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Coexistence of antiferromagnetism and unconventional superconductivity in a quasi-one-dimensional flat-band system: Creutz lattice

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We study the coexistence of antiferromagnetism and unconventional superconductivity on the Creutz lattice which shows strictly flat bands in the noninteracting regime. The famous renormalized mean-field theory is used to deal with strong electron–electron repulsive Hubbard interaction in the effective low-energy t–J model, the superfluid weight of the unconventional superconducting state has been calculated via the linear response theory. An unconventional superconducting state with both spin-singlet and staggered spin-triplet pairs emerges beyond a critical antiferromagnetic coupling interaction, while antiferromagnetism accompanies this state. The superconducting state with only spin-singlet pairs is dominant with paramagnetic phase. The A phase is analogous to the pseudogap phase, which shows that electrons go to form pairs but do not cause a supercurrent. We also show the superfluid behavior of the unconventional superconducting state and its critical temperature. It is proven directly that the flat band can effectively raise the critical temperature of superconductivity. It is implementable to simulate and control strongly-correlated electrons' behavior on the Creutz lattice in the ultracold atoms experiment or other artificial structures. Our results may help the understanding of the interplay between unconventional superconductivity and magnetism.

Keywords: flat-band unconventional superconductivity, antiferromagnetism, strong electron–electron interaction, superfluid weight

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1. Introduction

Antiferromagnetism (AFM) and unconventional superconductivity (SC) are two key phenomena that appeared in strongly correlated electronic materials such as cuprates and iron pnictide high temperature superconductors, and heavy fermion systems.[1–3] In particular, an unusual quantum state where both AF and superconducting orders coexist microscopically has been widely studied in many theoretical works.[4–10] Some experimental works have revealed that several quantum materials display unique AFM–SC coexistence states in their phase diagrams.[12–15] It is noteworthy that a staggered spin-triplet pairing superconducting order parameter is induced in the coexistence of AFM and singlet SC.[4,10] The spin-triplet superconductors are ideal materials that can keep SC under a strong magnetic field and are also important candidate materials for quantum computation.[11] Hence, it is of great interest to study the interplay between unconventional SC and AFM, in particular for systems with strong electron–electron correlations.[7]

The highly-degenerated Bloch band with constant energy, named flat band, with the quench of the kinetic energy is an ideal platform to study the behavior of strongly correlated electrons. As shown in the recent works, adding an attractive Hubbard interaction in a flat band can carry a nonzero superfluid weight.[16–19] This conclusion is universal for flat bands in the strongly correlated limit, in which the superfluid weight is proportional to effective attractive interaction.[20–22] Flat-band magnetism is another interesting topic, such as the famous ferromagnetism in the Lieb lattice, the spin-imbalanced effect on the superfluidity is also studied in this system.[17,44] The interplay of magnetism and SC in the flat-band system is a meaningful topic that deserves further study.

A quasi-one dimensional flat band system such as Creutz lattice has been of great interest because it is much easier to solve than two-dimensional models in the theory and is easy to simulate in an ultracold atom experiment.[18,19,23–25] It is essentially one-dimensional system with two-dimensional characteristics because of the interchain degrees of freedom that appear. Another quasi-one-dimensional system, named the two-leg square ladder, has been attracting attention for both theoretical and experimental reasons because it is directly related to the stripe-ordered cuprate superconductor $La_{2-x}Ba_xCuO_4$.[26,27]

Flat band systems have attracted attention since the experimental discovery of a superconducting state in twisted bilayer graphene related to dispersionless bands.[29–33] The original interest in the flat band has arisen from the prediction that it has a dramatically higher critical temperature in a flat band based on the Bardeen–Cooper–Schrieffer theory.[34,35] There are many systems that have flat bands, e.g., sawtooth, kagome, Lie lattice, and others. These systems have recently been stud-

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ied extensively, in both fermionic and bosonic language.[36–38] A superfluid state is supported by bosons paired on different sites or Cooper pairs using a fermionic language in these flat band systems.[39] There is a rich phase diagram exhibiting a condensate, a pair condensate, a supersolid and phase separation phases in the study of bosons on the flat band Creutz lattice.[23,24] The Creutz model with an attractive Hubbard interaction using the extended Bardeen–Cooper–Schrieffer theory and the density matrix renormalization group has been studied, which displays finite superfluid weight with lower bound related to the winding number.[18]

Here, we study unconventional SC on the Creutz model with strong repulsive electron–electron interaction using the renormalized mean-field method with Gutzwiller factors. This paper is organized as follows. In Section 2, we describe the basic theoretical model and renormalized mean-field theory. In Section 3, we present our numerical results and discuss their physical meanings. Finally, some conclusions are drawn in Section 4.

