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Parameter estimation in n-dimensional massless scalar field

Ying Yang(杨颖)[1,†] and Jiliang Jing(荆继良)[2]

[1] *Hunan Provincial Key Laboratory of Intelligent Sensors and Advanced Sensor Materials, School of Physics and Electronics, Hunan University of Science and Technology, Xiangtan* 411201, *China*

[2] *Department of Physics, Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha* 410081, *China*

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Quantum Fisher information (QFI) associated with local metrology has been used to parameter estimation in open quantum systems. In this work, we calculated the QFI for a moving Unruh–DeWitt detector coupled with massless scalar fields in n-dimensional spacetime, and analyzed the behavior of QFI with various parameters, such as the dimension of spacetime, evolution time, and Unruh temperature. We discovered that the QFI of state parameter decreases monotonically from 1 to 0 over time. Additionally, we noted that the QFI for small evolution times is several orders of magnitude higher than the QFI for long evolution times. We also found that the value of QFI decreases at first and then stabilizes as the Unruh temperature increases. It was observed that the QFI depends on initial state parameter θ, and F_θ is the maximum for $\theta = 0$ or $\theta = \pi$, F_ϕ is the maximum for $\theta = \pi/2$. We also obtain that the maximum value of QFI for state parameters varies for different spacetime dimensions with the same evolution time.

Keywords: quantum Fisher information, parameter estimation, open quantum systems

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1. Introduction

In the context of quantum measurements, the interaction between the detector and the system being measured leads to entanglement and information exchange. This results in the system being detected becoming an open quantum system.[1–3] In this context, the state of the detector is no longer described as a pure state, but rather as a mixed state. The evolution of the detector's state is described by the master equation, typically in the Lindblad form.[4,5] During the process of quantum measurement, interaction between the detector and the measured system causes dissipation and decoherence, leading to the loss of coherence and information of the quantum system. During the process of quantum measurement, the interaction between the detector and the measured system causes dissipation and decoherence. This leads to the loss of coherence and information in the quantum system. Strategies have been developed for quantum estimation by understanding the dynamics of open quantum systems.[6–11]

Accurately estimating parameters is a challenge, and improving measurement accuracy is a key objective. There is a lot of research involved in improving parameter estimation.[12–18] Two methods have been proposed: optimizing the measurement process or optimizing the detection conditions. To enhance the measurement process, precise measurement techniques are used, such as selecting accurate and reliable instruments. Optimizing the detection conditions involves identifying the best operating conditions under which the parameter of interest can be measured with utmost precision.[19] This can be achieved by changing environmental parameters, preparing the optimal detector state, or controlling the detection time. This work focuses on the second approach, discussing the detection conditions for improving parameter estimation from a theoretical perspective.

The concept of QFI is essential when discussing the accuracy of parameter estimation. It has numerous applications in both theoretical and experimental domains.[20–23] In recent years, QFI associated with local operators has been employed to parameter estimation in open quantum systems, such as Unruh effect,[9,24] entanglement,[25,26] phase transitions,[27,28] etc. For instance, QFI has been employed to observe multipartite entanglement in Ref. [29]. Besides, it has been a diagnostic for the nature of the quantum state of the system in a many-body quantum system in Ref. [30]. Additionally, it has been suggested that QFI can be used as a probe for Unruh thermality in Ref. [9]. Since QFI quantifies the precision of parameter estimation, several research works have focused on protecting the QFI of parameters in various ways.[31–33]

Quantum field theory commonly employs models that describe how atoms interact with quantum fields. Among these models, the Unruh–DeWitt detector is the most basic particle detector.[34] Initially proposed for studying the Unruh effect, it shows that for a uniformly accelerated observer, the vacuum of quantum fields in Minkowski spacetime is transformed into the thermal state.[35–37] The Unruh effect reveals that "vacuum" and "particle" depend on the observer. There are many applications in the model of Unruh–DeWitt detector, such as the connection of measurement uncertainty and quantum coherence for an inertial Unruh–DeWitt

†Corresponding author. E-mail: yingyanghnust@163.com

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detector,[38] the transition rate of the Unruh–DeWitt detector in curved spacetime,[39] entanglement dynamics and entanglement harvesting for Unruh–DeWitt detector,[40,41] and more. Our research aims to explore the effect of Unruh temperature on quantum estimation. Specifically, we will consider the quantum estimation of the state parameter with an Unruh–DeWitt detector in n-dimensional Minkowski spacetime and analyze the influence of Unruh temperature on the QFI of state parameters. It is worth mentioning that the previous studies mentioned have primarily focused on state parameter estimation in 4-dimensional spacetime.[7,11,42] However, it has been observed that the response function of the detector in vacuum varies depending on the number of spacetime dimensions.[43,44] In Ref. [9], the QFI is discussed as a probe to detect the Unruh effect in n-dimensional Minkowski spacetime. Effect of spacetime dimensions on quantum entanglement has been investigated in Ref. [45]. Inspired by these works, we are interested in generalizing the investigation of state parameter estimation to a more general case, namely a quantum field in n-dimensional Minkowski spacetime.

