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Dynamical behaviors in discrete memristor-coupled small-world neuronal networks

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The brain is a complex network system in which a large number of neurons are widely connected to each other and transmit signals to each other. The memory characteristic of memristors makes them suitable for simulating neuronal synapses with plasticity. In this paper, a memristor is used to simulate a synapse, a discrete small-world neuronal network is constructed based on Rulkov neurons and its dynamical behavior is explored. We explore the influence of system parameters on the dynamical behaviors of the discrete small-world network, and the system shows a variety of firing patterns such as spiking firing and triangular burst firing when the neuronal parameter α is changed. The results of a numerical simulation based on Matlab show that the network topology can affect the synchronous firing behavior of the neuronal network, and the higher the reconnection probability and number of the nearest neurons, the more significant the synchronization state of the neurons. In addition, by increasing the coupling strength of memristor synapses, synchronization performance is promoted. The results of this paper can boost research into complex neuronal networks coupled with memristor synapses and further promote the development of neuroscience.

Keywords: small-world networks, Rulkov neurons, memristor, synchronization

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1. Introduction

It is well known that the human brain consists of large numbers of neurons and synapses connecting neurons, and has a highly complex topology of neuronal networks. Because the acquisition of clinical experimental data is often restricted by technical considerations, using a network to simulate the nervous system has become an important means to observe the dynamic behavior of neurons and explore the pathogenesis and treatment of diseases. Thus, we can regard a neuronal network as a mathematical model that simulates the biological nervous system and reflects the function of the human brain. In order to have a deeper understanding of neuronal networks, researchers in various fields have explored the characteristics and dynamical behavior of neuron models from the perspective of different mathematical functions and analyses.[1–10] Memristors are the fourth basic passive circuit element after resistors, inductors and capacitors. It can be seen from Ref. [11] that memristors have unique nonlinearity and memory characteristics, and researchers have constructed different new chaotic systems based on discrete memristor models.[12–20] Neuronal networks are able to generate complex dynamical behavior when memristors are applied to the field of neuronal networks, which is beneficial for simulating real biological neuronal networks. A three-order neuron circuit with complex neuromorphic behavior was established by inserting a current-controlled Chua Corsage memristor into a resonator.[21] In Ref. [22], a memristor was used to simulate the external electromagnetic environment, producing multi-scroll chaotic and multi-scroll periodic strange attractors with a symmetric structure in the magnetized Hopfield neuronal network. Other research[23] has shown that the unique memory characteristics, low power consumption and high integration characteristics of memristors could largely meet the needs of computing in the field of neuroscience. These excellent properties provide a new idea for artificial simulation of neuronal synapses and construction of neuronal network models. Compared with traditional chemical or electrical synapses, the use of memristors to simulate synapses, establish connections between neurons and study neuronal networks represents a major breakthrough in neuroscience. Therefore, memristor-coupled neuronal networks have attracted the attention of many scholars, and memristors have also been applied to the analysis of dynamic behaviors of different types of neuronal networks. Ma *et al*. studied the multi-stability, firing patterns and synchronous coexistence of the Rulkov neuronal network model based on discrete active memristors.[24] Ding *et al*. analyzed the characteristics of the Hopfield neuronal network after coupling a memristor and the factors affecting the dynamical behavior of the neuronal network.[25] From the perspective of energy, Li *et al*. studied the dynamical behavior of memristor-coupled Hindmarsh–Rose (HR) neuronal networks.[26] The number of neurons in the human brain is large, morphological and functional, so it is also important to explore the firing behavior of hetero-

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geneous neuronal networks coupled from different models of neurons. Wang *et al*.[27] explored the firing patterns of neurons in small heterogeneous network coupled with a memristor. Li *et al*. studied the dynamical behavior of a heterogeneous neuronal network formed by the coupling of HR and FitzHugh–Nagumo neurons by memristors, which further promoted research into neuronal networks with memristive synapses.[28] Ma *et al*.[29] studied the dynamic behavior of heterogeneous discrete neuronal networks under synaptic crosstalk, and their results were helpful in deepening our understanding of complex neuronal networks.

