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Thermodynamics in a quantum corrected Reissner–Nordström–AdS black hole and its GUP-corrections

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We calculate the thermodynamic quantities in the quantum corrected Reissner–Nordström–AdS (RN-AdS) black hole, and examine their quantum corrections. By analyzing the mass and heat capacity, we give the critical state and the remnant state, respectively, and discuss their consistency. Then, we investigate the quantum tunneling from the event horizon of massless scalar particle by using the null geodesic method, and charged massive boson $W^{\pm}$ and fermions by using the Hamilton–Jacob method. It is shown that the same Hawking temperature can be obtained from these tunneling processes of different particles and methods. Next, by using the generalized uncertainty principle (GUP), we study the quantum corrections to the tunneling and the temperature. Then the logarithmic correction to the black hole entropy is obtained.

Keywords: black hole thermodynamics, quantum corrections, quantum tunneling, generalized uncertainty principle

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1. Introduction

In the last century, Stephen Hawking used the quantum field theory of curved spacetime to propose that black holes emit radiation that may eventually cause them to evaporate and disappear.[1,2] Hawking's radiation theory has led to a great deal of research and discussion, and has developed into black hole thermodynamics, which has become an important part of modern theoretical physics (see Ref. [3] as a review and references therein). Hence it has still received a lot of attention in recent years.[4–10] But it also poses many difficulties, for example, the problem of black hole information loss,[11–15] the statistical origin of black hole entropy,[16,17] and black hole temperature dispersion.[18–20] It is generally believed that to solve these problems, the unity of general relativity and quantum mechanics is required. However, the complete theory of spacetime quantization has not yet been established in current theoretical physics.[21]

At the end of the last century, Parikh and Wilczek proposed that Hawking radiation can be calculated by analyzing the semiclassical models of particle tunneling in the event horizon.[22,23] They believe that after the particles fall through the interface, they still have a chance to escape from the black hole according to the quantum tunneling effect. In this method, the emission rate is related to the changes in the black hole entropy before and after the emission, and the non-thermal spectrum is obtained. Thus, this method presents a unitary description of Hawking radiation and opens a way to a possible resolution of the information loss problem. Currently, two methods can be used to calculate the particle action, one is the null geodesic method[24–28] and the other is the Hamilton–Jacob method.[29–36]

There have been many studies on Hawking radiation corresponding to different particle's tunneling, including the scalar particles,[22,23,37,38] spin-1/2 particles,[32–34,39] spin-1 particles[35,40–45] and spin-3/2 particles.[46,47] In particle physics, the $W^{\pm}$ and Z^0 bosons are vector bosons, collectively known as weak bosons, or more generally as intermediate vector bosons, which are important in the Standard Model. The Z^0 boson is electrically neutral, and $W^{\pm}$ bosons have either a positive or negative electric charge of one elementary charge. They are particles with spin-1 and can be described by the Proca equation. For these three particles, this work focuses on the tunneling of the $W^{\pm}$ boson across the horizon. Fermions are also an important part of the Standard Model. Their spin is half-integer. Fermions with spin 1/2 can be described by the Dirac equation. In this work, we study charged fermions with spin-1/2.

Currently developing quantum gravity theory universally predicts the existence of a minimum length or the maximum momentum on the Planck scale.[20,48] On this basis, the discrete pictures of spacetime are analyzed, and several phenomenological quantum gravity models are put forward.[49,50] One such a model is called GUP.[51] It comes from a combination of quantum mechanics and general relativity,[20,48] which reveals that the basic physics laws may change under extreme conditions, such as extremely high energy[48,52,53] or extremely strong gravitational field.[8,54–58] Applying the GUP to black hole physics, the corrections to the thermodynamic quantities are derived and the resolution of the difficulties of black hole thermodynamics is promoted (see Ref. [51] as a review and references therein).

In addition, in the 1990s, Kazakov and Solodukhin ob-

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tained a quantum corrected Schwarzschild black hole by adding spherically symmetric quantum excitations to the spacetime.[59] Following this, in recent years, the quantum corrected RN-AdS black holes have been obtained by combining the quantum fluctuations with RN-AdS black holes.[60] In contrast to the usual RN-AdS black hole, the quantum corrected RN-AdS black hole introduces the spherically symmetric quantum fluctuations of metric.[60,61]

To discuss the quantum properties of black hole thermodynamics, we study quantum tunneling and consider the GUP effects in the quantum corrected RN-AdS black hole. Furthermore, to examine the influence of the type of radiating particles by the tunneling result, we calculate the tunneling rates of different kinds of particles by using different methods for obtaining particle action. The rest of this paper is organized as follows. In Section 2, we calculate the thermodynamic quantities and discuss the minimum radius of the quantum corrected RN-AdS black hole. In Section 3, we study the tunneling of massless scalar particles, massive vector particles, and fermions particles, respectively. In Section 4, considering GUP, we analyze the quantum corrections to the tunneling rate, temperature and entropy of the black hole. Section 5 is devoted to the conclusions drawn from the present study and discussion about some relevant points. The natural unit system with $G = k = c = \hbar = 1$ is used in this paper.

2. Thermodynamic quantities of quantum corrected RN-AdS black hole

The quantum corrected RN-AdS black hole is one of the solutions of general relativity with quantum excitations of spacetime, which describes a black hole possessing mass M and electric charge Q that exists in anti de-Sitter (AdS) spacetime with spherically symmetric quantum fluctuations of the metric. Its line element is[60]

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t_s^2 + f(r)^{-1}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2, \tag{1}$$

where

$$f(r) = \sqrt{1 - \frac{a^2}{r^2}} - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{L^2}. \tag{2}$$

Here, $a = 4L_{\mathrm{p}}$ represents the quantum excitation in spherically symmetric spacetime, and $L^2 = -3/\Lambda$, where Λ is the negative cosmological constant.

