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A novel complex-high-order graph convolutional network paradigm: ChyGCN

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In recent years, there has been a growing interest in graph convolutional networks (GCN). However, existing GCN and variants are predominantly based on simple graph or hypergraph structures, which restricts their ability to handle complex data correlations in practical applications. These limitations stem from the difficulty in establishing multiple hierarchies and acquiring adaptive weights for each of them. To address this issue, this paper introduces the latest concept of complex hypergraphs and constructs a versatile high-order multi-level data correlation model. This model is realized by establishing a three-tier structure of complexes-hypergraphs-vertices. Specifically, we start by establishing hyperedge clusters on a foundational network, utilizing a second-order hypergraph structure to depict potential correlations. For this second-order structure, truncation methods are used to assess and generate a three-layer composite structure. During the construction of the composite structure, an adaptive learning strategy is implemented to merge correlations across different levels. We evaluate this model on several popular datasets and compare it with recent state-of-the-art methods. The comprehensive assessment results demonstrate that the proposed model surpasses the existing methods, particularly in modeling implicit data correlations (the classification accuracy of nodes on five public datasets Cora, Citeseer, Pubmed, Github Web ML, and Facebook are 86.1±0.33, 79.2±0.35, 83.1±0.46, 83.8±0.23, and 80.1±0.37, respectively). This indicates that our approach possesses advantages in handling datasets with implicit multi-level structures.

Keywords: raph convolutional network, complex modeling, complex hypergraph

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1. Introduction

In the realm of natural data structures, graphs reveal intricate relationships and interactions, whereas traditional neural networks face limitations when dealing with such structured data.[1] The introduction of graph convolutional networks (GCNs) can remedy this inadequacy.[2,3] Through refined graph convolution operations, GCN's parameters efficiently learn node representations, and GCNs comprehensively consider the position and connections of nodes within the entire graph.[4] Consequently, GCN has become a robust tool for processing graph data and is widely implemented in the case such as social network node classification.[5] The sensitivity of GCN to node relationship and its flexible capturing capability have opened up new possibilities for deep learning across various domains.[6] With continuous innovation in the foundational aspects, the emergence of various variants has driven the further development of GCN.

Throughout the evolution of GCN, various models have emerged, collectively dedicated to enhancing the capability of modeling graph structures. In the process of aggregating nodes in GCN, some node information may be averaged or pooled, and this aggregation method leads some information to lose.[7] GraphSAGE improves scalability by sampling node neighbors and aggregating features.[8] However, because GraphSAGE fails to fully capture global information in certain cases, APPNP was developed.[9] APPNP employs approximate personalized propagation to enhance efficiency but has limitations in handling heterogeneous graphs. To better capture the structural information of heterogeneous graphs, HAN introduced a multi-level attention mechanism by changing the model structure.[10] GIN solves the computational challenges of massive graph data induced by the attention mechanism, where the aggregation operation of features from neighbouring nodes is defined as a summation, so that the information of each node is retained in a more balanced way.[11] For complex graph structures and datasets with substantial noise, GIN is prone to overfitting. GraphMix enhances model generalization and heterogeneous data processing by blending nodes or edges from different graphs.[12] While GraphMix faces the challenge of an overly large aggregation scale, GraphSAINT addresses this by constructing subgraphs that are more representative of the entire graph structure through hierarchical sampling.[13] Despite these variations continuously addressing the problems in previous models, such as the GraphZoom's introduction of hierarchical graph focusing mechanisms,[14] MixHop's incorporation of mixed hop counts in graph convolution,[15] and HGNN$^+$'s introduction of more complex graph neural network structures for different node types,[16] the demand of modeling multi-layer and heterogeneous networks is still rising.[17]

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When algorithms handle hierarchical relationships in graph data, GCN and its variants encounter common challenges. The construction of graphs leads the information to losing and blurring, thus the algorithm performs ineffectively the graphs with complex hierarchies.[18] Insufficient hierarchical depth hinders the model from providing sufficiently profound modeling capabilities for intricate multi-level structures.[19] Additionally, the lack of ability to comprehensively synthesize global context makes it difficult for the model to fully understand the overall structure within multi-level relationships.[20] Although some variants, such as GraphSAGE and Cluster-GCN,[21] attempt to address these problems through techniques like sampling[8] and hierarchical clustering,[21] modeling multi-level relationships is still a pressing challenge. When dealing with more complex graph structures, the designing of more flexible and deep composition models is needed in order to better capture multi-level relationship and structural information.[22]