2. Model and methodology

We study a quasi-one-dimensional system named Creutz lattice governed by the following Hamiltonian:

$$\begin{aligned}\hat{\mathcal{H}}_0 =& \sum_{i\sigma} t\hat{C}^{A\dagger}_{i\sigma}\hat{C}^{A}_{i+1\sigma} - t\hat{C}^{B\dagger}_{i\sigma}\hat{C}^{B}_{i+1\sigma} + t\hat{C}^{B\dagger}_{i\sigma}\hat{C}^{A}_{i+1\sigma} \\ & - t\hat{C}^{A\dagger}_{i\sigma}\hat{C}^{B}_{i+1\sigma} + \text{H.c.},\end{aligned} \tag{1}$$

where A and B label the two chains; the intrachain hopping energy on chain A (B) becomes t $(-t)$; interchain hopping energy between site i on chain A (B) and site $i+1$ on chain B (A) is given by $-t$ (t); and $\sigma = \uparrow, \downarrow$ is the fermion spin. For noninteracting cases, the system shows two strictly flat bands $E = \pm 2t$. Due to the flat band structure, electrons will be localized in the near lattice without correlation effects. To include correlation effects between electrons, the appropriate Hubbard model with repulsive potential is used, $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + U\sum_{i,\alpha}\hat{n}_{i,\alpha,\uparrow}\hat{n}_{i,\alpha,\downarrow}$, where $\alpha = A, B$ is the chain index. In the strong interacting limit when $U \gg t$, an effective t–J model in which the ground state is energetically confined to the singly occupied space is given by

$$\hat{\mathcal{H}} = P_{\text{G}}\left[\hat{\mathcal{H}}_0 + \sum_{\langle ij\rangle} J\left(\boldsymbol{S}_i\cdot\boldsymbol{S}_j - \frac{1}{4}n_i n_j\right)\right]P_{\text{G}}, \tag{2}$$

where $J = 4t^2/U$ is the AF coupling, $\boldsymbol{S}_i$ and n_i are the spin and number operators for electrons at i site, respectively, and $P_{\text{G}} = \prod_i(1 - n_{i\uparrow}n_{i\downarrow})$ is the Gutzwiller projection operator that projects out states with doubly-occupied sites and $\langle ij\rangle$ labels the nearest-neighbor bond.

In this paper, we use the renormalized mean-field theory to study the interplay of SC and AFM on the Creutz lattice. The no double occupancy constraint can be treated the same as in previous works, the projection operator is replaced by Gutzwiller factors leading to a renormalized Hamiltonian.[27,28] For simplicity, we use the link between $A_i - A_{i+1}$ as an example to get the renormalized Hamiltonian. The same procedure has also been done on the links between $A_i - B_{i+1}, B_i - B_{i+1}, A_i - B_{i+1}$. To study the interplay of SC and AFM, the local values of the magnetic order parameters are defined as follows:

$$m_i^{\alpha} = \frac{1}{2}\langle\Psi_0 \mid \hat{C}^{\alpha\dagger}_{i\uparrow}\hat{C}^{\alpha}_{i\uparrow} - \hat{C}^{\alpha\dagger}_{i\downarrow}\hat{C}^{\alpha}_{i\downarrow} \mid \Psi_0\rangle = m_0\exp(\mathrm{i}Qx_i), \tag{3}$$

where $\mid\Psi_0\rangle$ is the unprojected wavefunction and $Q = \pi$ for AFM. In addition, the superconducting pairing order parameters and the bond fields parameters and hole density are defined as

$$\begin{aligned}\chi_{i\sigma} &= \langle\Psi_0 \mid \hat{C}^{A\dagger}_{i\sigma}\hat{C}^{A}_{i+1\sigma} \mid \Psi_0\rangle, \\ \Delta_{i\sigma} &= \langle\Psi_0 \mid \sigma\hat{C}^{A}_{i\sigma}\hat{C}^{A}_{i+1-\sigma} \mid \Psi_0\rangle, \\ \delta_i &= 1 - \sum_{\sigma}\langle\Psi_0 \mid \hat{C}^{A\dagger}_{i\sigma}\hat{C}^{A}_{i\sigma} \mid \Psi_0\rangle,\end{aligned} \tag{4}$$

