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Preparation of squeezed light with low average photon number based on dynamic Casimir effect

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It is well known that squeezed states can be produced by nonlinear optical processes, such as parametric amplification and four wave mixing, in which two photons are created or annihilated simultaneously. Since the Hamiltonian of the dynamic Casimir effect contains a^2 and a^{+2}, photons in such a process are also generated or annihilated in pairs. Here we propose to get squeezed light through the dynamic Casimir effect. Specifically, we demonstrate it from the full quantum perspective and the semiclassical perspective successively. Different from previous work, we focus on generating squeezed states with the lowest average photon number, because such squeezed states have better quantum properties. For the full quantum picture, that is, phonons also have quantum properties, when the system is initially in the excited state of phonons, squeezed light cannot be generated during the evolution, but the light field can collapse to the squeezed state by measuring the state of phonons. When the phonon is treated as a classical quantity, that is, the cavity wall is continuously driven, squeezed light with the minimum average photon number will be generated in the case of off-resonance. This will play a positive role in better regulating the photon state generated by the dynamic Casimir system in the future.

Keywords: cavity quantum electrodynamics, quantum optics, squeezed light

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1. Introduction

Squeezed state[1] is a nonclassical state, which can reduce fluctuation of a measurement component below the shot noise limit. Squeezed state is an important quantum resource for quantum information processing and quantum measurement.[2,3] For example, squeezed light can be used to prepare continuous variable multi-component entangled state,[4,5] optical communication,[6] quantum teleportation,[7] gravitational wave detection,[8,9] and to realize high-precision quantum measurement,[10,11] etc. In 1985, Bell Laboratories obtained the squeezed light experimentally for the first time by means of four wave mixing.[12] Then, in 1986, the second-order nonlinear parametric down-conversion process was used to prepare and obtain the squeezed light.[13] So far there have been more and more methods to generate squeezed light, nevertheless the parametric down-conversion process has been proved to be the most effective way to prepare the squeezed light.[14–19]

Because the squeezed lights were generated from nonlinear optics in the past, they all have macro intensity. The development of quantum communication and quantum computing requires the light field to reach the single photon level. However, the single photon or Fock state is not a squeezed state, so we hope to produce the squeezed light with the minimum average photon number. It is well known that the squeezed state is the eigenstate of operator a^2, where a is the annihilated operator of photon. Therefore, in theory, as long as the Hamiltonian is related to a^2, such a system is able to generate squeezed states. Here, we find that the Hamiltonian of dynamic Casimir effect satisfies such a requirement.

The Casimir effect refers to the attractive force between two neutral perfectly conducting plates, which was first predicted by Casimir in 1948.[20] This surprising phenomenon has been attributed to the vacuum fluctuation of electromagnetic field, which is considered as a macroscopic manifestation of quantum mechanics.[21–23] The vacuum is not really empty, on the contrary, it contains many virtual photons with short life. Many researchers theoretically demonstrated that real particles can be produced from vacuum if injecting energy into fluctuations.[24–28] This is one of the most basic results of quantum field theory. For example, in 1970, Moore first predicted that a cavity with variable cavity length can convert virtual photons into real photons when its cavity wall moves back and forth under ideal conditions. This phenomenon was later referred to as the dynamic Casimir effect (DCE).[27] In 1976, Fulling and Davies[25] demonstrated that photons can be generated by moving a single cavity wall to make it undergo nonuniform acceleration.

Since the first prediction of DCE, many different experiments have been proposed, such as in 2005 Braggio *et al.*[29] proposed that photons can be generated and observed by adjusting the reflectivity of the semiconductor interface of the

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superconducting electromagnetic resonator. In 2009, Johansson *et al.*[30] suggested that DCE can be observed by changing the magnetic flux through the superconducting quantum interference device (SQUID) by parametrically modulating the boundary condition of the coplanar waveguide (CPW). Wilson *et al.*[31] reported the observation of photon generation in a microwave cavity whose electrical length is varied by using the tunable inductance of a superconducting quantum interference device in 2010. Further, in Refs. [32,33], the equivalent action of a quickly moving mirror is mimicked by an inductance variation of a SQUID controlled by a quickly oscillating magnetic flux. Unlike actual mirrors, the inductance of a SQUID can be driven at high frequencies (>10 GHz), which enables an experimentally detectable photon production.