This work is organized as follows. In Section 2, we introduce the dynamical evolution of a two-level atom in the framework of open quantum system. In Section 3, we review the QFI for a single-qubit system. The dynamical evolution of a two-level atomic probe in n-dimensional Minkowski spacetime is arrived in Section 4. In Section 5, we obtain the analytical results of QFI for state parameter, and the discussion on parameter estimation is provided. The summary and conclusions are given in Section 6. The units $c = \hbar = 1$ are used throughout this work.

2. The framework of open quantum system

A general parameter estimation consists of four processes: the preparation of the probe, the evolution of the probe state in the measured system, the measurement of the evolved probe state, and the estimation of the measured parameters from the measurement results. Here we use an Unruh–DeWitt detector as a probe which is modeled by a two-level atom. The detector is regarded as an open quantum system, which is coupled to a massless scalar field in n-dimensional Minkowski spacetime, and the Hamiltonian of the combined system reads

$$H = H_{\text{atom}} + H_{\text{field}} + \mu H_{\text{int}}, \tag{1}$$

where the atomic Hamiltonian $H_{\text{atom}} = (1/2)\omega_0\sigma_3$, H_{field} is the Hamiltonian of the scalar field. The interaction between the detector and the field is represented by the interaction Hamiltonian

$$H_{\text{int}} = (\sigma_+ + \sigma_-)\Phi(x), \tag{2}$$

with σ_+, σ_-, and ω_0 being the atomic raising, lowering operators, and the energy level spacing of the atom respectively.

The dynamic map of the whole system is introduced in the following. The state of atom–field combined system is approximated as $\rho_{\text{tot}}(0) = \rho(0)_{\text{atom}} \otimes \rho_{\text{field}}$ at the initial time $\tau = 0$, where $\rho(0)_{\text{atom}}$ is the atomic initial state and ρ_{field} represents the state of quantum field. Then the total density matrix ρ_{tot} is described by the von Neumann equation in the interaction picture as

$$\frac{\mathrm{d}\rho_{\text{tot}}(\tau)}{\mathrm{d}\tau} = -\mathrm{i}[H, \rho_{\text{tot}}(\tau)], \tag{3}$$

where τ is the proper time of the detector. By tracing over the field on both sides of Eq. (3), one can obtain

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\rho_{\text{atom}}(\tau) = -\mathrm{i}\,\mathrm{tr}_{\text{field}}[H, \rho_{\text{tot}}(\tau)].$$

For weak coupling between the atom and the field and the Markov approximation, the evolution of the system can be described by a simplified dynamic Kossakowski–Lindblad master equation[4,5]

$$\frac{\mathrm{d}\rho}{\mathrm{d}\tau} = -\mathrm{i}\,[H_{\text{eff}}, \rho(\tau)] + \mathcal{L}\,[\rho(\tau)]\,, \tag{4}$$

where

$$\mathcal{L}\,[\rho] = \sum_{i,j=1}^{3} C_{ij}\left[\sigma_j\rho\sigma_i - \frac{1}{2}\left\{\sigma_i\sigma_j, \rho\right\}\right]. \tag{5}$$

The density matrix of the detector is then governed by a master equation in Lindblad form in Eq. (4), which represents a dissipative evolution due to the interaction between the detector and the quantum fields, and C_{ij} is the Kossakowski matrix. Before giving the expression of C_{ij}, the Wightman function of scalar field should be introduced at

$$G^{+}\left(x, x'\right) = \left\langle \Phi(x)\Phi\left(x'\right)\right\rangle, \tag{6}$$

and the Fourier transform of the above Wightman function is

$$\mathcal{G}(\lambda) = \int_{-\infty}^{\infty} \mathrm{d}\tau\ \mathrm{e}^{\mathrm{i}\lambda\tau}G^{+}(\tau). \tag{7}$$

Then one obtains C_{ij} as[7,8,46]

$$C_{ij} = A_+\delta_{ij} - \mathrm{i}A_-\varepsilon_{ijk}\delta_{k3} - A_+\delta_{3i}\delta_{3j}, \tag{8}$$

with

$$\begin{aligned} A_+ &= \frac{1}{2}[\mathcal{G}(\omega) + \mathcal{G}(-\omega)], \\ A_- &= \frac{1}{2}[\mathcal{G}(\omega) - \mathcal{G}(-\omega)]. \end{aligned} \tag{9}$$

By considering the effect of scalar field on the detector, the effective Hamiltonian is derived as

$$H_{\text{eff}} = \frac{1}{2}\hbar\Omega\sigma_3 = \frac{\hbar}{2}\left\{\omega_0 + \frac{\mathrm{i}}{2}[\mathcal{K}(-\omega_0) - \mathcal{K}(\omega_0)]\right\}\sigma_3, \tag{10}$$

where

$$\mathcal{K}(\lambda) = \frac{1}{\mathrm{i}\pi}\mathrm{P}\int_{-\infty}^{\infty}\mathrm{d}\omega\frac{\mathcal{G}(\omega)}{\omega - \lambda},$$

which is the Hilbert transform of Wightman functions.