where $\sigma = \pm$ for $\uparrow\downarrow$. It is difficult to treat the strong coupling constraint of no double occupancy analytically, hence Zhang *et al.* first introduced Gutzwiller renormalization factors to approximate the constraint and did a lot of research work with great significance.[24,48,49] In this paper, we use the explicit renormalization factors as follows:

$$\begin{aligned}g^{A,t}_{i,\sigma} &= \sqrt{\frac{2\delta(1-\delta_i)}{1-\delta_i^2+4m_i^2}\frac{1+\delta_i+\sigma 2m_i}{1+\delta_i-\sigma 2m_i}}, \\ g^{A,t}_{i,i+1,\sigma} &= g^{A,t}_{i,\sigma}g^{A,t}_{i+1,\sigma}, \\ g^{A,s}_{i} &= \frac{2(1-\delta_i)}{1-\delta_i^2+4m_i^2}, \\ g^{A,s}_{i,i+1} &= g^{A,s}_{i}g^{A,s}_{i+1}.\end{aligned} \tag{5}$$

Thus, one obtains the following renormalized mean field Hamiltonian:

$$\begin{aligned}\hat{\mathcal{H}}_{\text{MF}} =& \sum_{i\sigma}\zeta_{i\sigma}\hat{C}^{A\dagger}_{i\sigma}\hat{C}^{A}_{i+1\sigma} + \eta_{i\sigma}\hat{C}^{A\dagger}_{j\sigma}\hat{C}^{A}_{i\sigma} \\ & + \sum_i \Lambda_{i\uparrow}\hat{C}^{A}_{i\uparrow}\hat{C}^{A}_{i+1\downarrow} - \Lambda_{i\downarrow}\hat{C}^{A}_{i\downarrow}\hat{C}^{A}_{i+1\uparrow} \\ & + \text{H.c.} + \text{constant}\end{aligned} \tag{6}$$

with

$$\begin{aligned}\zeta_{i\sigma} &= g^{A}_{t\sigma}t - J\left(\frac{g^{A}_{s}}{4} + \frac{g^{A}_{s}}{2}\frac{\chi_{i-\sigma}}{\chi_{i\sigma}}\right)\chi_{i\sigma}, \\ \Lambda_{i\sigma} &= -J\left(\frac{g^{A}_{s}}{4} + \frac{g^{A}_{s}}{2}\frac{\Delta_{i-\sigma}}{\Delta_{i\sigma}}\right)\Delta_{i\sigma}, \\ \eta_{i\sigma} &= \sigma g^{A}_{s}Jm_i.\end{aligned}$$

According to the previous works,[7,10] the superconducting pairing order parameter Δ_i can be rewritten in terms of the singlet and the staggered π-triplet components

$$\Delta_i = \frac{1}{2}(\langle\Psi_0 \mid \hat{C}_{i\uparrow}\hat{C}_{i+1\downarrow} - \hat{C}_{i\downarrow}\hat{C}_{i+1\uparrow} \mid \Psi_0\rangle$$

$$+\langle\Psi_0\mid\hat{C}_{i\uparrow}\hat{C}_{i+1\downarrow}+\hat{C}_{i\downarrow}\hat{C}_{i+1\uparrow}\mid\Psi_0\rangle)$$
$$=\Delta_i^{\mathrm{s}}+\Delta_i^{\mathrm{t}}\exp(\mathrm{i}Qx_i). \tag{7}$$

To obtain the optimum solution of the mean-field values, we perform a discrete Fourier transformation to the mean-field Hamiltonian with

$$\hat{C}_{i\sigma}^{\alpha}=\frac{1}{N}\sum_{\boldsymbol{k}}\hat{C}_{\boldsymbol{k}\sigma}^{\alpha}\exp(\mathrm{i}\boldsymbol{k}\boldsymbol{x}_i). \tag{8}$$

