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Quasi-anti-parity–time-symmetric single-resonator micro-optical gyroscope with Kerr nonlinearity

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Parity–time (PT) and quasi-anti-parity–time (quasi-APT) symmetric optical gyroscopes have been proposed recently which enhance Sagnac frequency splitting. However, the operation of gyroscopes at the exceptional point (EP) is challenging due to strict fabrication requirements and experimental uncertainties. We propose a new quasi-APT-symmetric micro-optical gyroscope which can be operated at the EP by easily shifting the Kerr nonlinearity. A single resonator is used as the core sensitive component of the quasi-APT-symmetric optical gyroscope to reduce the size, overcome the strict structural requirements and detect small rotation rates. Moreover, the proposed scheme also has an easy readout method for the frequency splitting. As a result, the device achieves a frequency splitting 10^5 times higher than that of a classical resonant optical gyroscope with the Earth's rotation. This proposal paves the way for a new and valuable method for the engineering of micro-optical gyroscopes.

Keywords: optical gyroscopes, quasi-anti-parity–time (quasi-APT) symmetric system, exceptional point (EP), the Kerr nonlinearity

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1. Introduction

Gyroscopes play a crucial role in inertial navigation and guidance systems, particularly in the commercial and military fields.[1] Among these, micro-optical gyroscopes with small ring resonators, including those in chip-based form, are highly promising due to their potential for high precision, low cost, low power consumption and miniaturization. Much attention has been devoted to improving the precision of micro-optical gyroscopes, including noise suppression,[2–4] boosting the quality factor of the resonator,[5,6] optimizing the optical structure[7–9] and exploring novel signal detection techniques.[10–12] However, the precision of micro-optical gyroscopes is still much lower than that of fiber-optic gyroscopes, which have attained remarkable levels of performance (bias instability $< 10^{-6}\,^{\circ}\cdot\mathrm{s}^{-1}$ with several kilometers of fiber).[6] This is because the sensitivity of chip-based Sagnac gyroscopes decreases with scaling down of the feature size of the resonator.

A non-Hermitian system has gain, loss or open boundary conditions, and generally has complex eigenvalues. Exceptional points (EPs) are non-Hermitian spectral degeneracies in which two or more eigenvalues and corresponding eigenstates coalesce. The splitting of eigenfrequencies near an EP is an exciting optical phenomenon that enhances weak perturbations and is widely used in various ultra-sensitive optical sensing fields, including temperature measurement,[13,14] biochemical sensors,[15,16] particle detection[17,18] and angular velocity detection.[19,20] Recently, Sagnac frequency splitting has been found to be enhanced near an EP.[21] The frequency splitting of a second-order EP based optical sensor is directly proportional to the square root of the intensity of the perturbation. For a given minimal rotation rate, large frequency splitting can be obtained near an EP. Enhanced Sagnac frequency splitting is typically studied in parity–time (PT)/anti-parity–time (APT)-symmetric systems. Optical gyroscopes utilizing coupled resonators with PT/APT symmetry have attracted significant interest recently by increasing the sensitivity of microscale sensors.[22] Lai *et al.* observed a four-fold increase in the Sagnac scale factor using a micro-resonator Brillouin laser gyroscope with EPs.[21] Hokmabadi *et al.* enhanced resonance splitting by up to a factor of 20 by exploiting the increased rotational sensitivity of ring laser gyroscopes in the vicinity of an EP.[23]

Generally, a PT-symmetric gyroscope comprises a pair of resonators that are either loss or gain, and is usually affected by different sources of noise. Moreover, the output spectral linewidth will be broadened with the enhancement of Sagnac frequency splitting, resulting in no improvement in its signal-to-noise ratio (SNR). Because in a PT-symmetric system the imaginary of the eigenfrequencies changes with the rotation rate, it may be difficult to measure the splitting between complex eigenfrequencies.[22] In contrast, quasi-APT-

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symmetric gyroscopes can exhibit completely real frequency splitting based on eigenfrequencies which can be read out easily and accurately.[24] An APT-symmetric system requires resonant frequencies to be opposite to each other, but there is no negative resonant frequency. Then, we just need the two resonant frequencies to be different and call it a quasi-APT-symmetric system. Also, the linewidth in a quasi-APT-symmetric gyroscope is a constant related to the total loss of the resonator. Then EP-enhanced Sagnac frequency splitting can emerge. However, to be quasi-APT-symmetric, a system should have the same gain or loss, two different resonant frequencies and two negative conjugate coupling coefficients. These parameters are non-adjustable when the resonator is designed and fabricated. A quasi-APT-symmetric gyroscope is difficult to realize and operate precisely at the EP. Therefore, it is important to shift a quasi-APT-symmetric gyroscope at the EP.