After resolving the master equation (4) with a general initial state

$$|\psi\rangle = \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle, \tag{11}$$

where the state of this single qubit is determined by the parameter θ and ϕ. Considering the effect of quantum field on quantum state, the density matrix $\rho(\tau)$ evolving over time can be expressed as

$$\rho(\tau) = \begin{pmatrix} \mathrm{e}^{-4A_+\tau}\cos^2\frac{\theta}{2} + \frac{A_- - A_+}{2A_+}(\mathrm{e}^{-4A_+\tau}-1) & \frac{1}{2}\mathrm{e}^{-2A_+\tau-\mathrm{i}\Omega\tau-\mathrm{i}\phi}\sin\theta \\ \frac{1}{2}\mathrm{e}^{-2A_+\tau+\mathrm{i}\Omega\tau+\mathrm{i}\phi}\sin\theta & 1-\mathrm{e}^{-4A_+\tau}\cos^2\frac{\theta}{2} - \frac{A_- - A_+}{2A_+}(\mathrm{e}^{-4A_+\tau}-1) \end{pmatrix}. \tag{12}$$

For a two-level atomic detector, the density matrix can be expressed in a Bloch form as

$$\rho(\tau) = \frac{1}{2}\Big(1 + \sum_{i=1}^{3}\omega_i(\tau)\sigma_i\Big). \tag{13}$$

Then the Bloch vectors $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)^{\mathrm{T}}$ reads

$$\begin{aligned} \omega_1(\tau) &= \sin\theta\cos(\Omega\tau+\phi)\,\mathrm{e}^{-2A_+\tau}, \\ \omega_2(\tau) &= \sin\theta\sin(\Omega\tau+\phi)\,\mathrm{e}^{-2A_+\tau}, \\ \omega_3(\tau) &= \cos\theta\,\mathrm{e}^{-4A_+\tau} - \frac{A_-}{A_+}(1-\mathrm{e}^{-4A_+\tau}). \end{aligned} \tag{14}$$

For a general initial state expressed in Eq. (11), the initial Bloch vector is $\boldsymbol{\omega} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. Assuming the atom is considered as a closed system, the Bloch vector of the state with atomic proper time τ becomes $\boldsymbol{\omega} = (\sin\theta\cos(\phi+\omega_0\tau), \sin\theta\sin(\phi+\omega_0\tau), \cos\theta)$. Due to the coupling of the two-level atomic system with the quantum field, the atom cannot be regarded as a closed system but an open quantum system, thus influence of the quantum fields will be encoded in the atomic state, and Bloch vector evolutes with time has an exponential decay factor due to decoherence in Eq. (14).

3. Quantum Fisher information for a single-qubit system

One of the basic characteristics of QFI is that we can get its Lower bound on the achievable mean-square error of the estimated parameter. The unbiased estimator for the parameter ϖ is called quantum Cramer–Rao (QCR) theorem, and the QCR bound is given in the following inequality:[47–49]

$$\Delta\varpi \geq \frac{1}{\sqrt{NF_\varpi}}. \tag{15}$$

In Eq. (15), F_ϖ is the QFI of ϖ, and $F_\varpi \equiv \mathrm{Tr}[\rho(\varpi)L(\varpi)^2]$, where $\rho(\varpi) = \sum_{i=1}^{N} p_i|\psi_i\rangle\langle\psi_i|$ is the density matrix of the probe and $L(\varpi)$ is the symmetric logarithmic derivative, and N is the number of experiments. From Eq. (15), we obtain that for a same N, the larger the QFI of the measured parameter, the smaller the deviation of the true value of the parameter and the measured value. Then the QFI can be read as[48]

$$F_\varpi = 2\sum_{m,n}^{N}\frac{|\langle\psi_m|\partial_\varpi\rho|\psi_n\rangle|^2}{p_m+p_n}. \tag{16}$$

With the expression of the Bloch vector in Eq. (13), the explicit form of QFI for a single-qubit system can be further expressed as[50]

$$F_\varpi = \begin{cases} |\partial_\varpi\boldsymbol{\omega}|^2 + \dfrac{(\boldsymbol{\omega}\cdot\partial_\varpi\boldsymbol{\omega})^2}{1-|\boldsymbol{\omega}|^2}, & |\boldsymbol{\omega}| < 1, \\ |\partial_\varpi\boldsymbol{\omega}|^2, & |\boldsymbol{\omega}| = 1. \end{cases} \tag{17}$$

For the mixed state, we calculate the QFI of parameter ϖ by the first line expression in Eq. (17), while for pure states, we use the second line expression in the above equation. Due to the interaction between the quantum system and the environment, the quantum state generally takes a mixed state after evolution, thus we will use the expression of QFI for the mixed state in the following parameter estimation.

