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Evolutionary game dynamics of combining two different aspiration-driven update rules in structured populations

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In evolutionary games, most studies on finite populations have focused on a single updating mechanism. However, given the differences in individual cognition, individuals may change their strategies according to different updating mechanisms. For this reason, we consider two different aspiration-driven updating mechanisms in structured populations: satisfied-stay unsatisfied shift (SSUS) and satisfied-cooperate unsatisfied defect (SCUD). To simulate the game player's learning process, this paper improves the particle swarm optimization algorithm, which will be used to simulate the game player's strategy selection, i.e., population particle swarm optimization (PPSO) algorithms. We find that in the prisoner's dilemma, the conditions that SSUS facilitates the evolution of cooperation do not enable cooperation to emerge. In contrast, SCUD conditions that promote the evolution of cooperation enable cooperation to emerge. In addition, the invasion of SCUD individuals helps promote cooperation among SSUS individuals. Simulated by the PPSO algorithm, the theoretical approximation results are found to be consistent with the trend of change in the simulation results.

Keywords: evolutionary game dynamics, aspiration-driven update, structured populations

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1. Introduction

Cooperative behavior has emerged in the daily interactions of living things.[1–3] When cooperating with others, people show extraordinary organizational, coordination, and communication abilities. Thus, cooperative behavior is indispensable in human science and technology development and innovation. Research by a large number of scholars has found that evolutionary game theory can be an excellent way to study behavior of populations. Darwin's[4] evolutionary theory shows that natural selection tends to favor defection in social dilemmas. Selfish behavior results in higher gains, so studying how cooperation develops in evolutionary game theory is considered to be a challenging problem.[5–7] Social dilemmas result from a conflict between individual interests and the long-term interests of society. The prisoner's dilemma[8] can be materialized as a two-versus-two game in which the players either choose to cooperate or defect, and the gains of the players are obtained by interacting with their opponents. Because the prisoner's dilemma has strong relevance, researchers from different disciplines analyze this framework from different perspectives.

To study how population structure affects evolutionary outcomes, we want to focus on the updating mechanisms that people adopt.[9–11] The updating mechanism specifies what information people use and how they use it to determine individual behavior. In evolutionary games, the information required for an update mechanism is the individual's strategy and payoffs, with a particular focus on the opponent's payoffs. Common updating mechanisms include Moran,[12–15] imitation,[16–19] pairwise comparisons,[20] and aspiration-driven.[21–25] It is well-known that differences in individuals' decisions to adopt renewal mechanisms can significantly influence the evolution of population cooperation. Considering that diversity is ubiquitous in nature, an individual's behavior towards social interactions may be different for each individual. Therefore, there is a great need to merge heterogeneous renewal mechanisms[26,27] to understand different realities better.

This paper is particularly interested in the coexistence of two different aspiration-based updating rules, namely satisfied-stay unsatisfied shift (SSUS) and satisfied-cooperate unsatisfied defect (SCUD). Unlike the imitation mechanism, the aspiration-based updating rule relies on self-assessment (level of aspiration). Individuals will change their strategy if their payoffs do not reach the aspiration level. Otherwise, they will keep their strategy unchanged. This rule is also known as the satisfied-stay unsatisfied shift (SSUS).[28] Instead, a new aspiration updating mechanism is proposed, which is different from the traditional one. Individuals will tend to maximize the collective good if their rewards are at the desired level. This rule is also known as the satisfied-cooperate unsatisfied defect (SCUD).[29] In addition to imitation of others, self-evaluation has always been present in human social behavior, so both behaviors exist in human and animal populations. Research experiments have shown that ants sometimes ignore social information when they have enough personal experience. Currently, the mechanism of heterogeneous updating is widely studied by scholars. For example, Wang *et al.*[30] proposed to

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study the cooperation level of participants in well-mixed and structured populations, in which each participant adopts imitation and aspiration updating rules with a certain probability. Liu *et al.*[31,32] discussed fixed time and fixed probability in evolutionary games based on Moran and Fermi processes.

A series of evolutionary algorithms based on dynamic search in organisms have been proposed, such as particle swarm algorithms, genetic algorithms, and evolutionary multi-objective algorithms. Because these algorithms originate from some evolutionary phenomena in living organisms, they are well suited for studying these phenomena.[33–38] Since the introduction of particle swarm algorithms in the last decade, many studies have proposed improved particle swarm algorithms. For example, Shi *et al.*[33] gave a particle swarm with inertia weights. This algorithm has strong convergence compared to the conventional particle swarm algorithm. Clerc[37] proposed a new PSO with a shrinkage factor, which may help ensure convergence. Compared with other algorithms, the particle swarm algorithm has the advantages of fast convergence, few parameters, and a simple and easy-to-implement algorithm. The particle swarm algorithm is based on observations of the feeding behavior of flocks of birds and how individuals in the group use the information to coexist to drive better problem-solving for the group as a whole. Therefore, based on the principle of the particle swarm algorithm, we improve this algorithm to simulate the strategic choices of the game players to get the realization path, which we call the population particle swarm algorithm (PPSO).

In this paper, we study the heterogeneous updating mechanisms: SSUS and SCUD, and we apply the pairwise approximation (PA) model to approximate the structure of heterogeneous models on irregular grids. Finally, we use PPSO to simulate the evolutionary process of the population. The rest of the paper is as follows: In Section 2, we introduce three models, namely the SSUS process, SCUD process, and SSUS-SCUD process. In Section 3, we introduce the particle swarm algorithm and an improved PPSO algorithm. Numerical examples of stable equilibrium point results and prisoner's dilemma are given in Sections 4 and 5, respectively. Section 6 draws conclusions.

2. Model

In the model, an individual has two alternative strategies: C (cooperation) and D (defection). When the individual's strategies interact two by two. Individuals receive benefits according to the benefit matrix, which is defined as

$$\begin{array}{c} \\ \mathrm{C} \\ \mathrm{D} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{C} & \mathrm{D} \end{array} \\ \begin{bmatrix} R & S \\ T & P \end{bmatrix} \end{array}, \tag{1}$$

where the cooperator gains R when the cooperator plays with another cooperator; the cooperator gains S when the cooperator plays with the defector; the defector gains T when the defector plays with the cooperator; and the defector gains P when the defector plays with another defector.

2.1. Satisfied-stay unsatisfied shift

A time step in which an individual in the population is randomly selected to compare their gains and aspiration levels and ultimately decide whether to change their strategy. We consider the traditional aspiration-driven updating mechanism, in which everyone in the group has the same level of aspiration. The aspiration-driven updating process is understandably more spontaneous and does not need to consider additional information on the strategic environment. The aspiration-driven updating mechanism is specified as follows: if an individual's payoff is above (or equal to) the aspiration level (i.e., $\pi \geq A$), then keep the current strategy, otherwise (i.e.,$\pi < A$) switch him/her to another strategy. Expectation level A can be interpreted as an individual's degree of satisfaction with the learning individual. This updating rule is called the satisfied-stay unsatisfied shift (SSUS). Such deterministic rules can become stochastic rules, which reflect errors and other uncertainties and drive probabilistic changes in the population's demographic composition. To observe stochastic switching in the strategies employed by the players, we use the following probability function based on the Fermi function:

$$q_{i,i+1}|_{\mathrm{SSUS}} = \frac{1}{1+\mathrm{e}^{-\omega(A-\pi_{\mathrm{D}})}}, \tag{2a}$$

$$q_{i,i-1}|_{\mathrm{SSUS}} = \frac{1}{1+\mathrm{e}^{-\omega(A-\pi_{\mathrm{C}})}}, \tag{2b}$$

where $q_{i,i+1}$ denotes the probability of switching defection to cooperation, $q_{i,i-1}$ denotes the probability of switching cooperation to defection, and A is the level of aspiration, with the extent to which the difference between the expected payoff and the aspiration level affects the individual's decision being controlled by the intensity of selection $\omega \geq 0$. When the payoff π is lower than the aspiration A, it is more likely to change strategies; the higher the payoff π is compared to the aspiration A, the less likely it is to switch strategies. The deterministic update rule is restored if $\omega \to \infty$. The selection is weak if $0 < \omega \leq 1$. (i) Weak selection arises when individuals do not clearly prefer different strategies. (ii) Due to noise or random interactions, individuals are uncertain about aspiration or aspiration-return differences.