A nonlinear quasi-APT-gyroscope has been achieved by spinning an optical resonator to improve the sensitivity compared with that of a conventional device without EPs.[25] However, this gyroscope scheme can only be used to detect a very fast rotation (more than 5×10^4 °·s^{-1}). Also, an EP shifted by the Kerr effect in an APT-symmetric system has been proposed with two resonators.[26]

In this paper, we propose a quasi-APT-symmetric micro-optical single-resonator gyroscope which can operate precisely at the EP by shifting the Kerr effect. By using a single resonator as the core sensing element instead of a pair of resonators, the complexity of manufacturing is greatly reduced. In addition, this proposal has easy to read real frequency splitting and an enhanced Sagnac scale factor near the EP with non-broadened linewidth. We simplify the structure of the EP-enhanced gyroscope, reduce the difficulty of micro-resonator fabrication and experiments and further improve the sensitivity of the gyroscope, which are all very significant for promoting engineering applications of micro-optical gyroscopes.

2. Structure design and working principle

The structure of the designed quasi-APT-symmetric micro-optical gyroscope with the Kerr effect is shown in Fig. 1(a). The resonator can be either a waveguide ring resonator or a whispering gallery-mode resonator. Clockwise (CW) and counterclockwise (CCW) beams are indirectly coupled with the resonator to achieve an imaginary coupling.[24] The input light $S_{\rm in}$ is coupled into the resonator after passing through the circulator (CIR) and travels in the CCW direction.[27] The light reflected by the reflector and incident upon the resonator travels in the CW direction. L_1 and L_2 are the optical distances of two waveguides coupled to the resonator in Fig. 1(a). L_1 is the round-trip distance from the second bus–resonator coupling point to the reflector. Extra pumping light $S_{\rm ink}$ is introduced to enhance the Kerr effect of the CW mode, resulting in a Kerr frequency shift. The operating wavelength of the $S_{\rm ink}$ signal is different from that of $S_{\rm in}$, and its intensity is far larger than that of $S_{\rm in}$. Then the Kerr effect caused by $S_{\rm in}$ can be ignored. A Fabry–Perot resonator filter (FPF) is used to block $S_{\rm ink}$ from injecting in the CCW direction. Figure 1(b) shows the operating principle of a quasi-APT-symmetric gyroscope compared with a PT-symmetric gyroscope. The frequency splittings of both are proportional to the square root of the rotation speed. However, the full width at half-maximum (FWHM) of output spectral broadening caused by the frequency splitting of complex eigenfrequencies makes readout of the PT-symmetric gyroscope difficult. In comparison, the real frequency-splitting found in the quasi-APT-symmetric gyroscope without output spectral broadening enhances the detection accuracy and makes it easy to read out.

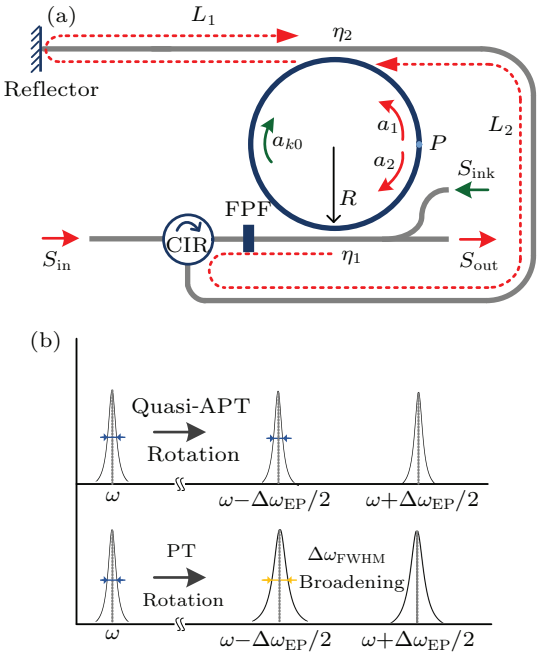


Fig. 1. (a) Structure of a quasi-APT-symmetric single-resonator gyroscope shifted by the Kerr effect. (b) Scheme of frequency splitting in rotation for quasi-APT-symmetric and PT-symmetric gyroscopes.