The research prospects of memristor-coupled neuronal networks are quite broad. Neuronal networks have self-learning and adaptive capabilities, and have good applications in the field of algorithm design. Based on the complex dynamical behavior of memristor systems and high dependence on the initial parameters, Wang *et al*. designed a memristor neuronal network with a selective supervised attention algorithm.[30] Peng combined an adaptive algorithm with discrete memristors to study its development in the field of parameter identification.[31] Yu *et al*. studied the application of memristor neuronal networks in the field of privacy protection and image encryption in media on the basis of multi-scroll memristor-coupled neuronal networks.[32–34] Similarly, He *et al*. proposed that discrete memristor neuronal networks could be applied to the field of character recognition[35] and Lai *et al*. constructed several memristive neuron models which improved security performance in the field of image recognition.[36–38] In the field of human–computer interaction, Sun *et al*. designed a three-dimensional pleasure–arousal–dominance emotional spatial memristor circuit, which provided reference data for human–computer emotional interaction for the study of bionic robots.[39] In summary, memristor synaptic neuronal networks have become a hot research topic in neurodynamics.

Compared with regular and random networks, a large number of real systems are complex networks with statistical characteristics. Scientists have applied the theoretical methods of complex networks to many fields, and have achieved results in fields such as transportation, disease transmission, electrical networks and so on. However, in neuroscience, there is little research using memristors to conduct synaptic connections and construct a neuronal complex network to explore the dynamic behavior of neurons. In order to simulate a more realistic neuronal network for the human brain, further analyze the dynamic mechanism of neurons and explore the factors affecting the synchronization performance and discharge mode of the nervous system, we take a small-world model as the research object, use memristors to simulate synapses and construct a new complex discrete neuronal network model on the basis of the theoretical method of complex networks. Its dynamic behaviors are also analyzed.

The main contributions of this article are as follows: (i) Based on the Rulkov neuron model and memristor synapse, a discrete small-world neuronal network model is constructed. (ii) The influence of neuron model parameter α on the dynamical behavior of the neuronal network is analyzed. (iii) The influence of network topology (the number of nearest neurons and reconnection probability) on dynamical behavior of the network is discussed. (iv) The influence of memristor synaptic parameters (coupling strength) on the dynamical behavior of neuronal networks is studied.

The layout of this paper is as follows: Section 2 introduces the local active memristor model, the discrete neuron model and the proposed discrete memristor-coupled small-world neuronal network (DMSNN) model and Section 3 analyzes in detail the effects of neuron model parameters, network topology and coupling strength on small-world neuronal network synchronization. Section 4 outlines our conclusions.

2. Construction of the DMSNN model

2.1. Memristor model

Based on the discrete memristor proposed in Ref. [40], the memristor synaptic model used in this paper is as follows:

$$\begin{cases} i(n) = W(q(n))\,v(n) = \tanh(q(n))\,v(n), \\ q(n+1) = \beta f(q(n)) + \gamma v(n), \\ f(q(n)) = -q(n)^3 + \delta q(n), \end{cases} \tag{1}$$

where $i(n)$ represents the input current of the memristor and $v(n)$ represents the voltage across the memristor, $W(q(n))$ is the memristor inductance value of the memristor. For memductance, $q(n)$ is the internal state variable of the memristor and $q(n+1)$ represents the next state of the memristor. β, γ, and δ represent the system parameters and we let $\beta = 0.1$, $\gamma = 0.1$, and $\delta = 11$.

2.2. Neuron model

The Rulkov neuron model[41] used in this paper is as follows:

$$\begin{cases} x_i(n+1) = \dfrac{\alpha}{1+x_i(n)^2} + y_i(n), \\ y_i(n+1) = y_i(n) - b(x_i(n) - \sigma), \end{cases} \tag{2}$$

where x represents the membrane potential, y represents the neuron recovery variable, α and b are the control parameters and σ denotes the externally exerted influence.

2.3. DMSNN model

It is surrounded by N neurons in a ring, and each neuron is connected to the nearest K neurons to its left and right ($K < N$). Then, for each neuron, at a certain probability p, we disconnect it from the neuron to which it is connected and

randomly establish a connection with another neuron. By this process a small-world network of neurons can be constructed. Figure 1 is a schematic diagram of a small-world neuronal network based on Matlab simulation (when $N = 100$, $K = 4$, the probability of reconnection is 0.1, 0.3, 0.5 respectively).

