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# Analytical solutions to the precession relaxation of magnetization with uniaxial anisotropy

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Based on the Landau–Lifshitz–Gilbert (LLG) equation, the precession relaxation of magnetization is studied when the external field $\boldsymbol{H}$ is parallel to the uniaxial anisotropic field $\boldsymbol{H}_{\rm k}$. The evolution of three-component magnetization is solved analytically under the condition of $H = nH_{\rm k}$ ($n = 3$, 1 and 0). It is found that with an increase of $\boldsymbol{H}$ or a decrease of the initial polar angle of magnetization, the relaxation time decreases and the angular frequency of magnetization increases. For comparison, the analytical solution for $H_{\rm k} = 0$ is also given. When the magnetization becomes stable, the angular frequency is proportional to the total effective field acting on the magnetization. The analytical solutions are not only conducive to the understanding of the precession relaxation of magnetization, but also can be used as a standard model to test the numerical calculation of LLG equation.

**Keywords:** precession relaxation, Landau–Lifshitz–Gilbert (LLG) equation, uniaxial anisotropy, analytical solutions

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## 1. Introduction

Magnetization relaxation processes in magnetic materials include magnetic after-effects, displacement of the domain wall and coherent domain rotation.[1] They reflect both the spatial route of magnetization from one steady state to another and the varying levels of time during different magnetization and reversal processes. The study of the relaxation process has become the key to improving the performance of magnetic functional devices, such as the switching speed of storage units in hard disk drives,[2,3] the read and write speed of magnetic random access memory,[4] and the response speed of magnetic sensors.[5]

With the miniaturization and high-frequency development of magnetic and spintronic devices, it has become inevitable to study the precession relaxation of magnetization (PRM) in single-domain particles at the nanoscale.[6–8] Since the PRM at this scale is mainly a coherent rotation of domain, one usually solves the Landau–Lifshitz–Gilbert (LLG) equation to gain an understanding of PRM.[9,10] It is well known that the LLG equation is a nonlinear differential equation

$$\frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}t} = -\gamma \boldsymbol{M} \times \boldsymbol{H}_{\rm eff} + \frac{\alpha}{M}\left(\boldsymbol{M} \times \frac{\mathrm{d}\boldsymbol{M}}{\mathrm{d}t}\right), \tag{1}$$

where $\boldsymbol{M}$ is the magnetization, $\gamma$ is the gyromagnetic ratio, $\boldsymbol{H}_{\rm eff}$ is the effective field and $\alpha$ is the Gilbert damping parameter. Due to the prevalence of angle-dependent anisotropic fields in real magnetic materials, it is usually difficult to solve Eq. (1) analytically.

Nevertheless, one can still understand the relaxation characteristics of single-domain particles through the LLG equation. Kikuchi first studied magnetization and reversal in an isotropic single-domain sphere and a thin sheet under a DC magnetic field.[11] Gillette and Oshima gave the LLG component equations for magnetization reversal by rotation in single-domain ferromagnets.[12] Doyle *et al.* used a numerical method to study the magnetization switching of Stoner–Wohlfarth particles.[13] Mallinson obtained the magnetization reversal time in a single domain uniaxial system.[14] The behavior of magnetization during reversal in a uniaxial system was discussed by Bertotti *et al.*[15] More studies have compared experimental results with the help of numerical calculations or simulations.[16–21] An analytical solution of PRM has not been found so far.

In this paper, the analytical solution of PRM in a simple uniaxial anisotropic system is given. Equation (1) can be solved strictly using a cylinder symmetry effective field. When the applied magnetic field is parallel to the anisotropic field, we give the analytical solution of PRM under specific conditions. On this basis, we study the evolution of the amplitude and angular frequency of PRM. The acquisition of the analytical solution of PRM not only provides the possibility of describing the exact position of magnetization, but also helps to check the reliability of the numerical calculation of the LLG equation.