To cope with these challenges, we introduce the concept of complex hypergraphs (Chygraphs) to extend graph construction schemes.[23] We propose a more flexible and deep learning representation model called Chygraph graph convolutional network (ChyGCN). This model aims to effectively handle the complex correlations and heterogeneity in systems that are expressed in the form of Chygraphs. To achieve this goal, we design strategies to construct Chygraphs from distance space, attribute space, and feature spectral space. Chygraph, as a unified representation of concepts such as simple graphs, hypergraphs, multi-layer networks, simple complexes (complexes are made of atoms and other complexes), and hyperstructures (high order structures defined using category theory concepts),[23] better describes the structure of complex graphs. To evaluate the performance of the proposed method, we conduct comparative experiments on five publicly available datasets. The results show that ChyGCN demonstrates outstanding effectiveness in handling the complex correlations and heterogeneity in systems. This research provides a new way for modeling the information in complex systems, with wide-ranging potential applications.

2. Methods

In this section, we discuss the definition and construction methods of Chygraph. In Subsection 2.1, we will present the definition of Chygraph's constituent components, followed by discussion on its transformation in relation to graph or hypergraph. After obtaining the transformation relationships among three graph structures, we further demonstrate examples in the real world. To further unveil its properties mathematically , Subsection 2.2 will delve into its network attributes, with notations and definitions outlined in Table 1. In Subsection 2.3, we will propose three parallel methods of constructing Chygraph. It is important to note that during usage, one of these methods can be flexibly chosen based on the actual data scenario. After constructing Chygraph, Subsection 2.4 elucidates the method of adaptive layer-wise learning and highlights the difference from traditional GCN.

Table 1. Notations and definitions.

Notation	Definition
V	the set of vertices (node)
v/u	vertex (node)
C	Chygraph
H	hypergraph
G	graph
π	vertex in complexes
ρ^{G}	network density of graphs
ρ^{H}	network density of hypergraphs
ρ^{C}	network density of Chygraphs
$\mathrm{Hop}(u,K)$	the nearest K neighboring vertices of vertex u
C_j^{dis}	complex j of distance space modeling method
Attr_i	I-th attribute
Rank_i	sort attributes with sequence number i
Q^{attr}	the leachable threshold of attribute space modeling method
C_i^{feature}	complex j of attribute space modeling method
Q^{feature}	the leachable threshold of feature spectral space modeling method
H_i^{feature}	the i-th hypergraph of feature spectrum space modeling method
Θ	learnable parameter matrix in ChyGCN

2.1. Definition of Chygraphs

2.1.1. Definitions of atoms and complexes

The smallest structural unit in Chygraphs is referred to as an atom. Atoms are considered indivisible and cannot be further divided, although they possess more intricate structures in the realm of physics. Another structural unit is the complex, which is comprised of atoms or other complexes. This approach yields a self-consistent definition of complex systems, wherein a complex system comprises a set of atoms and complexes.

2.1.2. Chygraph to graph or hypergraph

Building upon the foundational units of Chygraphs, which consist of atoms and complexes, these structures undergo a transformation into more general forms.

It should be noted that in order to highlight the key points, three simplified graphical structure representation methods are used here. The operation of converting Chygraph (C (complex, vertex)) into hypergraph (H (hyperedge, node)) is to break the complex structure and generate a hyperedge. When a complex contains only one hyperedge, it is equal to a hypergraph. Similarly, the operation of converting Chygraph (C (complex, vertex)) into graph (G (edge, node)) breaks both the complex structure and the hyperedge structure, generating G (edge, node). When the complex contains a case with only two nodes, the complex is equal to a graph. For ease of presentation, we illustratethem in Fig. 1(a) and Fig. 2 (Chygraph).

2.1.3. Chygraph in real-world

In the system of scientific publications (Fig. 1(b)), authors are represented as atoms (vertexes), while publications are represented as complexes. The publications contain hyperedges formed by a set of authors as one hyperedge, and a set of references as another hyperedge. Chygraphs offer a better representation for the intricate structures of many real-world scenarios. The flexible representation and interchangeability of various graph structures through these methods enhance the expressive power of Chygraph in complex systems.