2.2. Satisfied-cooperate unsatisfied defect

Recent research has shown that the traditional aspiration dynamics SSUS described above do not positively affect cooperation in the prisoner's dilemma.[39,40] Based on this consideration, this paper proposes a new aspiration learning rule: satisfied-cooperate unsatisfied defect (SCUD). This focuses on

choosing between individual and collective interests in different psychological states. It is clear that, in many cases, individuals' choices change according to their level of aspiration. Individuals tend to maximize collective interests when they are satisfied; otherwise, they tend to maximize individual interests. For example, in the process of demolition and compensation, if the developer can satisfy the heart expectations of the demolition households, they will generally cooperate with the demolition; otherwise, they will generally resist the demolition. Next, we give the individual's strategy updating rule, which is different from the traditional aspiration dynamics, and we call this updating rule the SCUD. It is assumed that the aspiration level of each individual in the game is the same, i.e., the aspiration level is also denoted by A. We use the following probability function based on the Fermi function and the transfer probability is defined as

$$q_{i,i+1}|_{\mathrm{SCUD}} = \frac{1}{1+\mathrm{e}^{-\omega(\pi_{\mathrm{D}}-A)}}, \tag{3a}$$

$$q_{i,i-1}|_{\mathrm{SCUD}} = \frac{1}{1+\mathrm{e}^{-\omega(A-\pi_{\mathrm{C}})}}. \tag{3b}$$

Note that it differs only on $q_{i,i+1}$ compared to the probability of SSUS. In addition, when the individual's payoff π_i $(i=\mathrm{C},\mathrm{D})$ is greater than the aspiration level, the individual has a greater probability of using cooperation. When the individual's payoff π_i $(i=\mathrm{C},\mathrm{D})$ is smaller than the aspiration level, the individual has a greater probability of using defection.

2.3. The mixed aspiration update process

In the currently proposed update process, individuals update their strategies through SSUS or SCUD. The probability that an individual adopts the SSUS updating rule is assumed to be γ. Conversely, the probability that the SCUD updating rule is adopted is $1-\gamma$. The $\gamma\in[0,1]$ indicates a trade-off factor. In particular, to update their strategies, players adopt the SSUS rule only when $\gamma=1$ and the SCUD update rule when $\gamma=0$. The transition probability of the mixed SSUS-SCUD process can be written as

$$q_{i,i+1} = \gamma\frac{1}{1+\mathrm{e}^{-\omega(A-\pi_{\mathrm{D}})}} + (1-\gamma)\frac{1}{1+\mathrm{e}^{-\omega(\pi_{\mathrm{D}}-A)}}, \tag{4a}$$

$$\begin{aligned} q_{i,i-1} &= \gamma\frac{1}{1+\mathrm{e}^{-\omega(A-\pi_{\mathrm{C}})}} + (1-\gamma)\frac{1}{1+\mathrm{e}^{-\omega(A-\pi_{\mathrm{C}})}} \\ &= \frac{1}{1+\mathrm{e}^{-\omega(A-\pi_{\mathrm{C}})}}. \end{aligned} \tag{4b}$$

2.4. Structured populations

In structured populations, nodes represent individuals and lines illustrate their interactions. Individuals in the structured population change their strategies through the SSUS update rule with probability γ and the SCUD update rule with probability $1-\gamma$. First, an individual x is randomly selected and referred to as the focal individual. Individual x gains by interacting with individual neighbors around her. Thus, the expected payoff of individual x can be defined as

$$\pi_x = \frac{1}{k}\sum_{y'\in H_x} a_{xy'}, \tag{5}$$

where H_x is the set of k neighbors of the individual x, and $a_{xy'}$ is the payoff from the interaction between the individual x and one of its neighbors y'. Subsequently, the individual x randomly selects one of its neighbors y ($y\in H_x$) from H_x as the model individual. Similarly, the same steps are applied to obtain the expected payoff π_y for y. With probability γ of SSUS and probability $1-\gamma$ of SCUD, Individual x considers whether to update his current strategy by comparing his average gain from interacting with his neighbors to the desired level A. We use a square network of size $L\times L$ with periodic boundary conditions. Figure 1 shows the mixed update model.

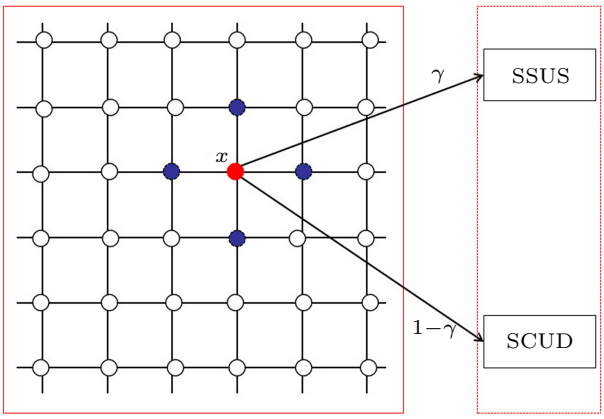


Fig. 1. A randomly selected individual x (defined as the focal individual) is located on a two-dimensional four-neighbor square network (on the left side) and x interacts with its four neighbors to come to a payoff. The focal individual x updates its strategy with an SSUS update rule with probability γ and an SCUD update rule with probability $1-\gamma$, where $\gamma\in[0,1]$.

2.5. Pair approximation model

In a structured population, a player's payoff depends on the configuration of its neighbors. The PA model must consider the frequency of strategies and the frequency of pairs of strategies. Suppose that p_{C} and p_{D} are the frequencies of cooperators and defectors on the structure population. Let p_{CC}, p_{CD}, p_{DC}, and p_{DD} denote the frequencies of CC, CD, DC, and DD pairs. Further, let $q_{X|Y}$ be the conditional probability of finding an individual of type X in the neighborhood of a focal type Y individual, where X and Y represent C or D. All these symbols follow the following constant equation:

$$p_{\mathrm{C}} + p_{\mathrm{D}} = 1, \tag{6a}$$

$$q_{\mathrm{C}|X} + q_{\mathrm{D}|X} = 1, \tag{6b}$$

$$p_{XY} = p_Y q_{X|Y}, \tag{6c}$$

$$p_{\mathrm{CD}} = p_{\mathrm{DC}}. \tag{6d}$$

This means that the whole system can be represented by p_{C} and $q_{\mathrm{C|C}}$. Assume that each gamer has k neighbors, where the number of C (D) participants is k_{C} (k_{D}) such that $k_{\mathrm{C}}+k_{\mathrm{D}}=k$. Thus, the expected payoffs of the cooperators and defectors in the structured population are, respectively,