## 2. Equation of PRM

Suppose the uniaxial anisotropic field $H_{\rm k}$ and the applied magnetic field $H$ are both along the $z$ direction. For a single-domain particle, the effective field $\boldsymbol{H}_{\rm eff} = (0, 0, H + H_{\rm k}\cos\theta)$ has cylinder symmetry, where $\theta$ is the angle between the magnetization $\boldsymbol{M}$ and the $z$ axis. To simplify the calculation pro-

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cess, we transform Eq. (1) into the following form:

$$\frac{\mathrm{d}m_x}{\mathrm{d}t} = -H_\mathrm{k}^* m_y (h+m_z) - \alpha H_\mathrm{k}^* (h+m_z) m_x m_z, \tag{2a}$$

$$\frac{\mathrm{d}m_y}{\mathrm{d}t} = H_\mathrm{k}^* m_x (h+m_z) - \alpha H_\mathrm{k}^* (h+m_z) m_y m_z, \tag{2b}$$

$$\frac{\mathrm{d}m_z}{\mathrm{d}t} = \alpha H_\mathrm{k}^* (h+m_z)\left(1-m_z^2\right), \tag{2c}$$

where $m_{i=x,y,z} = M_i/M$ are the components of reduced magnetization and satisfy $m_x^2+m_y^2+m_z^2=1$, $H_\mathrm{k}^* = \gamma H_\mathrm{k}/(1+\alpha^2)$ is the reduced anisotropic field, and $h = H/H_\mathrm{k}$ is the reduced applied magnetic field. Clearly, the component $m_z$ can be obtained by integrating Eq. (2c) directly. The possibility of obtaining an analytical solution depends entirely on the magnitude of $h$.

To obtain solutions for $m_x$ and $m_y$, let $m_{xy} = m_x + \mathrm{i}m_y$, from Eqs. (2a) and (2b) we obtain

$$\frac{\mathrm{d}m_{xy}}{\mathrm{d}t} = H_\mathrm{k}^* (h+m_z) m_{xy} (\mathrm{i}-\alpha m_z). \tag{3a}$$

Combining Eq. (2c), $m_{xy}$ can be derived as

$$m_{xy} = \sqrt{1-m_z^2}\,\mathrm{e}^{\frac{\mathrm{i}}{2\alpha}\left(\ln\frac{1+m_z}{1-m_z}+d_0\right)}, \tag{3b}$$

where $d_0$ is the integration constant. Obviously, the relaxation behaviors of $m_x$ and $m_y$ are the cosine and sine functions, respectively. If the analytical solution of $m_z(t)$ is obtained from Eq. (2c), the time evolutions of $m_x$ and $m_y$ can be obtained from Eq. (3b).

If Eq. (2c) is integrated, an algebraic equation in terms of $m_z$ can be obtained, whose highest power is determined by $h$. For cases where the applied field is greater than, equal to or less than the anisotropic field, the quadratic equation can be obtained only for $h = 3$, 1 and 0. The analytical solutions for these three special cases are discussed below.

## 3. Relaxation solution of magnetization

Let $x = 1+m_z$, $y = 1-m_z$ and $z = m_z$, when $h = 3$, 1 and 0, Eq. (2c) is rewritten as follows:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha H_\mathrm{k}^* x\left(4-x^2\right), \tag{4a}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha H_\mathrm{k}^* y (2-y)^2, \tag{4b}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \alpha H_\mathrm{k}^* z\left(1-z^2\right). \tag{4c}$$