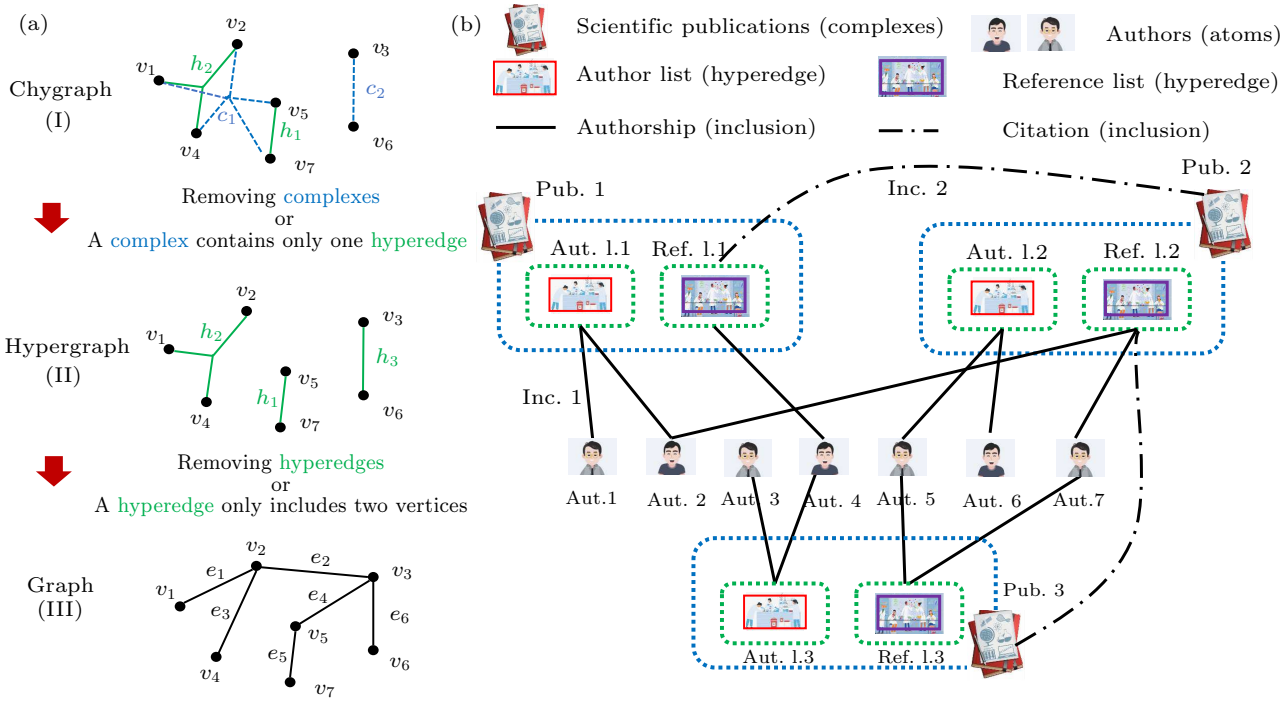


Fig. 1. Chygraph and the realistic scenario. (a) Differences in different graphic structures. Chygraph degenerates into hypergraph or graph after removing certain structures. The transition from Chygraph to hypergraph involves removing c_1 to obtain two sets of hyperedges (h_1 and h_2), or when the composite body only contains one hyperedge (such as c_2), it directly degenerates into a hyperedge (h_3). The operation of changing from Chygraph to graph is to remove hyperedges (h_1 and h_2) from the hypergraph, or to directly degenerate the hyperedge into a regular edge when it contains only two vertices. (b) In the scientific publishing system, individual authors represent atoms (such as Aut. 1), and author lists or reference lists represent complexes (green dashed boxes, such as Aut. l.1 or Ref. l.1). The publication represents Chygraph edges (blue dashed box, such as Pub. 1), black solid lines represent atomic membership relationships (such as Inc. 1), and black dashed lines represent citation relationships (such as Inc. 2). It should be noted that solid and dashed lines are used to represent two different relationships.

2.2. Properties of Chygraphs

2.2.1. Network degree

Chygraph (C) is comprised of multiple complexes. Given a complex j, it contains a hypergraph $H(v_j, E_j^h)$. Let $i \in A \cap C$ represent an atom or complex, where $i = 1, 2, \ldots, n$ and $n = A \cup C$. In other words, $i \in C_j$. The adjacency matrix of the Chygraph is an n-order matrix, with matrix elements defined as $\alpha_{ij} = \begin{cases} 1, & i \in C_i \\ 0, & i \notin C_i \end{cases}$. The degree of the associated Chygraph is defined as stated in $\text{Degree}^c = \sum_j \alpha_j$. In real systems, higher-level complexes are often employed to describe lower-level distinctions, such as population descriptions on location (city, province, country). This hierarchical structure leads to the definition of Chygraph length, denoted as $L(C)$, which represents the longest path in subpartitions in the graph, as defined in $\pi = \pi_1 \cup \cdots \cup \pi_i$ and $\pi_i \cap \pi_j = \emptyset$ for $i \neq j$.