The event horizon r_{h} can be obtained by letting $f(r) = 0$ and the mass M can be expressed by r_{h} as

$$M = \frac{1}{2}\left(\sqrt{r_{\mathrm{h}}^2 - a^2} + \frac{r_{\mathrm{h}}^3}{L^2} + \frac{Q^2}{r_{\mathrm{h}}}\right). \tag{3}$$

Considering that the mass should be a real number, the restriction on the values of r_{h} can be obtained from Eq. (3) and expressed as

$$r_{\mathrm{h}} \geq a. \tag{4}$$

This reflects the constraint of spacetime quantum fluctuations on a black hole scale. So,

$$r_{\mathrm{ch}} = a \tag{5}$$

is the smallest value of the black hole radius, which presents the critical state of the quantum corrected RN-AdS black hole. When $r_{\mathrm{h}} < a$, there will be complex singularities in the mass, which is unacceptable to physics. By substituting Eq. (5) into Eq. (3), the mass of the critical state can be calculated from

$$M = \frac{a^4 + L^2Q^2}{2aL^2}. \tag{6}$$

The surface gravity of the static black hole is given by

$$\kappa = -\frac{1}{2}\lim_{r\to r_{\mathrm{h}}}\frac{\partial_r g_{\mathrm{tt}}}{\sqrt{-g_{\mathrm{tt}}g_{\mathrm{rr}}}} = \frac{f'(r_{\mathrm{h}})}{2}. \tag{7}$$

Then, the temperature of the quantum corrected RN-AdS black hole can be calculated from

$$T_\kappa = \frac{\kappa}{2\pi} = \frac{a^2}{4\pi r_{\mathrm{h}}^2\sqrt{r_{\mathrm{h}}^2 - a^2}} + \frac{\sqrt{r_{\mathrm{h}}^2 - a^2}}{4\pi r_{\mathrm{h}}^2} + \frac{3r_{\mathrm{h}}}{4\pi L^2} - \frac{Q^2}{4\pi r_{\mathrm{h}}^3}. \tag{8}$$

Figure 1 shows that the temperature of quantum corrected RN-AdS black hole is higher than that of RN-AdS black hole. This suggests that the quantum fluctuations of spacetime cause the black hole temperature to rise. Furthermore, when $r_{\mathrm{h}} > 6$, the increase is slight, but near $r_{\mathrm{h}} = 4$, *i.e.*, $r_{\mathrm{h}} = a$, this increase is significant and has a fundamental change. In fact, substituting Eq. (5) into Eq. (8), we can obtain the temperature of the critical state expressed as

$$T_{\mathrm{ck}} \to \infty. \tag{9}$$

This means that when the quantum fluctuation of spacetime is taken into account, the singularity of the temperature in classical black hole is shifted from $r_{\mathrm{h}} = 0$ to $r_{\mathrm{h}} = a$.

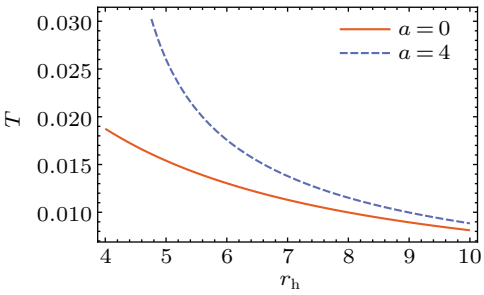


Fig. 1. Plot of T *versus* r_{h} with $L = 100$ and $Q = 4$. For RN-AdS black hole $a = 0$, for quantum corrected RN-AdS black hole, setting $L_{\mathrm{P}} = 1$ leads to $a = 4$. It is seen that the temperature of the quantum corrected RN-AdS black hole is higher than that of the RN-AdS black hole, suggesting that the temperature of the black hole will increase by taking into account the quantum fluctuations of spacetime. Furthermore, for the quantum corrected RN-AdS black hole, in the neighborhood of $r_{\mathrm{h}} = 4$, $T \to \infty$, implying that the temperature of the critical state is singularity.

It has been shown that the cosmological constant can be regarded as the thermodynamic pressure of a black hole as[62–64]

$$P=\frac{3}{8\pi L^2}. \tag{10}$$

Furthermore, for the quantum corrected RN-AdS black hole, the quantum excitation parameter a can also be considered as a thermodynamic variable.[65] Then, to determine and calculate the other thermodynamic quantities, we use the first law of thermodynamics

$$\mathrm{d}M=T\,\mathrm{d}S+\Phi\mathrm{d}Q+V\,\mathrm{d}p+\mu_a\mathrm{d}a. \tag{11}$$

The electric potential V can be calculated and expressed as

$$\Phi=\left(\frac{\partial M}{\partial Q}\right)_{T,V,\mu_a}=\frac{Q}{r_\mathrm{h}}, \tag{12}$$

and μ_a can be defined by Eq. (11) and obtained as follows:

$$\mu_a=\left(\frac{\partial M}{\partial a}\right)_{T,V,\Phi}=-\frac{a}{2\sqrt{r^2-a^2}}. \tag{13}$$

Substituting Eqs. (3) and (12) into Eq. (11) yields

$$\mathrm{d}S=\frac{\mathrm{d}M-\Phi\mathrm{d}Q-V\,\mathrm{d}p-\mu_a\mathrm{d}a}{T}=2\pi r_\mathrm{h}\,\mathrm{d}r_\mathrm{h}. \tag{14}$$

Integrating the above equation, the black hole entropy S for the black hole can be obtained as

$$S=\pi r_\mathrm{h}^2=\frac{A}{4}, \tag{15}$$

where $A=\int\sqrt{-g}\mathrm{d}\theta\,\mathrm{d}\varphi=4\pi r_\mathrm{h}^2$ is the area of the event horizon. It is seen that in the case of quantum excitation of spacetime, the corrected RN-AdS black hole satisfies the Bekenstein–Hawking area law.

