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Effects of cross-Kerr coupling on transmission spectrum of double-cavity optomechanical system

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We theoretically study the transmission spectrum of the cavity field in a double-cavity optomechanical system with cross-Kerr (CK) effect. The system consists of two tunneling coupling optomechanical cavities with a mechanical resonator as a coupling interface. By doping CK medium into the mechanical resonator, CK couplings between the cavity fields and the mechanical resonator are introduced. We investigate the effects of CK coupling strength on the transmission spectrum of the cavity field, including the transmission rate, nonreciprocity and four-wave mixing (FWM). We find that the transmission spectrum of the probe field can show two obvious transparent windows, which can be widened by increasing the CK coupling strength. For the transmission between the two cavity fields, the perfect nonreciprocity and reciprocity are present and modulated by CK coupling and phase difference between two effective optomechanical couplings. In addition, the effects of the optomechanical and CK couplings on FWM show that the single peak of FWM is split into three symmetrical peaks due to the introduction of the CK effect.

Keywords: cross-Kerr, transmission rate, nonreciprocity, four-wave mixing

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1. Introduction

In recent decades, as an important platform of light–matter interaction, cavity optomechanical systems have received wide attention due to their potential application in quantum communication and networking.[1–3] The different significant physical phenomena originating from the optomechanical coupling between an optical cavity and a mechanical resonator (MR) have been made in both theory and experiment, including slowing and storage of the light pulse,[4,5] perfect nonreciprocity,[6,7] information processing,[8,9] and optomechanical induced transparency (OMIT),[10–12] etc.

A cavity optomechanical system also provides a rich platform to study nonlinear optical effects.[13–15] Recently, the cross-Kerr (CK) interaction has been introduced into the optomechanical system to enhance nonlinear coupling.[16] The CK effect is a nonlinear optical effect that describes the interaction between two optical fields. Namely, when two optical fields pass through a medium with Kerr nonlinearity, one of them affects the refractive index of the other.[15,16] The CK interaction between two bosonic modes can be obtained by coupling these two modes to a common medium.[17] This interaction can be used to achieve photon blocking, phase modulation, and optical lattice.[18–20] In a two-tone driven cavity optomechanical system, the CK effect can induce the more stable behaviors of the average photon number in the cavity.[16,21] In addition, it can not only enhance the effective optomechanical coupling but can also cause strong nonlinearity.[22] As a typical nonlinear effect, four-wave mixing (FWM) has been widely studied and modulated in different optomechanical systems.[9,14,15,23] The optomechanically induced FWMs have been presented to generate strong and robust quantum entanglement,[24] optical amplifier,[25] optical switching and frequency conversion,[26,27] optical wavelength converter,[28,29] and so on. In addition, FWM has potential applications in quantum information processing and quantum communication.[30]

In such a rich platform, the nonreciprocity of optical transmission was studied based on optomechanical coupling. The basic principle of nonreciprocity in the optomechanical system is to explicitly break the time reversal symmetry by using the optomechanical interaction. Nonreciprocal responses were theoretically presented in the ring resonator,[31] and have recently been observed in experiment.[32–35]

OMIT, FWM, and nonreciprocity, which are important components of cavity output spectrum, have been seldom and simultaneously studied in an optomechanical cavity with the CK effect, especially the effects of CK coupling on them. In this work, we study the output spectrum of the optomechanical cavity including OMIT, nonreciprocity, and FWM in a double-cavity optomechanical system, where as an optomechanical

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coupling interface, the MR is coupled to the two cavity fields via the radiation pressure and the CK effect. The effects of the CK coupling strength and main system parameters on the OMIT, nonreciprocity, and FWM of the output field are shown. Such an effective scheme of CK coupling enhanced nonlinear optical effects is theoretically presented.

This paper is organized as follows. In Section 2, we present and introduce the model, including the Hamiltonian and dynamical equations of the system. In Section 3, we investigate the effects of CK coupling strength on the transmission spectrum of the system including the transmission rate and nonreciprocity. In Section 4, we discuss in detail the effects of the optomechanical coupling and CK coupling strength on FWM. Finally, a summary is given in Section 5.

2. Theoretical model and equations

The optomechanical system that we discuss here consists of an optical cavity a_1 tunneling coupled to another optical cavity a_2 with the tunneling coupling strength J by an MR as an interface, as shown in Fig. 1(a). The two cavity fields with the frequency ω_{0i} $(i=1,2)$ are simultaneously coupled to the MR via radiation pressure force, i.e., optomechanical coupling. By doping CK medium into the MR, the CK interaction with the coupling strength $g_{\rm ck}$ can be achieved. CK medium may be natural or artificial atoms[17] (e.g., superconducting qubits in the microwave domain or atoms with transition frequencies in the optical frequency domain). As shown in Fig. 1(b), three different couplings are present. To drive two cavity fields, the strong coupling field $\varepsilon_{\rm c1}$ $(\varepsilon_{\rm c2})$ with the frequency $\omega_{\rm c1}$ $(\omega_{\rm c2})$ and weak probe field $\varepsilon_{\rm p1}$ $(\varepsilon_{\rm p2})$ with the frequency $\omega_{\rm p1}$ $(\omega_{\rm p2})$ are injected into the cavity a_1 (a_2), respectively. Thus, the total Hamiltonian of the system can be determined by $H=H_0+H_{\rm I}+H_{\rm d}$.

The free Hamiltonian H_0 can be written as

$$H_0=\hbar\omega_{01}a_1^\dagger a_1+\hbar\omega_{02}a_2^\dagger a_2+\hbar\omega_{\rm m}b^\dagger b, \tag{1}$$

where $a_i^\dagger$ (a_i) with $i=1$ and 2 is the creation (annihilation) operator of the i-th optical cavity mode, and $b^\dagger$ (b) is the creation (annihilation) operator of phonon mode of MR. H_0 describes the free energies of the total system.