From Eq. (4) we have

$$\frac{x^2}{4-x^2} = a_0\,\mathrm{e}^{8\alpha H_\mathrm{k}^* t}, \tag{5a}$$

$$\left(\frac{y}{2-y}\right)\exp\left(\frac{y}{2-y}\right) = b_0\,\mathrm{e}^{-4\alpha H_\mathrm{k}^* t}, \tag{5b}$$

$$\frac{z^2}{1-z^2} = c_0\,\mathrm{e}^{2\alpha H_\mathrm{k}^* t}, \tag{5c}$$

where $a_0$, $b_0$ and $c_0$ are the integration constants. When $t = 0$, supposing $\boldsymbol{m} = (\sin\theta_0, 0, \cos\theta_0)$, then

$$a_0 = \cos^4\frac{\theta_0}{2}\Big/\left(1-\cos^4\frac{\theta_0}{2}\right), \tag{6a}$$

$$b_0 = \left(\tan^2\frac{\theta_0}{2}\right)\exp\left(\tan^2\frac{\theta_0}{2}\right), \tag{6b}$$

$$c_0 = \cot^2\theta_0. \tag{6c}$$

Substituting Eq. (6) into Eq. (5) and using the relationship between $m_z$ and $x$, $y$ and $z$, the time evolution of $m_z$ can be obtained as follows:

$$m_{z,h=3} = 2\left[1+f\left(\cos^4\frac{\theta_0}{2}\right)\mathrm{e}^{-8\alpha H_\mathrm{k}^* t}\right]^{-1/2} - 1, \tag{7a}$$

$$m_{z,h=1} = \frac{1-W\left[g\left(\tan^2\frac{\theta_0}{2}\right)\mathrm{e}^{-4\alpha H_\mathrm{k}^* t}\right]}{1+W\left[g\left(\tan^2\frac{\theta_0}{2}\right)\mathrm{e}^{-4\alpha H_\mathrm{k}^* t}\right]}, \tag{7b}$$

$$m_{z,h=0} = \left(1+\tan^2\theta_0\,\mathrm{e}^{-2\alpha H_\mathrm{k}^* t}\right)^{-1/2}, \tag{7c}$$

where $f(q) = (1-q)/q$, $g(q) = q\mathrm{e}^q$ and $W(q)$ is the Lambert $W$ function. The contour of $m_z$ is related not only to $h$, but also to $H_\mathrm{k}^*$, $\theta_0$ and $\alpha$.

Substituting Eq. (7) into Eq. (3b), $m_x$ and $m_y$ are as follows:

$$m_{x,h=3} = \sqrt{2z_3-z_3^2}\cos\left\{\frac{1}{2\alpha}\left[\ln\frac{z_3}{2-z_3} - \ln\cot^2\frac{\theta_0}{2}\right]\right\}, \tag{8a}$$

$$m_{y,h=3} = \sqrt{2z_3-z_3^2}\sin\left\{\frac{1}{2\alpha}\left[\ln\frac{z_3}{2-z_3} - \ln\cot^2\frac{\theta_0}{2}\right]\right\}, \tag{8b}$$

$$m_{x,h=1} = \sqrt{\frac{4z_1}{(1+z_1)^2}}\cos\left\{\frac{1}{2\alpha}\left[\ln\frac{1}{z_1} - \ln\cot^2\frac{\theta_0}{2}\right]\right\}, \tag{8c}$$

$$m_{y,h=1} = \sqrt{\frac{4z_1}{(1+z_1)^2}}\sin\left\{\frac{1}{2\alpha}\left[\ln\frac{1}{z_1} - \ln\cot^2\frac{\theta_0}{2}\right]\right\}, \tag{8d}$$

$$m_{x,h=0} = \sqrt{\frac{z_0}{1+z_0}}\cos\left\{\frac{1}{2\alpha}\left[\ln\frac{1+(1+z_0)^{-1/2}}{1-(1+z_0)^{-1/2}} - \ln\cot^2\frac{\theta_0}{2}\right]\right\}, \tag{8e}$$

$$m_{y,h=0} = \sqrt{\frac{z_0}{1+z_0}}\sin\left\{\frac{1}{2\alpha}\left[\ln\frac{1+(1+z_0)^{-1/2}}{1-(1+z_0)^{-1/2}} - \ln\cot^2\frac{\theta_0}{2}\right]\right\}, \tag{8f}$$