By substituting Eq. (5) into Eq. (15), we can obtain the entropy of the critical state as

$$S_\mathrm{c}=\pi a^2. \tag{16}$$

This is a non-zero value and means that some information is retained in the critical state of the black hole. This may provide inspiration for the problem of black hole information.

The heat capacity at constant volume of a black hole is defined as

$$C_v=T\frac{\partial S}{\partial T}=T\frac{\partial S}{\partial r_\mathrm{h}}\frac{\partial r_\mathrm{h}}{\partial T}. \tag{17}$$

Then, substituting Eqs. (8) and (15) into Eq. (17), the heat capacity of the quantum corrected RN-AdS black hole can be obtained as

$$C_v=\frac{2\pi r_\mathrm{h}^2\left(r_\mathrm{h}^2-a^2\right)\left(L^2\left(r_\mathrm{h}^3-Q^2\sqrt{r_\mathrm{h}^2-a^2}\right)+3r_\mathrm{h}^4\sqrt{r_\mathrm{h}^2-a^2}\right)}{3r_\mathrm{h}^4\left(r_\mathrm{h}^2-a^2\right)^{3/2}-L^2\left(-3Q^2r_\mathrm{h}^2\sqrt{r_\mathrm{h}^2-a^2}+3a^2Q^2\sqrt{r_\mathrm{h}^2-a^2}+r_\mathrm{h}^5\right)}. \tag{18}$$

In the limit $a\to 0$, equation (18) reduces to the heat capacity of an RN-AdS black hole,[61] that is

$$\lim_{a_1\to 0}C_v=\frac{2\pi r_\mathrm{h}^2\left(L^2\left(r_\mathrm{h}^2-Q^2\right)+3r_\mathrm{h}^4\right)}{L^2\left(3Q^2-r_\mathrm{h}^2\right)+3r_\mathrm{h}^4}. \tag{19}$$

Considering at the end of black hole evaporation, there is[66]

$$C_v=0. \tag{20}$$

Then from Eqs. (18) and (20), we have

$$r_\mathrm{rh}=a. \tag{21}$$

This is the radius of the radiation remnant of the quantum corrected black hole. Furthermore, there may exist another solution for Eq. (20), specifically for the case of

$$\left(L^2\left(r_\mathrm{h}^3-Q^2\sqrt{r_\mathrm{h}^2-a^2}\right)+3r_\mathrm{h}^4\sqrt{r_\mathrm{h}^2-a^2}\right)=0. \tag{22}$$

However, this equation cannot be solved analytically. Therefore, in the subsequent discussion, we first utilize numerical methods to ascertain whether a solution exists under certain conditions. We assume that $L=100$, $Q=100$, and $a=4$, and depict the variation relationship between

$$f=\left(L^2\left(r_\mathrm{h}^3-Q^2\sqrt{r_\mathrm{h}^2-a^2}\right)+3r_\mathrm{h}^4\sqrt{r_\mathrm{h}^2-a^2}\right)$$

and event horizon r_h radius in Fig. 2.

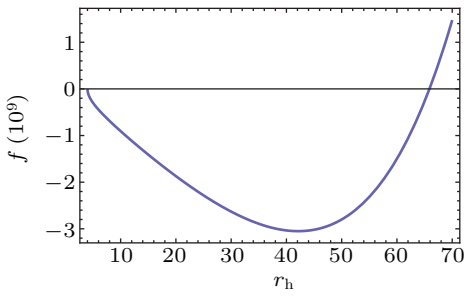


Fig. 2. The f as a function of horizon radius with $L=100$, $Q=100$, $a=4$, showing that the curve intersects with $f=0$, implying that there are other values of the event horizon radius for which the black hole's heat capacity equals zero.

It can be observed that the curve intersects with $f=0$, indicating that under certain conditions, there exist situations where the black hole heat capacity is zero when $r_\mathrm{h}\neq a$. However, this scenario is different from the case when $r_\mathrm{h}\neq a$. In

the latter case, even if the black hole's heat capacity is zero, the black hole, as a unique thermodynamic system, can absorb the matters with different charge-to-mass ratios, thereby changing its heat capacity and allowing the evaporation process to continue.

Comparing Eq. (5) with Eq. (21), one can see that for the quantum corrected RN-AdS black hole, the critical state is just the remnant state. Then, when the black hole reaches a critical state, it will not evaporate anymore. This can avoid the complex singularity of mass through the actual radiation process. On the other hand, the remnant state is a critical state. This can ensure that there will be no further evaporation after having reached the remnant state due to the non-singularity of black hole mass.

3. Hawking radiation as quantum tunneling

First, in the quantum corrected RN-AdS black hole, we derive the Hawking radiation by using the quantum tunneling method. In the method, for the particle to pass though the event horizon, the tunneling rate Γ can be obtained by the imaginary part of the action I, that is[22,23]

$$\Gamma \sim \exp(-2\,\mathrm{Im}I). \tag{23}$$

Next, we perform the specific calculations of the action. To investigate the universality of tunneling results, we study the tunneling of different kinds of particles and use different methods of obtaining particle action.