However, it is meaningful to generate and observe dynamic Casimir effect through resonators with oscillating cavity walls, whereas it is still difficult in experiments because the method of generating a large number of photons by moving the cavity mirror requires very fast movement. In 1996, Lambrecht *et al.*[34] made a quantitative estimation of the photon flux radiated by the optomechanical system consisting of a cavity with an oscillating mirror for the first time. Using the resonance enhancement effect, they showed that in a high Q cavity, a large number of microwave photons can be generated enough to detect. However, the condition for generating photons is that the mechanical oscillation frequency $\omega_{\rm m}$ is required to be at least twice the frequency of the cavity mode $\omega_{\rm c}$. Such resonance condition is the main obstacle to the DCE experiment.

To overcome these difficulties, Ricardo *et al.*[35] proposed a classical analog of the dynamical Casimir effect in engineered photonic lattices in 2017, where the propagation of classical light emulates the photon generation from the quantum vacuum of a single-mode tunable cavity. In 2018, Macrì *et al.*[36] took both the cavity field and the moving cavity wall as quantum mechanical systems, that is, photons and phonons, and extended the study of DCE to the ultra-strong coupling limit of cavity optomechanics. They found that this can successfully observe photons generated by vacuum cavity mode resonance.

In view of the generation mechanism of squeezed states and the Hamiltonian of dynamic Casimir effect, it is reasonable to predict that the photons generated by dynamic Casimir effect will appear in pairs, and the generated photon state will possess the squeezing characteristics to a certain degree. In this article we plan to mainly verify the above conjecture, and propose to generate the squeezed state with the lowest average photon number through the dynamic Casimir effect.

The article is organized as follows. We introduce the physical model and define the squeezed state with the minimum average photon number in Section 2. In Section 3 we analyze the squeezing properties of photon states triggered by initial excited phonon states in the full-quantum dynamic Casimir effect, and find that the squeezed state can only be obtained when the phonon is measured. In order to get a stable squeezed state, in Section 4 we study the properties of light field produced by dynamic Casimir effect from a semi-classical perspective, in which the cavity wall is continuously driven with certain frequency, and find that the squeezed light with the lowest average photon number can be obtained in the case of detuning driving. Conclusions and future perspectives are finally given in Section 5.

2. Model and the squeezed state constructed

It is well known that coherent state is the minimum uncertainty state of orthogonal components. The orthogonal component or orthogonal operator of the optical field can be expressed as $X_1 = \frac{1}{2}(a+a^+)$ $X_2 = \frac{1}{2\rm i}(a-a^+)$, where $\hat{a}^+$ and $\hat{a}$ are the creation operator and annihilation operator of the optical field, respectively. For a long time, the standard deviation of the orthogonal operator $\Delta X_{\rm i}$ $(i=1,2)$ would satisfy the uncertainty principle, and there is the minimum value $\Delta X_i = 1/2$ and $\Delta X_1 \Delta X_2 = 1/4$ for coherent state, which is called the quantum noise limit. Then some researchers proposed that, without violating the uncertainty principle $\Delta X_1 \Delta X_2 \geqslant 1/4$, the fluctuation of one orthogonal component of the optical field can be smaller than the quantum fluctuation limit, that is, $\Delta X_i < 1/2$ ($i=1$ or 2), and such a quantum state is called the squeezed state.