4. Dynamical evolution of an Unruh–DeWitt detector in n-dimensional Minkowski spacetime

In order to arrive the dynamic evolution of a two-level atom, we need to discuss specific trajectories. In this section we will talk about the following uniformly accelerated trajectory:

$$x^0(\tau) = a^{-1}\sinh a\tau, \quad x^1(\tau) = a^{-1}\cosh a\tau, \quad \boldsymbol{x}(\tau) = \text{const.} \tag{18}$$

In a free massless scalar field, the field function $\Phi(x)$ is expended in n-dimensional Minkowski spacetime as

$$\Phi(x) = \int \mathrm{d}^{n-1}k\left\{a_k U_k(x) + a_k^+ U_k^*(x)\right\}, \tag{19}$$

and the positive field mode reads

$$U_k = \left[2\Omega_k(2\pi)^{n-1}\right]^{-1/2}\mathrm{e}^{\mathrm{i}\left(k_1x^1 + \boldsymbol{k}\cdot\boldsymbol{x} - \Omega_k x^0\right)}, \tag{20}$$

where Ω_k is the frequency, $\boldsymbol{k}$ is the wave number, and k_1 is the wave number in the x^1 direction, and $\Omega_k^2 = k_1^2 + |\boldsymbol{k}|^2$. The creation operator a_k^+ and annihilation operator a_k in Eq. (19) obey the canonical communication relation $[a_k, a_{k'}^\dagger] = \delta^{n-1}(k-k')$.

For a detector with uniformly accelerated trajectory, the Wightman function fulfills the KMS condition, *i.e.*, $G^+(\tau) = G^+(\tau+\mathrm{i}\beta)$, where $\beta = 1/T_U$, and $T_U = a/(2\pi)$ denotes the Unruh temperature. Substituting Eqs. (19) and (18) into the definition of Wightman function, an analytical form is obtained as

$$\mathcal{G}_n(\omega_0) = \frac{\pi^{(n-5)/2}\beta^{3-n}}{2\Gamma\left(\frac{n-1}{2}\right)}\left|\Gamma\left(\frac{n}{2}-1+\frac{\beta\omega_0}{2\pi}\mathrm{i}\right)\right|^2 \frac{D_n(\omega_0)}{\mathrm{e}^{-\beta\omega_0}-(-1)^n}, \tag{21}$$

with

$$D_n(\omega_0) = \begin{cases} -\sinh(\beta\omega_0/2), & \text{if } n \text{ is even}, \\ \cosh(\beta\omega_0/2), & \text{if } n \text{ is odd}. \end{cases} \tag{22}$$

Substituting Eqs. (21) and (22) into Eq. (9), one can obtain the Kossakowski coefficients for massless scalar field

$$A_{+,n} = \frac{\pi^{(n-5/2)\beta^{3-n}}}{4\Gamma\left(\frac{n-1}{2}\right)}\left|\Gamma\left(\frac{n}{2}-1+\frac{\beta\omega_0}{2\pi}\mathrm{i}\right)\right|^2 \cosh(\beta\omega_0/2), \tag{23}$$

$$A_{-,n} = \frac{\pi^{(n-5/2)\beta^{3-n}}}{4\Gamma\left(\frac{n-1}{2}\right)}\left|\Gamma\left(\frac{n}{2}-1+\frac{\beta\omega_0}{2\pi}\mathrm{i}\right)\right|^2 \sinh(\beta\omega_0/2), \tag{24}$$

where

$$\Gamma\left(\frac{n}{2}-1+\frac{\beta\omega_0}{2\pi}\mathrm{i}\right)$$

in Eqs. (21), (23), and (24) represents the conventional gamma function. From Euler's reflection formula,[51] we have $|\Gamma(\mathrm{i}x)|^2 = \pi/x\sinh(\pi x)$, $|\Gamma(1/2+\mathrm{i}x)|^2 = \pi/\cosh(\pi x)$, and recurrence relation $\Gamma(z+1) = z\Gamma(z)$. Then we obtain the Bloch vectors $\omega = (\omega_1, \omega_2, \omega_3)^{\mathrm{T}}$ in n-dimensional Minkowski spacetime as

$$\begin{aligned} \omega_1(\tau) &= \sin\theta\cos(\Omega\tau+\phi)\,\mathrm{e}^{-2A_{+,n}\tau}, \\ \omega_2(\tau) &= \sin\theta\sin(\Omega\tau+\phi)\,\mathrm{e}^{-2A_{+,n}\tau}, \\ \omega_3(\tau) &= \cos\theta\,\mathrm{e}^{-4A_{+,n}\tau} - \frac{A_{-,n}}{A_{+,n}}(1-\mathrm{e}^{-4A_{+,n}\tau}). \end{aligned} \tag{25}$$

The information about the evolution of quantum states is encoded in Eq. (25), thus we can calculate QFI by using the above Bloch vectors.

5. Discussion on quantum Fisher information

In this section, we will explore the behavior of QFI with different dimensions, and analysis how spacetime dimension, Unruh temperature and evolution time affect QFI, and then analyze the estimated accuracy of the initial parameter estimation via QFI.