3.1. Massless scalar particle tunneling

For the massless scalar particle, we use the null geodesic method.[24–28] To perform the calculations of the particles tunneling through the event horizon, we use the Painlevé coordinate transformation to remove the coordinate singularities of element (1), that is,[67]

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + 2\sqrt{1-f(r)}\,\mathrm{d}t\,\mathrm{d}r + \mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2. \tag{24}$$

For the massless scalar particle tunneling, we use the null geodesic method and set $\mathrm{d}s^2 = \mathrm{d}\theta = \mathrm{d}\varphi = 0$. By using Eq. (24), we can obtain the null geodesic equation

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = \pm 1 - \sqrt{1-f(r)}, \tag{25}$$

where the symbol "+" represents outgoing particles, while the symbol "−" refers to incoming particles. To study the radiation outside the horizon, we take "+". A Taylor expansion of Eq. (25) at $r = r_\mathrm{h}$ gives

$$\begin{aligned}\dot{r} &= \frac{1}{2}f'(r)(r-r_\mathrm{h}) + o(r-r_\mathrm{h})^2 \\ &= \kappa(M)(r-r_\mathrm{h}) + o(r-r_\mathrm{h})^2.\end{aligned} \tag{26}$$

The imaginary part of the action of the particle passing through the event horizon is

$$\mathrm{Im}I = \mathrm{Im}\int_{r_\mathrm{i}}^{r_\mathrm{f}} p_r\,\mathrm{d}r = \mathrm{Im}\int_{r_\mathrm{i}}^{r_\mathrm{f}}\int_0^{p_r} \mathrm{d}p_r'\,\mathrm{d}r, \tag{27}$$

where I represents the zeroth-order action in the WKB approximation wave function in the tunneling process. According to the Hamiltonian equation, we have $\dot{r} = \mathrm{d}M/\mathrm{d}p_r$ and we obtain that

$$\mathrm{Im}I = \mathrm{Im}\int_0^{\omega}\int_{r_\mathrm{i}}^{r_\mathrm{f}} \frac{\mathrm{d}r}{f'(r_\mathrm{h})(r-r_\mathrm{h})}\left(-\mathrm{d}\omega'\right). \tag{28}$$

Using the Residue theorem, we have

$$\mathrm{Im}I = \mathrm{Im}\int_0^{\omega} \frac{2\pi}{f'(r_\mathrm{h})}\,\mathrm{d}\omega' = \frac{\pi\omega}{\kappa}. \tag{29}$$

Substituting Eq. (29) into Eq. (23) leads to

$$\Gamma \sim \exp\left(-\frac{2\pi\omega}{\kappa(M)}\right). \tag{30}$$

Compared Eq. (30) with the Boltzmann factor $\exp(-\omega/T)$, we can obtain the temperature of the black hole by the tunneling method as

$$T_\mathrm{t} = \frac{a^2}{4\pi r_\mathrm{h}^2\sqrt{r_\mathrm{h}^2-a^2}} + \frac{\sqrt{r_\mathrm{h}^2-a^2}}{4\pi r_\mathrm{h}^2} + \frac{3r_\mathrm{h}}{4\pi L^2} - \frac{Q^2}{4\pi r_\mathrm{h}^3}, \tag{31}$$

which is the same as Eq. (8) obtained by the surface gravity of the black hole.

3.2. Tunneling of charged vector particles

For the tunneling of charged vector particles, we use the Hamilton–Jacobi method.[29–36] By using the Glashow–Weinberg–Salam model,[68] the Lagrangian of the W-bosons in a background electromagnetic can be written as

$$\begin{aligned}\mathcal{L} =\ & -\frac{1}{2}(D_\mu^+ W_\nu^+ - D_\nu^+ W_\mu^+)(D^\nu W^{-\mu} - D^\mu W^{-\nu}) \\ & + m_W^2 W_\mu^+ W^{-\mu} - \mathrm{i}eF^{\mu\nu}W_\mu^+ W_\nu^-,\end{aligned} \tag{32}$$

where $D_{\pm\mu} = \nabla_\mu \pm \mathrm{i}eA_\mu$, with ∇_μ denoting the geometrically covariant, $A_\mu = (A_0, 0, 0, 0)$ referring to the electromagnetic potential of the black hole, e being the charge of the $W^\pm$ boson; $F^{\mu\nu} = \nabla^\mu A^\nu - \nabla^\nu A^\nu$. According to the Euler–Lagrange equation, the equation of motion of the W boson field can be obtained by using Eq (32) as

$$\begin{aligned}&\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\left(D^{\pm\mu}W^{\pm\nu} - D^{\pm\nu}W^{\pm\mu}\right)\right] + m_W^2 W^{\pm\nu} \\ &\pm \mathrm{i}eA_\mu\left(D^{\pm\mu}W^{\pm\nu} - D^{\pm\nu}W^{\pm\mu}\right) \pm \mathrm{i}eF^{\mu\nu}W_\mu^\pm = 0.\end{aligned} \tag{33}$$

From Eq. (33), we can see that W^+ and W^- have the same equation of motion, hence they have the same tunneling process too. Therefore, it is sufficient to study only the W^+ boson case. As the metric (1) is diagonal, the equation of motion for the W boson can be reformulated as

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\frac{\sqrt{-g}}{g_{\mu\mu}g_{\nu\nu}}\left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right.\right.$$

$$+\mathrm{i}eA_\mu W_\nu^+ - \mathrm{i}eA_\nu W_\mu^+\big)\Big] + \frac{m_W^2}{g_{\nu\nu}} W_\nu^+$$
$$+\frac{\mathrm{i}eA_\mu}{g_{\mu\mu}g_{\nu\nu}}\left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + \mathrm{i}eA_\mu W_\nu^+ - \mathrm{i}eA_\nu W_\mu^+\right)$$
$$-\mathrm{i}eF^{\mu\nu}W_\mu^+ = 0, \tag{34}$$

where $\nu = 0,1,2,3$.