The squeezing intensity S can be defined by the variance of the orthogonal operator of the cavity field, it can be written as $S_i = (\Delta X_i)^2 - 1/4 = \langle X_i^2\rangle - \langle X_i\rangle^2 - 1/4$ $(i=1,2)$. If certain squeezing intensity satisfies $S_i < 0$ ($i=1$ or 2), there will appear squeezing in the cavity field, otherwise there is no squeezing. The squeezing intensities S_1 and S_2 can be expressed in detail by the creation (annihilation) operator, i.e.,

$$\begin{aligned} S_1 &= \frac{1}{4}(2\langle a^+a\rangle + \langle a^2\rangle + \langle a^{+2}\rangle) - \frac{1}{4}\langle a+a^+\rangle^2, \\ S_2 &= \frac{1}{4}(2\langle a^+a\rangle - \langle a^2\rangle - \langle a^{+2}\rangle) + \frac{1}{4}\langle a-a^+\rangle^2. \end{aligned} \tag{1}$$

The squeezed light produced in previous experiments has macroscopic intensity, its properties hardly change when they interact with matter. Now the manipulation of matter has entered the single quantum level, and the demand of light with low photon numbers in quantum communication and quantum computing is growing. Therefore, we hope to know what constitutes the squeezed state with the minimum average photon number.

To obtain squeezed light with low average photon number, we should know what constitutes the squeezed state with the minimum average photon number. Therefore, we list the

possible superposition states of various Fock states with low photon numbers, as given in Table 1. The first column in Table 1 provides various possible superposition states, the second column indicates whether each superposition state is squeezing state, and the maximum squeezing intensity $S_{i\max}$ $(i=1,2)$ of all squeezing state is calculated as listed in the third column. At the same time, the fourth and fifth columns list the probability amplitudes corresponding to the maximum squeezing intensities $S_{1\max}$ and $S_{2\max}$, respectively.

It can be clearly seen from Table 1 that although several different coherent superposition of Fock states composed of $|0\rangle$, $|1\rangle$, $|2\rangle$, $|3\rangle$ and $|4\rangle$ can produce squeezed states, the $|0\rangle$, $|2\rangle$ and $|4\rangle$ states will constitute the maximum squeezing with the minimum average photon number.

Table 1. Squeezing of various superposition states.

Coherent superposition states	Squeezing state	Maximum squeezing $S_{i\max}$ $(i=1,2)$	Probability amplitude of $S_{1\max}$	Probability amplitude of $S_{2\max}$
$\Psi_{01}=c_0\|0\rangle+c_1\|1\rangle$	yes	$S_1=-0.062$ $S_2>0$	$c_0=0.866$ $c_1=0.500$	
$\Psi_{02}=c_0\|0\rangle+c_2\|2\rangle$	yes	$S_1=-0.112$ $S_2=-0.112$	$c_0=-0.953$ $c_2=0.303$	$c_0=0.953$ $c_2=0.303$
$\Psi_{03}=c_0\|0\rangle+c_3\|3\rangle$	no			
$\Psi_{04}=c_0\|0\rangle+c_4\|4\rangle$	no			
$\Psi_{012}=c_0\|0\rangle+c_1\|1\rangle+c_2\|2\rangle$	yes	$S_1=-0.112$ $S_2=-0.112$	$c_0=-0.953$ $c_1=0$ $c_2=0.303$	$c_0=0.953$ $c_1=0$ $c_2=0.303$
$\Psi_{013}=c_0\|0\rangle+c_1\|1\rangle+c_3\|3\rangle$	yes	$S_1=-0.063$ $S_2>0$	$c_0=0.865$ $c_1=0.501$ $c_3=0$	
$\Psi_{014}=c_0\|0\rangle+c_1\|1\rangle+c_4\|4\rangle$	yes	$S_1=-0.063$ $S_2>0$	$c_0=0.867$ $c_1=0.498$ $c_4=0.001$	
$\Psi_{023}=c_0\|0\rangle+c_2\|2\rangle+c_3\|3\rangle$	yes	$S_1=-0.112$ $S_2=-0.112$	$c_0=-0.953$ $c_2=0.302$ $c_3=0.004$	$c_0=0.953$ $c_2=0.302$ $c_3=0.004$
$\Psi_{024}=c_0\|0\rangle+c_2\|2\rangle+c_4\|4\rangle$	yes	$S_1=-0.155$ $S_2=-0.155$	$c_0=0.904$ $c_2=-0.396$ $c_4=0.159$	$c_0=0.904$ $c_2=0.396$ $c_4=0.159$
$\Psi_{034}=c_0\|0\rangle+c_3\|3\rangle+c_4\|4\rangle$	no			
$\Psi_{0123}=c_0\|0\rangle+c_1\|1\rangle+c_2\|2\rangle+c_3\|3\rangle$	yes	$S_1=-0.112$ $S_2=-0.112$	$c_0=0.950$ $c_1=-0.002$ $c_2=-0.314$ $c_3=0$	$c_0=0.950$ $c_1=0.002$ $c_2=0.314$ $c_3=0$
$\Psi_{01234}=c_0\|0\rangle+c_1\|1\rangle+c_2\|2\rangle+c_3\|3\rangle+c_4\|4\rangle$	yes	$S_1=-0.155$ $S_2=-0.155$	$c_0=0.903$ $c_1=0$ $c_2=-0.396$ $c_3=-0.001$ $c_4=0.160$	$c_0=0.903$ $c_1=0$ $c_2=0.396$ $c_3=0.001$ $c_4=0.160$

