &\times \left\langle \sqrt{1-\xi}z \middle| \rho \middle| \sqrt{1-\xi}z \right\rangle, \end{aligned} \tag{9}$$

where ξ is the quantum efficiency of the detector, the photocount distribution is related to the average value $\langle\sqrt{1-\xi}z|\rho|\sqrt{1-\xi}z\rangle$, which builds the relation from the Q distribution function to the photocount distribution. Considering that the average value of the density operator $\rho_{\rm out}$ is

$$\begin{aligned} &\left\langle \sqrt{1-\xi}z \middle| \rho_{\rm out} \middle| \sqrt{1-\xi}z \right\rangle \\ &= \sum_{n=0}^{m} D_m^{-1} \mathcal{N}_{\rm c} \exp\left[\left({\rm e}^{\lambda}\,{\rm sech}^2 r - 1\right)(1-\xi)\,|z|^2\right] \\ &\times {\rm H}_{m,m-n}\left(\sqrt{1-\xi}z^{*}\varkappa, \sqrt{1-\xi}z\sigma\right) \\ &\times {\rm H}_{m,m-n}\left(\sqrt{1-\xi}z\varkappa, \sqrt{1-\xi}z^{*}\sigma\right), \end{aligned} \tag{10}$$

and using the relations between the mth-order Laguerre polynomials and the two-variable Hermitian polynomials $\mathrm{L}_m(|z|^2) = \frac{(-1)^m}{m!}\mathrm{H}_{m,m}(z,z^*)$, we obtain the photocount distribution $\mathcal{P}(\varpi)$ as

$$\begin{aligned}\mathcal{P}(\varpi) = & \left(\frac{\xi}{\xi-1}\right)^{\varpi}\sum_{n=0}^{m}\frac{\mathcal{N}_{\mathrm{c}}D_m^{-1}(-1)^{\varpi}}{\varpi!\left[1-\mathrm{e}^{\lambda}\operatorname{sech}^2 r(1-\xi)\right]}\\ &\times\frac{\partial^{2m-n}}{\partial s^m\partial t^{m-n}}\frac{\partial^{2m-n}}{\partial s'^m\partial t'^{m-n}}\frac{\partial^{2\varpi}}{\partial\gamma^{\varpi}\partial\varepsilon^{\varpi}}\exp\left(-st-s't'-\gamma\varepsilon\right)\\ &\times\exp\left[\frac{[\sqrt{1-\xi}\,(t\sigma+s'\varkappa)+\gamma]\cdot[\sqrt{1-\xi}\,(s\varkappa++t'\sigma)+\varepsilon]}{1-\mathrm{e}^{\lambda}\operatorname{sech}^2 r(1-\xi)}\right]\Bigg|_{s=t=s'=t'=\gamma=\varepsilon=0}.\end{aligned}\tag{11}$$

Obviously, for $\xi = 1$, the photocount distribution $\mathcal{P}(\varpi)$ simplifies to the PND of the multiphoton catalysis output state. In Fig. 4, we plot the photocount distribution $\mathcal{P}(\varpi)$ as a function of the detected photons number ϖ for different values of m, $\bar{n}$, r and ξ. It can be seen from Fig. 4 that the photocount distributions $\mathcal{P}(\varpi)$ are prominent only for certain photon counting numbers ϖ. For a fixed $\xi = 0.5$, with the increase of m and $\bar{n}$ the maximum probability of counting photons decreases, while the position of the maximum value remains in the range of smaller average thermal photon number $\bar{n}$. However, with the increase of r the maximum probability of counting photons increases [refer to Figs. 4(e) and 4(f)]. The oscillatory behavior becomes more pronounced with increase in either m or r, indicating that multiphoton catalysis has a more enhancing effect on nonclassical features compared to single-photon catalysis. Thus, the photocount distribution and nonclassical features can be modulated by an average thermal photon number, catalyzed photon number and squeezing parameter.