Utilizing WKB approximation, W_μ^+ can be expressed as

$$W_\mu^+ = C_\mu(t,r,\theta,\phi)\exp(\mathrm{i}I_0(t,r,\theta,\phi)), \tag{35}$$

where I_0 is zeroth-order of the action. Then, based on the symmetry of spacetime, we can use the separation of variables for I_0 to obtain the following form:

$$I_0 = -Et + W(r) + \mathrm{j}\phi + \Theta(\theta) + C_\mathrm{I}. \tag{36}$$

By substituting Eqs. (35) and (36) into Eq. (34), the following four equations can be obtained:

$$C_0\left(-(\partial_1 I_0)^2 - \frac{(\partial_2 I_0)^2}{r^2 f(r)} - \frac{(\partial_3 I_0)^2}{r^2 f(r)\sin^2\theta} - \frac{m_W^2}{f(r)}\right)$$
$$+C_1(\partial_2 I_0)(eA_0 + \partial_0 I_0) + C_2\frac{\partial_2 I_0}{r^2 f(r)}(eA_0 + \partial_0 I_0)$$
$$+C_3\frac{(\partial_1 I_0)}{r^2 f(r)\sin^2\theta}(eA_0 + \partial_0 I_0) = 0, \tag{37}$$
$$C_0\left(-(\partial_2 I_0)(eA_0 + \partial_0 I_0)\right) + C_2\frac{f(r)(\partial_1 I_0)(\partial_1 I_0)}{r^2}$$
$$+C_3\frac{f(r)(\partial_1 I_0)(\partial_3 I_0)}{r^2\sin^2\theta}$$
$$+C_1\left(-\frac{f(r)(\partial_1 I_0)^2}{r^2} - \frac{f(r)(\partial_3 I_0)^2}{r^2\sin^2\theta}\right.$$
$$\left.+\left(-(\partial_2 I_0)(eA_0 + \partial_0 I_0)\right)^2 - m_W^2 f(r)\right) = 0, \tag{38}$$
$$C_0\left(-(\partial_2 I_0)\frac{eA_0 + \partial_0 I_0}{f(r)}\right) + C_1(f(r)(\partial_1 I_0)(\partial_2 I_0))$$
$$+C_3\frac{(\partial_2 I_0)(\partial_3 I_0)}{r^2\sin^2\theta} + C_2\left(-f(r)(\partial_1 I_0)^2 - \frac{(\partial_3 I_0)^2}{r^2\sin^2\theta}\right.$$
$$\left.+\frac{(eA_0 + \partial_0 I_0)^2}{f(r)} - m_W^2\right) = 0, \tag{39}$$
$$C_0\left(-(\partial_3 I_0)\frac{eA_0 + \partial_0 I_0}{f(r)}\right) + C_1(f(r)(\partial_1 I_0)(\partial_3 I_0))$$
$$+C_2\frac{(\partial_2 I_0)(\partial_3 I_0)}{r^2} + C_3\left(-f(r)(\partial_1 I_0)^2 - \frac{(\partial_2 I_0)^2}{r^2}\right.$$
$$\left.+\frac{(eA_0 + \partial_0 I_0)^2}{f(r)} - m_W^2\right) = 0, \tag{40}$$

where we retain to the zeroth order of WKB approximation too, as we did for Eq. (35). Equations (37)–(40) can be regarded as a matrix

$$\mathrm{C}(C_0, C_1, C_2, C_3)^\mathrm{T} = 0, \tag{41}$$

where C is a 4×4 matrix. If there is a non-trivial solution to Eq. (41), then the determinant of the matrix is equal to zero.

Due to the spherical symmetry of space-time, for convenience, we let $\theta = \pi/2$ in the following calculations. Then, by substituting Eq. (36) into Eq. (41) and solving the determinant of the matrix C, we obtain

$$(f(r)W'^2 r^2 + (r^2 m^2 + (\partial_2 H)^2)f(r)$$
$$-r^2(E - eA_0)^2)^3 m_W^2 = 0, \tag{42}$$

where $W' = \mathrm{d}W/\mathrm{d}r$. By solving Eq. (42), we can obtain

$$W_\pm(r)$$
$$= \pm\int\frac{1}{f(r)}\sqrt{E^2 - 2EeA_0 + e^2A_0^2 - f(r)\left(m_W^2 + \frac{(\partial_2 H)^2}{r^2}\right)}, \tag{43}$$

where $W_+(r)$ denotes the radial function associated with outgoing particles, whereas $W_-(r)$ represents the radial function associated with ingoing particles. From Eq. (43), one obtains

$$W_+(r) = -W_-(r). \tag{44}$$

Integrating around the pole at the event horizon r_h, we obtain

$$W_\pm(r) = \pm\frac{\mathrm{i}\pi(E - eA_0)}{f'(r_\mathrm{h})}. \tag{45}$$

So, for the quantum corrected RN-AdS black hole, the tunneling rate of charged vector particles is

$$\Gamma = \frac{P_+}{P_-} = \frac{\exp(-2\,\mathrm{Im}W_+ + \mathrm{Im}C_I)}{\exp(-2\,\mathrm{Im}W_- + \mathrm{Im}C_I)}, \tag{46}$$

where P_+ is the probability of the outgoing massive vector particle, and P_- is the probability of the ingoing massive vector particle. Near the outer edge of horizon, particles have a 100% chance of entering the black hole, that is,

$$P_- = 1. \tag{47}$$

So

$$\mathrm{Im}W_- = -\mathrm{Im}C_I. \tag{48}$$

Substituting Eqs. (44) and (48) into Eq. (46), we obtain

$$\Gamma = \exp(-4\,\mathrm{Im}W_+) = \exp\left(\frac{-4\pi(E - eA_0)}{f'(r_\mathrm{h})}\right). \tag{49}$$

By comparing with the Boltzmann factor $\exp(-\omega/T)$, the Hawking temperature can be obtained as

$$T_\mathrm{t} = \frac{\kappa}{2\pi} = \frac{a^2}{4\pi r_\mathrm{h}^2\sqrt{r_\mathrm{h}^2 - a^2}} + \frac{\sqrt{r_\mathrm{h}^2 - a^2}}{4\pi r_\mathrm{h}^2} + \frac{3r_\mathrm{h}}{4\pi L^2} - \frac{Q^2}{4\pi r_\mathrm{h}^3}, \tag{50}$$

which is the same as Eq. (31), and also Eq. (8).