In order to observe the squeezing of several superposition states more clearly, we use the following three coherent superposition states as examples to illustrate, here assuming c_0, c_2, and c_4 to be positive real number, which are

$$\psi_{02}=c_0|0\rangle+c_2|2\rangle, \tag{2a}$$

$$\psi_{24}=c_2|2\rangle+c_4|4\rangle, \tag{2b}$$

$$\psi_{024}=c_0|0\rangle+c_2|2\rangle+c_4|4\rangle. \tag{2c}$$

The squeezing intensities S_1 and S_2 of the above three initial states as functions of c_0 and c_2 are plotted in Fig. 1. The normalization condition of probability amplitude is automatically satisfied.

For the state $\psi_{02}=c_0|0\rangle+c_2|2\rangle$, it is obvious from Fig. 1(a) that there appears squeezing in S_2 only when $c_0>0.83$. Notice $c_2=\sqrt{1-c_0^2}$. The squeezing of the photon state reaches the maximum $S_{2\max}=-0.112$ at $c_0=0.953$.

For the state $\psi_{24}=c_2|2\rangle+c_4|4\rangle$, it is clear from Fig. 1(b) that there is no squeezing no matter what value c_2 takes.

For the state $\psi_{024}=c_0|0\rangle+c_2|2\rangle+c_4|4\rangle$, see Figs. 1(c) and 1(d), it is clear that there are some parameter regions with squeezing marked by blue and surrounded with thick black curves. Compared with ψ_{02}, the maximum squeezing can be promoted to $S_{1\max}=S_{2\max}=-0.155$ with the addition of $|4\rangle$.

It can be concluded from the above discusstion that $|0\rangle$ is very important to get squeezed for the superposition of Fock state. If we can construct coherent superposition states of $|0\rangle$, $|2\rangle$, $|4\rangle$ or more photons in the evolution of the system, we can reach squeezing.