The interaction Hamiltonian $H_{\rm I}$ can be described by

$$\begin{aligned}H_{\rm I}=&\sum_{i=1,2}\hbar g_i a_i^\dagger a_i\left(b^\dagger+b\right)\\&+\sum_{i=1,2}\hbar g_{\rm ck}a_i^\dagger a_i b^\dagger b+\hbar J\left(a_1^\dagger a_2+a_2^\dagger a_1\right),\end{aligned} \tag{2}$$

where g_i $(i=1,2)$ is the optomechanical coupling intensity between the cavity mode a_i and the mechanical mode b. The first term is the optomechanical interactions between the i-th cavity fields and MR. The second term is introduced by the CK effect. The third term shows the tunneling interaction between two cavity fields.

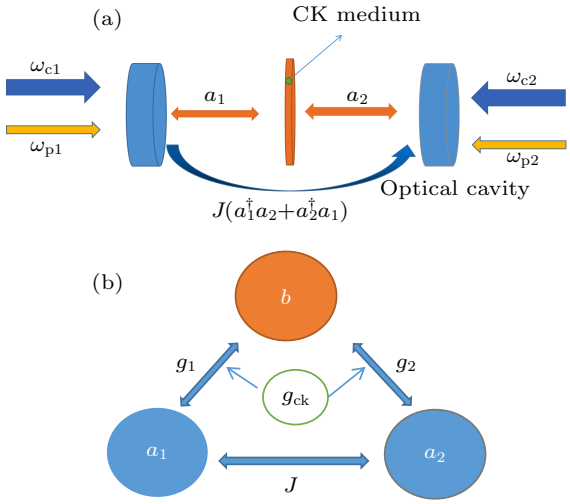


Fig. 1. Schematic diagram of a double-cavity optomechanical system. (a) It is composed of two tunneling coupling optical cavities and an MR. Two strongly (weak) coupled fields (probe fields) with amplitudes of $\varepsilon_{\rm c1}$ and $\varepsilon_{\rm c2}$ ($\varepsilon_{\rm p1}$ and $\varepsilon_{\rm p2}$) are injected into two cavities, respectively. CK effect is introduced by doping CK medium into MR. The two cavity fields are coupled to the MR by the optomechanical and CK interactions. (b) Equivalent diagram of (a).

The driving Hamiltonian $H_{\rm d}$ represents the driving interactions of two optical cavity fields and can be written as

$$\begin{aligned}H_{\rm d}=&\,{\rm i}\hbar\varepsilon_{\rm c1}\left(a_1^\dagger {\rm e}^{-{\rm i}\omega_{\rm c1}t}-a_1\,{\rm e}^{{\rm i}\omega_{\rm c1}t}\right)\\&+{\rm i}\hbar\varepsilon_{\rm p1}\left(a_1^\dagger {\rm e}^{-{\rm i}\omega_{\rm p1}t}-a_1\,{\rm e}^{{\rm i}\omega_{\rm p1}t}\right)\\&+{\rm i}\hbar\varepsilon_{\rm c2}\left(a_2^\dagger {\rm e}^{-{\rm i}\omega_{\rm c2}t}-a_2\,{\rm e}^{{\rm i}\omega_{\rm c2}t}\right)\\&+{\rm i}\hbar\varepsilon_{\rm p2}\left(a_2^\dagger {\rm e}^{-{\rm i}\omega_{\rm p2}t}-a_2\,{\rm e}^{{\rm i}\omega_{\rm p2}t}\right).\end{aligned} \tag{3}$$

After the rotating frame with the coupling frequency ω_{ci} $(i=1,2)$,[36] the total Hamiltonian can be rewritten as

$$\begin{aligned}H=&\,\hbar\Delta_{\rm c}\left(a_1^\dagger a_1+a_2^\dagger a_2\right)+\hbar\omega_{\rm m}b^\dagger b\\&+\sum_{i=1,2}\hbar g_i a_i^\dagger a_i\left(b^\dagger+b\right)\\&+\sum_{i=1,2}\hbar g_{\rm ck}a_i^\dagger a_i b^\dagger b+\hbar J\left(a_1^\dagger a_2+a_2^\dagger a_1\right)\\&+{\rm i}\hbar\varepsilon_{\rm c1}(a_1^\dagger-a_1)+{\rm i}\hbar\varepsilon_{\rm p1}(a_1^\dagger {\rm e}^{-{\rm i}\delta_{\rm p}t}-a_1\,{\rm e}^{{\rm i}\delta_{\rm p}t})\\&+{\rm i}\hbar\varepsilon_{\rm c2}(a_2^\dagger-a_2)+{\rm i}\hbar\varepsilon_{\rm p2}(a_2^\dagger {\rm e}^{-{\rm i}\delta_{\rm p}t}-a_2\,{\rm e}^{{\rm i}\delta_{\rm p}t}),\end{aligned} \tag{4}$$

where $\Delta_{\rm c}=\Delta_{ci}=\omega_{0i}-\omega_{ci}$ $(i=1,2)$ is the frequency detuning of the i-th cavity field and the corresponding strong coupling field, and $\delta_{\rm p}=\delta_{pi}=\omega_{pi}-\omega_{ci}$ represents the frequency detuning of the i-th weak probe field and the corresponding strong coupling field. By introducing dissipation and fluctuation terms, the nonlinear Heisenberg–Langevin equations of the system can be derived as