3.3. Tunneling of charged fermion particles

Like the case of charged vector particles, we use the Hamilton–Jacobi method[29–36] to study the charged fermions tunneling. The Dirac equation with electric charge in curved spacetime is

$$\mathrm{i}\gamma^{\mu}\left(\nabla_{\mu}-\frac{\mathrm{i}q}{\hbar}A_{\mu}\right)\Psi+\frac{m_{\mathrm{f}}}{\hbar}\Psi=0,\quad \mu=0,1,2,3. \tag{51}$$

Here, m_{f} is the mass of a fermion particle, q is the electric charge, and Ψ is the wave function, while

$$\nabla_{\mu}=\partial_{\mu}-\Gamma_{\mu}^{\alpha\beta}\frac{1}{4}[\gamma^{\alpha},\gamma^{\beta}], \tag{52}$$

where γ^{α} is Dirac gamma matrices, which satisfy$\{\gamma^{\alpha},\gamma^{\beta}\}=2g^{\alpha\beta}$. There are many kinds of gamma matrices, but for the sake of convenience, we take the following form[33,34]

$$\begin{aligned}
\gamma^{t}&=\frac{1}{\sqrt{f(r)}}\begin{pmatrix}\mathrm{i}&0\\0&-\mathrm{i}\end{pmatrix},\\
\gamma^{r}&=\sqrt{g(r)}\begin{pmatrix}0&\sigma^{3}\\\sigma^{3}&0\end{pmatrix},\\
\gamma^{\theta}&=\frac{1}{r}\begin{pmatrix}0&\sigma^{1}\\\sigma^{1}&0\end{pmatrix},\\
\gamma^{\phi}&=\frac{1}{r\sin\theta}\begin{pmatrix}0&\sigma^{2}\\\sigma^{2}&0\end{pmatrix}.
\end{aligned} \tag{53}$$

Here, σ^{μ} are the Pauli matrices, the positive r-direction is assumed to be the spin-up, and the negative r-direction is assumed to be the spin-down. Considering the similarity between the scenarios of spin-up and spin-down, we shall limit our analysis to the case of spin-up.[33,34] The solution to the Dirac equation has the following form:

$$\begin{aligned}
\psi_{\uparrow}(t,r,\theta,\phi)&=\begin{bmatrix}A(t,r,\theta,\phi)\xi_{\uparrow}\\B(t,r,\theta,\phi)\xi_{\uparrow}\end{bmatrix}\exp\left[\frac{\mathrm{i}}{\hbar}I_{\uparrow}(t,r,\theta,\phi)\right]\\
&=\begin{bmatrix}A(t,r,\theta,\phi)\\0\\B(t,r,\theta,\phi)\\0\end{bmatrix}\exp\left[\frac{\mathrm{i}}{\hbar}I_{\uparrow}(t,r,\theta,\phi)\right],
\end{aligned} \tag{54}$$

where $\psi_{\uparrow}(t,r,\theta,\phi)$ represents the wave function for spin-up. Substituting Eq. (54) into Eq. (51), we can obtain

$$-\left(\frac{\mathrm{i}A}{\sqrt{f(r)}}\partial_{t}I_{\uparrow}+B\sqrt{f(r)}\partial_{r}I_{\uparrow}\right)+Am=0, \tag{55}$$

$$-\frac{B}{r}\left(\partial_{\theta}I_{\uparrow}+\frac{1}{\sin\theta}\mathrm{i}\partial_{\phi}I_{\uparrow}\right)=0, \tag{56}$$

$$\left(\frac{\mathrm{i}B}{\sqrt{f(r)}}\partial_{t}I_{\uparrow}-A\sqrt{f(r)}\partial_{r}I_{\uparrow}\right)+Bm=0, \tag{57}$$

$$-\frac{A}{r}\left(\partial_{\theta}I_{\uparrow}+\frac{1}{\sin\theta}\mathrm{i}\partial_{\phi}I_{\uparrow}\right)=0. \tag{58}$$

Again, according to the WKB approximation, we keep to the leading order. Like Eq. (35), we also assume that

$$I_{\uparrow}=I_{\uparrow 0}=-Et+W(r)+j\phi+\Theta(\theta)+C_{I}. \tag{59}$$

Employing the ansatz (29), equations (55)–(58) can be rewritten as

$$\left(\frac{\mathrm{i}AE}{\sqrt{f(r)}}-B\sqrt{f(r)}W'(r)\right)+mA=0, \tag{60}$$

$$-\frac{B}{r}\left(J_{\theta}+\frac{1}{\sin\theta}\mathrm{i}J_{\phi}\right)=0, \tag{61}$$

$$-\left(\frac{\mathrm{i}BE}{\sqrt{f(r)}}+A\sqrt{f(r)}W(r)\right)+Bm=0, \tag{62}$$

$$-\frac{A}{r}\left(J_{\theta}+\frac{1}{\sin\theta}\mathrm{i}J_{\phi}\right)=0. \tag{63}$$

When $m=0$, equations (60) and (62) have two possible solutions as follows:

$$W'(r)=W'_{+}(r)=\frac{E}{f(r)}\ \text{ and }\ A=\mathrm{i}B, \tag{64}$$

$$W'(r)=W'_{-}(r)=\frac{-E}{f(r)}\ \text{ and }\ A=-\mathrm{i}B. \tag{65}$$

Then we can calculate that

$$W_{\pm}(r)=\int\frac{\pm E\,\mathrm{d}r}{f(r)}. \tag{66}$$

Applying the residue theorem and integrating around the pole at the outer horizon r_{h}, and substituting Eq. (66) into Eq. (49), after that by comparing with the Boltzmann factor $\exp(-\omega/T)$, the Hawking temperature can be obtained as

$$T_{\mathrm{t}}=\frac{\kappa}{2\pi}=\frac{a^{2}}{4\pi r_{\mathrm{h}}^{2}\sqrt{r_{\mathrm{h}}^{2}-a^{2}}}+\frac{\sqrt{r_{\mathrm{h}}^{2}-a^{2}}}{4\pi r_{\mathrm{h}}^{2}}+\frac{3r_{\mathrm{h}}}{4\pi L^{2}}-\frac{Q^{2}}{4\pi r_{\mathrm{h}}^{3}}. \tag{67}$$

When $m\neq 0$, we can solve

$$\left(\frac{A}{B}\right)^{2}=\frac{-\mathrm{i}E+\sqrt{f(r)}m}{\mathrm{i}E+\sqrt{f(r)}m}. \tag{68}$$

In the limit $r\to r_{\mathrm{h}}$, $(A/B)^{2}=-1$. In the vicinity of the horizon, it is the same as in the case of $m=0$.

