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Citation: Chin. Phys. B, 2024, 33 (**4**): 040401. DOI: 10.1088/1674-1056/ad0624
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Harmonic balance simulation of the influence of component uniformity and reliability on the performance of a Josephson traveling wave parametric amplifier

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(Received 12 July 2023; revised manuscript received 19 October 2023; accepted manuscript online 24 October 2023)

A Josephson traveling wave parametric amplifier (JTWPA), which is a quantum-limited amplifier with high gain and large bandwidth, is the core device of large-scale measurement and control systems for quantum computing. A typical JTWPA consists of thousands of Josephson junctions connected in series to form a transmission line and hundreds of shunt LC resonators periodically loaded along the line for phase matching. Because the variation of these capacitors and inductors can be detrimental to their high-frequency characteristics, the fabrication of a JTWPA typically necessitates precise processing equipment. To guide the fabrication process and further improve the design for manufacturability, it is necessary to understand how each electronic component affects the amplifier. In this paper, we use the harmonic balance method to conduct a comprehensive study on the impact of nonuniformity and fabrication yield of the electronic components on the performance of a JTWPA. The results provide insightful and scientific guidance for device design and fabrication processes.

Keywords: Josephson traveling wave parametric amplifier (JTWPA), harmonic balance method, yields, uniformity

PACS: 04.30.Nk, 03.65.–w, 03.67.–a **DOI:** 10.1088/1674-1056/ad0624

1. Introduction

Superconducting qubits are a promising physical implementation of quantum computing. Nevertheless, they are vulnerable and can be easily disturbed by noise.[1] Their signals are extremely weak, with a few microwave photons. Therefore, quantum-limited amplifiers with noise temperatures close to the quantum limit[2] are required for the measurement of superconducting qubits. The Josephson parametric amplifier (JPA)[3] and impedance-transformed Josephson parametric amplifier (IMPA)[4] are commonly used quantum-limited amplifiers, with typical bandwidths of tens and hundreds of megahertz, respectively. As the number of qubits increases, there is an urgent need for quantum-limited amplifiers with wider bandwidths.

Both JPAs and IMPAs are resonant parametric amplifiers. They put a nonlinear medium into a resonator to increase the interaction time of pump, signal and nonlinear medium. They are both limited by the gain–bandwidth trade-off.[5] To overcome this limitation and achieve larger gain and bandwidth simultaneously, the traveling wave parametric amplifier (TWPA) was designed and fabricated.[6] This type of amplifier is equivalent to a nonlinear transmission line, where the pump and signal interact with the nonlinear medium during their transmission to achieve signal amplification. Two types of nonlinear medium are commonly used in a TWPA. One type uses the nonlinear effect of the kinetic inductance[7] of a superconductor. It usually requires a large pump power,[8] which can result in a large thermal effect that presents engineering challenges for qubit reading. The other type is based on Josephson junctions.[9–12] Due to their high nonlinearity, this type of amplifier requires less pump power and the chip size is smaller. JTWPAs meet the needs of multi-qubit measurement and control systems and can also be applied in astronomical detection.[13]

A JTWPA uses a Josephson junction as the nonlinear element. It typically contains a transmission line formed by thousands of Josephson junctions in series, accompanied by thousands of shunt capacitors to maintain impedance matching as well as hundreds of resonators to provide phase matching. This means that each repeated part needs to be consistent. The parameters of each capacitor, Josephson junction and resonator on the chip are greatly affected by the fabrication process, which in turn affects the performance of the JTWPA. A JTWPA with 20 dB gain in 3 GHz bandwidth[9] requires precise manufacturing equipment. Based on design optimization and a simplified process, a gain of 20 dB with 1 GHz bandwidth was achieved using a wafer-level fabrication pro-