In the presence of the Kerr effect, the energy exchange between CW and CCW resonant modes of the system can be described as

$$\begin{aligned}
{\rm i}\frac{{\rm d}a_1}{{\rm d}t} &= \omega a_1+\omega h(2|a_{k0}|^2+|a_1|^2)a_1\\
&\quad -{\rm i}ga_1-{\rm i}\eta_1\eta_2{\rm e}^{-{\rm i}\phi_1}a_2,\\
{\rm i}\frac{{\rm d}a_2}{{\rm d}t} &= \omega a_2+\omega h|a_2|^2a_2-{\rm i}ga_2-{\rm i}\eta_2^2{\rm e}^{-{\rm i}\phi_2}a_1, \qquad (1)
\end{aligned}$$

where $a_{1/2}$ represents the energy amplitude of the CW and CCW modes, $|a_{k0}|^2$ represents the light intensity of $S_{\rm ink}$ coupled in the resonator, g is the total loss of the resonator (an external gain can be introduced to keep g small and negative[24]).

$\omega h|a_{k0}|^2$ represents the nonlinear frequency shift of the resonance frequency due to cross-phase modulation (XPM), where ω is the resonant frequency, $h = 2\pi\chi^{(3)}/n^2$ represents the Kerr coefficient, n is the linear refractive index and $\chi^{(3)}$ is the third-order nonlinear susceptibility and is considered as a constant in our scheme. $\omega h|a_1|^2$ and $\omega h|a_2|^2$ are nonlinear frequency shifts of resonance frequency due to self-phase modulation (SPM). $\eta_{1/2}$ is the real direct coupling coefficient between the bus and the resonator at the ith coupling point and $\phi_{1/2}$ represents the phase shifts

$$\phi_{1/2} = \frac{2\pi}{\lambda} n_{\mathrm{g}} L_{1/2} + \frac{2\pi}{\lambda} n_{\mathrm{g}} \frac{\pi R}{2}, \tag{2}$$

where R and n_{g} represent the radius and refractive index of the resonator, respectively, and λ represents the wavelength of the laser S_{in}. Since the coupling mode of a quasi-APT-symmetric system is an imaginary coupling, it is imperative to satisfy the condition $k_1 = -k_2^*$ ($k_1 = \mathrm{i}\eta_1\eta_2\mathrm{e}^{-\mathrm{i}\phi_1}$, $k_2 = \mathrm{i}\eta_2^2\mathrm{e}^{-\mathrm{i}\phi_2}$), so $\mathrm{e}^{-\mathrm{i}\phi_1} = \mathrm{e}^{\mathrm{i}\phi_2}$ and $\eta_1 = \eta_2$ are needed. A possible solution is given by $\phi = \phi_1 = \phi_2 = m\pi$ ($m \in N$) and $k = k_1 = k_2$. Then Eq. (1) can be solved to obtain the eigenfrequencies of the system

$$\omega'_{\mathrm{EP}1,2} = \frac{\omega\left(2 + h|a_k|^2\right)}{2} - \mathrm{i}g \pm \sqrt{\frac{\left(\omega h|a_k|^2\right)^2}{4} + k^2}, \tag{3}$$

where $\omega h|a_k|^2 = \omega h(2|a_{k0}|^2 + |a_1|^2 - |a_2|^2) \approx \omega h 2|a_{k0}|^2$ (SPM is totally submerged by XPM, so SPM can be neglected). For $\omega h|a_k|^2 > -2\mathrm{i}k$, the system is considered to be in a regime where quasi-APT symmetry is not broken. For $\omega h|a_k|^2 < -2\mathrm{i}k$, quasi-APT symmetry is broken. For $\omega h|a_k|^2 = -2\mathrm{i}k$, at the EP, as the system transitions from the unbroken to the broken regime both the eigenvalues and their corresponding eigenvectors coalesce. From that, $\omega h|a_k|^2 = -2\mathrm{i}k$ is a necessary condition for the quasi-APT-symmetric system to operate at the EP. However, this condition is hard to meet due to errors in resonator fabrication or experimental uncertainties in ω and k. Modulating the light intensity $|a_k|^2$ is a good solution to achieve the EP.