Obviously, like Eqs. (31) and (50), the tunneling temperature (67) is the same result as Eq. (8) from the surface gravity. Then, it is seen that in the quantum corrected RN-AdS spacetime, the type and the property of the tunneling particle do not affect the black hole temperature. Considering the temperature of a black hole determined by the spacetime structure of the event horizon, this implies that in the quantum tunneling method, the detected different tunneling particles possess the same spacetime nature. But what would happen if we consider the quantum gravity effect of tunneling particles? In the next section, we calculate the black hole temperature by using the GUP.

4. Effect of GUP on the quantum tunneling

Due to the quantum nature of the radiation particle system, the influence of the uncertainty principle cannot be overlooked. The GUP is an extension of Heisenberg's principle under extreme conditions. Therefore, we should consider the influence of the GUP on Hawking radiation.[49–51]

Considering the quantum effects of gravity, the Heisenberg uncertainty principle $\Delta x\Delta p \geq 1/2$ can be generalized to the GUP[20,69]

$$\Delta x\Delta p \geq (1+\lambda^2\Delta p), \tag{69}$$

where the uncertainty of position and the uncertainty of momentum are represented by Δx and Δp respectively, and λ is a dimensionless quantum gravity parameter. From Eq. (69) we obtain

$$\Delta p \geq \frac{\Delta x}{2\lambda^2}\left(1-\sqrt{1-\frac{4\lambda^2}{\Delta x^2}}\right). \tag{70}$$

The above equation can be expanded into the Taylor series, and considering the first-order approximation, we obtain

$$\Delta p \geq \frac{1}{\Delta x}\left(1+\frac{\lambda^2}{\Delta x^2}\right). \tag{71}$$

The correction to the energy of the particle after having taken into account the GUP can be given by Eq. (71) as indicated in Ref. [69]

$$E_{\rm GUP} = E\left(1+\frac{\lambda^2}{\Delta x^2}\right). \tag{72}$$

Taking the uncertainty of the particle as $\Delta x = 2r_{\rm h}$,[70] the energy of the tunneling particle corrected by the GUP can be obtained as

$$E_{\rm GUP} = E\left(1+\frac{\lambda^2}{4r_{\rm h}^2}\right). \tag{73}$$

Substituting Eq. (72) into Eq. (30), tunneling rate of the particle corrected by the GUP can be obtained as

$$\Gamma \sim \exp\left(-\frac{2\pi E_{\rm GUP}}{\kappa(M)}\right) = \exp\left(-\frac{2\pi E}{\kappa(M)}\left(1+\frac{\lambda^2}{4r_{\rm h}^2}\right)\right). \tag{74}$$

Comparing Eq. (74) with the Boltzmann factor $\exp(-\omega/T)$ gives

$$T_{\rm GUP} = T_k\left(1+\frac{\lambda^2}{4r_{\rm h}^2}\right)^{-1}. \tag{75}$$

It is seen that the temperature of the black hole decreases when the GUP is taken into account. Since λ is a Planck-scale magnitude quantity, this correction is significant for small black hole on the Planck scale. For large black holes, this correction can be ignored, and when $\lambda = 0$, equation (75) degenerates to Eq. (8).

Substituting Eq. (21) into Eq. (73) and considering Eq. (9), we can calculate the GUP corrected temperature for the critical state from

$$T_{\rm cGUP} = T_{ck}\left(1+\frac{\lambda^2}{4a^2}\right)^{-1} \to \infty. \tag{76}$$

This suggests that the GUP changes the temperature of the black hole, but fails to remove the singularity of the critical state temperature.

Using the first law of thermodynamics, we can obtain the entropy modified by the GUP as

$$\begin{aligned} S_{\rm GUP} &= \int \frac{{\rm d}M-\Phi\,{\rm d}Q-V\,{\rm d}p-\mu_a\,{\rm d}a}{T_{\rm GUP}} \\ &= \int 2\pi r_{\rm h}\left(1+\frac{\lambda^2}{\Delta x^2}\right){\rm d}r_{\rm h}. \end{aligned} \tag{77}$$

Neglecting the integration constant, we have

$$S_{\rm GUP} = \pi\left(r_{\rm h}^2+\frac{\lambda^2}{2}\ln r_{\rm h}\right) = \frac{A}{4}+\frac{\lambda^2}{4\pi}\ln A. \tag{78}$$

Comparing this with Eq. (15), it can be found that the black hole entropy appears to add a logarithmic correction to the Bekenstein–Hawking area law after having considered the GUP. The increase is significant for small black hole on the Planck scale, but not for large hole. When $\lambda = 0$, equation (78) then degenerates to Eq. (15). This is a consistent result with the quantum corrected entropy of black hole.[70–73] However, when reviewing the literature, it can be found that GUP does not necessarily only lead to logarithmic correction terms. Depending on the specific black hole, there might be other correction terms or no logarithmic corrections at all.[74–77] Furthermore, it can be noted that employing the GUP can also yield corrected temperatures different from Eq. (75).[74,76]

The forms of the modified entropy calculated for different black holes vary. The reason for this situation should be attributed to whether the entropy satisfies the Bekenstein–Hawking area law. When the Bekenstein–Hawking area law is not satisfied on the black hole, we cannot obtain a modified entropy expression like Eq. (77), and it may not be possible to derive a modified entropy with logarithmic corrections.