2.2.2. Network density

Similarly, network density describes the density of social relationships or network edges. An undirected graph consists of t vertices. Network density of graph $\rho^N = M/(1/2)t(t-1)$ is obtained, where M represents the actual number of edges. Since the construction rules of both hypergraphs and Chygraphs are more complex than those of graphs, we consider the following ideal cases. In a hypergraph, if only one type of single hyperedge is formed,

$$\rho^{\mathrm{H}} = \frac{M}{\frac{1}{\frac{N}{S}!}\prod_{i=1}^{S}\left(N - \frac{N}{i}\right)}$$

is derived, with a maximum of $N/2$ types of hyperedges and a minimum of N/s types (s being the number of nodes included in a hyperedge). It is observed that when the number of nodes included in a hyperedge is fixed at two, the equation degenerates into a density formula of the graph. When the network structure becomes more heterogeneous, we assume that the nodes in Chygraph are uniformly distributed as S_i,

$$\rho^c = \frac{M}{\prod_{i=4}^{h} \mathrm{Com}_{(N/h)}^{(N/h)s_i}}$$

is obtained, where $S_i = 1, \ldots, h$ and Com represents combination method in Mathematics.

2.2.3. Network-path-length

By introducing network density, average network-path-length in $\bar{L} = \frac{1}{M/\rho^k} \cdot \sum_{i \geq j} d_{ij}^k$, $k = G, H, C$, network paths are compared regardless of the order of magnitude of the vertices, as exemplified by protein interaction networks or karate club networks.

2.2.4. Global efficiency

Global efficiency $GE = 1/L$ with node strength $NS_i = \sum_j \alpha_{ij}$ were employed to describe the network's information transmission or routing capabilities. For example, the efficiency of traffic flow or the identification of critical intersections in a transportation network is assessed by GE.

2.3. Methods of Chygraph construction

In this subsection, we describe how to construct a complete Chygraph from raw data, both when the data naturally exhibits a Chygraph structure or when it does not. To better modeling the higher-order representation, it is important to generate a well-defined hypergraph structure. It should be noted that in most cases, there is no explicit Chygraph structure present. Therefore, three generation strategies of Chygraph are proposed, which are formed on distance-space (Fig. 2(I)), attribute-space (Fig. 2(II)), and feature-spectrum-space (Fig. 2(III)). These strategies complement each other, and model various complex real-world scenarios.