Because the gyroscope is in a frame rotating with a rotation rate Ω, the Sagnac shift $\varepsilon = 2R\Omega/(n_{\mathrm{g}}\lambda)$ in the resonance frequency is added to the system. The rotation applied to the gyroscope shifts the resonance frequency of the CW/CCW mode from ω to $\omega \pm \varepsilon$. Then the energy exchange equations can be rewritten as

$$\begin{aligned}
\mathrm{i}\frac{\mathrm{d}a_1}{\mathrm{d}t} &= \omega a_1 + \omega h\left(2|a_{k0}|^2 + |a_1|^2\right)a_1 \\
&\quad - \mathrm{i}g a_1 - k a_2 - \varepsilon a_1, \\
\mathrm{i}\frac{\mathrm{d}a_2}{\mathrm{d}t} &= \omega a_2 + \omega h|a_2|^2 a_2 - \mathrm{i}g a_2 - k a_1 + \varepsilon a_2.
\end{aligned} \tag{4}$$

If the system is at the EP when it is at rest, the rotation forces the system away from the EP. By solving Eq. (4), two eigenfrequencies become

$$\omega_{\mathrm{EP}1,2} = \frac{\omega\left(2 + h|a_k|^2\right)}{2} - \mathrm{i}g \pm \sqrt{\frac{\left(\omega h|a_k|^2 + 2\varepsilon\right)^2}{4} + k^2}. \tag{5}$$

It can be observed that the rotation-induced frequency splitting is

$$\Delta\omega_{\mathrm{EP}} = \sqrt{4k^2 + \left(\omega h|a_k|^2\right)^2 + 4\omega h|a_k|^2\,\varepsilon}, \tag{6}$$

where $\Delta\omega_{\mathrm{EP}}$ becomes proportional to the square root of ε if $\omega h|a_k|^2 = 2\mathrm{i}k$. By manipulating the external light intensity $|a_k|^2$, the gyroscope can operate at the EP, meeting the strict requirements on fabrication errors or experimental uncertainties. The EP can be shifted by the Kerr effect and the sensitivity of the gyroscope can be enhanced. A larger h will have a larger Kerr frequency shift, providing more space for moving the EP. In addition, unlike the frequency splitting of complex eigenfrequencies in a PT-symmetric system, the frequency splitting in a quasi-APT-symmetric system is real, which does not cause broadening of the FWHM and a change in power.[24] More importantly, the light intensity becomes an additional control parameter in the system while considering the Kerr effect.

3. Simulation and result analysis

The effect of a small detuning ε caused by rotation on the eigenfrequency ω_{EP} is analyzed by numerically solving Eq. (6). In Fig. 2(a), it is shown that the real part of the frequency splitting $\mathrm{Re}(\Delta\omega_{\mathrm{EP}})$ is a function of both ε and k. For the quasi-APT-symmetric gyroscope, before the EP, the eigenfrequencies become locked in the quasi-APT-symmetric regime. This leads to a dead band, preventing the measurement of rotation. In the quasi-APT symmetry-broken regime, the rotation leads to frequency splitting between the unlocked eigenfrequencies, as shown in Eq. (5). Figure 2(b) shows the imaginary part $\mathrm{Im}(\Delta\omega_{\mathrm{EP}})$ as a function of both ε and k. In the quasi-APT symmetry-broken regime, $\mathrm{Im}(\Delta\omega_{\mathrm{EP}})$ is unchanged with increase in ε, which indicates that the spectral power and FWHM of the spectrum do not change with the rotation. Therefore, the sensitivity enhancement is not compensated, which is a significant advantage of a quasi-APT-symmetric system. The real part of the difference between two eigenfrequencies $\mathrm{Re}(\Delta\omega_{\mathrm{EP}})$ is plotted in Fig. 2(c) as a function of ε at the EP $\left(k = \omega h|a_k|^2/(2\mathrm{i})\right)$. It can be observed that frequency splitting is enhanced in the quasi-APT broken regime compared with a classic gyroscope. The logarithmic behavior of this curve is shown in Fig. 2(d), in which the slope of the response is $1/2$. This illustrates that perturbation around the second-order EP can experience an enhancement of $\varepsilon^{1/2}$. This feature can be used to greatly improve the sensitivity of the gyroscope.