where

$$z_3 = 2\left[1+f\left(\cos^4\frac{\theta_0}{2}\right)\mathrm{e}^{-8\alpha H_\mathrm{k}^* t}\right]^{-1/2},$$

$$z_1 = W\left[g\left(\tan^2\frac{\theta_0}{2}\right)\mathrm{e}^{-4\alpha H_\mathrm{k}^* t}\right],$$

$$z_0 = \tan^2\theta_0\,\mathrm{e}^{-2\alpha H_\mathrm{k}^* t}.$$

Figure 1 shows the evolution of the reduced magnetization in the uniaxial anisotropic system. The contour plots of

$\boldsymbol{m}$ for $h = 0$, 1 and 3 are shown in Figs. 1(a), 1(c) and 1(e), respectively. The time evolutions of the components are shown in Figs. 1(b), 1(d) and 1(f), respectively. All calculations presented in the following are performed for $\gamma$-$Fe_2O_3$ as $\alpha = 0.003$, $H_k = 9.07 \times 10^4$ A/m, $\gamma = 2.21 \times 10^5$ (rad/s)/(A/m) and $\theta_0 = (\pi/2 - 0.01)$ rad.[22–24] In order to clearly see the evolution pattern of the magnetization, the calculational time intervals of Figs. 1(a), 1(c) and 1(e) are from 0 to $2 \times 10^{-8}$ s, and those of Figs. 1(b), 1(d) and 1(f) are from 0 to $1 \times 10^{-7}$ s. It is found that with the increase of $h$, $\boldsymbol{m}$ approaches stability more quickly and the precession angular frequencies of $m_x$ and $m_y$ increase gradually.