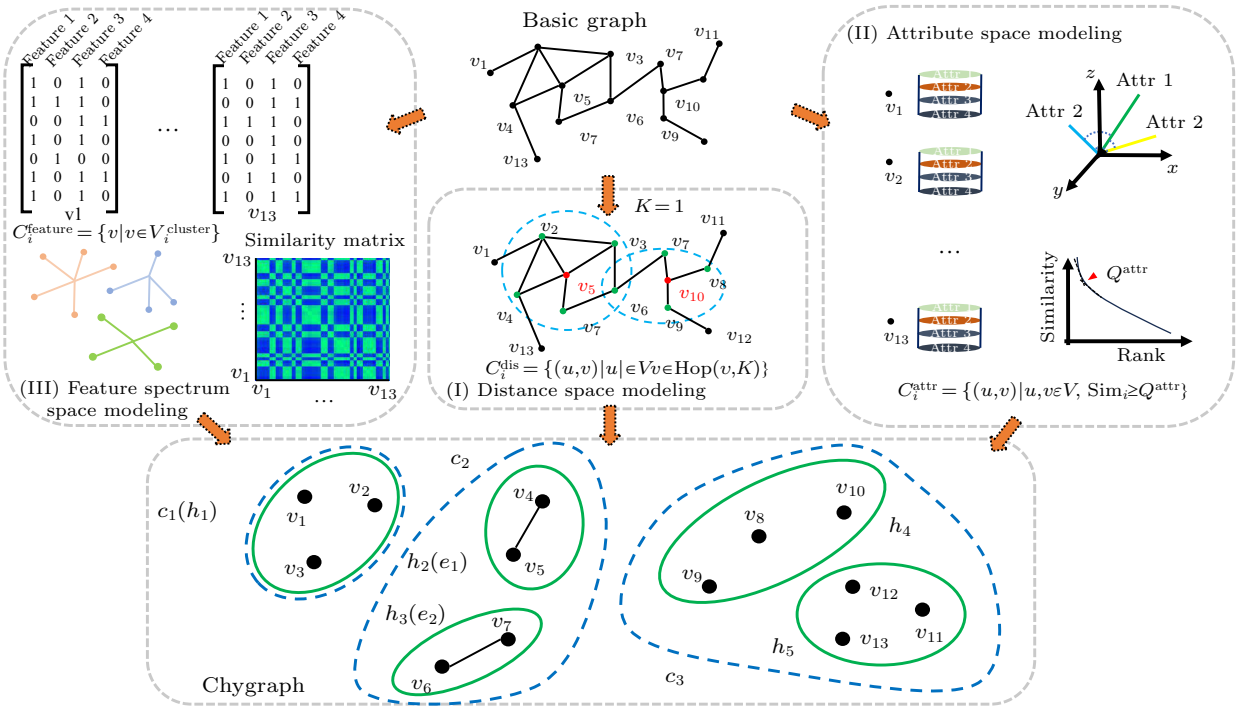


Fig. 2. Construction process of Chygraph involving three key approaches: distance space modeling (I), attribute space modeling (II), and feature spectrum space modeling (III). Panel (I) shows a composite graph created by using distance learning parameters with $K = 1$. A specified distance target set (v_2, v_3, v_4, v_6, and v_7) is retrieved by sequentially searching each node based on the target node v_1. Panel (II) displays the attribute vectors of each node are mapped to Euclidean space, with cosine similarity calculated, and cosine similarity sequence is truncated based on a threshold to create the composite graph. Panel (III) exhibits and models vertex 0–1 feature data based on similarity affinity. Through different data structure representations, Chygraph is ultimately obtained. For Chygraph in panels, the difference among three types of graphs: graph, hypergraph, and Chygraph, are elucidated. For example, c_1 and h_1 denote equivalence in the containment relationship with vertices when the composite structure contains only one hyperedge. Similarly, h_2 and e_1 represent equivalence between hyperedges and ordinary edges when the hyperedge contains only two vertices. Furthermore, when the composite structure contains only one hyperedge and the hyperedge includes only two vertices, the composite structure is equivalent to an ordinary edge in terms of representation.

2.3.1. Distance-space modeling

When data is represented as a graph structure, each data element serves as a vertex in the graph, and the relationships between elements are represented by edges. The construction of Chygraph relies on the reachable distances of each vertex, determined by calculating these distances to establish the correlation between vertices. Based on this correlation, we obtain the K-nearest neighbors for each vertex, integrating them into a first-layer complex. Further recombination, guided by a learnable threshold, results in a different set from the first layer, constructing the second-layer hypergraph.

First, we perform the first step of obtaining the K-nearest neighbors for each vertex $\text{Hop}(u,k) = \{v \in V | v \notin u, (u,V) \in E\}$, v is one of the K vertices closest to u. By considering the distance and similarity between vertices, we identify the K-nearest neighboring nodes for each vertex. The collection of neighboring vertices $C_i^{\text{dis}} = \{(u,v) | u \in V, v \in \text{Hop}(u,K)\}$ is referred to as the first-layer complex, where each vertex

forms a complex relationship with its corresponding neighbors. Next, we further construct the second-layer hypergraph by thresholds. In the first-layer complex, vertices are grouped into different sets on whether degree of association meets the threshold criteria. If the degree of association between vertices exceeds the threshold, they are considered to be part of the second-layer hypergraph. Through this construction process, we can represent the data as a multilevel complex-graph-structure, i.e., a Chygraph. The construction of the Chygraph is relied on the reachable distance with degree of association between vertices. This manipulation can model the complex relationships or patterns among data elements (such as social networks).

2.3.2. Attribute-space modeling

There exists a class of real-world data which possesses multiple attributes but does not exhibit an obvious graph structure, such as various parameters during vehicle operation. For this kind of data, we attempt to model its correlations with patterns within the attribute-space. First, the cosine similarity is calculated between different attributes, namely,

$$\mathrm{Sim}_{ij} = \frac{\sum_{i,j=1}^{n} (\mathrm{Attr}_i \times \mathrm{Attr}_j)}{\sqrt{\sum_{i=1}^{n} \mathrm{Attr}_i^2} \cdot \sqrt{\sum_{j=1}^{n} \mathrm{Attr}_j^2}}.$$