The mean-field Hamiltonian matrix can be expressed as

$$\hat{\mathcal{H}}_{\mathrm{MF}}=\sum_{\boldsymbol{k}}\psi^{\dagger}(\boldsymbol{k})h_{\mathrm{MF}}(\boldsymbol{k})\psi(\boldsymbol{k})+\mathrm{constant}, \tag{9}$$

where

$$\psi^{+}=(\hat{C}_{\boldsymbol{k}\uparrow}^{A\dagger},\hat{C}_{\boldsymbol{k}\uparrow}^{B\dagger},\hat{C}_{\boldsymbol{k}+Q\uparrow}^{A\dagger},\hat{C}_{\boldsymbol{k}+Q\uparrow}^{B\dagger},\hat{C}_{-\boldsymbol{k}\downarrow}^{A},\hat{C}_{-\boldsymbol{k}\downarrow}^{B},\hat{C}_{-\boldsymbol{k}-Q\downarrow}^{A},\hat{C}_{-\boldsymbol{k}-Q\downarrow}^{B}).$$

The full renormalized mean field Hamiltonian consists of four links between sublattice A,B and the chemical potential term $-\mu\sum_{\alpha,\boldsymbol{k},\sigma}\hat{C}_{\boldsymbol{k}\sigma}^{\alpha\dagger}\hat{C}_{\boldsymbol{k}\sigma}^{\alpha}$, $\alpha=A,B$ is added to the Hamiltonian to control the hole-doping concentration. Then, the self-consistent mean field values of the operator $\hat{O}$ can be found as follows:

$$O(T)=\frac{1}{N}\sum_{\boldsymbol{k}}\langle\hat{O}\rangle=\frac{1}{N}\sum_{n\boldsymbol{k}}\langle\psi_{n\boldsymbol{k}}\mid\hat{O}\mid\psi_{n\boldsymbol{k}}\rangle f(E_{n\boldsymbol{k}}), \tag{10}$$

where $\psi_{n\boldsymbol{k}}$ and $E_{n\boldsymbol{k}}$ are the eigenfunctions and eigenvalues of the mean-field Hamiltonian $\hat{\mathcal{H}}_{\mathrm{MF}}$, $f(E_{n\boldsymbol{k}})=\frac{1}{1+\exp(E_{n\boldsymbol{k}}/(k_{\mathrm{B}}T))}$ is the Fermi–Dirac distribution function, k_{B} is the Boltzmann constant, and T is temperature. In our numerical calculations, the singlet and triplet physical superconducting order and bond order parameters in the mean-field level can be found as

$$\begin{aligned}
\Delta^{\mathrm{s}} &= \frac{1}{2N}g_t\sum_i\langle\Psi_0\mid\hat{C}_{i\uparrow}\hat{C}_{i+1\downarrow}-\hat{C}_{i\downarrow}\hat{C}_{i+1\uparrow}\mid\Psi_0\rangle\\
&= \frac{1}{N}g_t\sum_{n\boldsymbol{k}}\cos(k_x)\langle\psi_{n\boldsymbol{k}}\mid\hat{C}_{\boldsymbol{k}\uparrow}\hat{C}_{-\boldsymbol{k}\downarrow}\mid\psi_{n\boldsymbol{k}}\rangle f(E_{n\boldsymbol{k}}),\\
\Delta^{\mathrm{t}} &= \frac{1}{2N}g_t\sum_i\exp(-\mathrm{i}Qx_i)\langle\Psi_0\mid\hat{C}_{i\uparrow}\hat{C}_{i+1\downarrow}+\hat{C}_{i\downarrow}\hat{C}_{i+1\uparrow}\mid\Psi_0\rangle\\
&= \frac{1}{N}g_t\sum_{n\boldsymbol{k}}\sin(k_x)\langle\psi_{n\boldsymbol{k}}\mid\hat{C}_{\boldsymbol{k}+Q\uparrow}\hat{C}_{-\boldsymbol{k}\downarrow}\mid\psi_{n\boldsymbol{k}}\rangle f(E_{n\boldsymbol{k}}),\\
\chi &= \frac{1}{N}g_t\sum_{i,\sigma}\langle\Psi_0\mid\hat{C}_{i\sigma}^{\dagger}\hat{C}_{i+1\sigma}\mid\Psi_0\rangle\\
&= \frac{1}{N}g_t\sum_{n\boldsymbol{k}\sigma}\cos(k_x)\langle\psi_{n\boldsymbol{k}}\mid\hat{C}_{\boldsymbol{k}\sigma}^{\dagger}\hat{C}_{\boldsymbol{k}\sigma}\mid\psi_{n\boldsymbol{k}}\rangle f(E_{n\boldsymbol{k}}).
\end{aligned} \tag{11}$$