$$\begin{aligned}\dot{a}_1=&-\left(\frac{\kappa_1}{2}+{\rm i}\Delta_{\rm c}\right)a_1-{\rm i}g_1a_1\left(b^\dagger+b\right)-{\rm i}g_{\rm ck}a_1b^\dagger b\\&-{\rm i}Ja_2+\varepsilon_{\rm c1}+\varepsilon_{\rm p1}\,{\rm e}^{-{\rm i}\delta_{\rm p}t}+\sqrt{\kappa_1}a_{\rm 1in},\\\dot{a}_2=&-\left(\frac{\kappa_2}{2}+{\rm i}\Delta_{\rm c}\right)a_2-{\rm i}g_2a_2\left(b^\dagger+b\right)-{\rm i}g_{\rm ck}a_2b^\dagger b\end{aligned}$$

$$
\begin{aligned}
&-\mathrm{i}Ja_1+\varepsilon_{\mathrm{c}2}+\varepsilon_{\mathrm{p}2}\,\mathrm{e}^{-\mathrm{i}\delta_{\mathrm{p}}t}+\sqrt{\kappa_2}a_{2\mathrm{in}},\\
\dot{b}\;=&-\left(\frac{\gamma}{2}+\mathrm{i}\omega_{\mathrm{m}}\right)b-\sum_{i=1,2}\mathrm{i}g_i a_i^{\dagger}a_i\\
&-\sum_{i=1,2}\mathrm{i}g_{\mathrm{ck}}a_i^{\dagger}a_i b+\sqrt{\gamma}b_{\mathrm{in}},
\end{aligned}
\tag{5}
$$

where κ_i $(i=1,2)$ and γ are the decay coefficients of the i-th cavity and the MR, respectively. From Eq. (5), it can be seen that there are two main kinds of noise in the system. One is the input noise $a_{i\mathrm{in}}$ of the i-th cavity. The other is the thermal noise b_{in} of mechanical mode. The mean values of the input noise terms are considered to be zero, i.e., $\langle a_{1\mathrm{in}}\rangle=\langle a_{2\mathrm{in}}\rangle=\langle b_{\mathrm{in}}\rangle=0$. Under the Markov approximation, two-time correlation functions of these input noise operators are given by $\langle a_{i\mathrm{in}}(t)a_{i\mathrm{in}}^{\dagger}(t')\rangle=\delta(t-t')$, $\langle b_{\mathrm{in}}^{\dagger}(t)b_{\mathrm{in}}(t')\rangle=n_{\mathrm{th}}\delta(t-t')$ and $\langle b_{\mathrm{in}}(t)b_{\mathrm{in}}^{\dagger}(t')\rangle=(n_{\mathrm{th}}+1)\delta(t-t')$, where the average thermal excitation number[16,37] is $n_{\mathrm{th}}=[\mathrm{e}^{(\frac{\hbar\omega_{\mathrm{m}}}{k_{\mathrm{B}}T})}-1]^{-1}$, here k_{B} is the Boltzmann constant and T is the temperature of the MR. Under the condition of a sufficiently strong coupling field, each operator in Eq. (5) is rewritten as a sum of its steady-state solution plus a fluctuation term by linearization method, i.e., $O=O_{\mathrm{s}}+\delta O$ $(O=a_i,b)$, where the fluctuation term is much smaller than the steady-state solution. By the approximation of $\langle ab\rangle=\langle a\rangle\langle b\rangle$[15,38] and neglecting the second-order small quantities,[39] the steady-state solutions can be written as

$$
\begin{aligned}
a_{1\mathrm{s}}&=\frac{\varepsilon_{\mathrm{c}1}-\mathrm{i}Ja_{2\mathrm{s}}}{\frac{\kappa_1}{2}+\mathrm{i}\Delta_1},\\
a_{2\mathrm{s}}&=\frac{\varepsilon_{\mathrm{c}2}-\mathrm{i}Ja_{1\mathrm{s}}}{\frac{\kappa_2}{2}+\mathrm{i}\Delta_2},\\
b_{\mathrm{s}}&=-\frac{\mathrm{i}g_1|a_{1\mathrm{s}}|^2+\mathrm{i}g_2|a_{2\mathrm{s}}|^2}{\frac{\gamma}{2}+\mathrm{i}\omega},
\end{aligned}
\tag{6}
$$

where $\Delta_i=\Delta_{\mathrm{c}}+g_i(b_{\mathrm{s}}^{*}+b_{\mathrm{s}})+g_{\mathrm{ck}}|b_{\mathrm{s}}|^2$ $(i=1,2)$ is the effective detuning of the i-th cavity due to the optomechanical interaction and CK effect, $\omega=\omega_{\mathrm{m}}+g_{\mathrm{ck}}(|a_{1\mathrm{s}}|^2+|a_{2\mathrm{s}}|^2)$ is the effective mechanical frequency. Then, keeping only the linear terms of fluctuation operators in the interaction picture by introducing $\delta a_i=\delta a_i\,\mathrm{e}^{-\mathrm{i}\Delta_i t}$ and $\delta b=\delta b\,\mathrm{e}^{-\mathrm{i}\omega t}$, the linear quantum Langevin equation of the fluctuation terms can be given as

$$
\begin{aligned}
\delta\dot{a}_1&=-\frac{\kappa_1}{2}\delta a_1-\mathrm{i}G_1\,\mathrm{e}^{\mathrm{i}\theta}\delta b-\mathrm{i}J\delta a_2+\varepsilon_{\mathrm{p}1}\,\mathrm{e}^{-\mathrm{i}xt},\\
\delta\dot{a}_2&=-\frac{\kappa_2}{2}\delta a_2-\mathrm{i}G_2\delta b-\mathrm{i}J\delta a_1+\varepsilon_{\mathrm{p}2}\,\mathrm{e}^{-\mathrm{i}xt},\\
\delta\dot{b}&=-\frac{\gamma}{2}\delta b-G_1\,\mathrm{e}^{-\mathrm{i}\theta}\delta a_1-\mathrm{i}G_2\delta a_2,
\end{aligned}
\tag{7}
$$