$$\pi_{\mathrm{C}}=\frac{k_{\mathrm{C}}R+k_{\mathrm{D}}S}{k}, \tag{7a}$$

$$\pi_{\mathrm{D}}=\frac{k_{\mathrm{C}}T+k_{\mathrm{D}}P}{k}. \tag{7b}$$

3. Particle swarm optimization algorithm

The particle swarm optimization algorithm (PSO) is an evolutionary computation proposed by James Kennedy and Russell Eberhart in 1995,[41] stemming from behavioral studies of bird flock feeding. The algorithm was initially inspired by the regularity of a flock of flying birds' activities, which leads to a simplified model using group intelligence. Individual information sharing is utilized to make the motion of the whole flock evolve from disorder to order in the problem solution space and to find the global optimum by following the currently searched optimum. It starts from a random solution and searches for the optimal solution through iteration. In the particle swarm algorithm, the feasible solution of each optimization problem can be regarded as a point in the d-dimensional search space, called a particle. Let the i-th particle be denoted as $s_i=(s_{i1},s_{i2},\ldots,s_{id})$, the velocity as $v_i=(v_{i1},v_{i2},\ldots,v_{id})$, and let the best position (i.e., the part with the best value of gain) experienced by it be denoted as $p_i=(p_{i1},p_{i2},\ldots,p_{id})$, also known as p_{best}. The overall best position experienced by all particles in the population is denoted by g_{best}. In each iteration, the velocity and position of the particles change according to the following equation:

$$v_i^{k+1}=wv_i^k+c_1r_1\left(p_{\mathrm{best}}^k(i)-s_i^k\right)+c_2r_2\left(g_{\mathrm{best}}^k-s_i^k\right), \tag{8a}$$

$$s_i^{k+1}=s_i^k+v_i^{k+1}, \tag{8b}$$

where c_1 is the individual learning factor, c_2 is the group learning factor; r_1 and r_2 are random numbers in the interval $[0,1]$; w is the inertia weight, and k is the number of iterations.

3.1. Implementation principle

Since both particle swarm algorithms and evolutionary games are based on the principle of biological iteration to find an optimum, we make the following analogy:

small groups in a population $\rightarrow$	particles in swarm,
strategy space $\rightarrow$	search space,
behavioral diffusion $\rightarrow$	iterative process,
payoff function $\rightarrow$	benefit function,
optimal payoff $\rightarrow$	iterative optimal payoff,
evolutionarily stable strategy $\rightarrow$	iterative optimal point.

Thus, all individual strategy changes in the population are analogous to the particles' search for an optimal solution. The idea of the evolutionary game can be used to improve the particle swarm optimization algorithm to simulate the population evolution process. Considering the rationality of mixed-strategy particle selection, it is assumed that the overall population consists of small groups, each of which collectively selects a strategy. Therefore, in the particle swarm algorithm, the small population can be regarded as a particle, and the particle can adopt the player's strategy.

3.2. Evolutionarily stable strategy

In a two-player symmetric game, we denote the common set of pure strategies by $K=\{1,2,\ldots,k\}$, where k is the number of strategies that the two players can implement. The corresponding set of mixed strategies is denoted as $\Delta=\{x\in R_+^k:\sum_{i\in K}x_i=1\}$.

For populations with an overall presentation strategy state of x, let the individuals in the total population adopt the (pure or mixed) original strategy $x\in\Delta$. After that, the strategies of some individuals change, and these individuals with strategy changes are defined as mutants. Individuals with no strategy change are defined as non-mutated individuals, assuming all mutants adopt the other (pure or mixed) mutated strategies $y\in\Delta$. The share of mutants in the population is denoted as ε, where $\varepsilon\in(0,1)$. In this bimodal aggregate, pairs of individuals are randomly and repeatedly sampled to play the game, where the probability that his opponent adopts the mutated strategy is ε, and the probability that he adopts the existing strategy is $1-\varepsilon$. Here, we assume that the sign of the payoff is μ. Thus, the payoff of the original strategy is $\mu[x,\varepsilon y+(1-\varepsilon)x]$, while the payoff of the variant strategy is $\mu[y,\varepsilon y+(1-\varepsilon)x]$. Evolution will choose the original strategy if the payoff of the corresponding individual is higher than that of the individual who chooses the variant strategy. The specific situation is as follows:

$$\mu[x,\varepsilon y+(1-\varepsilon)x]>\mu[y,\varepsilon y+(1-\varepsilon)x]. \tag{9}$$

In this case, the original strategy $x\in\Delta$ is considered to be evolutionarily stable, and the inequality holds for any mutant strategy $y\neq x$ if the mutant's share of the overall in the overall is sufficiently small.

In the classical 2×2 game, the strategy of the player is a two-dimensional vector with the sum of the vectors departing from 1. The second component can be expressed in terms of the first component. That is, there is only one variable in the game party 1, defined as x in the fitness function. Similarly, the strategy of the adversary is a two-dimensional vector that can be expressed in terms of a single variable, defined as the second variable y in the fitness function. Thus, the entire payoff function is binary.

3.3. PPSO algorithm

The PPSO algorithm implements stable equilibrium points based on the PSO algorithm framework and introduces the theory of social learning and group imitation into the PSO algorithm. In the PPSO algorithm, each stable equilibrium point represents a solution to a problem, and each participant in the group is considered as a particle of the PPSO algorithm. Participants with different strategies reflect the differences in positions between particles, and different gains reflect the differences in expected gains between particles, both of which constitute particle diversity.

Based on Eqs. (8a) and (8b), the PPSO algorithm iteration function is

$$v_i^{k+1} = i_{\text{best}}^k - s_i^k, \tag{10a}$$

$$s_i^{k+1} = \begin{cases} s_i^k + v_i^{k+1}, & \pi_i < A, \\ s_i^k, & \pi_i \geq A, \end{cases} \tag{10b}$$

where i_{best}^k is the particle with the highest expected payoff, the traditional idea of expectation dynamics is borrowed from the above particle iteration formula. When the gain exceeds the expectation, the particle position changes; otherwise, the particle position remains unchanged. The above equation draws on the original iterative process Eqs. (8a) and (8b) an idea derived from group imitation and social learning theory.

3.4. Algorithmic structure

(a) Algorithm initialization parameters. These parameters include the learning factor c, the lower bound of the search space pop min, the upper bound of the search space pop max, the number of iterations gen max, and the population size n.

(b) Create a population with n particles, and the algorithm randomly generates strategy $x^i = 0$ or 1 for the population of n particles.

(c) Use Eqs. (7a) and (7b) to compute the expected returns of all particles in the population and find the particle strategy with the highest expected return, denoted as i_{best}^k.

(d) All particles update their positions according to Eqs. (10a) and (10b).

(e) Decide whether to end the iteration according to the number of iterations gen max, and when the iteration ends, the optimal gain and particle position map in the algorithm's output. Otherwise, the algorithm turns to (c).