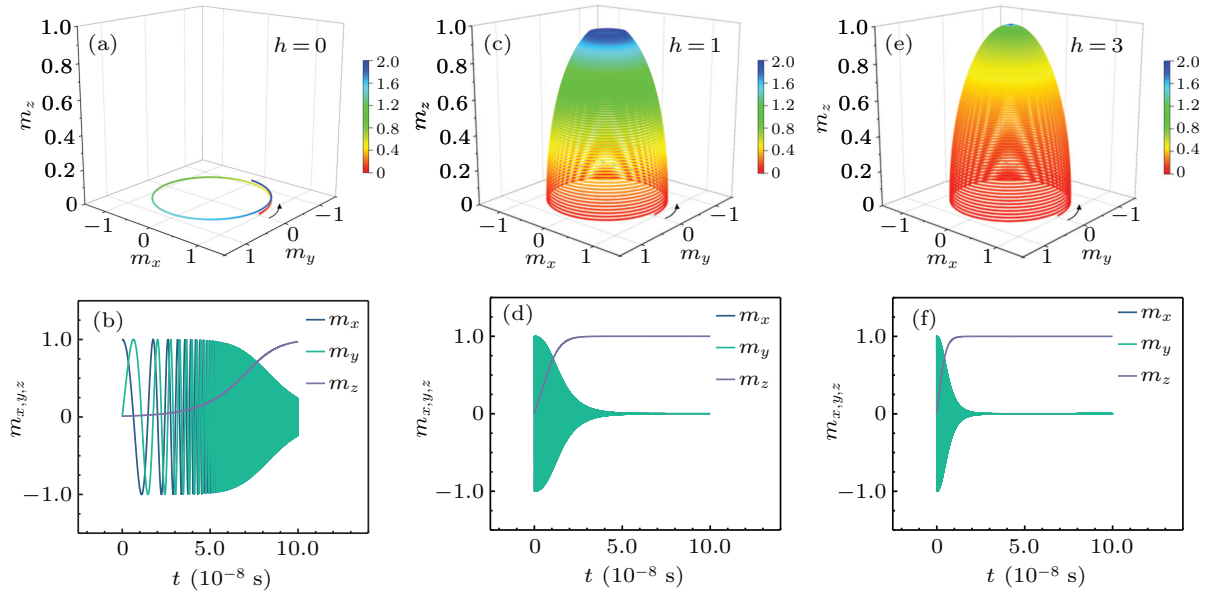



**Fig. 1.** Evolution of reduced magnetization $\boldsymbol{m}$. The initial direction of magnetization is along the positive $x$ direction and at an angle of $(\pi/2 - 0.01)$ rad with the positive $z$ direction, the anisotropic field $H_k = 9.07 \times 10^4$ A/m, the easy axis direction is along the positive $z$ direction, the Gilbert damping parameter $\alpha = 0.003$, the calculational time interval of panels (a), (c) and (e) are from 0 to $2 \times 10^{-8}$ s, and those for panels (b), (d) and (f) are from 0 to $1 \times 10^{-7}$ s. (a) Contour of $\boldsymbol{m}$ in space at $h = 0$. (b) Time evolution of components at $h = 0$. (c) Contour of $\boldsymbol{m}$ in space at $h = 1$. (d) Time evolution of components at $h = 1$. (e) Contour of $\boldsymbol{m}$ in space at $h = 3$. (f) Time evolution of components at $h = 3$.

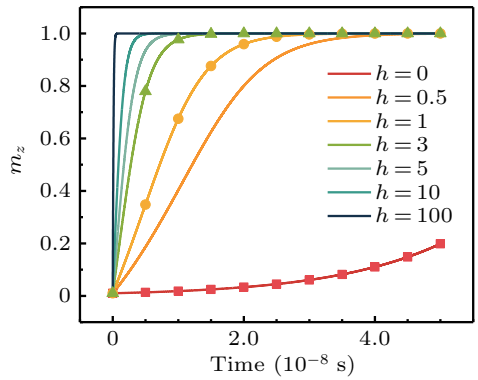



**Fig. 2.** Evolution of the $m_z$ component with time under different reduced magnetic fields $h$. The material parameters are the same as those in Fig. 1, the anisotropic filed $H_k = 9.07 \times 10^4$ A/m, $h = 0$, 0.5, 1, 3, 5, 10 and 100. The solid lines are the results of numerical calculation, while the solid symbols show the results of the analytical solution.

In order to better analyze the relaxation process of magnetization under different external fields and also to verify the correctness of the obtained analytical solutions, the fourth-order Runge–Kutta method was used to directly numerically calculate the LLG equation. The variation of the relaxation trajectory of the $m_z$ component with respect to the reduced magnetic field $h$ was obtained, as shown in Fig. 2, where $H_k = 9.07 \times 10^4$ A/m. The material parameters are the same as those in Fig. 1 and $h = 0$, 0.5, 1, 3, 5,10 and 100. The final time of the calculation is $5 \times 10^{-8}$ s. The solid lines are the results of numerical calculation, while the solid symbols show the results of the analytical solution. By observing the position of magnetization at the same time, it is found that the relaxation time of magnetization is obviously shortened by the increase of the external magnetic field, which is consistent with the conclusion of the previous analytical solution. Meanwhile, the analytical solutions in the three cases are in perfect agreement with the numerical solutions, which shows the correctness of the analytical solutions.

For the evolution of magnetization at $h = H/H_k \to \infty$, although an analytic solution cannot be found, it is equivalent to the case $H_k \to 0$. In this case, using the same idea of solving Eqs. (7) and (8), it is obtained that the change of magnetization components with an applied DC field satisfies the following relation:

$$m_{x,H_k=0} = \left(\tan^2\frac{\theta_0}{2} + 1\right)\sin\theta_0 \exp\left[\alpha H^* t - \ln\left(\tan^2\frac{\theta_0}{2} + e^{2\alpha H^* t}\right)\right]\cos H^* t, \tag{9a}$$