Then, we resort to the linear response theory and compute the current–current correlation function to determine the superfluid weight as follows:

$$\frac{D_{\mathrm{s}}(T)}{\pi e^2}=\langle-K_x\rangle-\Lambda_{xx}(k_x\to0,\mathrm{i}\omega_m=0)\equiv\frac{n_{\mathrm{s}}(T)}{m}, \tag{12}$$

where $\langle-K_x\rangle$ is the kinetic energy density and Λ_{xx} is the current–current correlation function.[40–42] The superfluid weight in the flat-bands systems has been widely considered, especially the geometric origins of superfluidity in topologically nontrivial flat bands.[16] According to the Bardeen–Cooper–Schrieffer theory, the superfluid weight should go to zero due to the fermi velocity being zero in the single flat band. However, the geometric contributions of energy bands will lead to nonzero superfluid weight with a well-defined lower bound, even for an isolated and strictly flat band.[17–19]

3. Results and discussion

We first show physical orders with the increase of the hole-doping concentration δ in Fig. 1 under the zero temperature limit with $J=0.5t$. The notable point is that the flat band can extend the region of AFM and SC in the holes-doping case. The spin-singlet superconducting order parameter exists unless the system is nearly empty. The spin-triplet superconducting order and the AFM order also exist until $\delta\approx0.19$ is much higher than the result from another Gutzwiller approximation scheme on a normal system, in which AFM and SC are coexisting up to the doping $\delta\approx0.1$.[43] It also clearly shows a staggered spin-triplet component of superconducting order parameter accompanied with AF order, which has been discussed and analyzed in the previous works.[4,10] The staggered spin-triplet superconducting order parameter has a spatial oscillation in the real space, and can then be seen as Cooper pairs with center-of-mass momentum Q in the momentum space. This unconventional superconducting state now is named as pair density wave state, in an analogy to the Fulde–Ferrell–Larkin–Ovchinnikov state.[46] The staggered spin-triplet pairing is induced in the coexistence state of AFM and singlet SC and its free energy based on Ginzburg–Landau analysis has been shown in previous works, and some similar discussions have been done from the point of view of the symmetry of systems.[8,10,47] The staggered spin-triplet superconducting order parameter shows a small dome shape. It increases with hole-doping near half-filled with small δ and reaches a maximum value. It then decreases until it disappears. In other words, the spin-singlet SC is a dominating phase in the ground state of the flat band system, while spin-triplet SC and AFM are accompanied in the lightly-doped region.

We show that the physical order parameters vary with the AF coupling interaction J with fixed hole-doping $\delta=0.125$ under the zero-temperature limit in Fig. 2. This result indicates that the emergence of staggered spin-triple superconducting order and AF order need strong effective AF interaction, which is also the effective attractive potential between Cooper electrons pairs. There is a quantum critical point near $J\approx0.28t$ — if the AF coupling interaction J is larger than the critical value, then both the staggered spin-triple superconducting order and AF order emerge. At the same time, the spin-singlet superconducting order always exists from $J=0.15t$ to $0.65t$. No matter

how weak the effective attractive potential is, the spin-singlet Copper pairs can be formed. In the flat band system, the electrons' kinetic energy is quenched and the electrons go to spin-singlet pairs with tiny any effective attraction, but staggered spin-triple pairs and AFM need a certain effective interaction in this hole doping level.

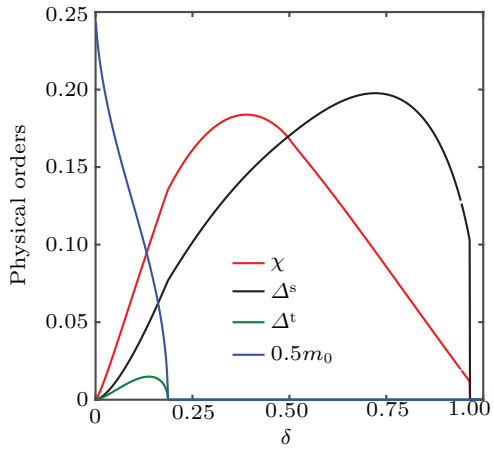


Fig. 1. The physical orders vary with the hole-doping concentration δ. The spin-singlet superconducting orders Δ^s increase with the hole-doping concentration until $\delta \simeq 0.75$ and then descend. The spin-triple superconducting orders coexist with magnetic order parameters in the lightly doped region, and are shown as a dome. The parameters are chosen as $J = 0.5t$, $T = 10^{-5}t$.