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cess compatible with superconducting qubits.[6] In this paper we employ the harmonic balance method to conduct a comprehensive investigation regarding the impact of component survival rate and uniformity on the performance of a JTWPA. The results provide reliable and scientific guidance for device design and fabrication processes compatible with superconducting qubits. The studies reveal that inadequate uniformity of the Josephson junctions can not only increase the gain ripple but also reduce the average gain, in contrast to a previous report that the uniformity of Josephson junctions has negligible impact on average gain.[6]

2. Method

As shown in Fig. 1(a), a JTWPA typically consists of two main parts. The first part comprises thousands of series Josephson junctions and shunt capacitors C_0 to form a nonlinear transmission line with an impedance of 50 Ω. Due to the Kerr-like nonlinearity caused by the Josephson current–phase relationship,[14] four-wave mixing occurs when the signal and the pump are transmitted on the nonlinear transmission line, resulting in signal amplification. However, the gain and bandwidth of the nonlinear transmission line are small due to the phase mismatch caused by self- and cross-phase modulation effects. To address this issue, another crucial part, the dispersion engineering part, needs to be introduced. It typically comprises hundreds or thousands of shunted LC resonators, compensating for the phase mismatch by increasing the pump wave vector[15] to obtain greater gain and bandwidth.

The coupled mode equation method[6] is often used to simulate the performance of a JTWPA. The dispersion characteristics are obtained through numerical calculation or simulation, and the results are then input into coupled mode equations to calculate the gain. Higher-order mixing products can be obtained by extending the equations.[16] However, this method does not consider the change in pump current caused by the standing wave due to impedance variation. To solve this problem, the finite difference time domain model[17] was proposed. Nevertheless, neither of these methods can account for the impact of the JTWPA component yield on device performance. WRspice is a circuit simulator that can simulate Josephson circuits,[16] and can solve the aforementioned problems. However, it can only solve the JTWPA in transient mode, which can make the simulation task onerous. We use the harmonic balance method to simulate the JTWPA, an approach that has been successfully used to simulate kinetic inductance TWPAs.[18] This method enables us to easily study and analyze the device's performance.

The Josephson junctions provide the JTWPA with the nonlinearity required for amplification, which can be described by the nonlinear inductive element. The current-dependent nonlinear inductance can be obtained from the Josephson equation

$$L_{\mathrm{J}} = \frac{L_{\mathrm{J0}}}{\sqrt{1-\left(\dfrac{I}{I_0}\right)^2}}. \tag{1}$$

Here, $L_{\mathrm{J0}} = \Phi_0/(2\pi I_0)$ is the minimum inductance of the Josephson junction, Φ_0 is the magnetic flux quantum and I_0 is the critical current. The Taylor expansion of formula (1) can be obtained as

$$L_{\mathrm{J}} = L_{\mathrm{J0}}\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\cdots\right), \tag{2}$$

where $x = (I/I_0)^2$. Using the circuit parameters in Ref. [15], gain curves at different pump powers were obtained (see Fig. 1(b)), and the results are consistent with the literature. Figure 1(c) shows the variation of the gain with pump power at a signal frequency of 5 GHz. The gain increases with increasing pump power, and the slope of the curve also increases. Assuming perfect phase matching and zero initial idle amplitude, the amplitude of the signal at position z can be approximated as $a_{\mathrm{s}}(z) \approx a_{\mathrm{s}}(0)\,\mathrm{e}^{gz}/2$. The amplifier gain can be approximated as exponential gain,[15] and the gain factor g is directly related to the pump power $g \propto k_{\mathrm{p}} I_{\mathrm{p}}^2/I_0^2$, where k_{p}, I_{p} and I_0 are the pump wave vector, pump current and critical current of the junction, respectively.