5.1. Quantum Fisher information of parameter θ

By substituting Eq. (25) into Eq. (17), we obtain the analytical results of QFI for state parameter θ as follows:

$$F_\theta = \frac{2\,\mathrm{e}^{-4A_{+,n}\tau}\left[\left(A_{+,n}^2-A_{-,n}^2\right)\mathrm{e}^{4A_{+,n}\tau}\cos^2\theta + \left(A_{-,n}\cos\theta + A_{+,n}\right)^2\right]}{A_{+,n}^2\left(2\,\mathrm{e}^{4A_{+,n}\tau}+\cos(2\theta)+1\right) + 4A_{+,n}A_{-,n}\cos\theta - 2A_{-,n}^2\left(\mathrm{e}^{4A_{+,n}\tau}-1\right)}. \tag{26}$$

It is interesting to find that F_θ is independent of initial phase parameter ϕ from Eq. (26). To analyze how spacetime dimension influences parameter estimation, we discuss several cases with the dimensions $n = 4, 5, 6, 7$. In the following context, we would like to discuss the variation of QFI with several parameters, such as evolution time, Unruh temperature, and initial state parameter.

As is shown in Fig. 1, the QFI exhibits a monotonically decreasing behavior from 1 to 0 over time. For fixed Unruh temperature $T_U = 0.1, 1, 5$, we find that the time it takes for the QFI to decay from 1 to 0 varies significantly at different Unruh temperatures. For example, for $n = 5$ the decay time τ is around 100 for QFI to decay from 1 to 0 with $T_U = 0.1$, while the decay time τ is around 6 for QFI to decay from 1 to 0 with $T_U = 1$, and the decay time τ is around 0.3 for QFI to decay from 1 to 0 with $T_U = 5$, which present that for different Unruh temperatures, the decay time differs by 3 orders of magnitude. Hence the behavior of the QFI with evolution time is strongly influenced by the Unruh temperature. We find that for $T_U = 0.1$, in the case of high dimensions, the QFI of state parameters is larger. However, with the increase of Unruh temperature, the measurement advantage of high dimensions gradually disappears, and the measurement accuracy in the case of 4-dimensional spacetime is higher than other dimensions. When the Unruh temperature T_U and initial state parameter θ are the same, we find that the attenuation rate of QFI corresponding to different dimensions over time is not consistent, and even there is a big difference. Therefore, may be we can distinguish different spacetime dimensions via QFI.

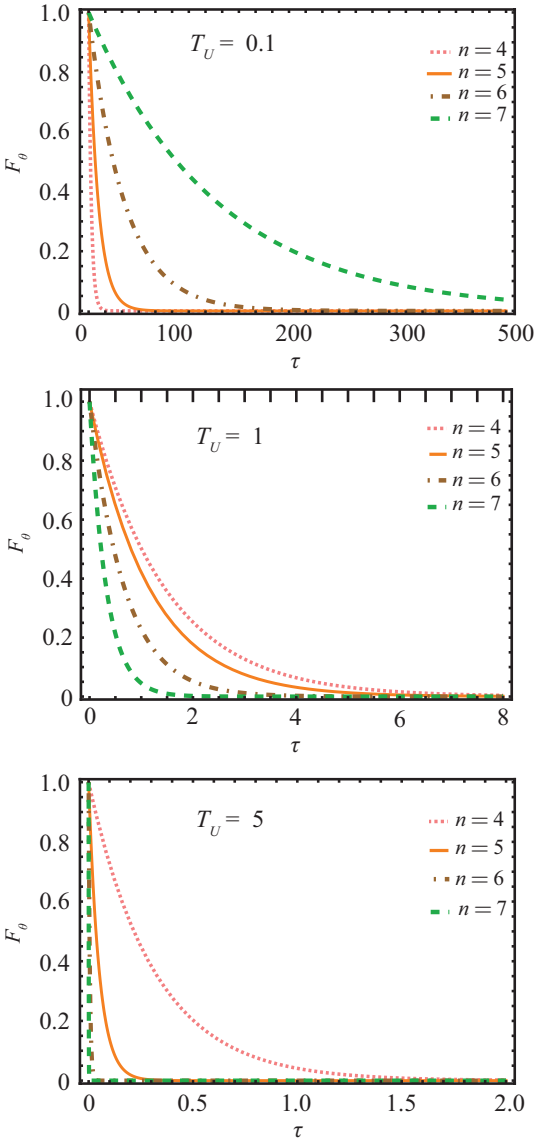


Fig. 1. The F_θ as a function of the evolution time τ with fixed values of the initial state parameter $\theta = 0$. We take $\omega_0 = 1$. From the top panel to the bottom panel, as indicated in the figure, we take the Unruh temperature $T_U = 0.1, 1$, and 5 respectively.

Now we analyze the overall behavior of the quantum Fisher information with respect to the Unruh temperature T_U. As is shown in Fig. 2, as the Unruh temperature T_U increases, the value of the quantum Fisher information decreases apparently at first, and then gradually trends to 0. As the Unruh temperature increases, the thermal fluctuations cause a decrease in the quantum Fisher information of the state parameter. As the temperature increases to a sufficiently high temperature, the quantum Fisher information reaches to a stable value. In Fig. 2, we fixed $\theta = 0$, thus the above equation (26) can be simplified as $F_\theta = \mathrm{e}^{-4A_{+,n}\tau}$. Then we obtain that there is a decay rate factor in the expression of F_θ. As the Unruh temperature increases, the thermal fluctuations cause a increase of the decay rate which is demonstrated by the factor $A_{+,n}$, thus cause a decrease in the quantum Fisher information of the parameter θ. By comparing to different spacetime dimensions, we find that at $\tau = 1, 3, 5$, $F_\theta^{n=7} > F_\theta^{n=6} > F_\theta^{n=5} > F_\theta^{n=4}$ for small value of $T_U = 0.1$, while we find that $F_\theta^{n=4} > F_\theta^{n=5} > F_\theta^{n=6} > F_\theta^{n=7}$ with a higher Unruh temperature.