4. Result

4.1. The $\hat{p}_{\text{C}}$ of SSUS

For SSUS updating, the focal individual gains from interacting with its neighbors and then compares it with its expected level to decide whether to switch strategies. Under weak selection, we find that strategy C is better than strategy D if $R+S>T+P$, i.e., $\langle x_{\text{C}}\rangle > 1/2$ (where $\langle x_{\text{C}}\rangle$ denotes the average abundance of strategy C), and strategy D is better than strategy C if $R+S<T+P$, i.e., $\langle x_{\text{D}}\rangle > 1/2$ (where $\langle x_{\text{D}}\rangle$ denotes the average abundance of strategy D). The probability that C and CC change within a time step is (see Appendix A for the calculation procedure):

$$\begin{aligned}\dot{p}_{\text{C}}|_{\text{SSUS}} = {} & \frac{p_{\text{D}}-p_{\text{C}}}{2} + \frac{\omega}{4}\{(p_{\text{D}}-p_{\text{C}})A-(T-P)p_{\text{CD}} \\ & -Pp_{\text{D}}+(R-S)p_{\text{CC}}+Sp_{\text{C}}\}+O(\omega^2),\end{aligned} \tag{11a}$$

$$\begin{aligned}\dot{p}_{\text{CC}}|_{\text{SSUS}} = {} & p_{\text{CD}}-p_{\text{CC}}+\frac{\omega p_{\text{CD}}}{2} \\ & \times\left[A-\frac{T}{k}-(Tq_{\text{C|D}}+P-Pq_{\text{C|D}})\frac{k-1}{k}\right] \\ & -\frac{\omega p_{\text{CC}}}{2}\left[A-\frac{R}{k}-(Rq_{\text{C|C}}+S-Sq_{\text{C|C}})\right. \\ & \left.\times\frac{k-1}{k}\right]+O(\omega^2).\end{aligned} \tag{11b}$$

Similarly, here you can get the expression for $\dot{p}_{\text{DD}}|_{\text{SSUS}}$. By $\omega\to 0$, it is possible to obtain a balance of Eqs. (11a) and (11b), i.e., $(\hat{p}_{\text{C}},\hat{p}_{\text{CC}})=(\frac{1}{2},\frac{1}{4})$. Further, $\hat{p}_{\text{D}}=\frac{1}{2}$, $\hat{p}_{\text{CD}}=\hat{p}_{\text{DD}}=\frac{1}{4}$. According to perturbation theory, there exists f_{XY}, such that

$$\hat{p}_{\text{CC}} = \frac{1}{4}+\omega\times f_{\text{CC}}, \tag{12a}$$

$$\hat{p}_{\text{CD}} = \frac{1}{4}+\omega\times f_{\text{CD}}, \tag{12b}$$

$$\hat{p}_{\text{DC}} = \frac{1}{4}+\omega\times f_{\text{DC}}, \tag{12c}$$

$$\hat{p}_{\text{DD}} = \frac{1}{4}+\omega\times f_{\text{DD}}. \tag{12d}$$

Substituting the above expresses into the equation, when choosing intensity $\omega\to 0$, let $\dot{p}_{\text{C}}|_{\text{SSUS}}=0$, we have

$$\begin{aligned}&\frac{1-2p_{\text{C}}}{2}+\frac{\omega}{4}\left[(1-2p_{\text{C}})A-(T-P)\frac{1}{4}-P(1-p_{\text{C}})\right.\\ &\left.+(R-S)\frac{1}{4}+Sp_{\text{C}}\right]=0,\end{aligned} \tag{13}$$

such that

$$\hat{p}_{\text{C}}(\omega)|_{\text{SSUS}} = \frac{2+\frac{\omega}{4}(R+S-T-P)+\omega(A-P)}{4-\omega(S+P-2A)}. \tag{14}$$

In addition, it is able to determine the stability of $\hat{p}_{\text{C}}$ under a weak selection of $(\omega\leq 1)$, such that $V(p_{\text{C}})=\dot{p}_{\text{C}}$, then $V'(p_{\text{C}})|_{p_{\text{C}}=\hat{p}_{\text{C}}}=-1<0$. Also, being Taylor-expanded at equilibrium $\hat{p}_{\text{C}}$, and taking the linear term only once, then we have $V(p_{\text{C}})=V'(\hat{p}_{\text{C}})(p_{\text{C}}-\hat{p}_{\text{C}})$. Correspondingly, due to $V'(p_{\text{C}})|_{p_{\text{C}}=\hat{p}_{\text{C}}}=-1<0$, then $\hat{p}_{\text{C}}$ is the stabilization point.

4.2. The $\hat{p}_{\text{C}}$ of SCUD

For the SCUD update, under weak selection, we find that strategy C is better than strategy D if $R+S+T+P-4A>0$, i.e., $\langle x_{\text{C}}\rangle > 1/2$; strategy D is better than strategy C if $R+S+T+P-4A<0$, i.e., $\langle x_{\text{D}}\rangle > 1/2$; and the probability that C and CC change in a time step is (see Appendix A for the calculation procedure):

$$\dot{p}_{\text{C}}|_{\text{SCUD}} = \frac{p_{\text{D}}-p_{\text{C}}}{2}+\frac{\beta}{4}\{-A-[(d-c)p_{\text{CD}}$$

$$-dp_{\mathrm{D}}+(b-a)p_{\mathrm{CC}}-bp_{\mathrm{C}}]\}+O(\beta^2), \tag{15a}$$

$$\dot{p}_{\mathrm{CC}}|_{\mathrm{SCUD}}=p_{\mathrm{CD}}-p_{\mathrm{CC}}+\frac{\omega p_{\mathrm{CD}}}{2}\times\left[\frac{T}{k}+(Tq_{\mathrm{C|D}}+P-Pq_{\mathrm{C|D}})\frac{k-1}{k}-A\right]-\frac{\omega p_{\mathrm{CC}}}{2}\left[A-\frac{R}{k}-(Rq_{\mathrm{C|C}}+S-Sq_{\mathrm{C|C}})\times\frac{k-1}{k}\right]+O(\omega^2). \tag{15b}$$

Similarly, when selecting intensity $\omega\to 0$, let $\dot{p}_{\mathrm{C}}=0$, then

$$\frac{1-2p_{\mathrm{C}}}{2}+\frac{\omega}{4}\left[-A-(P-T)\frac{1}{4}+P(1-p_{\mathrm{C}})-(S-R)\frac{1}{4}+Sp_{\mathrm{C}}\right]=0. \tag{16}$$

Therefore,

$$\hat{p}_{\mathrm{C}}(\omega)|_{\mathrm{SCUD}}=\frac{2-\omega A-\frac{\omega}{4}(P-T+S-R)+\omega P}{4+\omega P-\omega S}, \tag{17}$$

and it is easy to show that $\hat{p}_{\mathrm{C}}$ is a stable equilibrium under a weak selection ($\omega\leq 1$).

4.3. The mixed aspiration updating process (SSUS-SCUD)

Under the mixed SSUS-SCUD rule (SSUS is used with a probability of γ, SCUD is used with a probability of $1-\gamma$, and the probability that the C and CC of the mixed aspiration updating process change within a time step is

$$\dot{p}_{\mathrm{C}}=\gamma\dot{p}_{\mathrm{C}}|_{\mathrm{SSUS}}+(1-\gamma)\dot{p}_{\mathrm{C}}|_{\mathrm{SCUD}}, \tag{18a}$$

$$\dot{p}_{\mathrm{CC}}=\gamma\dot{p}_{\mathrm{CC}}|_{\mathrm{SSUS}}+(1-\gamma)\dot{p}_{\mathrm{CC}}|_{\mathrm{SCUD}}. \tag{18b}$$

This result stays within the limit of weak selection ($\omega\leq 1$). Under weak selection, the mixed update rules are all the same as the zeroth order in ω, but the first-order terms differ.