Further, by substituting Eqs. (8) and (78) into Eq. (17), the corrected heat capacity can be calculated from

$$\begin{aligned} &(C_v)_{\rm GUP} \\ &= \frac{\left(r^2-a^2\right)\left(\lambda^2+4r^2\right)\left(\lambda^2+4\pi^2r^2\right)\left(L^2\left(r^2-Q^2\sqrt{1-\frac{a^2}{r^2}}\right)+3r^4\sqrt{1-\frac{a^2}{r^2}}\right)}{2\pi L^2r^2\left(Q^2\sqrt{1-\frac{a^2}{r^2}}\left(\lambda^2+12r^2\right)-4r^4+\lambda^2r^2\right)-2\pi a^2\left(L^2\left(Q^2\sqrt{1-\frac{a^2}{r^2}}\left(\lambda^2+12r^2\right)+2\lambda^2r^2\right)+3r^4\sqrt{1-\frac{a^2}{r^2}}\left(3\lambda^2+4r^2\right)\right)+6\pi r^6\sqrt{1-\frac{a^2}{r^2}}\left(3\lambda^2+4r^2\right)}. \end{aligned} \tag{79}$$

When $\lambda \to 0$, the heat capacity (79) degenerates to Eq. (18), that is,

$$\lim_{a\to 0}(C_v)_{\rm GUP} = C_v. \tag{80}$$

By substituting Eq. (79) into Eq. (20), the remnant state can likewise be obtained as

$$r_{\rm rh} = a. \tag{81}$$

It can be seen that the GUP does not affect the remnant state of the black hole. This implies that the remnant state of the quantum corrected black hole is determined by the quantum fluctuations of spacetime and is not affected by the quantum properties of tunneling particle.

Substituting Eq. (21) into Eq. (78), we obtain the entropy of the remnant state as

$$S_{\rm GUP} = \pi\left(a^2 + \frac{\lambda^2}{2}\ln a\right). \tag{82}$$

Comparing Eq. (82) with Eq. (16), it is found that the entropy of the remnant state increases after the GUP has been taken into account, although the remnant state itself is not changed. This implies that less information can be stored when the black hole evaporation ends. It may be explained as a result of the GUP suppressing quantum state density.[78]

5. Discussion and conclusion

In this work, we studied the black hole thermodynamics of the quantum corrected RN-AdS black hole with quantum fluctuations of spacetime. We mainly focused on the influence of the quantum gravity effects on the quantum tunneling and thermodynamics of black holes. First, we calculated the thermodynamic quantities and examined their quantum correction effects. Considering the regularity of the black hole mass, we obtained the critical state of the black hole. This is the limit radius of the black hole due to the quantum fluctuations of spacetime. If the radius of the black hole is smaller than the critical value, there is a complex singularity in the mass of the black hole. Letting the heat capacity equal zero, the remnant state is obtained. In the process of black hole evaporation, the radius of the horizon decreases gradually, and the evaporation stops when the radius decreases to the remnant radius. It is found that the critical radius is equal to the remnant radius. Then, in the evaporation process of the black hole, the presence of the remnant state ensures that there is no mass singularity. That is, the bound property of the critical state can be actually ensured by the remnant state. On the other hand, the presence of critical state avoids the further evaporation of black hole due to the mass regularity. That is, the property of black hole remnant as the final stage of radiation is ensured by the critical state.

For the quantum corrected RN-AdS black hole, we find that the quantum fluctuations of spacetime increase the temperature of the black hole. For the black hole near the critical state, the increase is significant, but away from the critical state, it is not. This can be understood that as the scale increases, the effect of spacetime quantization decreases and the temperature gradually degenerates back to the temperature in the case of the usual RN-AdS black hole. In addition, the temperature of the remnant state is infinite. Then, the divergence of the black hole temperature in classical black hole is not eliminated by the quantum fluctuations of spacetime. Only, the singularity of the temperature at $r_{\rm h} = 0$ is transferred to $r_{\rm h} = a$. Furthermore, the black hole ceases to radiate at the high temperature, making it difficult to understand. On this basis, more quantum gravity effects should be considered. Anyway, the quantum fluctuation effect of spacetime can exert a significant influence on the nature of black hole thermodynamics.

Next, we studied the tunneling processes on the event horizon for different particles by using different methods. For the massless scalar particle, we use the null geodesic method, for the charged vector and fermion particles, we use the Hamilton–Jacob method. The results show that the same emission rate and Hawking temperature are always obtained for different cases. This implies that in the quantum corrected RN-AdS black hole, there is no influence of the type of radiated particles on the Hawking radiation as tunneling.

Last, taking the GUP effect of radiation particle into the tunneling process, we studied the quantum corrections to the thermodynamic quantities. Firstly, by using the quantum tunneling method, we obtained the corrected temperature for the quantum corrected RN-AdS black hole. The result is that when considering the GUP there is a decrease in temperature. This reduction is particularly significant for black holes on the order of the Planck scale. After that, by using the corrected temperature, we obtained the black hole entropy, which has the logarithm corrected term to the Bekenstein–Hawking area law. Moreover, due to the GUP effects, the black hole entropy increases. In this way, we find that unlike the types of radiating particles, the GUP effect affects the temperature and entropy of the quantum corrected black hole. Finally, using the corrected temperature and entropy, we calculated the heat capacity and obtained the remnant state. The obtained heat capacity is changed, but the remnant state of the black hole remains the same as $r_{\rm h} = a$ without being affected by GUP.

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