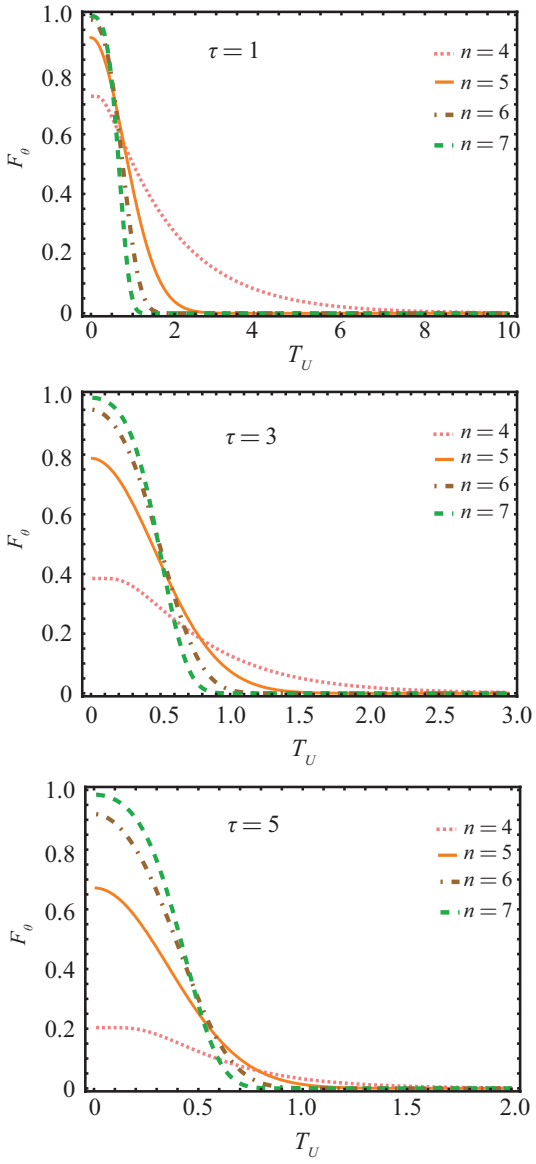


Fig. 2. F_θ as a function of the Unruh temperature T_U with fixed values of the initial state parameter $\theta = 0$. We take $\omega_0 = 1$. From the top panel to the bottom panel, as indicated in the figure, we take $\tau = 1, 3$, and 5 respectively.

In Fig. 3, we can see that the quantum Fisher information of the state parameter is dependent on θ itself. The quantum Fisher information is maximum for $\theta = 0$ (excited state) or $\theta = \pi$ (ground state), which highlights that these states are highly sensitive to changes in the parameter being studied. When we compare the quantum Fisher information for the same Unruh temperature, state parameter θ, and spacetime dimension, we find that the quantum Fisher information for $\tau = 1$ is over ten times higher than that of $\tau = 5$. Besides, we also find that $F_\theta^{n=4} > F_\theta^{n=5} > F_\theta^{n=6} > F_\theta^{n=7}$ for the same fixed state parameter θ, Unruh temperature T_U, and evolution time τ.

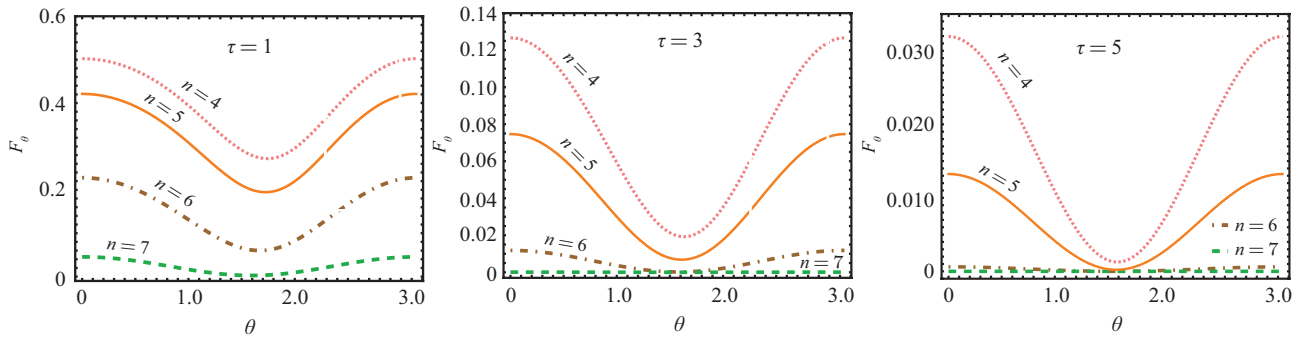


Fig. 3. The F_θ as a function of the initial state parameter θ with fixed values of Unruh temperature $T_U = 1$. We take $\omega_0 = 1$. From the left panel to the right panel, as indicated in the figure, we take $\tau = 1$, 3, and 5 respectively.