$$m_{y,H_k=0} = \left(\tan^2\frac{\theta_0}{2} + 1\right)\sin\theta_0 \exp\left[\alpha H^* t - \ln\left(\tan^2\frac{\theta_0}{2} + e^{2\alpha H^* t}\right)\right]\sin H^* t, \tag{9b}$$

$$m_{z,H_k=0} = \frac{\exp\left[2\alpha H^* t + \ln\cot^2\frac{\theta_0}{2}\right] - 1}{\exp\left[2\alpha H^* t + \ln\cot^2\frac{\theta_0}{2}\right] + 1}, \tag{9c}$$

where $H^* = \gamma H/\left(\alpha^2+1\right)$ and $\boldsymbol{m}(t=0) = (\sin\theta_0, 0, \cos\theta_0)$.

Figure 3 shows the evolution of the $m_z$ component obtained by analytical solution with the applied field when there is only $H$ or only $H_k$ in the effective field. In the calculation, $\alpha = 0.003$, $\gamma = 2.21\times10^5$ (rad/s)/(A/m), $\theta_0 = (\pi/2 - 0.01)$ rad, and the calculational time interval is from 0 to $1\times10^{-8}$ s. The red solid square symbol in the figure is the result where only $H$ exists in the effective field, and the blue solid circle symbol is the result where only $H_k$ exists. It is found that with the increase of $H$ or $H_k$, the $m_z$ component approaches to the equilibrium position more quickly. If $H$ and $H_k$ have the same magnitude, magnetization relaxation occurs more quickly under the action of $H$.

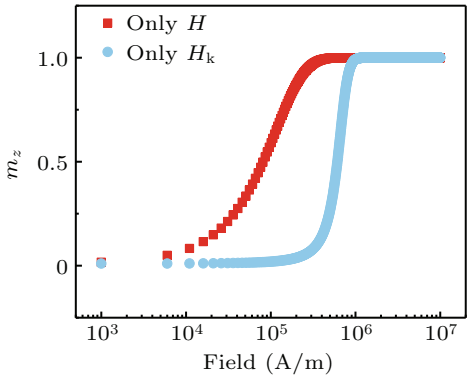



**Fig. 3.** Evolution of the $m_z$ component with an applied field. The initial position of the magnetization is along the positive $x$ direction and at an angle of $(\pi/2-0.01)$ rad with the positive $z$ direction, the field increases from $10^3$ A/m to $10^7$ A/m, and the Gilbert damping parameter $\alpha = 0.003$. The time interval in the calculation is from 0 to $1\times10^{-8}$ s. In the figure, the red solid square symbol represents the result of the case that only $H$ exists in the effective field and the blue solid circle symbol represents the case where only $H_k$ exists.

## 4. Relaxation time and angular frequency

By transforming Eq. (3c) into the form $m_{xy} \propto \exp[\mathrm{i}\omega(t)t]$, the phase $\omega(t)t$ can be written as follows:

$$\omega(t)t = \frac{1}{2\alpha}\ln\left(\frac{\tan^2\frac{\theta_0}{2}}{\tan^2\frac{\theta}{2}}\right). \tag{10}$$

As seen in Eq. (10), the angular frequency $\omega(t)$ during PRM does not have a form of a constant multiplied by $t$. The angular frequency can only be shown if the form of $t$ is determined. With $x = 1+\cos\theta$, $y = 1-\cos\theta$, $z = \cos\theta$, it can be seen from Eq. (7) that

$$t_{h=3} = \frac{1}{8\alpha H_k^*}\ln\frac{\left(1-\cos^4\frac{\theta_0}{2}\right)\cos^4\frac{\theta}{2}}{\left(1-\cos^4\frac{\theta}{2}\right)\cos^4\frac{\theta_0}{2}}, \tag{11a}$$

$$t_{h=1} = -\frac{1}{4\alpha H_k^*}\ln\frac{\tan^2\frac{\theta}{2}\exp\left(\tan^2\frac{\theta}{2}\right)}{\tan^2\frac{\theta_0}{2}\exp\left(\tan^2\frac{\theta_0}{2}\right)}, \tag{11b}$$

$$t_{h=0} = \frac{1}{2\alpha H_k^*}\ln\frac{\cot^2\theta}{\cot^2\theta_0}. \tag{11c}$$