5. Numerical example

In this section, we study the stable equilibrium point theoretical approximation results for the SSUS process, the SCUD process and the mixed SSUS-SCUD process, respectively, and simulate the stable equilibrium points using the PPSO algorithm.

This section considers the prisoner's dilemma, then the payoff matrix of the game is

$$\begin{array}{c} \\ \mathrm{C} \\ \mathrm{D} \end{array}\begin{array}{c} \begin{array}{cc}\mathrm{C} & \mathrm{D}\end{array} \\ \begin{bmatrix}1-p & -p\\ 1 & 0\end{bmatrix}\end{array}, \tag{19}$$

where $p\in(0,1)$ denotes the cost of cooperation. Next, to validate the theoretical approximation results for the evolution of facilitated cooperation under weak selection in the prisoner's dilemma, this paper investigates the effects of the cost p, the aspiration level A, and the trade-off coefficient γ on the proportion of cooperators in the outcome group. For the structural population, degree $k=4$ and $L\times L$ ($L=100$) square lattice network is selected for simulation here. That is, each individual performs a prisoner's dilemma with four surrounding neighbors, the expected payoff of the individual is calculated by Eqs. (7a) and (7b), the individual performs a strategy update by SSUS rules (2a) and (2b) or SCUD rules (3a) and (3b), respectively, and then the equilibrium point of the individual's strategy is simulated by a PPSO algorithm. In the algorithm, the learning factor $c=0.5$, the search space from $pop\min=0$ to $pop\max=1$, the number of iterations $gen\max=68$ and the population size $n=100$.

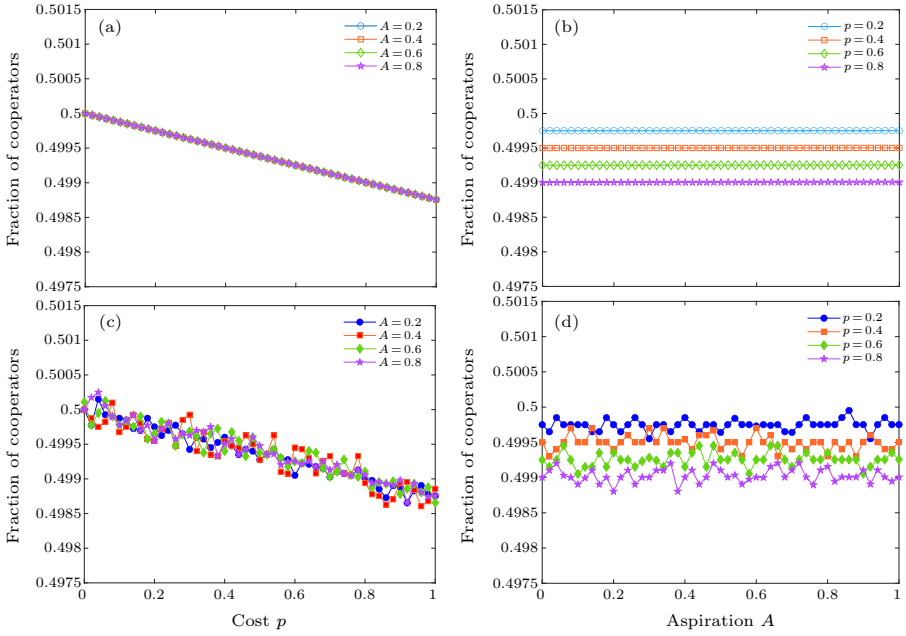


Fig. 2. Proportion of cooperators in the SSUS process varying with the cost p and the aspiration level A: (a) and (b) are theoretical approximations computed from Eq. (14), (c) and (d) are the results of simulations using the PPSO algorithm in a square lattice network with $k=4$ and all individuals at $L\times L$ ($L=100$). Here $\omega=0.01$.

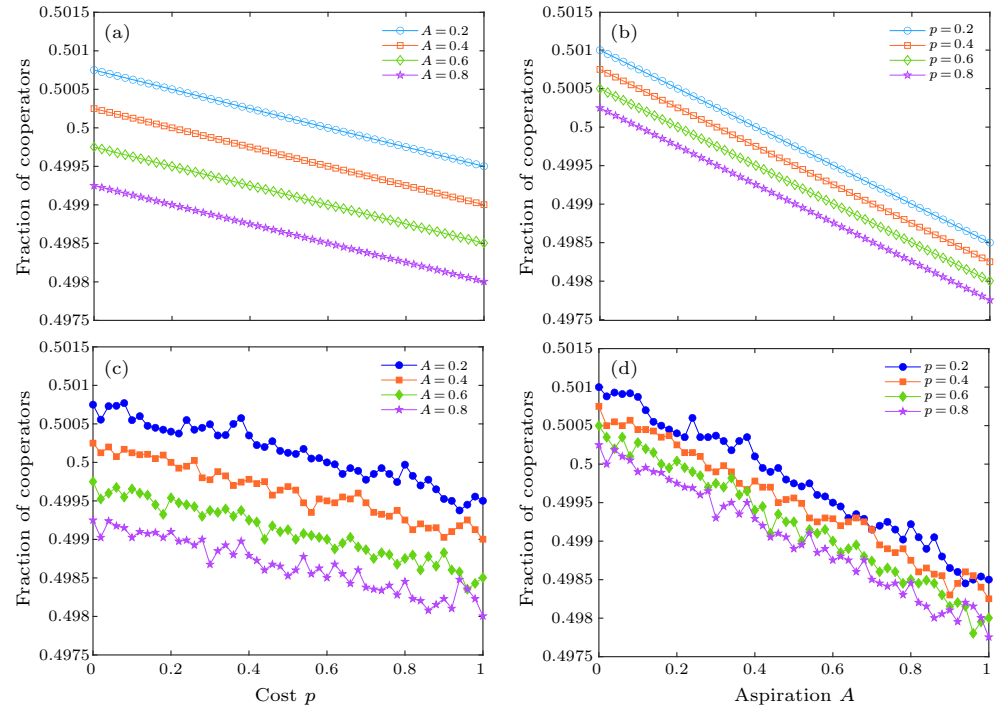


Fig. 3. Proportion of cooperators in the SCUD process varying with cost p and aspiration level A: (a) and (b) are theoretical approximations computed from Eq. (17), (c) and (d) are results of simulations using the PPSO algorithm in a square lattice network with $k=4$ and all individuals at $L\times L$ ($L=100$). Here $\omega=0.01$.