The cosine similarity between attributes provides the foundation for modeling. After calculating the cosine similarity for all attribute- pairs, similarities are sorted, identify the inflection point of the sorted data, $Q^{\mathrm{attr}} = (\mathrm{rank}_1 + \mathrm{rank}_2)/2$, $f(\lambda \mathrm{rank}_1 + (1-\lambda)\mathrm{rank}_2) \leq \lambda f(\mathrm{rank}_1) + (1-\lambda) f(\mathrm{rank}_2)$. The inflection point is considered as the threshold for establishing connections. We designate the inflection point as a similarity threshold. This threshold aids in identifying strong correlations between attributes, thereby constructing the first layer of complexes, $C_i^{\mathrm{attr}} = \{(u,v) | u,v \varepsilon V, \mathrm{Sim}_i \geq Q^{\mathrm{attr}}\}$. Next, the intersections are examined between the first-layer complexes whether they exceed a predefined threshold or not. If the intersection between certain complexes exceeds the threshold (Z), $H_i^{\mathrm{attr}} = \{(c_i, c_j) | c_i, c_j \varepsilon C, c_i \cap c_j \geq Z\}$, those complexes are considered part of the second-layer hypergraph. Through the above process, data with multiple attributes is transformed into a multilevel complex-graph-structure. This modeling approach, which relies on similarity with intersection relationships between attributes, helps uncover hidden patterns or correlations within the data. This method is used in various domains, including the mining of vehicle operation data.

2.3.3. Feature-spectrum-space modeling

In the data collection process, the data subject often has a complex-spatial-structure that is not directly reflected in the collected data, especially when Chygraph deals with graph data. To reconstruct the data's complex structure as accurately as possible, a feature-spectrum-space approach is used. The core of the feature-spectrum-space methodlies in the analysis of both attraction and attribution information to reveal the intrinsic patterns. In this process, the affinity propagation (AP) algorithm is used to partition the features. This clustering algorithm, known for uncovering clustering structures in the data, facilitates the identification of patterns within the partitioned features.

First, AP algorithm is used to cluster the data, thereby obtaining the first-layer complexes $C_i^{\mathrm{feature}} = \{v | v \in V_i^{\mathrm{cluster}}\}$. These complexes are seen as subsets or clusters. Next, we calculate the ratio of the number of vertices in each set to the total number of vertices in the first-layer complex, $\mathrm{ratio}_i^{\mathrm{feature}} = |V(C_i)|/|V|$. By comparing the ratio with a predefined threshold, a set is determined, whether it exceeds the threshold in terms of its size or not. Due to this determination, the second-layer hypergraph is generated, $H_i^{\mathrm{feature}} = \{v_i | \mathrm{ratio}_i^{\mathrm{feature}} \geq Q^{\mathrm{feature}}\}$, among sets that surpass the threshold, with each hyperedge representing the relationship between first-layer complexes. Through the above steps, the data is represented as a multilevel complex structure, where the first-layer complexes represent clusters within the data, and the second-layer hypergraph models the associations between different clusters.

2.4. Constraction of ChyGCN

This subsection focuses on the details of ChyGCN (Fig. 3). Dataset is divided into training, validation and testing vertices, where each data entry contains multiple inherent attributes or features of vertices. The Chygraph structure is constructed by correlations to form the association matrix H. The corresponding adjacency matrix is obtained from $A = HWH^{\mathrm{T}} - D$, where D or W represents the degree matrix or weight matrix of vertices or complexes, respectively. It should be noted that the process of generating the Chygraph in this step involves learnable parameters (the thresholds which are mentioned in different methods). The introduction of learnable parameters aims to enhance the ability of algorithm to model complex information. The adjacency matrix of Chygraph A with the features (attributes) of vertex are inputted into the ChyGCN to produce the output labels. As described in the previous section, the ChyGCN convolutional layer $f(XW\Theta)$ is built on the following equation $X^{l+1} = \sigma(D^{-1/2}AD^{-1/2}X^l\Theta)$, where X^l represents the signal of the l-th layer of ChyGCN and Θ denotes the learnable parameter.