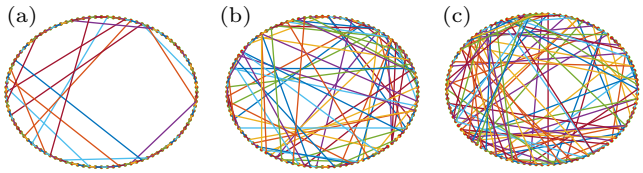


Fig. 1. Schematic diagram of the small-world neuronal network, $N = 100$ and $K = 4$, and (a), (b) and (c) correspond to reconnection probabilities of 0.1, 0.3, and 0.5, respectively

In this paper, a new complex neuronal network is established by using a memristor to simulate synapses and couple neurons. The model of the memristor-coupled discrete small-world neuronal network is as follows:

$$\begin{cases} x_i(n+1) = \dfrac{\alpha}{1+x_i(n)^2} + y_i(n) \\ \qquad\qquad -k\sum \varepsilon_{i,j}\cdot(x_j(n)-x_i(n))\cdot\tanh(q(n)), \\ y_i(n+1) = y_i(n) - b(x_i(n)-\sigma), \\ q(n+1) = -0.1q(n)^3 - 0.1(x_j(n)-x_i(n)) + 1.1q(n), \end{cases} \tag{3}$$

where k represents the coupling strength between neurons and all $\varepsilon_{i,j}$ form a connection matrix. When the i-th neuron and the j-th neuron are coupled together by memristor synapses, $\varepsilon_{i,j}$ is equal to 1, otherwise it is equal to 0.

3. Analysis of dynamical behaviors

The study of synchronous firing of neurons is essential for understanding the mechanism of perception, learning, memory and other behaviors of the human brain. Abnormal firing of neurons can cause many neurological diseases. This section will explore the effects of neuron model parameters, network topology and memristor coupling strength between neurons on the synchronous firing behavior of neuronal networks. In this process, we will take the number of nearest neurons K and the probability of reconnection p as variables and change the topology of the memristor-coupled small-world neuronal network composed of 100 neurons by changing the values of the variables.

3.1. Effect of neuron model parameters on firing patterns

The neuron model parameter α is considered as the control coefficient, and in this section other small-world neuronal network parameters are as follows: $K = 4$, $p = 0.1$ and the coupling strength between neurons $k = 0.005$. When the parameter α is set to 2.3, 3.0 and 4.0 respectively, figure 2 shows the firing behavior of the neuronal network through the spatio–temporal pattern and the isolated neuron timing diagram.

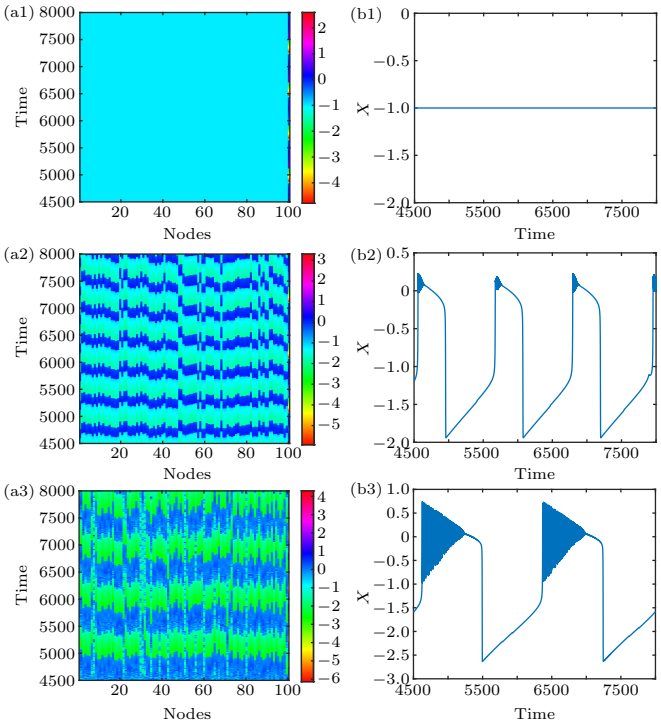


Fig. 2. Set the neuron control parameter α to 2.3, 3.0 and 4.0. (a1)–(a3) The spatio–temporal patterns in the corresponding case, and (b1)–(b3) timing diagrams of a neuron.