where $G_i=(g_i+g_{\mathrm{ck}}b_{\mathrm{s}})\,|a_{i\mathrm{s}}|$ $(i=1,2)$ is the effective optomechanical coupling strength, and $x=\delta_{\mathrm{p}}-\Delta_i$ is the nonlinear detuning. Therefore, we can find that the effective optomechanical coupling strength can be enhanced by the CK coupling g_{ck}, the steady-state phonon number $\langle b_{\mathrm{s}}^{\dagger}b_{\mathrm{s}}\rangle$, and photon number $\langle a_{i\mathrm{s}}^{\dagger}a_{i\mathrm{s}}\rangle$. According to Eq. (6), the phase difference θ between effective optomechanical coupling G_1 and G_2 can be introduced and controlled by adjusting the coupling field $\varepsilon_{\mathrm{c}1}$ and $\varepsilon_{\mathrm{c}2}$.[39] It can be seen from Eq. (7) that θ is a key factor to achieve nonreciprocity, which will be discussed later. To calculate the first order amplitude, we further analyze that

$$
\delta a_i=\delta a_{i+}\,\mathrm{e}^{-\mathrm{i}xt}+\delta a_{i-}\,\mathrm{e}^{\mathrm{i}xt},\quad \delta b=\delta b_{+}\,\mathrm{e}^{-\mathrm{i}xt}+\delta b_{-}\,\mathrm{e}^{\mathrm{i}xt}.
\tag{8}
$$

Here, the parameter $\langle\delta a_{i\pm}\rangle$ $(\langle\delta b_{\pm}\rangle)$ is much smaller than the steady-state value $a_{i\mathrm{s}}$ (b_{s}). By substituting Eq. (8) into Eq. (7), we can obtain

$$
\begin{aligned}
\delta a_{1+}&=\frac{2\,\mathrm{e}^{\mathrm{i}\theta}\left(4G_2^2+\kappa_x\gamma_x\right)\varepsilon_{\mathrm{p}1}-4\,\mathrm{e}^{\mathrm{i}\theta}\left(\mathrm{i}J\gamma_x+2G_1G_2\,\mathrm{e}^{\mathrm{i}\theta}\right)\varepsilon_{\mathrm{p}2}}{-8\mathrm{i}G_1G_2J+4G_1^2\kappa_x\,\mathrm{e}^{\mathrm{i}\theta}+4G_2^2\kappa_x\,\mathrm{e}^{\mathrm{i}\theta}+4J^2\gamma_x\,\mathrm{e}^{\mathrm{i}\theta}+\kappa_x^2\gamma_x\,\mathrm{e}^{\mathrm{i}\theta}-8\mathrm{i}G_1G_2J\,\mathrm{e}^{2\mathrm{i}\theta}},\\
\delta a_{1-}&=\frac{(4G_2^2+\tilde{\kappa}_x\tilde{\gamma}_x)\varepsilon_{\mathrm{p}1}-(\mathrm{i}2J\tilde{\gamma}_x+4G_1G_2\,\mathrm{e}^{\mathrm{i}\theta})\varepsilon_{\mathrm{p}2}}{-4\mathrm{i}G_1G_2J(\mathrm{e}^{-\mathrm{i}\theta}+\mathrm{e}^{\mathrm{i}\theta})+2\tilde{\kappa}_x(G_1^2+G_2^2)+(2J^2+0.5\tilde{\kappa}_x{}^2)\tilde{\gamma}_x}\,\mathrm{e}^{-2\mathrm{i}xt},\\
\delta a_{2+}&=\frac{-4\left(2G_1G_2+\mathrm{i}J\gamma_x\theta^{\mathrm{i}\theta}\right)\varepsilon_{\mathrm{p}1}+2\,\mathrm{e}^{\mathrm{i}\theta}\left(4G_1^2+\kappa_x\gamma_x\right)\varepsilon_{\mathrm{p}2}}{-8\mathrm{i}G_1G_2J+4G_1^2\kappa_x\,\mathrm{e}^{\mathrm{i}\theta}+4G_2^2\kappa_x\,\mathrm{e}^{\mathrm{i}\theta}+4J^2\gamma_x\,\mathrm{e}^{\mathrm{i}\theta}+\kappa_x^2\gamma_x\theta^{\mathrm{i}\theta}-8\mathrm{i}G_1G_2J\,\mathrm{e}^{2\mathrm{i}\theta}},\\
\delta a_{2-}&=\frac{-\left(4G_1G_2\,\mathrm{e}^{-\mathrm{i}\theta}+\mathrm{i}2J\tilde{\gamma}_x\right)\varepsilon_{\mathrm{p}1}+\left(\tilde{\kappa}_x\tilde{\gamma}_x+4G_1^2\right)\varepsilon_{\mathrm{p}2}}{-4\mathrm{i}G_1G_2J(\mathrm{e}^{-\mathrm{i}\theta}+\mathrm{e}^{\mathrm{i}\theta})+2\tilde{\kappa}_x(G_1^2+G_2^2)+(2J^2+0.5\tilde{\kappa}_x{}^2)\tilde{\gamma}_x}\,\mathrm{e}^{-2\mathrm{i}xt},\\
\delta b_{+}&=\frac{-4\left(\mathrm{i}G_1\kappa_x+2JG_2\,\mathrm{e}^{\mathrm{i}\theta}\right)\varepsilon_{\mathrm{p}1}-4\left(2JG_1+\mathrm{i}G_2\kappa_x\,\mathrm{e}^{\mathrm{i}\theta}\right)\varepsilon_{\mathrm{p}2}}{-8\mathrm{i}G_1G_2J+4G_1^2\kappa_x\,\mathrm{e}^{\mathrm{i}\theta}+4G_2^2\kappa_x\,\mathrm{e}^{\mathrm{i}\theta}+4J^2\gamma_x\theta^{\mathrm{i}\theta}+\kappa_x^2\gamma_x\theta^{\mathrm{i}\theta}-8\mathrm{i}G_1G_2J\,\mathrm{e}^{2\mathrm{i}\theta}},\\
\delta b_{-}&=\frac{-(2\mathrm{i}G_1\tilde{\kappa}_x\,\mathrm{e}^{-\mathrm{i}\theta}+4JG_2)\varepsilon_{\mathrm{p}1}-(4JG_1\,\mathrm{e}^{-\mathrm{i}\theta}+2\mathrm{i}G_2\tilde{\kappa}_x)\varepsilon_{\mathrm{p}2}}{-4\mathrm{i}G_1G_2J(\mathrm{e}^{-\mathrm{i}\theta}+\mathrm{e}^{\mathrm{i}\theta})+2\tilde{\kappa}_x(G_1^2+G_2^2)+(2J^2+0.5\tilde{\kappa}_x{}^2)\tilde{\gamma}_x}\,\mathrm{e}^{-2\mathrm{i}xt},
\end{aligned}
\tag{9}
$$