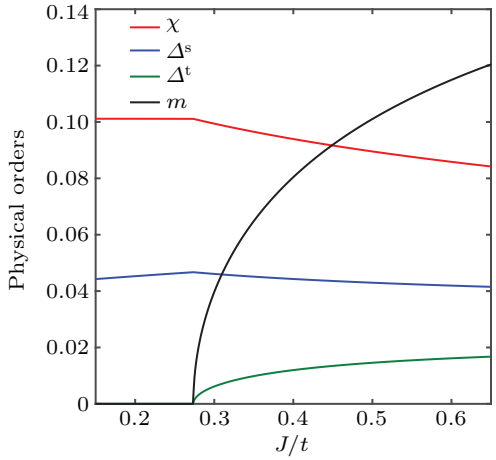


Fig. 2. The physical orders vary with the AF coupling interaction J. The spin-triple superconducting order and AF order increase with J and emerge reach the critical value, the spin-singlet superconducting order always exists in spite of weak effective attractive potential between the electrons. The parameters are chosen as $\delta = 0.125$, $J = 0.15t$–$0.65t$, $T = 10^{-5}t$.

The thermodynamics of physical orders are shown in Fig. 3. It is interesting and confirmed that the staggered spin-triplet superconducting order is induced by both spin-singlet and AF order, $\Delta^t \propto \Delta^s m_0$, Δ^t vanishes regardless of Δ^s or m_0 disappears. An unconventional superconducting state with both spin-singlet and spin-triplet Cooper pairs coexists with AFM. The AF order has a much higher critical temperature than the superconducting order, even though it has a small initial value near the critical hole-doping $\delta = 0.187$. Without AF and staggered spin-triplet superconducting order, the spin-singlet superconducting order vanishes at a relatively low temperature. The superfluid weight is unchanged near zero temperature because when the temperature increases, the energy gap declines to a critical point. Near the critical temperature, the superfluid weight shows linear behavior. The disparity between the critical temperature of superfluid weight and superconducting order shows that the electrons go to preformed pairs but do not cause supercurrent.

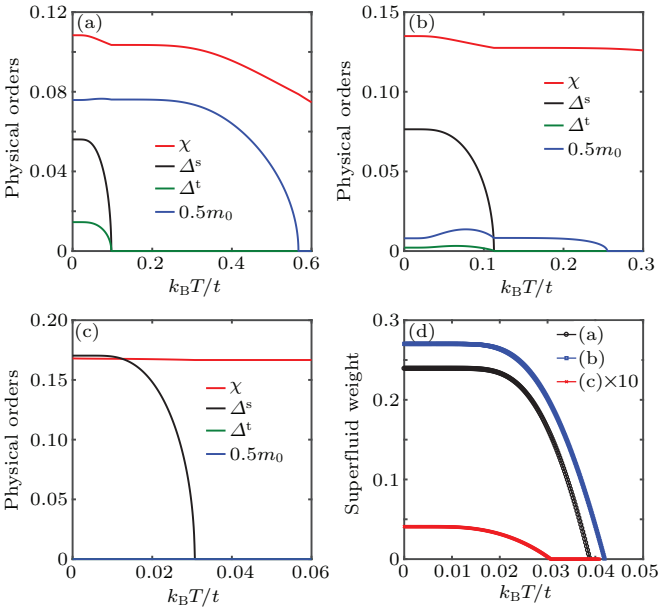


Fig. 3. The superconducting orders, AF order, and the bond order evolution as a function of temperature with fixed hole-doping level with $J = 0.5t$, (a) $\delta = 0.125$, (b) $\delta = 0.186$, (c) $\delta = 0.5$. The corresponding superfluid weight is shown in subfigure (d). It is clear that the spin-triple superconducting order exists on the premise of both spin-singlet superconducting order and the AF order. The superfluid weight stays changeless near zero temperature and linearly declines near the critical temperature.