When the parameter $\alpha = 2.3$, it can be seen from Fig. 2(a1) that the color distribution in the spatio–temporal pattern is uniform at this time, and as shown in Fig. 2(b1) the membrane potential of isolated neurons stabilizes to a specific potential, indicating that all neurons are in a silent state. When the parameter $\alpha = 3.0$, as shown in Figs. 2(a2) and 2(b2), the neuron begins to fire and exhibits a triangular burst peak firing pattern. Finally, when the parameter $\alpha = 4.0$, figure 2(a3) shows that the neuronal network is in an asynchronous state; the timing diagram of Fig. 2(b3) shows that the firing mode of the neuron is triangular burst firing with a larger oscillation amplitude. With the increase in the parameter system of the neuron model, the synchronization state and firing mode of the neuronal network change significantly. In summary, changes in system parameter α can affect the dynamical behavior of DMSNNs.

4. Effect of reconnection probability p on the synchronization of DMSNNs

When the number of nearest neurons $K = 4$, the coupling strength $k = 0.005$ and the system parameter $\alpha = 3 + \text{rand}(N,1)/5$. As shown in Fig. 3, by comparing the spatio–temporal patterns and the node's instantaneous state diagrams we find that the synchronization states of neurons are different under different reconnection probabilities p.

When the probability of reconnection $p = 0.1$, figure 3(b1) shows that the scatter points are roughly in the interval $[-2, -1.45]$ and the distribution is messy. Figure 3(a1)

shows that the neuronal network is basically synchronized. When the probability of reconnection $p = 0.3$, as shown in Fig. 3(b2), the scatter distribution is more uniform than in the former case. Figure 3(a2) shows that the synchronization performance of the neuronal network is better. When the reconnection probability $p = 0.5$, figure 3(b3) shows that the scatter points are closely distributed in the interval $[-1.9, -1.5]$, and the synchronization state of the neurons is more pronounced. In summary, in small-world neuronal networks, the higher the reconnection probability p, the better the synchronization effect of the DMSNN.

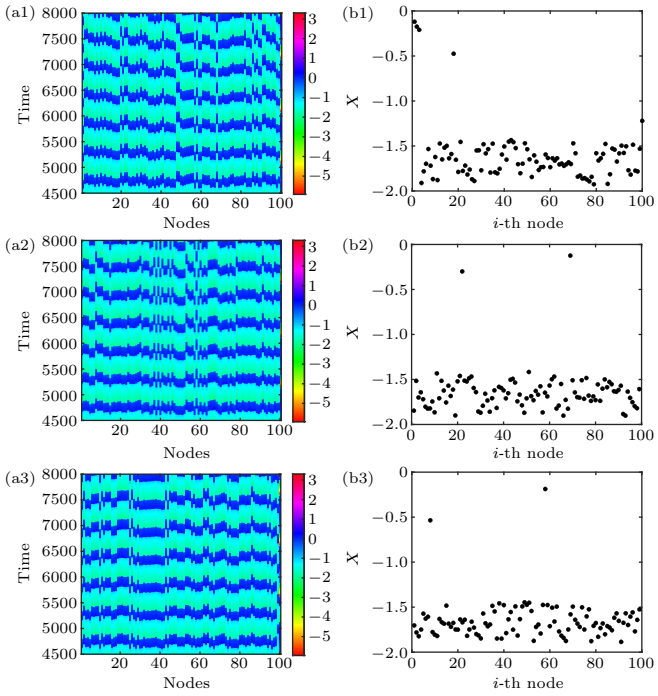


Fig. 3. When $K = 4$, the probability of reconnection is 0.1, 0.3, 0.5 from top to bottom. (a1)–(a3) Spatio–temporal patterns. (b1)–(b3) Node instantaneous state diagrams.

4.1. Influence of the number of nearest neurons on the synchronization of DMSNNs

We set the coupling strength to 0.005 and the neuronal network reconnection probability to 0.1. Parameter α is set to $\alpha = 3 + \mathrm{rand}\,(N, 1)/5$. The spatio–temporal patterns and node's instantaneous state diagram for the neuronal network system are shown in Fig. 4, and the number of nearest neurons is 4, 10 and 14, respectively.