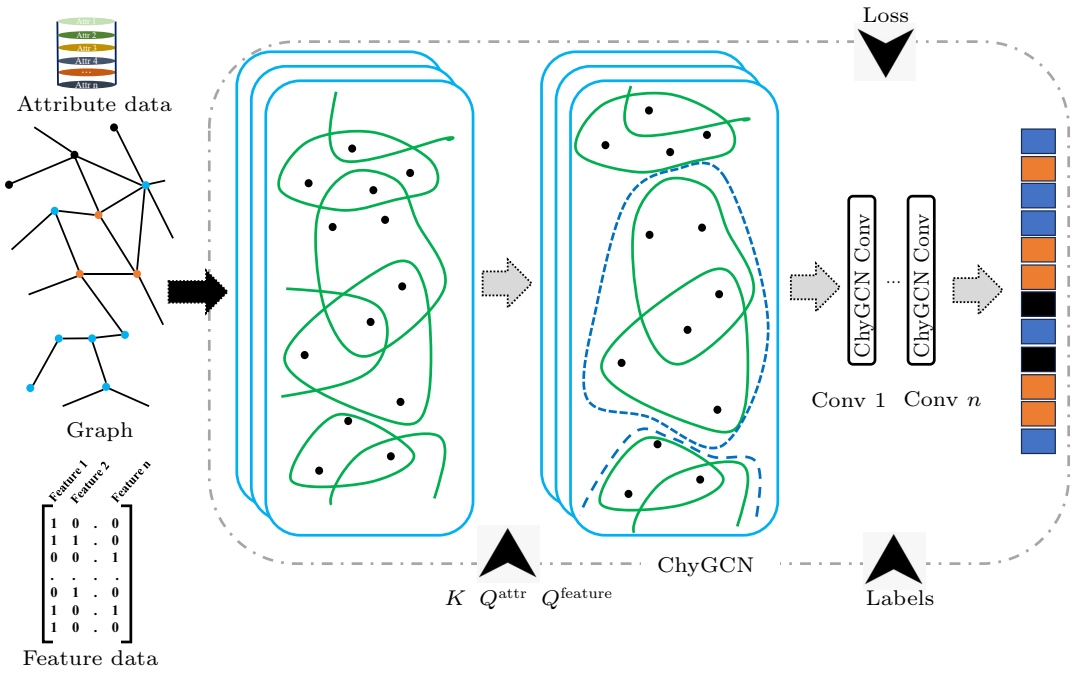


Fig. 3. Algorithm flowchart of ChyGCN built on the basis of graph structure of Chygraph.

The main distinction between ChyGCN and GCN lies in their graph construction approaches. ChyGCN, as previously stated, delves more in depth into the relationships between features by adopting a three-tier hierarchical structure, namely, complex-hyperedge-vertex. Additionally, during the training phase, ChyGCN introduces learning parameters that can be utilized to adjust the graph structure. This innovation makes ChyGCN more flexible and tunable in the process of graph construction. In other aspects, ChyGCN remains consistent with the GCN algorithm, such as in message propagation. These features collectively endow ChyGCN with enhanced expressive capabilities and performance advantages in the application of GCN.

3. Experiment and discussion

The classification task of ChyGCN is evaluated by comparing other existing techniques. Five widely tested public datasets are used as target datasets for this task. Datasets are divided into two categories: public-citation-networks (Cora,[14] Citeseer, and Pubmed[15]) and social-media-networks (Github Web ML and Facebook[16]), as shown in Table 2. These five datasets represent intricate relational networks across diverse domains, i.e., researchers have opportunities to explore academic, technological, and social landscapes. The academic citation networks of Cora and Citeseer facilitate both the analysis and the comprehension of citation relationships among papers. The incorporation of additional text features, such as keywords and abstracts, enhances the context for tasks like topic modeling and information retrieval. Pubmed, focusing on the biomedical field, furnishes valuable resources for biomedical informatics, literature mining, and disease network research. Github Web ML reflects collaboration and project interactions within the machine learning community, offering robust support for understanding machine learning engineering practices, technological trends, and collaborative innovation within the community. Additionally, Facebook's social network dataset serves as a foundation for research on social relationships. This dataset enables sociologists and computational social scientists to in depth investigate users' interactions, information propagation, and social network dynamics.

Table 2. Detailed information of five datasets with graph structure.

	Cora	Citeseer	Pubmed	Github Web ML	Facebook
Nodes	2708	3327	19717	37700	22470
Edges	5429	4732	44338	289003	171002
Classes	7	6	3	2	4
Features	1433	3703	500	4005	4714
Training node	300	350	450	300	900
Validation node	500	500	1000	2000	3500
Testing node	1000	2000	5000	10000	9000

The diversity of these five datasets provides researchers with extensive experimental scenarios and prompts cutting-edge research in graph neural networks, machine learning, and social network analysis. Whether exploring academic cooperation, medical research, technological innovation or understanding social behavior, these datasets provide a rich and comprehensive perspective for in-depth understanding of complex network structures and features.

In the experiments, a two-layer ChyGCN is constructed with a hidden layer feature dimension set to 32. The dropout

strategy ($p = 0.5$) is adopted to prevent overfitting. Relu is selected as an activation function. In the training processes of ChyGCN, the Adam optimizer is adopted to minimize the cross-entropy loss function, with a learning rate set to 0.001. The comparison methods all use default parameters for comparisons. The experiments are conducted 100 times, and the results of each run are averaged, as shown in the following Table 3.

Table 3. Experimental results with mean (%) and standard deviation (%) over the trials. The best results are displayed in bold, the second-best ones are underlined, and the third-best scores are displayed in italics.