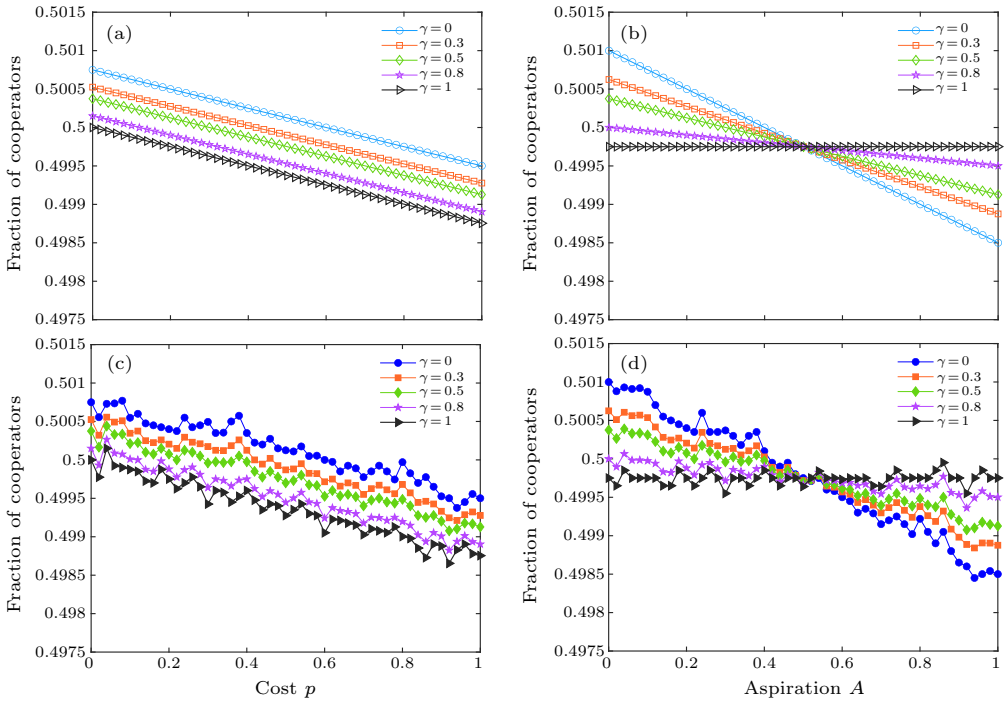


Fig. 4. The proportion of collaborators in the hybrid aspiration updating process according to different trade-off coefficients γ and thus to analyze the proportion of cooperators that change with the cost p and the aspiration level A: (a) and (c) are for a fixed aspiration level $A=0.2$, (b) and (d) are for a fixed cost $p=0.2$. Also, both (a) and (b) are theoretical approximations computed via Eq. (18a), (c) and (d) are the results of simulations using the PPSO algorithm in a square lattice network with $k=4$ and all individuals at $L\times L$ ($L=100$). The green and blue lines show the SSUS process and the SCUD process, respectively. The red, yellow, and purple lines show our proposed mixed processes of $\gamma=0.3$, $\gamma=0.5$, and $\gamma=0.8$, respectively. Here $\omega=0.01$.

Figures 2(a) and 2(b) illustrate the theoretical approximation results of SSUS, which find that for a fixed A, the proportion of cooperators decreases as the cost p increases. The ratio of cooperators is the same regardless of any change in A. Also, for fixed p, the proportion of cooperators is constant with increasing aspiration level A and cooperation is more dominant for small costs. Figures 2(c) and 2(d) demonstrate the simulation results, which show that the theoretical approximation results are consistent with the trend of changes in the simulation results. Therefore, the PPSO algorithm can accurately

simulate the particle positions and benefits with its stable equilibrium point and counterpart. It completely records the stable equilibrium point path of the SSUS process. The condition for promoting cooperation in SSUS is $R+S>T+P$. These results confirm that cooperation is no more dominant than defection for weak selection in the prisoner's dilemma.

Figures 3(a) and 3(b) illustrate the theoretical approximation results for SCUD, where for fixed A, the proportion of cooperators decreases with increasing cost p and cooperation is more dominant for small aspiration levels. Alternatively, for fixed p, the proportion of cooperators still decreases with increasing aspiration level A and cooperation is more prevalent for small costs. Figures 3(c) and 3(d) demonstrate the simulation results, which show that the theoretical approximation results are consistent with the trend of the simulation results. Therefore, the PPSO algorithm can accurately simulate the particle positions and benefits with its stable equilibrium point and counterpart. It completely records the stable equilibrium point path of the SCUD process. The condition that promotes cooperation in SSUS is $R+S+T+P-4A>0$. This result confirms that in the prisoner's dilemma for weak selection, then $A<(1-p)/2$, cooperation is more advantageous than defection.

Figures 4(a) and 4(b) illustrate the results of the theoretical approximation of the mixed aspiration update, where the proportion of cooperators decreases with increasing cost p, regardless of γ, for a fixed aspiration level $A=0.2$ in Fig. 4(a). The proportion of cooperators is lowest in the SSUS process $(\gamma=1)$ (black line). Therefore, they have a slow update rate, and it can also be seen that the SSUS process has the opposite update rate to the SCUD process, with the highest proportion of cooperators in the SCUD process $(\gamma=0)$ (blue line), and therefore a faster update rate. The proportions of cooperators in the mixed update processes $(\gamma=0.3)$, $(\gamma=0.5)$ and $(\gamma=0.8)$ are between the proportions of cooperators in the two single update processes. It can be obtained that adding the SCUD process to the SSUS process can promote the generation of cooperation as the cost p increases. Fixing the cost $p=0.2$ in Fig. 4(b), a phase channg point $A=0.5$ can be obtained. Before the phase transition point, the SSUS process has lowest cooperators and the SCUD process has the highest cooperators. However, after the phase transition point, the results for single and mixed processes are completely opposite. Figures 4(c) and 4(d) demonstrate the simulation results, which show that the theoretical approximation results are consistent with the trend of the simulation results. Therefore, the PPSO algorithm is able to accurately simulate the particle positions and benefits that want to correspond to the stable equilibrium point. It completely records the stable equilibrium point path of the mixed aspiration updating process.

6. Conclusion

This paper considers two aspiration-based learning rules, SSUS and SCUD, and investigates how such updating rules affect the evolutionary dynamics of cooperation in structured populations. Under weak selection, this paper considers the homogeneity of individual strategy types and gives analytic conditions under which cooperation prevails. In addition, compared with the traditional aspiration dynamics SSUS, the new aspiration dynamics SCUD can effectively solve the cooperation problem in the prisoner's dilemma game.

Previous work has demonstrated that traditional aspiration dynamics play an active role in the evolution of cooperation. For example, in the spatial prisoner's dilemma, Liu *et al.*[42] proposed an aspiration-based learning approach and showed that appropriate aspirations can lead to high levels of cooperation. In addition, Shen *et al.*[43] proposed an evolutionary approach based on aspiration-based connection weights in a spatial prisoner's dilemma game, where they found that appropriate aspirations promote high levels of cooperation, yet too high or too low aspirations inhibit the evolution of cooperation. In contrast, when the cost of cooperation is fixed, the new aspiration updating rule SCUD proposed in this paper demonstrates that low aspirations favor the evolution of cooperation. Further, for the traditional aspiration dynamics SSUS, previous work[44] has presented a common condition that facilitates the development of cooperation, namely $R+S>T+P$. However, this condition does not enable cooperation to emerge in the prisoner's dilemma. The new aspiration updating rule SCUD gives the condition for cooperation to evolve as $R+S+T+P-4A>0$ and finds that cooperation can be made to spring up in the prisoner's dilemma game with the right aspiration. For the mixed aspiration updating rule, under certain conditions, introducing the new aspiration updating rule SCUD can promote an increase in the proportion of cooperators in the traditional aspiration dynamics SSUS.