When $K = 4$, as shown by the spatio–temporal pattern corresponding to Fig. 4(a1), the neuronal synchronization effect is not obvious. The scatter range is wide $[-1.95, -1.45]$ and the distribution is scattered and not concentrated, as shown in Fig. 4(b1). When $K = 10$, according to analysis of the spatio–temporal pattern in Fig. 4(a2), the synchronization of neurons is significantly improved. As shown in Fig. 4(b2), the scatter distribution is reduced to $[-1.5, -1.8]$. When $K = 14$, figures 4(a3) and 4(b3) show that the distribution of the spatio–temporal pattern is uniform and the scatter distribution range is further narrowed to the range of $[-1.7, -1.5]$; the neurons are basically completely synchronized. Therefore, we can learn that an increase in the number of nearest neurons can also promote the synchronization of DMSNNs.

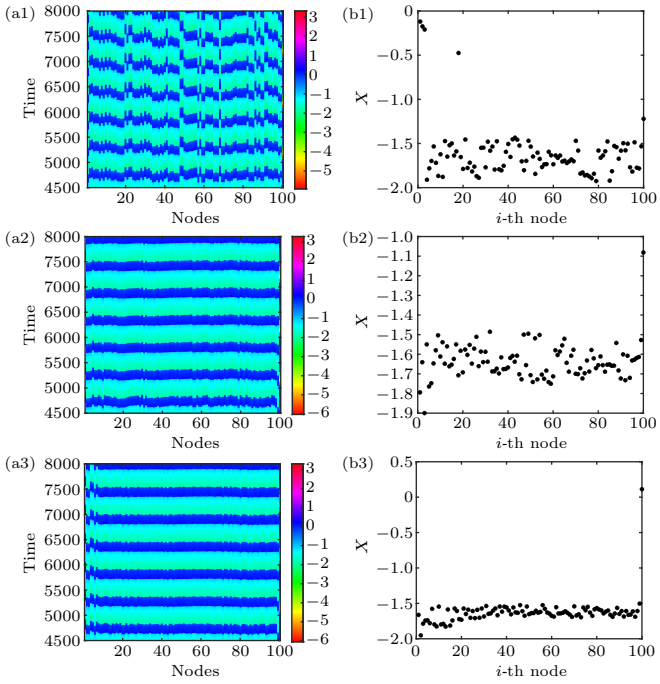


Fig. 4. When $p = 0.1$, the number of nearest neurons from top to bottom is 4, 10, 14. (a1)–(a3) Spatio–temporal patterns. (b1)–(b3) Node instantaneous state diagrams.

4.2. Effect of memristor coupling strength on the synchronization of DMSNNs

In this section we set the small-world model topology parameters $K = 4$, $p = 0.1$ and the neuron control parameters $\alpha = 3 + \mathrm{rand}\,(N, 1)/5$. The values of the coupling strength k between neurons are set to 0.002, 0.003, 0.004 and 0.006 in turn, and the synchronous firing states of the neuronal network are observed when the coupling strength of memristor gradually increases.

First, when $k = 0.002$, the coupling strength is small. As shown in Figs. 5(a1) and 5(b1), the color distribution state of the spatio–temporal pattern fluctuates greatly and the spots are scattered in the interval of $[-2.5, -1.7]$. When the coupling strength $k = 0.003$, figure 5(b2) shows that the scatter distribution is in the interval $[-2.3, -1.6]$, and the distribution range is reduced. When the coupling strength k further increases to $k = 0.004$, the scatter interval is further reduced to $[-2.1, -1.5]$ and the scatter distribution is more compact than before, as shown in Fig. 5(b3). Finally, when $k = 0.006$, as shown in Figs. 5(a4) and 5(b4), the color distribution of the spatio–temporal pattern is only a small ripple and the scatter is concentrated in $[-1.8, -1.4]$, basically showing the shape of a straight line. In summary, with increase in the memristor synaptic coupling strength, the spatio–temporal pattern gradually becomes striped and the distribution interval of scatter be-

comes smaller and smaller, so increasing the memristor coupling strength promotes the synchronization performance of DMSNNs.