Equations (11a), (11b) and (11c) are the relaxation times from initial polar angle $\theta_0$ to final polar angle $\theta$ for $h = 3$, 1 and 0, respectively. It can be seen that the relaxation time for a certain $h$ is only related to $\alpha$, $H_k^*$, $\theta_0$ and $\theta$.

Figure 4 shows the change of relaxation time with $\theta_0$ and $\theta$ at different $h$. We find that the relaxation time increases monotonically with increasing $\theta_0$ when $\theta$ is fixed, as shown in Fig. 4(a). When $\theta_0$ is fixed, the relaxation time decreases monotonically with increasing $\theta$ as shown in Fig. 4(b). With the fixed $\theta_0$ and $\theta$, the relaxation time decreases monotonically with increasing $h$, as shown by the intersection of the dashed and solid lines in Fig. 4(b).

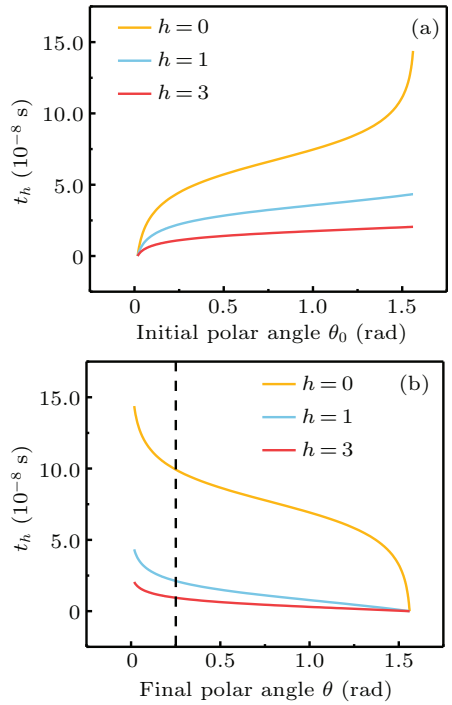



**Fig. 4.** Variation law of relaxation time $t_h$. The material parameters are the same as those in Fig. 1, $\theta_0$ or $\theta$ increasing from 0.0175 rad to $(\pi/2-0.01)$ rad. (a) The variation with initial polar angle $\theta_0$, while $\theta$ is fixed at 0.0175 rad. (b) The variation with final polar angle $\theta$, while $\theta_0$ is fixed at $(\pi/2-0.01)$ rad.

From Eqs. (10) and (11), the time evolution of angular frequency $\omega(t)$ can be obtained as follows:

$$\omega_{h=3}(t) = 4H_k^*\left[\ln\frac{\left(1-\cos^4\frac{\theta_0}{2}\right)\cos^4\frac{\theta}{2}}{\left(1-\cos^4\frac{\theta}{2}\right)\cos^4\frac{\theta_0}{2}}\right]^{-1} \times \ln\left(\frac{\tan^2\frac{\theta_0}{2}}{\tan^2\frac{\theta}{2}}\right), \tag{12a}$$

$$\omega_{h=1}(t) = -2H_k^*\left(\tan^2\frac{\theta}{2} - \tan^2\frac{\theta_0}{2} + \ln\frac{\tan^2\frac{\theta}{2}}{\tan^2\frac{\theta_0}{2}}\right)^{-1} \times \ln\left(\frac{\tan^2\frac{\theta_0}{2}}{\tan^2\frac{\theta}{2}}\right), \tag{12b}$$

$$\omega_{h=0}(t) = H_k^*\left(\ln\frac{\cot^2\theta}{\cot^2\theta_0}\right)^{-1}\ln\left(\frac{\tan^2\frac{\theta_0}{2}}{\tan^2\frac{\theta}{2}}\right). \tag{12c}$$

It is clear that the angular frequency during PRM is related to $H_k^*$, $\theta_0$ and $\theta$ when the reduced magnetic field $h$ is fixed.