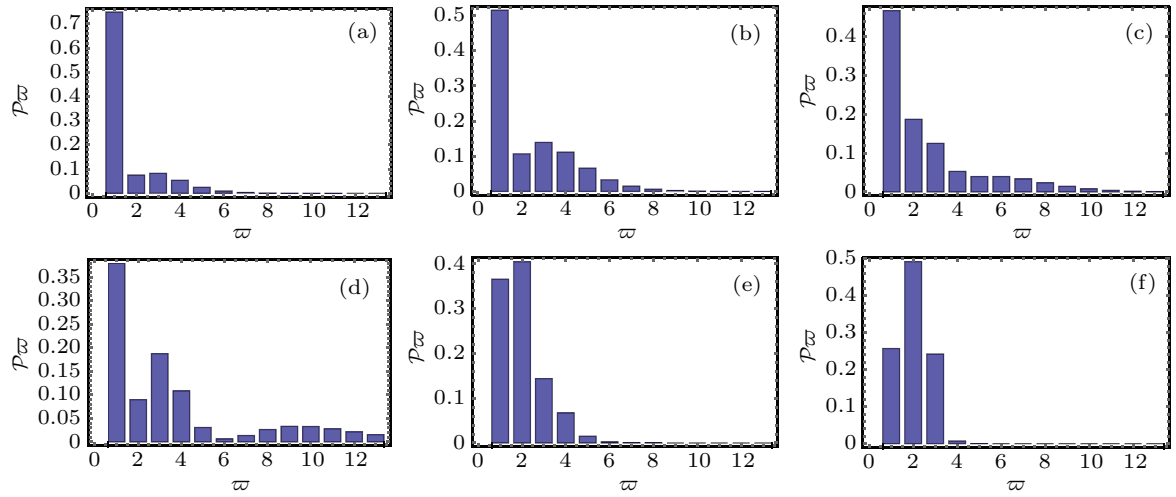


Fig. 4. The photocount distribution $\mathcal{P}(\varpi)$ as a function of the detected photon number ϖ with several different values of $(m,\bar{n},\xi,r)$: (a) $(1,2,0.5,0.8)$, (b) $(1,5,0.5,0.8)$, (c) $(2,5,0.5,0.8)$, (d) $(2,5,0.9,0.8)$, (e) $(2,2,0.5,1.6)$, (f) $(2,2,0.5,3)$.

5. Wigner function

In quantum optics, a roundabout yet efficacious method for illustrating quantum states revolves around the characteristics of their WFs in phase space. This is due to the fact that the nonclassical features of quantum states can be quantitatively assessed through the partial negativity of the WF. Therefore, in this section, we will examine the partial negativity of the WF by utilizing operator ordering products of ρ_{out}. Using the coherent state representation of the Wigner operator $\Delta(\alpha,\alpha^*)$,[29,30] the WF of the state ρ is characterized as $W(\alpha) = \mathrm{tr}[\Delta(\alpha,\alpha^*)\rho]$. Thus, through numerical analysis, we have obtained the WF of the multiphoton catalysis output state as

$$\begin{aligned}W(\alpha) = & \sum_{n=0}^{m}\frac{D_m^{-1}\mathcal{N}_{\mathrm{c}}}{\pi\left(1+\mathrm{e}^{\lambda}\operatorname{sech}^2 r\right)\mathfrak{f}^{2m-n}}\\ &\times\exp\left[\frac{-2|\alpha|^2\left(1-\mathrm{e}^{\lambda}\operatorname{sech}^2 r\right)}{1+\mathrm{e}^{\lambda}\operatorname{sech}^2 r}\right]\\ &\times\mathfrak{D}_{m,m-n,m,m-n}\left(\mathfrak{g},\mathfrak{h},\mathfrak{g}^*,\mathfrak{h}^*;\frac{-\varkappa^2}{1+\mathrm{e}^{\lambda}},\frac{-\sigma^2}{1+\mathrm{e}^{\lambda}}\right),\end{aligned}\tag{12}$$

where

$$\mathfrak{f} = \frac{1+\mathrm{e}^{\lambda}\operatorname{sech}^2 r}{1+\mathrm{e}^{\lambda}},\quad \mathfrak{g} = \frac{2\alpha^*\sinh r\sqrt{\mathfrak{f}}}{1+\mathrm{e}^{\lambda}\operatorname{sech}^2 r},$$
$$\mathfrak{h} = \frac{2\alpha\,\mathrm{e}^{\lambda}\operatorname{sech} r\tanh r\sqrt{\mathfrak{f}}}{1+\mathrm{e}^{\lambda}\operatorname{sech}^2 r}.$$