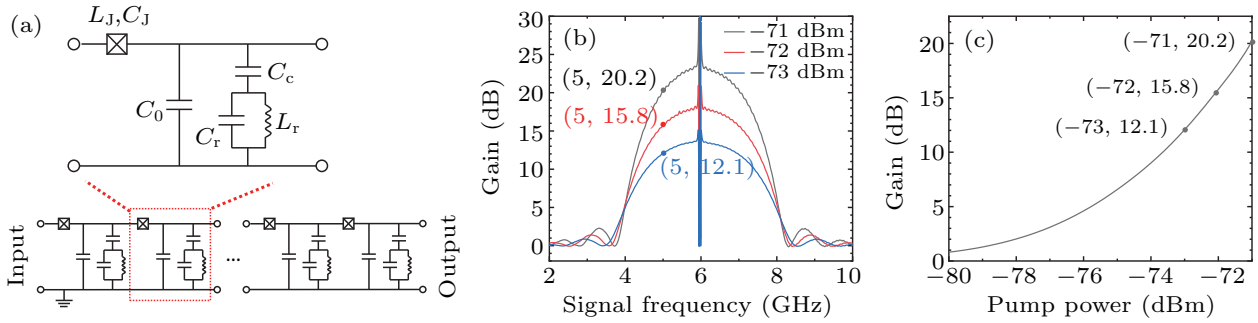


Fig. 1. (a) Equivalent circuit model of a JTWPA. (b) Using the circuit parameters in Ref. [15], the gain curves at different pump powers simulated by the harmonic balance method are shown. The pump frequency was set to 5.97 GHz. (c) Gain versus pump power with a signal frequency of 5 GHz.

3. Results and discussion

3.1. Component yields

The shunt capacitor C_0 and the coupling capacitor C_c (see Fig. 1(a)) are used to maintain impedance matching and couple the resonator to the nonlinear transmission line, respectively. They are typically in the femtofarad range. In order to reduce the size of the device, these capacitors will be designed as parallel plate capacitors with the dielectric layer tens to hundreds of nanometers thick. As these capacitors are small and within the thin dielectric layer, the dielectric layer can be easily broken down, leading to the loss of insulating properties, depending on the degree of breakdown.[19] Based on experimental data, we set the resistance of the breakdown of C_0 and C_c to 200 Ω and 500 Ω, respectively. The breakdown of C_0 or C_c can significantly reduce the gain and cause gain ripple (see Figs. 2(a) and 2(b)), and the influence on the gain worsens as the problematic capacitor gets closer to the input port. Figures 2(d) and 2(e) correspond to the transmission characteristic curves in Figs. 2(a) and 2(b), respectively. It is clear that the decrease in S_{21} caused by the problematic capacitor is small. This suggests that the reduction in gain is primarily due to a decrease in the efficiently transmitted pump energy, which is consistent with the conclusion in Fig. 1(c). The closer the problematic capacitor is to the input port, the more Josephson junctions that receive a small pump energy, and the more the gain decreases.