where $\kappa_1=\kappa_2=\kappa$, $\kappa_x=\kappa-2\mathrm{i}x$, $\gamma_x=\gamma-2\mathrm{i}x$, $\tilde{\kappa}_x=\kappa+2\mathrm{i}x$ and $\tilde{\gamma}_x=\gamma+2\mathrm{i}x$. Then, by making use of the standard input–output theory $\langle\varepsilon_i^{\mathrm{out}}\rangle+\varepsilon_{\mathrm{c}i}+\varepsilon_{\mathrm{p}i}\,\mathrm{e}^{-\mathrm{i}\delta_{\mathrm{p}i}t}=\sqrt{\kappa_i}\langle a_i\rangle$,[15] the output field of the i-th cavity can be given by

$$
\langle\varepsilon_i^{\mathrm{out}}\rangle=\left(\sqrt{\kappa_i}a_{i\mathrm{s}}-\varepsilon_{\mathrm{c}i}\right)\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{c}i}t}+\left(\sqrt{\kappa_i}\delta a_{i+}-\varepsilon_{\mathrm{p}i}\right)\mathrm{e}^{-\mathrm{i}(\omega_{\mathrm{c}i}+\delta_{\mathrm{p}i})t}+\sqrt{\kappa_i}\delta a_{i-}\,\mathrm{e}^{-\mathrm{i}(\omega_{\mathrm{c}i}-\delta_{\mathrm{p}i})t}
$$

$$= \underbrace{\left(\sqrt{\kappa_i}a_{is} - \varepsilon_{ci}\right)\mathrm{e}^{-\mathrm{i}\omega_{ci}t}}_{\text{Coupling field}} + \underbrace{\left(\sqrt{\kappa_i}\delta a_{i+} - \varepsilon_{pi}\right)\mathrm{e}^{-\mathrm{i}\omega_{pi}t}}_{\text{Probe field}} + \underbrace{\sqrt{\kappa_i}\delta a_{i-}\,\mathrm{e}^{-\mathrm{i}\left(2\omega_{ci}-\omega_{pi}\right)t}}_{\text{FWM field}}. \tag{10}$$

From Eq. (10), we can find that the output fields contain three frequency components, i.e., the coupling field frequency at ω_{ci}, the probe field frequency ω_{pi}, and one FWM field frequency at $2\omega_{ci}-\omega_{pi}$. FWM is generated by Stokes scattering of optical fields from the optomechanical coupling, i.e., interaction between two coupling photons and a probe photon via the mechanical mode.[6] For the probe frequency field, we can get

$$\varepsilon_{pi}^{\mathrm{out}} + \varepsilon_{pi}^{\mathrm{in}}\,\mathrm{e}^{-\mathrm{i}xt} = \sqrt{\kappa_i}\delta a_i, \tag{11}$$

where $\varepsilon_{pi}^{\mathrm{in}} = \varepsilon_{pi}/\sqrt{\kappa_i}$. Still, following the assumption $\delta O = \delta O_+\,\mathrm{e}^{-\mathrm{i}xt} + \delta O_-\,\mathrm{e}^{\mathrm{i}xt}$, the output fields at the frequency ω_{pi} can be obtained as[39]

$$\varepsilon_{pi+}^{\mathrm{out}} = \sqrt{\kappa_i}\delta a_{i+} - \frac{\varepsilon_{pi}}{\sqrt{\kappa_i}}. \tag{12}$$

3. The transmission rate and nonreciprocity of the cavity field

In this section, we discuss the influence of CK effect on the transmission rate of the probe frequency field and nonreciprocity between two cavity fields. First, in order to study the transmission rate of the probe field ε_{p1}, we set $\varepsilon_{p2}=0$ and investigate the transmission rate of the probe field with the tunneling and effective optomechanical force couplings.[40] Then, we study the nonreciprocity relationship between two cavity fields.