	Cora	Citeseer	Pubmed	Github Web ML	Facebook
GCN[2,3]	72.4±0.51	63.8±0.33	68.9±0.41	73.5±0.82	62.7±0.54
GraphSAGE[8]	78.3±0.64	65.2±0.38	75.7±0.54	79.3±0.74	65.2±0.46
APPNP[9]	80.1±0.35	71.4±0.43	78.5±0.62	78.6±0.64	65.6±0.39
HAN[10]	79.8±0.34	75.1±0.52	77.4±0.45	80.2±0.57	68.3±0.22
GIN[11]	81.4±0.37	76.2±0.47	80.5±0.53	81.7±0.42	71.8±0.42
GraphMix[12]	83.3±0.72	*76.8±0.52*	78.4±0.35	*81.8±0.12*	73.4±0.36
GraphSAINT[13]	82.7±0.62	75.4±0.58	79.3±0.32	79.5±0.31	74.3±0.42
GraphZoom[14]	73.2±0.12	70.1±0.51	75.3±0.39	75.9±0.27	69.4±0.55
MixHop[15]	*83.9±0.51*	77.5±0.33	*80.3±0.52*	77.5±0.46	78.6±0.48
HGNN$^+$[16]	84.2±0.26	72.8±0.62	78.4±0.36	82.8±0.3	*75.1±0.31*
Ours	**86.1±0.33**	**79.2±0.35**	**83.1±0.46**	**83.8±0.23**	**80.1±0.37**

In comparisons with state-of-the-art methods, our proposed ChyGCN has demonstrated optimal performance across five datasets, achieving accuracies of 86.1%, 79.2%, 83.1%, 83.8%, and 80.1% respectively. The exceptional performance of our model stems from several key factors. Unlike basic graphs that establish only pairwise associations or higher-order hypergraphs representing the coexistence of multiple nodes, the Chygraph contains more comprehensive structural information. It encompasses both types of information mentioned above and provides the more precise modeling of heterogeneity within system components. Experimental results show that MixHop outperforms models using conventional sampling by adopting long-distance sampling to comprehensively capture both local and global contexts of nodes. In contrast, HGNN$^+$ constructs a hypergraph model, and its hierarchical structure aids in capturing features at different abstraction levels, resulting in superior performance compared to MixHop and conventional sampling models. Moreover, our proposed ChyGCN goes beyond these, with more in-depth exploring the relational structure information in graph data. For example, in scientific publication systems like Cora, Citeseer, and Pubmed, our model treats publications as complexes and makes a distinction between author lists and reference lists.

As demonstrated by the construction methods employed in Chygraph, the three complementary approaches significantly enhance the effective representation of complex modeling relationships within both graph structure and non-graph structure. In the distance-space modeling approach, the multi-hop reachable neighbors and their interrelationships are taken into consideration, thus enabling the flexible handling of potential relationships, which surpasses traditional block concepts. The attribute-space modeling method involves mapping the vertex set to a multi-dimensional space. After a comprehensive assessment of the interdependence strength between vertex attributes, the differences among divided vertex sets are further taken into account. In the feature-spectral-space modeling approach, attraction and ownership information on the graph are utilized to partition vertices.

Additionally, our observations indicate that social media networks exhibit a higher level of complexity. This complexity is evident in the simplicity of feature or attribute coding observed in scientific citation networks, especially when compared with the inclusion of location, interests, etc., in social media networks. The latter contains much richer latent information than the word coding information present in the former. It is essential to note that the proposed Chygraph generation method represents a forward-looking technology. There may be even more effective and convenient methods for future practical applications that surpass the current approach.

4. Conclusions

In this study, we introduced ChyGCN, an innovative graph-structured adaptive learning method based on Chygraph. The Chygraph, encompassing the concepts of hypergraph and graph in its definition, provides ChyGCN with a remarkable advantage in modeling high-order data correlations. In the method section, we elaborate three adaptive fusion strategies related to graph data structures, collectively forming a Chygraph generation approach. The definition of Chygraph adopts a three-tier structure of complex-hyperedge-vertex, effectively modeling a multi-level graph structure and using ChyGCN. By integrating hierarchical adaptive learning methods with GCN, the ChyGCN enhances its ability to comprehensively capture complex relationships within the data.

Through experiments on five diverse datasets and comparisons with state-of-the-art methods, we validate the effec-

tiveness of the proposed approach. The results indicate that our framework achieves new state-of-the-art performance, particularly in datasets with known multi-level structures. The ChyGCN is compared with existing GCN models and their variants, particularly emphasizing its advantages in graph construction and handling complex data correlations. This work not only introduces a novel method into the field of graph representation learning but also demonstrates substantial prospects for practical applications, particularly in scenarios involving multi-level structured data.

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