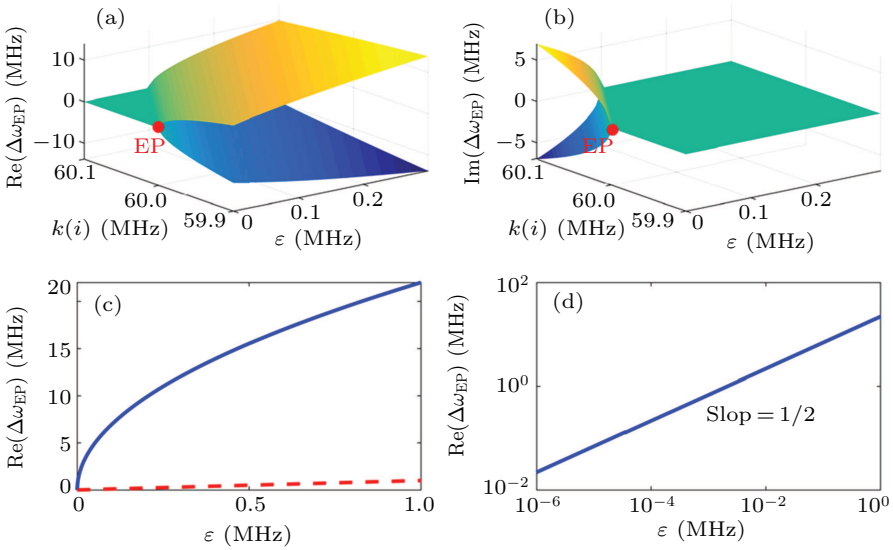


Fig. 2. (a) The real part $\mathrm{Re}\,(\Delta\omega_{\mathrm{EP}})$ and (b) the imaginary part $\mathrm{Im}\,(\omega_{\mathrm{EP}})$ of the frequency splitting as a function of ε and k. (c) Numerical solution of $\mathrm{Re}\,(\Delta\omega_{\mathrm{EP}})$ for $k=\omega h|a_k|^2/(2\mathrm{i})$. The dashed red curve shows the profile of a classic gyroscope for comparison. (d) The results from (c) on a logarithmic scale. The slope of $1/2$ confirms the square-root response of ε. The parameters used are $\lambda=1550$ nm, $h=3\times10^{-18}$ m^2/W, $|a_k|^2=2\times10^{11}$ W/m^2, $\omega=2\times10^{14}$ Hz and $R=2$ mm.

Based on the above discussion, the gyroscope can be operated in the quasi-APT symmetry-broken regime to improve the sensitivity. However, the frequency splitting of typical quasi-APT symmetry will not emerge at a low rotation rate.[25] Also, when the resonator is designed and fabricated, the coupling coefficient k and the resonant frequency are determined. Due to fabrication errors and experimental uncertainties, the system is hard to operate at the EP. Then, the presence of Kerr nonlinearity caused by an external light field S_{ink} is used to keep the system operating at the EP, leading to a high sensitivity. Figure 3 displays the plot of the real part of the eigenfrequency $\mathrm{Re}\,(\Delta\omega_{\mathrm{EP}})$ as a function of $|a_k|^2$ and k. The light intensity of $|a_k|^2$ becomes an additional item of freedom for controlling the system's EP. Hence, the Kerr effect caused by an external light field S_{ink} is used to keep the system operating at the EP, leading to high sensitivity.

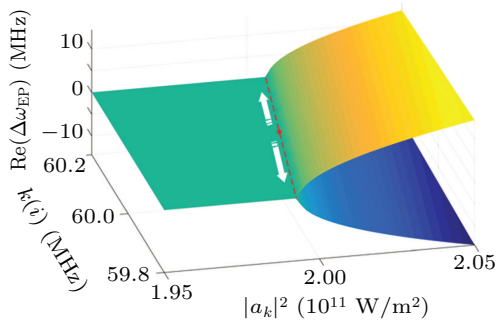


Fig. 3. The real part of the frequency splitting $\mathrm{Re}\,(\Delta\omega_{\mathrm{EP}})$ is a function of k and $|a_k|^2$ with $\varepsilon=0$. The other parameters are the same as those in Fig. 2.