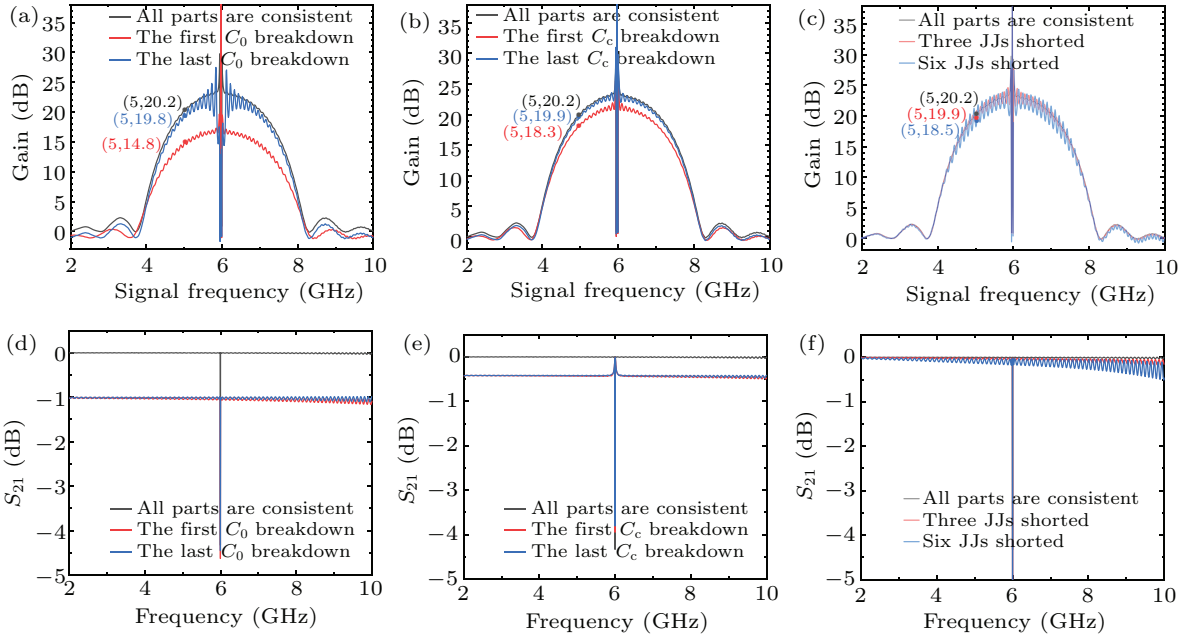


Fig. 2. The impact of component yield on JTWPA performance, with pump power at the input set to −71 dBm. (a), (d) The effects of C_0 breakdown on gain and transmission characteristics. The first and last cases represent the closest and farthest C_0 breakdown from the input, respectively. (b), (e) The effects of C_c breakdown on gain and transmission characteristics. The first and last cases represent the closest and farthest C_c breakdown from the input, respectively. (c), (f) The impact of randomly shorted Josephson junctions (JJs) on gain and transmission characteristics.

In the process of preparing superconducting qubits, the yield of wafer-level Josephson junctions can reach 99.9%.[20] Some Josephson junctions are shorted or open. Since the JTWPA consists of thousands of Josephson junctions connected in series, an open Josephson junction can disable the amplifier. Figures 2(c) and 2(f), respectively, show the effect of shorted Josephson junctions on the performance of the JTWPA. Six randomly shorted Josephson junctions (approximately 0.3% of the total) have little impact on the average gain but significantly increase the gain ripple.

3.2. Component uniformity

Due to the superconducting energy gap of Josephson junctions, when the current flowing through the junction exceeds its critical current, the Josephson junction will fail and behave as a resistor with resistance equal to its normal resistance.[21] As shown in Fig. 3(a), the failure of a Josephson junction significantly reduces the amplifier gain. If the failed junction is close to the input port, the JTWPA essentially has no gain. This is mainly because the failed junction significantly reduces the pump and signal waves passing through the nonlinear transmission line (see Fig. 3(b)). In the nonuniform condition of Josephson junctions, for example if the minimum critical current $I_{0,\min}$ for the Josephson junctions on the nonlinear transmission line is $0.9\langle I_0\rangle$, the maximum pump power has to be reduced by approximately 0.9 dBm for its operation and the gain will drop by about 4 dB according to Fig. 1(c). Another situation caused by nonuniform Josephson junctions is the significant standing waves formed on the transmission line (Fig. 3(c)), which makes some Josephson junc-

tions located at the peaks of the waves more prone to failure. It is noted that, in the device fabrication process, nonuniformity arising from different processing conditions for various components could lead to impedance mismatches and standing waves on the transmission line. In the preparation process, the equivalent evaporation rate affects the uniformity of the grains, and the equivalent evaporation angle affects the junction area. These factors ultimately contribute to the nonuniformity of Josephson junctions on the wafer. Optimized evaporation steps[22] and evaporation rates[23] can improve the uniformity of Josephson junctions.