3.1. Transmission rate of the probe field

In order to study the transmission rate of the probe field ε_{p1}, according to the input–output relation of Eq. (12), we can define the transmission rate of probe field ε_{p1} in the first optical cavity as[41] (the case of ε_{p2} in the optical cavity is similar)

$$\left|T_{p_1}\right|^2 = \left|\frac{\varepsilon_{p1}^{\mathrm{out}}}{\varepsilon_{p1}^{\mathrm{in}}}\right|^2 = \left|1 - \frac{\kappa\delta a_{1+}}{\varepsilon_{p1}}\right|^2. \tag{13}$$

For simplicity, by setting $\theta=\pi/2$ and $G_1=G_2=G=(g+g_{ck}|b_s|)|a_s|$ with $g_1=g_2=g$ and $a_{1s}=a_{2s}=a_s$, we can get the transmission rate of ε_{p1} as

$$\left|T_{p_1}\right|^2 = \left|1 - \frac{2\kappa(4G^2+\kappa_x\gamma_x)}{8G^2\kappa + 4J^2\gamma_x + \kappa_x^2\gamma_x}\right|^2. \tag{14}$$

By Eq. (14), we can study the effects of CK coupling strength g_{ck} on the transmission rate of ε_{p1}, as shown in Fig. 2. We can find that there are three dip peaks and two windows for the evolution of $|T_{p_1}|^2$ versus the detuning x with the different CK coupling strengths. Without the CK effect ($g_{ck}=0$), two windows are preliminarily presented, the mechanism of which is derived from quantum interference, i.e., the mechanism of OMIT[10] and tunnel induced transparency, as shown in Fig. 3.[42] OMIT is present due to quantum interference between the transition paths $|n_1,n_2,n_b\rangle \leftrightarrow |n_1+1,n_2,n_b\rangle$ and $|n_1,n_2,n_b+1\rangle \leftrightarrow |n_1+1,n_2,n_b\rangle$, where the transition between $|n_1,n_2,n_b+1\rangle \leftrightarrow |n_1+1,n_2,n_b\rangle$ depends on the optomechanical and CK couplings. The tunneling induced transparency comes from quantum interference between the transition paths $|n_1,n_2,n_b\rangle \leftrightarrow |n_1+1,n_2,n_b\rangle$ and $|n_1+1,n_2,n_b\rangle \leftrightarrow |n_1,n_2+1,n_b\rangle$. In addition, the transmission curve is symmetric with respect to $x=0$. With the increase of CK coupling strength, there are two characteristics. First, the depth of non-absorbing transparent window becomes deeper than the case of $g_{ck}=0$. Second, when g_{ck} increases, the width of the transparent window becomes wider. The reason for these two phenomena is due to the mechanism of OMIT, where the effective optomechanical coupling strength G becomes stronger as the CK coupling strength increases. This is similar to increasing the strong coupling field strength for EIT in atomic systems.

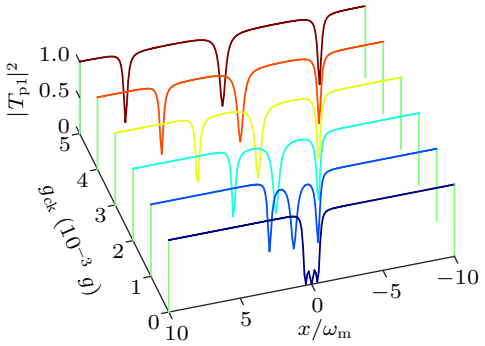


Fig. 2. The transmission rate of $|T_{p_1}|^2$ versus the detuning amount x with the different CK coupling strengths in the case of $\kappa=5$ MHz, $g=1$ kHz, $\omega_m/2\pi=10$ MHz and $\gamma=1$ MHz.[7]

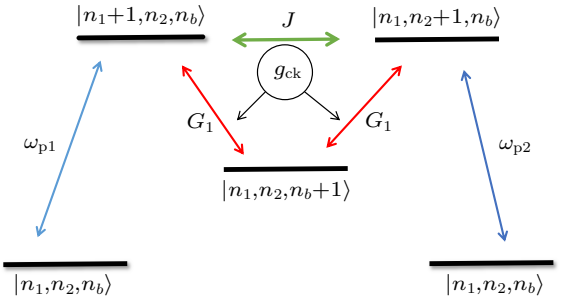


Fig. 3. Energy-level diagram of the hybrid system. The number states of the cavity i-th photons and phonon are denoted by n_i and n_b.

3.2. Non-reciprocal relationship

According to Eq. (12), perfect optical nonreciprocity relationship can be achieved, when the transmission amplitude T_{ij} meets the following conditions:[7]

$$T_{12} = \left|\frac{\varepsilon_{p2+}^{\mathrm{out}}}{\varepsilon_{p1}^{\mathrm{in}}}\right|_{\varepsilon_{p2}^{\mathrm{in}}=0} = 1, \quad T_{21} = \left|\frac{\varepsilon_{p1+}^{\mathrm{out}}}{\varepsilon_{p2}^{\mathrm{in}}}\right|_{\varepsilon_{p1}^{\mathrm{in}}=0} = 0, \tag{15}$$

or

$$T_{12}=\left|\frac{\varepsilon_{\mathrm{p2}+}^{\mathrm{out}}}{\varepsilon_{\mathrm{p1}}^{\mathrm{in}}}\right|_{\varepsilon_{\mathrm{p2}}^{\mathrm{in}}=0}=0,\quad T_{21}=\left|\frac{\varepsilon_{\mathrm{p1}+}^{\mathrm{out}}}{\varepsilon_{\mathrm{p2}}^{\mathrm{in}}}\right|_{\varepsilon_{\mathrm{p1}}^{\mathrm{in}}=0}=1. \tag{16}$$