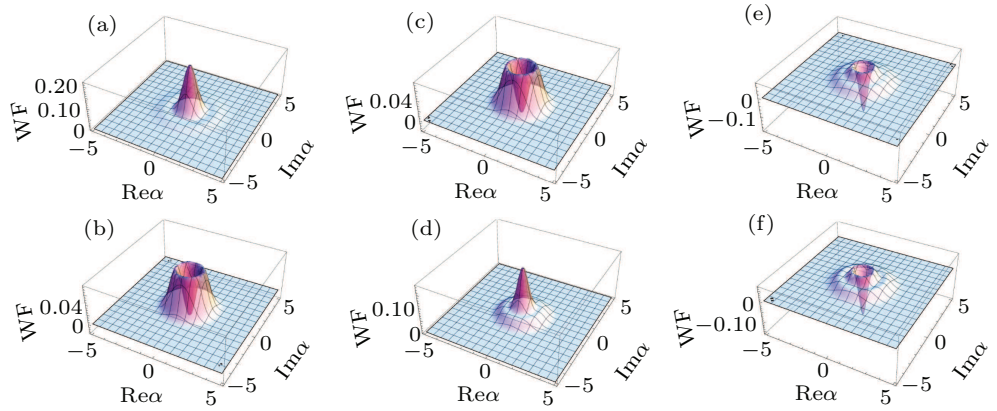


Fig. 5. The WF $W(\alpha)$ as a function of Re and Im for different values of $(m,r,\overline{n})$: (a) $(1,0.8,2)$, (b) $(1,1.6,2)$, (c) $(1,1.6,3)$, (d) $(2,0.8,2)$, (e) $(2,1.6,2)$, (f) $(2,1.6,3)$.

In Fig. 5, we plot the WF of the multiphoton catalysis output state as a function of Reα and Imα for several different values of catalyzed photon number m, average thermal photon number $\overline{n}$ and squeezing r. From Fig. 5 we can see that the structure of the WF can be modulated by the catalyzed photon number, thermal parameter and squeezing parameter. There are distinct regions of negativity within the WF at specific parameter values, indicating the presence of nonclassical features in the quantum state under investigation. Specifically, the partial negative region is larger for $m=2$ than for $m=1$. With a fixed $m=2$, the negative region of the WF becomes larger as $\overline{n}$ and r increase. For equal values of m and r, an increase in $\overline{n}$ causes the negative region to expand. Thus, after performing the multiphoton catalysis operation, the resultant multiphoton catalysis output state ρ_{out} may exhibit a more pronounced nonclassical and non-Gaussian characteristics. These attributes of the output state are ascribed to the multiphoton catalysis operation.

6. Conclusions

In summary, we propose a theoretical scheme for generating non-Gaussian quantum states through the use of multiphoton measurements and two-mode squeezing, with a thermal state as the initial input. We discuss the probabilities of successful detection of the proposed state in relation to the normalization factor. Additionally, we extensively analyze both the PND and the photocount distribution based on two-variable Hermite polynomials. Furthermore, the nonclassical behavior of the output state resulting from multiphoton catalysis is investigated using multi-variable special polynomials, with the negativity of the WF serving as a quantifier. Our findings indicate that the success probability for the output state of multiphoton catalysis increases as the number of catalyzed photons m decreases and can be further enhanced within a region characterized by a smaller squeezing r. We also observe that fixing the catalyzed photon number results in an expanded negative region of the WF when both the thermal and squeezing parameters are increased, reflecting a substantial enhancement in the nonclassicality of the states. This nonclassicality is contingent upon the modulation of the thermal parameter, catalyzed photon number and squeezing parameter of the two-mode squeezed operator.

We conclude that employing multiphoton catalysis operation with the two-mode squeezing transform is beneficial in enhancing the nonclassicality of quantum states. Such operations facilitate the preparation of highly nonclassical and non-Gaussian quantum states from various initial conditions. Moreover, quantum catalysis operations can impart non-Gaussianity and also improve the fidelity of quantum teleportation as well as the entanglement of quantum states. Our research findings offer theoretical references for forthcoming experimental investigations. In 2001, Kniil, Laflamme and Milburn discovered a technique using only linear optical devices, single-photon sources and detectors to generate nonlinear processes. For the entire quantum catalysis system, although there is no addition or subtraction of photons, the quantum catalysis exhibits significant nonlinearity, thereby presenting an effective method for preparing non-Gaussian quantum states. Our current work provides a comprehensive theoretical analysis, suggesting that the proposed scheme holds potential applications in the realms of quantum computing and quantum metrology.

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