5.2. Quantum Fisher information of parameter ϕ

The analytic expression of parameter ϕ is obtained after calculation as

$$F_\phi = \mathrm{e}^{-4A_{+,n}\tau}\sin^2\theta. \tag{27}$$

From the above equation, we find that F_ϕ decays from 1 to 0 with increasing proper time for $\theta = \pi/2$, and the decay rates depend on the function $4A_{+,n}$. Besides, we obtain that $F_\phi = 0$ when $\theta = 0, \pi$, and here $\theta = 0, \pi$ correspond to the initial excited state. Since the variation of QFI for parameters ϕ with evolution time τ and Unruh temperature T_U is similar to that of the case of parameters θ which we have discussed above, then we will not analyze the variation of phase parameters with time and Unruh temperature in detail. Here we just gives the figures of F_ϕ as a function of initial parameter θ.

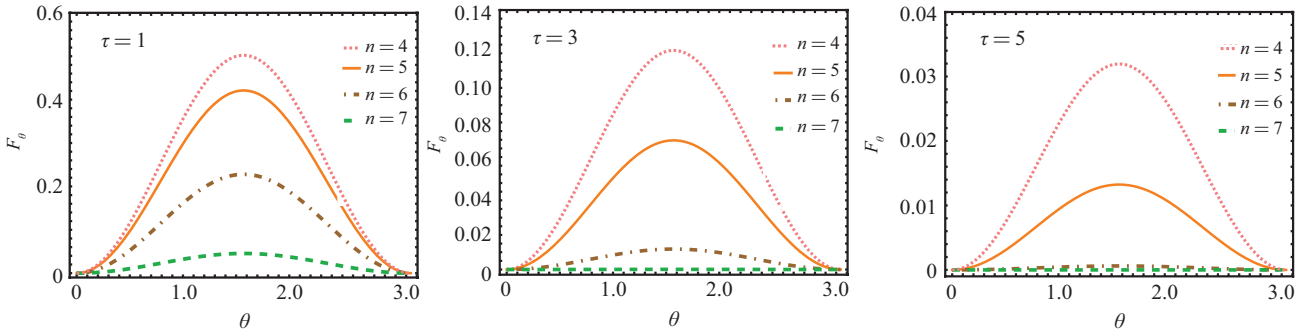


Fig. 4. F_ϕ as a function of initial state parameter θ with fixed values of the Unruh temperature $T_U = 1$. We take $\omega_0 = 1$. From the left panel to the right panel, as indicated in the figure, we take $\tau = 1$, 3, and 5 respectively.

As is shown in Fig. 4, it is obviously to see that the value of F_ϕ is the maximum for $\theta = \pi/2$, and it is the minimum for $\theta = 0, \pi$. When other parameters (Unruh temperature, initial parameter, and spacetime dimension) are the same, only the evolution time is different, we find that the QFI for $\tau = 1$ is about one order of magnitude higher than the case of $\tau = 5$. For Unruh temperature $T_U = 1$, the initial state parameter $\theta = \pi/2$ and the evolution time $\tau = 1, 3, 5$, we obtain that $F_\phi^{n=4} > F_\phi^{n=5} > F_\phi^{n=6} > F_\phi^{n=7}$.

6. Conclusions

We conducted a study on parameter estimation using local quantum estimation. Our focus was on an Unruh–DeWitt detector as an open quantum system that interacts with a massless scalar background in n-dimensional spacetime. The detector's dynamics are described by a Lindblad master equation that governs the evolution of its density matrix. We discovered that the QFI of state parameters θ and ϕ depends on various factors, including evolution time, Unruh temperature, and scalar field dimensionality. By studying the QFI's behavior under different parameters, we aimed to improve the accuracy of parameter estimation. Our results show that the QFI exhibits a monotonically decreasing behavior over time, decaying from 1 to 0 at a varying rate in different Unruh temperatures. It is observed that the QFI of the state parameter depends on θ, and F_θ is the maximum for $\theta = 0$ or $\theta = \pi$, F_ϕ is the maximum for $\theta = \pi/2$. We also found that the attenuation rate of QFI corresponding to different dimensions over time is not consistent. This indicates that we may be able to distinguish different spacetime dimensions using QFI. As the Unruh temperature increases, the QFI value first decreases, then gradually trends to 0. We attribute this trend to the thermal fluctuations caused by the increasing Unruh temperature, which makes the system more disordered, thus decreasing the QFI of the state parameter. We also observed that the QFI for small evolution time is about several orders of magnitude higher than that of long evolution time.