Next, as an example, we will focus on the non-reciprocal relation in Eq. (16), where $\varepsilon_{\mathrm{p1/p2}}^{\mathrm{in}}=0$ indicates that there is no optical fields input from the left/right cavity. From Eqs. (9) and (12), we can obtain

$$\frac{\varepsilon_{\mathrm{p2}+}^{\mathrm{out}}}{\varepsilon_{\mathrm{p1}}^{\mathrm{in}}}=\kappa\frac{-4(2G_1G_2+\mathrm{i}J\gamma_x\mathrm{e}^{\mathrm{i}\theta})}{-8\mathrm{i}G_1G_2J+4G_1^2\kappa_x\mathrm{e}^{\mathrm{i}\theta}+4G_2^2\kappa_x\mathrm{e}^{\mathrm{i}\theta}+4J^2\gamma_x\mathrm{e}^{\mathrm{i}\theta}+\kappa_x^2\gamma_x\mathrm{e}^{\mathrm{i}\theta}-8\mathrm{i}G_1G_2J\mathrm{e}^{2\mathrm{i}\theta}}, \tag{17}$$

$$\frac{\varepsilon_{\mathrm{p1}+}^{\mathrm{out}}}{\varepsilon_{\mathrm{p2}}^{\mathrm{in}}}=\kappa\frac{-4\mathrm{e}^{\mathrm{i}\theta}(2G_1G_2\mathrm{e}^{\mathrm{i}\theta}+\mathrm{i}J\gamma_x)}{-8\mathrm{i}G_1G_2J+4G_1^2\kappa_x\mathrm{e}^{\mathrm{i}\theta}+4G_2^2\kappa_x\mathrm{e}^{\mathrm{i}\theta}+4J^2\gamma_x\mathrm{e}^{\mathrm{i}\theta}+\kappa_x^2\gamma_x\mathrm{e}^{\mathrm{i}\theta}-8\mathrm{i}G_1G_2J\mathrm{e}^{2\mathrm{i}\theta}}. \tag{18}$$

If $\theta=n\pi$ (n is an integer), we find that $T_{12}=T_{21}$, which shows two cavity field is reciprocity. When $\theta\neq n\pi$ time-reversal symmetry is broken, i.e., the system exhibits a non-reciprocal response.[39] The perfect nonreciprocity can be obtained when $\theta=n\pi/2$. The dependence of the nonreciprocity on the phase difference θ is plotted in Fig. 4 and we can find that the transformation between reciprocity and perfect non-reciprocity is modulated by changing the phase difference θ. For simplicity, by setting $\theta=\pi/2$ and $G_1=G_2=G$, Eqs. (17) and (18) can be simplified to

$$\frac{\varepsilon_{\mathrm{p2}+}^{\mathrm{out}}}{\varepsilon_{\mathrm{p1}}^{\mathrm{in}}}=\kappa\frac{4\mathrm{i}(2G^2-J\gamma_x)}{8G^2\kappa_x+4J^2\gamma_x+\kappa_x^2\gamma_x}, \tag{19}$$

$$\frac{\varepsilon_{\mathrm{p1}+}^{\mathrm{out}}}{\varepsilon_{\mathrm{p2}}^{\mathrm{in}}}=\kappa\frac{-4\mathrm{i}(2G^2+J\gamma_x)}{8G^2\kappa_x+4J^2\gamma_x+\kappa_x^2\gamma_x}. \tag{20}$$

From Eqs. (19) and (20), the perfect nonreciprocity can be achieved, i.e., $T_{12}=0$, $T_{21}=1$, when

$$x=0,\quad J=\frac{\kappa}{2},\quad G=\frac{\sqrt{\kappa\gamma}}{2}. \tag{21}$$

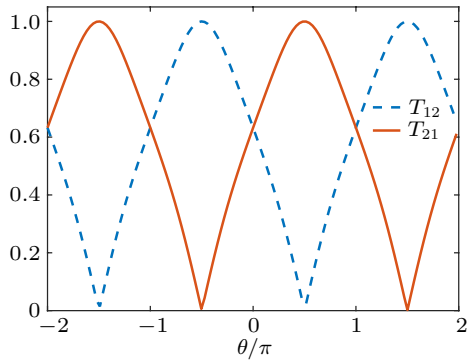


Fig. 4. Transmission amplitude T_{12} (blue dotted line) and T_{21} (red solid line) are plotted as a function of θ. $x=0$, $J=\kappa/2$, $G=\sqrt{\kappa\gamma}/2$ and the other parameters are the same as in Fig. 2.

Based on Eqs. (19) and (20), we plot the transmission amplitude T_{12} (blue dotted line) and T_{12} (red solid line) as a function of the detuning amount x/ω_{m} in Fig. 5(a) and the CK coupling strength g_{ck}/g in Fig. 5(b). Nonreciprocity results from the interference between two possible transition paths, i.e., the tunneling coupling between $|n_1+1,n_2,n_b\rangle\leftrightarrow|n_1,n_2+1,n_b\rangle$ and the effect optomechanical coupling with CK effect $|n_1+1,n_2,n_b\rangle\leftrightarrow|n_1,n_2,n_b+1\rangle\leftrightarrow|n_1,n_2+1,n_b\rangle$. Here we choose the parameters as $\kappa=5$ MHz, $\gamma=1$ MHz, $J=\kappa/2$ and $G_1=G_2=G=\sqrt{\kappa\gamma}/2$.[39] It can be clearly seen that when the detuning amount $x=0$, perfect nonreciprocity appears, i.e., the probe field can be transmitted completely from the right cavity to the left cavity, but not vice versa. In addition, the two sides of two curves basically overlap, i.e., the reciprocity of the optical transmission. As shown in Fig. 5(b), when the detuning amount $x=0$, transmission amplitude T_{12} depends the CK coupling efficient g_{ck}. With the increasing of g_{ck}, the transmission amplitude T_{12} decreases first and then increases. When $g_{\mathrm{ck}}=g_{\mathrm{ck0}}$, perfect reciprocity appears, as shown in Fig. 5(a) with $x=0$. Figure 5(b) further proves that CK coupling can effectively adjust the transmission amplitude.