Due to the input and output relationship $a_{\mathrm{out}}=a_{\mathrm{in}}-\mathrm{i}\eta_1 a_1$, the frequency splitting is real. Figure 4 shows the normalized output spectrum as a function of ε near the EP of a quasi-APT-symmetric gyroscope. When the system is at rest ($\varepsilon=0$), the transmission spectrum exhibits a resonance peak since the resonator supports only one traveling mode. A quasi-APT-symmetric gyroscope provides a large frequency splitting even with a small rotation rate. The rotation rate can be obtained by measuring the frequency difference directly. Figure 5 shows the enhancement of the frequency splitting of a quasi-APT-symmetric gyroscope at the EP (red solid line), with the classic Sagnac frequency splitting without an EP (blue dashed line) shown for comparison. It can be seen that the sensitivity of a quasi-APT-symmetric gyroscope can be significantly improved, especially at a low rotation rate. The sensitivity can be improved by five orders of magnitude with the Earth's rotation, showing the superiority of the square root response to the rotation rate.

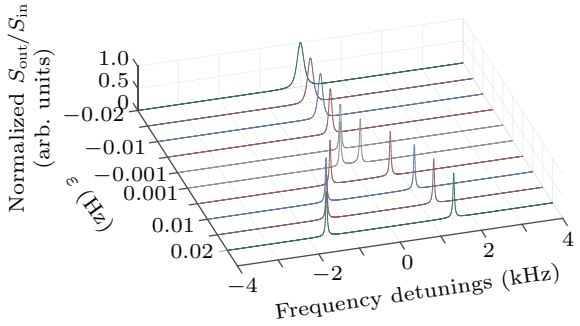


Fig. 4. Normalized output spectrum in the proximity of the EP of a quasi-APT-symmetric gyroscope with $g=200$ Hz (4×10^{-8} dB). The other parameters are the same as those in Fig. 2.

Figure 6 further shows the dependence of the frequency splitting $\mathrm{Re}\,(\Delta\omega_{\mathrm{EP}})$ on the coupling rate k at the EP ($\omega h|a_k|^2=-\mathrm{i}2k$). It is found that $\mathrm{Re}\,(\Delta\omega_{\mathrm{EP}})$ is reduced from a value of about 9×10^{-3} MHz to a value of about 2×10^{-3} MHz for a rotation rate of $1\,^\circ\cdot\mathrm{h}^{-1}$ when k is decreased from $\mathrm{i}1.2\times10^9$ Hz to $\mathrm{i}6\times10^7$ Hz. Thus, the sensitivity of the quasi-APT-symmetric

gyroscope can be manipulated by properly manipulating the coupling rate k when designing the resonator. Additionally, the fabrication error of k can be compensated for by manipulating the light intensity $|a_k|^2$, which will reduce the fabrication difficulty.

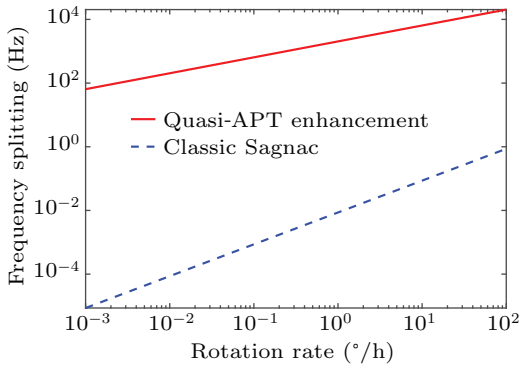


Fig. 5. Spectral splitting of a quasi-APT-symmetric gyroscope at the EP compared with the classical Sagnac effect. The parameters are the same as those in Fig. 2.

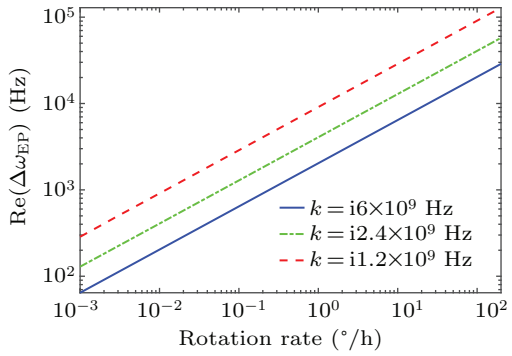


Fig. 6. Real part of frequency splitting $\mathrm{Re}(\Delta\omega_{\mathrm{EP}})$ as a function of the rotation rate Ω for different k values at the EP. $h = 3\times10^{-18}$ m^2/W, and the other parameters are the same as those in Fig. 2.