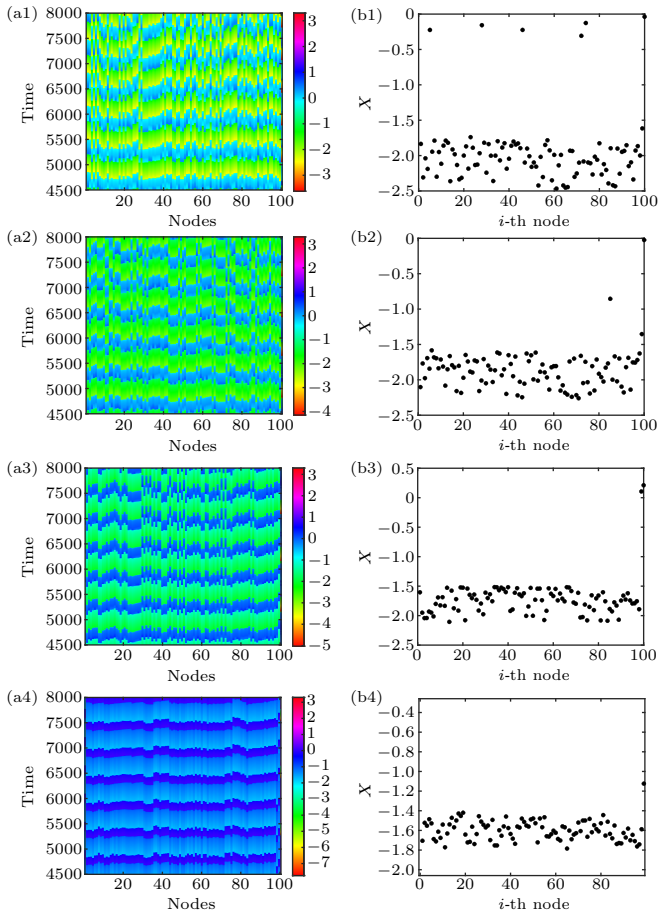


Fig. 5. Set the coupling strength k to 0.002, 0.003, 0.004 and 0.006. (a1)–(a4) Spatio–temporal pattern. (b1)–(b4) Node instantaneous state diagrams.

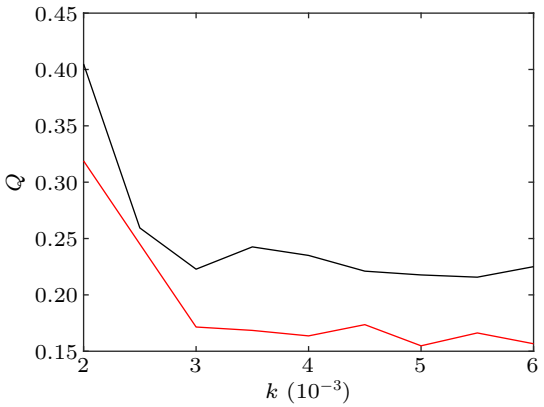


Fig. 6. Diagram of synchronization coefficient with $p = 0.1$, $\alpha = 3 + \mathrm{rand}(N,1)/5$, different $k \in [0, 0.006]$, when the number of the nearest neurons K is set to 14 in red and 10 in black, respectively.

In addition to the above work, we also simulated the relationships between coupling strength k, the number of the nearest neurons K in the neighborhood and the synchronization coefficient. The synchronization coefficient is equal to the average standard deviation of X values of each neuron and is inversely proportional to the synchronization performance. As shown in Fig. 6, on the one hand the synchronization coefficient decreases with increase in k. On the other hand, in the whole range of k values the synchronization coefficient for $K = 14$ is smaller than for $K = 10$.

5. Conclusion

In this paper, a DMSNN is constructed for the first time by simulating neuronal synapses with discrete memristor synapses, and the dynamical behaviors of DMSNNs with complex topologies are studied. The results show that increasing the number of nearest neurons, reconnection probability of DMSNNs and the coupling strength between neurons can promote the synchronous firing of DMSNNs. In addition, changing the parameters of the Rulkov neuron model will also affect the dynamical behavior of the network, so that the neurons in the neuronal network show different firing patterns, such as resting state, triangular burst peak firing, etc. Next, we will try to simulate synapses using different memristors to further study the influence of memristor synapses on the dynamical behavior of complex neuronal networks. We will also try to analyze the synchronization mechanism of DMSNNs from the perspective of Hamiltonian energy.

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