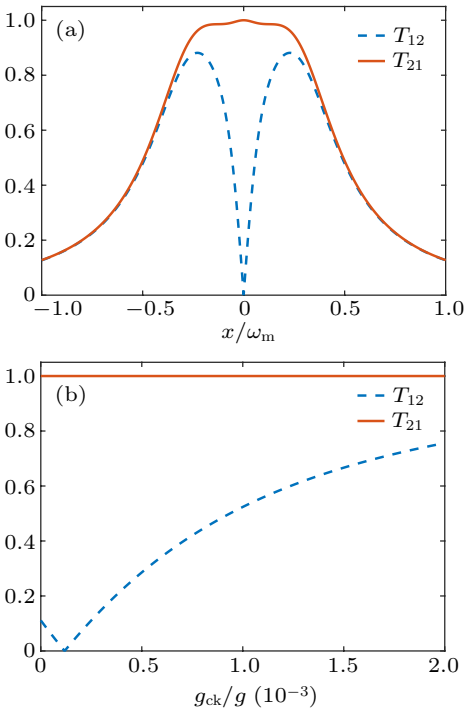


Fig. 5. Transmission amplitude T_{12} (blue dotted line) T_{21} (red solid line) as a function of the detuning x/ω_{m} with $G=\sqrt{\kappa\gamma}/2$ in (a) and the CK coupling strength g_{ck}/g with $x=0$ in (b). The other parameters are the same as in Fig. 2.

4. Four-wave mixing

In this section, we discuss FWM in the system. From Eq. (10), we know that there are three frequency components in the output field, where the frequency $2\omega_c - \omega_p$ is related to FWM due to the Stokes process.[43] The FWM intensity can be defined as[15]

$$I_{i\mathrm{FWM}} = \left| \frac{\sqrt{\kappa_i}\delta a_{i-}}{\varepsilon_{pi}} \right|^2. \tag{22}$$

Here, we mainly study the FWM intensity in the second cavity, i.e., $I_{2\mathrm{FWM}} = \left| \frac{\sqrt{\kappa_2}\delta a_{2-}}{\varepsilon_{p2}} \right|^2$ with $\varepsilon_{p1} = 0$. According to Eqs. (9) and (22), the FWM intensity can be rewritten as

$$I_{2\mathrm{FWM}} = \left| \frac{\sqrt{\kappa}(2\tilde{\kappa}_x\tilde{\gamma}_x + 8G_1^2)}{4\tilde{\kappa}_x(G_1^2+G_2^2) + \tilde{\gamma}_x(4J^2+\tilde{\kappa}_x{}^2)} \right|^2 \tag{23}$$

with $\theta = \pi/2$. From Eq. (23), we can find that the intensity of FWM depends on the effective optomechanical coupling intensity, the tunneling coupling intensity, the CK coupling strength, the decays of the second cavity, and mechanical modes. In order to reveal the effects of the main physical parameters on the strength of FWM, we discuss the results as follows. When CK effect is absent, the FWM spectrum versus the optomechanical coupling strength g and the detuning x with $J = \kappa/2$ is plotted in Fig. 6. It is clear that there is only a single peak of FWM intensity with the fixed g. With the increase of the optomechanical coupling strength g, the width of peak becomes narrow.

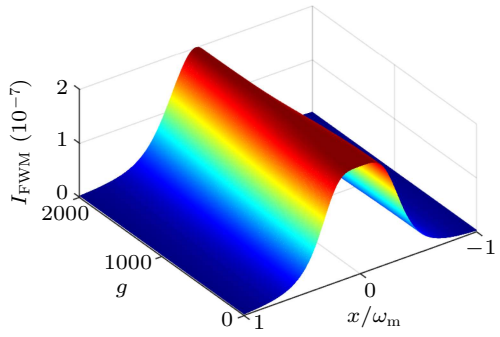


Fig. 6. The FWM spectrum versus the detuning amount x and optomechanical coupling strength g with $g_{ck} = 0$. The other parameters are the same as in Fig. 2.

In Fig. 7, the FWM intensity is plotted as a function of the detuning x with different CK coupling strengths and the fixed $g = 1$ kHz. From Fig. 7, we can clearly see that without the CK effect the curve of the FWM shows only a wide peak, which is similar to a Gaussian distribution, as shown in Fig. 6. The FWM spectrum will change from one peak to three peaks and the total width of peaks becomes wide as the CK coupling strength increases. Meanwhile, the gap between peaks gradually increases. It is worth noting that with the different CK coupling strengths, the maximum value of FWM signal is unchanged and is reached at $x = 0$. In addition, the two peaks on the two sides are symmetric with respect to $x = 0$. Therefore, from Fig. 7, we can get that the FWM signal depends obviously on the CK effect.

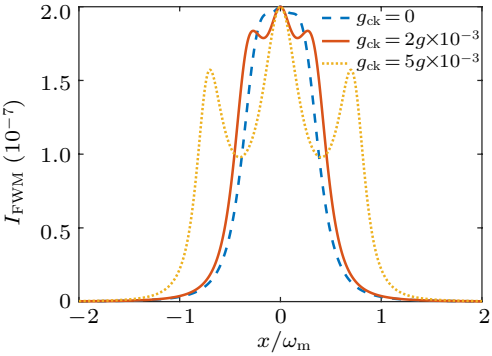


Fig. 7. FWM intensity as a function of the detuning amount x in the different cases of CK coupling strength. The other parameters are the same as in Fig. 2.

5. Conclusion

We studied the effects of CK coupling on the transmission spectrum of double-cavity optomechanical system. By introducing and increasing the CK coupling, we found that the CK coupling can enhance the transmission rate and double-transparency windows can be present based on OMIT and tunneling induced transparency. The width of the transparent window can be effectively adjusted by the CK coupling. For the transmission of optical signals, the perfect nonreciprocity and reciprocity can be shown and effectively modulated by the CK coupling strength and the phase difference θ. In addition, the peak number of FWM components in the output field is changed from one to three due to the introduction of CK coupling. Furthermore, the width of peaks becomes wider as the CK coupling increases. Therefore, the results of this research show that the CK effect can effectively modulate the transmission spectrum of the double optomechanical cavity.

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