Using the PPSO algorithm, the stable equilibrium point realization process for three aspiration processes is simulated. The simulation results show that the PPSO algorithm demonstrates the realization path of the equilibrium point. By analyzing the effects of cost p and aspiration A on the equilibrium point realization, the results show that the theoretical approximation results are consistent with the changes in the simulation results. It is better verified that the aspiration dynamics of the unification SSUS conditions for promoting cooperation do not enable cooperation to spring up in the prisoner's dilemma. On the contrary, the new aspiration updating rule SCUD facilitating cooperation conditions can make cooperation emerge in the prisoner's dilemma.

Appendix A

We next compute the probability that the C and CC pairs change within a time step during SSUS process. If the probability of selecting a D individual is $p_{\rm D}$, then the probability of this configuration occurring is $\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}$ and $p_{\rm C}$ is increased by $1/N$,

$$\begin{aligned}\text{Prob}\Big(\Delta p_{\rm C}=\frac{1}{N}\Big)=p_{\rm D}\sum_{k_{\rm C}=0}^{k}\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}\\ \times\frac{1}{1+{\rm e}^{-\omega(A-\pi_{\rm D})}}.\end{aligned}\tag{A1}$$

In addition, as the number of CC pairs increases with increasing $k_{\rm C}$, the probability that $p_{\rm CC}$ increases by $\frac{2k_{\rm C}}{kN}$ is

$$\begin{aligned}\text{Prob}\Big(\Delta p_{\rm CC}=\frac{2k_{\rm C}}{kN}\Big)=p_{\rm D}\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}\\ \times\frac{1}{1+{\rm e}^{-\omega(A-\pi_{\rm D})}}.\end{aligned}\tag{A2}$$

If the probability of selecting a C individual is $p_{\rm C}$, then the probability of this configuration occurring is $\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}$ and $p_{\rm C}$ is increased by $1/N$,

$$\begin{aligned}\text{Prob}\Big(\Delta p_{\rm C}=-\frac{1}{N}\Big)=p_{\rm C}\sum_{k_{\rm C}=0}^{k}\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|C}^{k_{\rm C}}q_{\rm D|C}^{k_{\rm D}}\\ \times\frac{1}{1+{\rm e}^{-\omega(A-\pi_{\rm C})}}.\end{aligned}\tag{A3}$$

In addition, as the number of CC pairs decreases with decreasing $k_{\rm C}$, the probability that $p_{\rm CC}$ decreases by $\frac{2k_{\rm C}}{kN}$ is

$$\begin{aligned}\text{Prob}\Big(\Delta p_{\rm DD}=-\frac{2k_{\rm C}}{kN}\Big)=p_{\rm C}\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|C}^{k_{\rm C}}q_{\rm D|C}^{k_{\rm D}}\\ \times\frac{1}{1+{\rm e}^{-\omega(A-\pi_{\rm C})}}.\end{aligned}\tag{A4}$$

Here, assuming that a substitution event occurs in one unit of time, the time derivatives of $p_{\rm C}$ and $P_{\rm CC}$ are, respectively,

$$\begin{aligned}\dot{p}_{\rm C}|_{\rm SSUS}&=\Big[\frac{1}{N}\text{Prob}\Big(\Delta p_{\rm C}=\frac{1}{N}\Big)\\&\quad+\Big(-\frac{1}{N}\Big)\text{Prob}\Big(\Delta p_{\rm C}=-\frac{1}{N}\Big)\Big]\Big/\Big[\frac{1}{N}\Big]\\&=\dot{p}_{\rm C}|_{\rm SSUS}(\omega=0)+\omega\times\frac{\partial\dot{p}_{\rm C}|_{\rm SSUS}}{\partial\omega}|_{\omega=0}+O(\omega^2)\\&=\frac{p_{\rm D}-p_{\rm C}}{2}+\frac{\omega}{4}\{(p_{\rm D}-p_{\rm C})A-(T-P)p_{\rm CD}\\&\quad-Pp_{\rm D}+(R-S)p_{\rm CC}+Sp_{\rm C}\}+O(\omega^2),\end{aligned}\tag{A5}$$

$$\begin{aligned}\dot{p}_{\rm CC}|_{\rm SSUS}&=\Big[\sum_{k_{\rm C}=0}^{k}\frac{2k_{\rm C}}{kN}\text{Prob}\Big(\Delta p_{\rm CC}=\frac{2k_{\rm C}}{kN}\Big)+\sum_{k_{\rm C}=0}^{k}\\&\quad\times\Big(-\frac{2k_{\rm C}}{kN}\Big)\text{Prob}\Big(\Delta p_{\rm CC}=-\frac{2k_{\rm C}}{kN}\Big)\Big]\Big/\Big[\frac{1}{N}\Big]\\&=\dot{p}_{\rm CC}|_{\rm SSUS}(\omega=0)+\omega\times\frac{\partial\dot{p}_{\rm CC}|_{\rm SSUS}}{\partial\omega}|_{\omega=0}\\&\quad+O(\omega^2)\\&=p_{\rm CD}-p_{\rm CC}+\omega\times\frac{\partial\dot{p}_{\rm CC}}{\partial\omega}|_{\omega=0}+O(\omega^2),\end{aligned}\tag{A6}$$

where

$$\begin{aligned}\frac{\partial\dot{p}_{\rm CC}}{\partial\beta}|_{\beta=0}&=\sum_{k_{\rm C}=0}^{k}\frac{2kc}{k}p_{\rm D}\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}\frac{\pi_{\rm D}-A}{4}\\&\quad-\sum_{k_{\rm C}=0}^{k}\frac{2kc}{k}p_{\rm C}\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}\frac{A-\pi_{\rm C}}{4}\\&=\sum_{k_{\rm C}=0}^{k}\frac{2kc}{k}p_{\rm D}\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}\\&\quad\times\frac{\frac{k_{\rm C}T+k_{\rm D}P}{k}-A}{4}-\sum_{k_{\rm C}=0}^{k}\frac{2kc}{k}p_{\rm C}\frac{k!}{k_{\rm C}!k_{\rm D}!}\\&\quad\times q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}\frac{A-\frac{k_{\rm C}R+k_{\rm D}S}{k}}{4}\\&=\frac{p_{\rm D}}{2}\sum_{k_{\rm C}=0}^{k}\frac{kc}{k}\frac{k!}{k_{\rm C}!k_{\rm D}!}q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}\\&\quad\times\Big(\frac{k_{\rm C}T+k_{\rm D}P}{k}-A\Big)-\frac{p_{\rm C}}{2}\sum_{k_{\rm C}=0}^{k}\frac{kc}{k}\frac{k!}{k_{\rm C}!k_{\rm D}!}\\&\quad\times q_{\rm C|D}^{k_{\rm C}}q_{\rm D|D}^{k_{\rm D}}\Big(A-\frac{k_{\rm C}R+k_{\rm D}S}{k}\Big)\\&=\frac{p_{\rm D}}{2}\Big[(-A+P)q_{\rm C|D}-\frac{(P-T)q_{\rm C|D}^2}{k}\\&\quad+(P-T)q_{\rm C|D}^2-\frac{(T-P)q_{\rm C|D}}{k}\Big]\\&\quad-\frac{p_{\rm C}}{2}\Big[(A-S)q_{\rm C|C}+\frac{(R-S)q_{\rm C|C}}{k}\\&\quad-(R-S)q_{\rm C|C}^2+\frac{(R-S)q_{\rm C|C}^2}{k}\Big]\\&=\frac{p_{\rm CD}}{2}\Big[A-\frac{T}{k}-(Tq_{\rm C|D}+P-Pq_{\rm C|D})\frac{k-1}{k}\Big]\\&\quad-\frac{p_{\rm CC}}{2}\Big[A-\frac{R}{k}-(Rq_{\rm C|C}+S-Sq_{\rm C|C})\frac{k-1}{k}\Big].\end{aligned}\tag{A7}$$