We show the critical temperature of AF order T_N, superconducting order T_Δ, and superfluid weight T_s with the increase of hole doping with $J = 0.5t$ in Fig. 4. The critical temperature of AF order T_N is much higher than superconducting order T_Δ and superfluid weight T_s, the Neel temperature is much higher than the superconducting critical temperature, which has been established in both theoretical and experimental works. A thermodynamics state with both AFM and unconventional SC is present at the low-temperature region, which is an unconventional superconducting state with both spin-singlet and spin-triplet pairs. The optimal hole-doping level near the $\delta = 0.187$ is the transition point at which the AF order and the spin-triplet superconducting order vanish. The difference between the critical temperature of superconducting order T_Δ and the superfluid weight T_s is different from the gap between pseudogap phase and superconducting phase, the critical temperature of superconducting order T_Δ increases with hole doping level in the lightly doped region. The critical temperature of the superconducting state with only spin-singlet pairs decreases with the hole doping level, as well as the critical temperature of superfluid weight. It is notable that

after $\delta = 0.6$, the spin-singlet superconducting order still exists but cannot support superfluid weight and T_s goes to zero in this doped region.

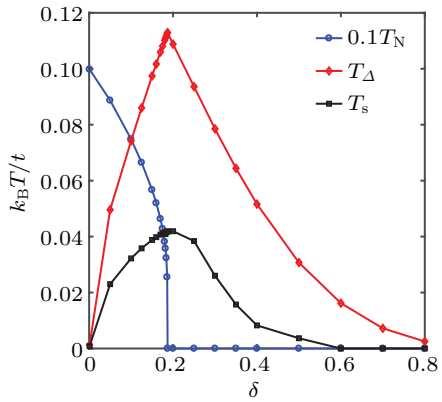


Fig. 4. The critical temperature of AF order T_N, superconducting order T_Δ and superfluid weight T_s versus hole-doping δ with parameter $J = 0.5t$.

The original research motivation of flat-band SC is to get a higher temperature superconductor. We compare the superconducting orders and their superfluid weight as a function of temperature in the flat band system to the normal dispersed system in Fig. 5. We add $t' = 0.2t$ between inter-chain hoping energy to get a normal dispersed system in the hole-doped case. It is clearly shown that the flat band enhances the critical temperature of the superconducting state, and simultaneously the superfluid weight in the flat band system is bigger than the corresponding ones in the normal dispersed system with higher critical temperature.

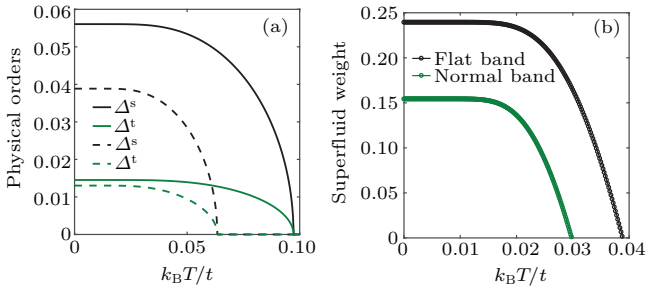


Fig. 5. The superconducting orders as a function of temperature with $\delta = 0.125$, $J = 0.5t$ in the flat band system (full line) compared to the corresponding ones in the normal dispersed system (dotted line). The right-hand subgraph is the superfluid weight with the change of temperature.

4. Conclusions

We study the coexistence of AFM and unconventional SC on the Creutz lattice under the strong electron–electron correlation limit using the renormalized mean-field theory. An extraordinary quantum thermodynamic state with both AFM and unconventional SC is present, which also includes both the spin-singlet and spin-triplet pairs. The staggered spin-triplet pairs are induced by both the spin-singlet pairs and AFM, but AFM and the spin-triplet pairs need strong effective AF interaction to support them. If the AF coupling interaction J goes beyond a quantum critical point, then AFM and the spin-triplet pairs both emerge with the spin-singlet SC. The flat band extends the hole-doped region of AFM and SC, the spin-singlet SC is found in most regions of the hole-doping case. The critical temperature of the AF order, superconducting order, and superfluid weight versus hole doping level show that although electrons go to preformed pairs, a real superconducting state leads to the Meissner effect and dissipationless transport emerges at the lower temperature. It is natural to consider that are there other non-uniform charge-ordered states in this flat band system, such as a charge density wave or pair density wave with another momentum Q. We will study the coexistence of charge density wave, spin density wave and pair density wave, and novel SC in the doped flat-band system in future work.

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