4. Conclusion and perspectives

In conclusion, a single-resonator quasi-APT-symmetric optical gyroscope based on the Kerr effect is proposed. To overcome the strict conditions for achieving a quasi-APT-symmetric system, a single resonator is used to reduce the fabrication difficulty and the Kerr effect is used to accurately locate the system operating at the EP. In addition, a quasi-APT-symmetric system is adopted to measure the frequency difference directly due to the real frequency splitting having a large measuring range. The sensitivity of this proposed gyroscope can be enhanced by five orders of magnitude compared with classical Sagnac frequency splitting at the rotation rate of the Earth. This work not only provides a high-sensitivity optical micro-gyroscope, but also provides a new platform for research into non-Hermitian physics based on APT symmetry.

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References

[1] Sun D X, Zhang D L, Lu L D, Xu T, Zheng X T, Zhou Z H and Zhu L Q 2023 *Chin. Phys. B* **32** 044212
[2] Khial P P, White A D and Hajimiri 2018 *Nat. Photonics* **12** 671
[3] Zhang J Z, Ma H L, Li H Z and Jin Z H 2017 *Opt. Lett.* **42** 3658
[4] Li H Z, Lin Y, Liu L, Ma H L and Jin Z H 2020 *Opt. Express* **28** 18103
[5] Jiang X S, Gu J X, Cheng X Y, Li G Y, Bai Z Q, Shi Q and Xiao M 2021 *Photonics Res.* **9** 722
[6] Lai Y H, Suh M G, Lu Y K, Shen B, Yang Q F, Wang H, Li J, Lee S H, Yang K Y and Vahala K 2020 *Nat. Photonics* **14** 345
[7] Liang W, llchenko V S, Savchenkov A A, Dale E, Eliyahu D, Matsko A B and Maleki L 2017 *Optica* **4** 114
[8] Geng J T, Yang L, Zhao S H and Zhang Y G 2020 *Opt. Express* **28** 32907
[9] Geng J T, Yang L and Zhang Y G 2022 *IEEE T. Instrum. Meas.* **71** 7004204
[10] Li H Z, Lin Y, Liu L, Ma H L and Jin Z H 2020 *Opt. Express* **28** 18103
[11] Liang J T, Yang L, Liu Z, Liu S L, Geng J T and Zhang Y G 2022 *Opt. Commun.* **520** 128423
[12] Gao W, Wang Z, Wang G C and Miao W Q 2019 *IEEE Photonics J.* **11** 7102313
[13] Zhao H, Chen Z W, Zhao R G and Feng L 2018 *Nat. Commun.* **9** 1764
[14] Hodaei H, Hassan A U, Wittek S, Gracia H G, Ganainy R E, Christodoulides D N and Khajavikhan M 2017 *Nature* **548** 187
[15] Sakhdari M, Farhat M and Chen P Y 2017 *New J. Phys.* **19** 065002
[16] Liu Q J, Wang B, Ke S L, Long H, Wang K and Lu P X 2017 *Opt. Express* **25** 7203
[17] Zhong Q, Ren J, Khajavikhan M, Christodoulides D N, Özdemir Ş K and Ganainy R E 2019 *Phys. Rev. Lett.* **122** 153902
[18] Chen W J, Özdemir Ş K, Zhao G M, Wiersig J and Yang L 2017 *Nature* **548** 192
[19] Smith D D, Chang H, Horstman L and Diels J C 2019 *Opt. Express* **27** 34169
[20] Horstman L, Hsu N, Hendrie J, Smith D and Diels J C 2020 *Photonics Res.* **8** 252
[21] Lai Y H, Lu Y K, Suh M G, Yuan Z Q and Vahala K 2019 *Nature* **576** 65
[22] Ren J, Hodaei H, Harari G, Hassan A U, Chow W, Soltani M, Christodoulides D and Khajavikhan M 2017 *Opt. Lett.* **42** 1556
[23] Hokmabadi M P, Schumer A, Christodoulides D N and Khajavikhan M 2019 *Nature* **576** 70
[24] Carlo M D, Leonardis F D, Lamberti L and Passaro V M N 2019 *Opt. letters* **44** 3956
[25] Zhang H L, Peng M Y, Xu X W and Jing H 2022 *Chin. Phys. B* **31** 014215
[26] Chen T Y, Xiao Z S, Jiang S, Li W S, Li J C, Wang Y F, Wang X C, Huang A P and Zhang H 2022 *J. Appl. Phys.* **131** 243101
[27] Huang D, Pintus P and Bowers J E 2018 *Opt. Mater. Express* **8** 2471