Similarly, we next compute the probability that the C and CC pairs change within a time step during SCUD process. Here, assuming that a substitution event occurs in one unit of time, the time derivatives of $p_{\rm C}$ and $p_{\rm CC}$ are, respectively,

$$\begin{aligned}\dot{p}_{\rm C}|_{\rm SCUD}&=\Big[\frac{1}{N}\text{Prob}\Big(\Delta p_{\rm C}=\frac{1}{N}\Big)\\&\quad+\Big(-\frac{1}{N}\Big)\text{Prob}\Big(\Delta p_{\rm C}=-\frac{1}{N}\Big)\Big]\Big/\Big[\frac{1}{N}\Big]\\&=\dot{p}_{\rm C}|_{\rm SCUD}(\omega=0)+\omega\times\frac{\partial\dot{p}_{\rm C}|_{\rm SCUD}}{\partial\omega}|_{\omega=0}\\&\quad+O(\omega^2)\\&=\frac{p_{\rm D}-p_{\rm C}}{2}+\frac{\omega}{4}\{-A-[(P-T)p_{\rm CD}-Pp_{\rm D}\\&\quad+(S-R)p_{\rm CC}-Sp_{\rm C}]\}+O(\omega^2),\end{aligned}\tag{A8}$$

$$\begin{aligned}\dot{p}_{\rm CC}|_{\rm SCUD}&=\Big[\sum_{k_{\rm C}=0}^{k}\frac{2k_{\rm C}}{kN}\text{Prob}\Big(\Delta p_{\rm CC}=\frac{2k_{\rm C}}{kN}\Big)+\sum_{k_{\rm C}=0}^{k}\\&\quad\times\Big(-\frac{2k_{\rm C}}{kN}\Big)\text{Prob}\Big(\Delta p_{\rm CC}=-\frac{2k_{\rm C}}{kN}\Big)\Big]\Big/\Big[\frac{1}{N}\Big]\end{aligned}$$

$$
\begin{aligned}
&= \dot{p}_{\mathrm{CC}}|_{\mathrm{SCUD}}(\omega=0)+\omega\times\frac{\partial \dot{p}_{\mathrm{CC}}|_{\mathrm{SCUD}}}{\partial\omega}|_{\omega=0}\\
&\quad +O(\omega^2)\\
&= p_{\mathrm{CD}}-p_{\mathrm{CC}}+\omega\times\frac{\partial \dot{p}_{\mathrm{CC}}}{\partial\omega}|_{\omega=0}+O(\omega^2), \qquad \text{(A9)}
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial \dot{p}_{\mathrm{CC}}}{\partial\beta}|_{\beta=0} &= \sum_{k_{\mathrm{C}}=0}^{k}\frac{2kc}{k}p_{\mathrm{D}}\frac{k!}{k_{\mathrm{C}}!k_{\mathrm{D}}!}q_{\mathrm{C}|\mathrm{D}}^{k_{\mathrm{C}}}q_{\mathrm{D}|\mathrm{D}}^{k_{\mathrm{D}}}\frac{\pi_{\mathrm{D}}-A}{4}\\
&\quad -\sum_{k_{\mathrm{C}}=0}^{k}\frac{2kc}{k}p_{\mathrm{C}}\frac{k!}{k_{\mathrm{C}}!k_{\mathrm{D}}!}q_{\mathrm{C}|\mathrm{D}}^{k_{\mathrm{C}}}q_{\mathrm{D}|\mathrm{D}}^{k_{\mathrm{D}}}\frac{A-\pi_{\mathrm{C}}}{4}\\
&= \sum_{k_{\mathrm{C}}=0}^{k}\frac{2kc}{k}p_{\mathrm{D}}\frac{k!}{k_{\mathrm{C}}!k_{\mathrm{D}}!}q_{\mathrm{C}|\mathrm{D}}^{k_{\mathrm{C}}}q_{\mathrm{D}|\mathrm{D}}^{k_{\mathrm{D}}}\\
&\quad \times\frac{\frac{k_{\mathrm{C}}T+k_{\mathrm{D}}P}{k}-A}{4}-\sum_{k_{\mathrm{C}}=0}^{k}\frac{2kc}{k}p_{\mathrm{C}}\frac{k!}{k_{\mathrm{C}}!k_{\mathrm{D}}!}\\
&\quad \times q_{\mathrm{C}|\mathrm{D}}^{k_{\mathrm{C}}}q_{\mathrm{D}|\mathrm{D}}^{k_{\mathrm{D}}}\frac{A-\frac{k_{\mathrm{C}}R+k_{\mathrm{D}}S}{k}}{4}\\
&= \frac{p_{\mathrm{D}}}{2}\sum_{k_{\mathrm{C}}=0}^{k}\frac{kc}{k}\frac{k!}{k_{\mathrm{C}}!k_{\mathrm{D}}!}q_{\mathrm{C}|\mathrm{D}}^{k_{\mathrm{C}}}q_{\mathrm{D}|\mathrm{D}}^{k_{\mathrm{D}}}\\
&\quad \times\left(\frac{k_{\mathrm{C}}T+k_{\mathrm{D}}P}{k}-A\right)-\frac{p_{\mathrm{C}}}{2}\sum_{k_{\mathrm{C}}=0}^{k}\frac{kc}{k}\frac{k!}{k_{\mathrm{C}}!k_{\mathrm{D}}!}\\
&\quad \times q_{\mathrm{C}|\mathrm{D}}^{k_{\mathrm{C}}}q_{\mathrm{D}|\mathrm{D}}^{k_{\mathrm{D}}}\left(A-\frac{k_{\mathrm{C}}R+k_{\mathrm{D}}S}{k}\right)\\
&= \frac{p_{\mathrm{D}}}{2}\left[(-A+P)q_{\mathrm{C}|\mathrm{D}}+\frac{(P-T)q_{\mathrm{C}|\mathrm{D}}^2}{k}\right.\\
&\quad \left.-(P-T)q_{\mathrm{C}|\mathrm{D}}^2+\frac{(P-T)q_{\mathrm{C}|\mathrm{D}}}{k}\right]\\
&\quad -\frac{p_{\mathrm{C}}}{2}\left[(A-S)q_{\mathrm{C}|\mathrm{C}}+\frac{(R-S)q_{\mathrm{C}|\mathrm{C}}}{k}\right.\\
&\quad \left.-(R-S)q_{\mathrm{C}|\mathrm{C}}^2+\frac{(R-S)q_{\mathrm{C}|\mathrm{C}}^2}{k}\right]\\
&= \frac{p_{\mathrm{CD}}}{2}\left[\frac{T}{k}+(Tq_{\mathrm{C}|\mathrm{D}}+P-Pq_{\mathrm{C}|\mathrm{D}})\frac{k-1}{k}-A\right]\\
&\quad -\frac{p_{\mathrm{CC}}}{2}\left[A-\frac{R}{k}-(Rq_{\mathrm{C}|\mathrm{C}}+S-Sq_{\mathrm{C}|\mathrm{C}})\frac{k-1}{k}\right]. \qquad \text{(A10)}
\end{aligned}
$$

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