Figure 5 shows the variation of angular frequency with $\theta_0$ or $\theta$ under different $h$. As shown in Figs. 5(a) and 5(b), we find that the angular frequency increases monotonically when both $\theta_0$ and $\theta$ decrease, respectively. This indicates that PRM is an accelerated process. With fixed $\theta_0$ and $\theta$, the angular frequency increases monotonically with increasing $h$, as shown by the intersection of the dashed and solid lines in Fig. 5(b).

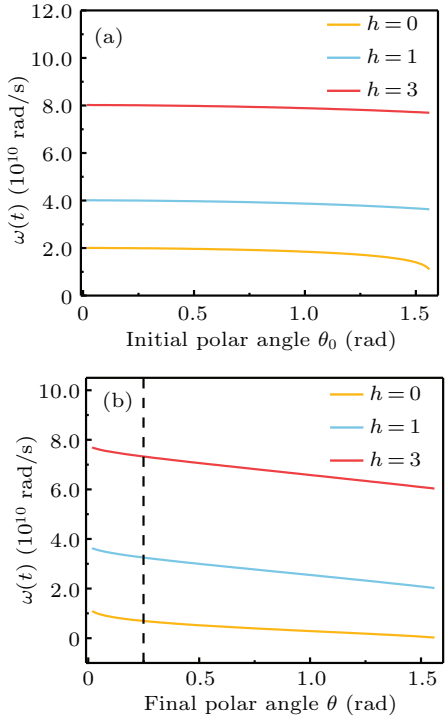



**Fig. 5.** Variation of angular frequency. The material parameters are the same as those in Fig. 1, $\theta_0$ or $\theta$ increasing from 0.0175 rad to $(\pi/2-0.01)$ rad. (a) The variation with initial polar angle $\theta_0$, while $\theta$ is fixed at 0.0175 rad. (b) The variation with final polar angle $\theta$, while $\theta_0$ is fixed at $(\pi/2-0.01)$ rad.

In the limit of $\theta \to 0$, the magnetization tends to the direction of uniaxial anisotropy and the three angular frequencies described by Eq. (12) can be expressed as

$$\omega_0 = \frac{\gamma(H_k + H)}{1+\alpha^2}. \tag{13}$$

It can be seen that the angular frequency at which the magnetization tends to the direction of uniaxial anisotropy is proportional to the total effective field acting on the magnetization and is also related to the $\gamma$ and $\alpha$ of the magnetic materials.

## 5. Conclusions

In this paper, PRM is studied by solving the LLG equation under the action of a DC applied magnetic field and uniaxial anisotropy. We obtain the analytical solutions of the relaxation position, relaxation time and angular frequency of magnetization in three special cases where the applied magnetic field is greater than, equal to or less than the anisotropic field. We also obtain the analytical solution of the relaxation position where only an applied DC field exists in the effective field. It is found that the magnetization makes a right-hand precession with its angular frequency increasing with time and the magnitudes of both the relaxation time and angular frequency are positively related to the $h$. When $\theta$ is fixed, the relaxation time and angular frequency increase or decrease, respectively, with the increase of $\theta_0$. The angular frequency is proportional to the total effective field acting on the magnetization when the magnetization becomes stable. For uniaxial anisotropic systems, an applied field will influence the trajectory of magnetization relaxation. When the total field magnitude remains constant, an increase in the applied field results in faster relaxation. The analytical solution obtained in this paper is helpful to accurately understand the relaxation characteristics of magnetization in a system containing anisotropy.

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