As well as the impact of junction uniformity, the effect of capacitor uniformity on amplifier gain without considering Josephson junction failure (see Fig. 4) was also studied. Compared with other capacitors, the uniformity of C_r can significantly reduce the device gain, which is consistent with existing reports,[6] because it can significantly affect the dispersion engineering. In experiments, the use of atomic layer deposition to prepare the dielectric layer results in better capacitor uniformity than using plasma enhanced chemical vapor deposition.[24] Although the uniformity of C_0 has a tiny impact on the average gain it can cause impedance mismatch and lead to standing waves. In the fabrication process this should be also minimized as much as possible. It is believed that with the optimization of design and fabrication, the performance of JTWPAs is bound to be greatly improved.

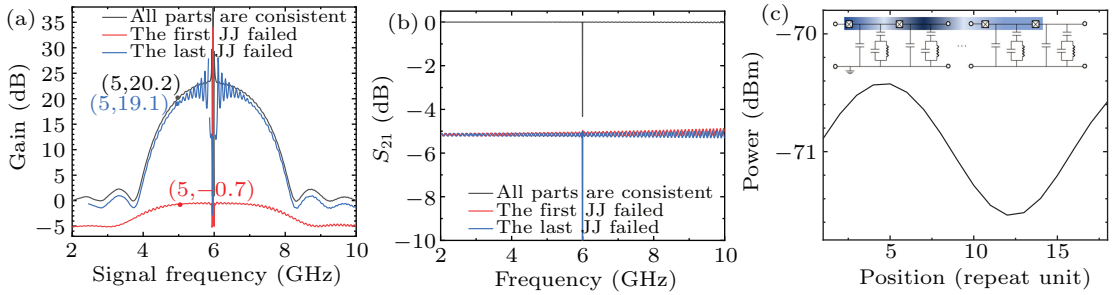


Fig. 3. Impact of the number of failed Josephson junctions (JJs), with pump power at the input being −71 dBm. (a) The effect of JJ failure on gain. (b) The effect of JJ failure on transmission characteristics. (c) The critical current of a random JJ is equal to half the design value, which results in standing waves on the nonlinear transmission line.

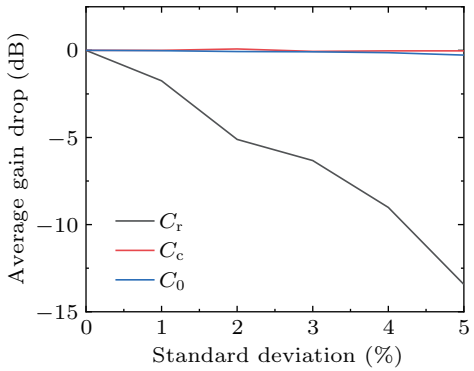


Fig. 4. The effect of capacitor nonuniformity on gain without considering the failure of the Josephson junction.

4. Conclusion

In summary, by using a harmonic balance method, a comprehensive investigation regarding the impact of component (capacitors and Josephson junctions) survival rate and uniformity on the performance of JTWPAs has been conducted. To achieve sufficient gain, it is necessary to ensure that the signal and pump waves have enough energy to pass through the nonlinear transmission line. The breakdown or shorting of C_0 or C_c will reduce the amplitude of S_{21} and thus the gain, which should be avoided in device design and fabrication. The uniformity of C_r is crucial, because it has the most significant impact on the gain of the JTWPA. Although the uniformity of the Josephson junction does not directly lead to a significant drop in gain, poorer uniformity usually means a wider critical current distribution. It can easily lead to failure of the Josephson junction, leading to an increase in gain ripple and influence the average gain. These discoveries provide scientific guidance for the design and fabrication of JTWPAs.

Acknowledgements

KLX acknowledges support from the Youth Innovation Promotion Association of Chinese Academy of Sciences (Grant No. 2019319). JGF acknowledges support from the Start-up Foundation of Suzhou Institute of Nano-Tech and Nano-Bionics, CAS, Suzhou (Grant No. Y9AAD110).

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