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References

[1] Isar A, Sandulescu A, Scutaru H, Stefanescu E and Scheid W 1994 *Int. J. Mod. Phys. E* **3** 635
[2] Viola L, Knill E and Lloyd S 1999 *Phys. Rev. Lett.* **82** 2417
[3] Rivas A and Huelga S F 2012 *Open quantum systems* (Berlin: Springer)
[4] Gorini V, Kossakowski A and Surdarshan E C G 1976 *J. Math. Phys.* **17** 821
[5] Lindblad G 1976 *Commun. Math. Phys.* **48** 119
[6] Lu X M, Wang X and Sun C P 2010 *Phys. Rev. A* **82** 042103
[7] Jin Y and Yu H W 2015 *Phys. Rev. A* **91** 022120
[8] Liu X, Jing J, Tian Z and Yao W 2021 *Phys. Rev. D* **103** 125025
[9] Feng J and Zhang J J 2022 *Phys. Lett. B* **827** 136992
[10] Yang Y, Jing J L and Tian Z H 2022 *Eur. Phys. J. C* **82** 1
[11] Yang Y, Zhang Y Q, Jia C X and Jing J L 2023 *Quantum Inf. Process.* **22** 1
[12] Giovannetti V, Lloyd S and Maccone L 2004 *Science* **306** 1330
[13] Giovannetti V, Lloyd S and Maccone L 2006 *Phys. Rev. Lett.* **96** 010401
[14] Chin A W, Huelga S F and Plenio M B 2012 *Phys. Rev. Lett.* **109** 233601
[15] Demkowicz-Dobrza'nski R and Maccone L 2014 *Phys. Rev. Lett.* **113** 250801
[16] Liu J, Zhang M, Chen H, Wang L and Yuan H 2022 *Advanced Quantum Technologies* **5** 2100080
[17] Barbieri M 2022 *PRX Quantum* **3** 010202
[18] Yang J, Pang S, Chen Z, Jordan A N and Del Campo A 2022 *Phys. Rev. Lett.* **128** 160505
[19] Giovannetti V, Lloyd S and Maccone L 2011 *Nat. Photonics* **5** 222
[20] Fisher R A 1925 *Theory of statistical estimation* (Cambridge: Cambridge University Press)
[21] Petz D and Ghinea C 2011 *Introduction to quantum Fisher information* (Singapore: World Scientific)
[22] Mathew G, Silva S L L, Jain A, *et al.* 2020 *Phys. Rev. Res.* **2** 043329
[23] Liu J, Yuan H, Lu X M and Wang X 2020 *J. Phys. A* **53** 023001
[24] Wang J, Tian Z, Jing J and Fan H 2014 *Sci. Rep* **4** 7195
[25] Hyllus P, Laskowski W, Krischek R, *et al.* 2012 *Phys. Rev. A* **85** 022321
[26] Li N and Luo S 2013 *Phys. Rev. A* **88** 014301
[27] Wang T L, Wu L N, Yang W, Jin G R, Lambert N and Nori F 2014 *New J. Phys.* **16** 063039
[28] Marzolino U and Prosen T 2017 *Phys. Rev. B* **96** 104402
[29] Dell'Anna F, Pradhan S, Boschi C D E and Ercolessi E 2023 *Phys. Rev. B* **108** 144414
[30] Hauke P, Heyl M, Tagliacozzo L and Zoller P 2016 *Nat. Phys.* **12** 778
[31] Zhang Y M, Li X W, Yang W and Jin G R 2013 *Phys. Rev. A* **88** 043832
[32] Hu M L and Wang H F 2020 *Annalen der Physik* **532** 1900378
[33] Ban M 2023 *Phys. Lett. A* **468** 128749
[34] Martin-Martinez E, Montero M and del Rey M 2013 *Phys. Rev. D* **87** 064038
[35] Fulling S A 1973 *Phys. Rev. D* **7** 2850
[36] Unruh W G 1976 *Phys. Rev. D* **14** 870
[37] Unruh W G 1984 *Phys. Rev. D* **29** 1047
[38] Yang Y Y, Ye L and Wang D 2020 *Annalen der Physik* **532** 2000062
[39] Louko J and Satz A 2008 *Class. Quantum Grav.* **25** 055012
[40] Zhou Y, Hu J and Yu H 2021 *JHEP* **2021** 88
[41] Zhang J and Yu H 2020 *Phys. Rev. D* **102** 065013
[42] Zhao Z, Pan Q and Jing J 2020 *Phys. Rev. D* **101** 056014
[43] Takagi S 1986 *Prog. Theor. Phys. Suppl.* **88** 1
[44] Sriramkumar L 2002 *Mod. Phys. Lett. A* **17** 1059
[45] Yan J and Zhang B 2022 *JHEP* **2022** 51
[46] Breuer H P and Petruccione F 2002 *The theory of open quantum systems* (Oxford: Oxford University Press)
[47] Cramér H 2016 *Mathematical Methods of Statistics* (Princeton: Princeton University)
[48] Braunstein S L and Caves C M 1994 *Phys. Rev. Lett.* **72** 3439
[49] Helstrom C W 1967 *Phys. Lett. A* **25** 101
[50] Zhong W, Sun Z, Ma J, Wang X and Nori F 2013 *Phys. Rev. A* **87** 022337
[51] Jeffrey A and Zwillinger D 2007 *Table of integrals, series, and products* (Burlington: Elsevier)