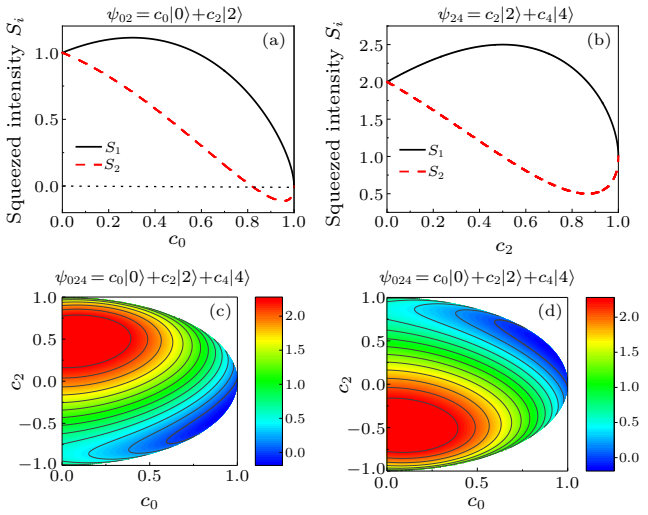


Fig. 1. (a) The squeezing intensity S_i $(i=1,2)$ as a function of c_0 for the photon state ψ_{02}. (b) The squeezing intensity S_i $(i=1,2)$ as a function of c_2 for the photon state ψ_{24}. [(c), (d)] The squeezing intensities (c) S_1 and (d) S_2 as functions of c_0 and c_2 for the photon state ψ_{024}. Here c_0, c_2, and c_4 are assumed to be positive real numbers.

As the Hamiltonian of the dynamic Casimir effect contains the square term a^2, it is an ideal physical system that can produce photon pairs. In this work, we consider a single-mode cavity, and one of the cavity mirrors is fixed while the other is oscillating with tiny amplitude, as shown in Fig. 2. It is the simplest model that describes the DCE in the absence of dissipation.

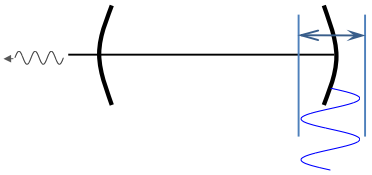


Fig. 2. Scheme of dynamic Casimir effect. A single mode cavity with frequency ω_c, while one wall oscillates with frequency ω_m.

It is worth noting that the elementary analysis in Ref. [35] shows if the mechanical and photonic loss rates are much lower than the optmechanical coupling rate, mechanical energy can be converted, at least in principle, into light with near 100% efficiency. It follows that when the system loss is very small, including the mechanical and photonic loss rates, and the Q factor of cavity is very high, the system loss has little influence on the photon number generated by the dynamic Casimir effect. Therefore, the resonator with high Q factor and low system loss is selected in this study, and the influence of the loss on the average photon number generated by the dynamic Casimir effect can be ignored.

Next, we study the squeezing characteristic of the cavity field due to the cavity wall vibration in the dynamic Casimir effect.

3. The full quantum theory

In order to generate the squeezed cavity field with the minimum average photon number, we firstly resort to the full quantum dynamic Casimir effect, in which the oscillating cavity wall can be modeled as phonon. As shown in Fig. 2, a single-mode cavity has frequency ω_c, while its oscillating wall can be seen as a phonon with frequency ω_m. The Hamiltonian of the system can be written as $(\hbar=1)$[36]

$$H=\hbar\omega_c\hat{a}^+\hat{a}+\hbar\omega_m\hat{b}^+\hat{b}+\hbar g\hat{a}^+a(\hat{b}+\hat{b}^+)+\frac{1}{2}\hbar g(\hat{a}^2+\hat{a}^{+2})(\hat{b}+\hat{b}^+), \tag{3}$$

where a and a^+ are the annihilation and creation operators of cavity mode, while b and b^+ are the annihilation and creation operator of phonon (oscillating wall). Photon and phonon are both the Bosons. The third term refers to the optmechanical interaction with the coupling coefficient g, in which the oscillating cavity wall changes the frequency of the cavity mode. The last term refers to the dynamic Casimir effect, in which energy exchange occurs between photon and phonon. Checking the Hamiltonian of DCE, it is clear that energy is not conserved in DCE, because some energy comes from vacuum. As the photon are created or annihilated in pair, the dynamic Casimir effect can be used to generate squeezed light in principle.

Here, we set $\omega_m=2\omega_c$, which is the resonant condition of DCE, and $g=0.04\omega_m$. Initially, there is one phonon for the cavity wall, and it is vacuum in the cavity, that is, $\Psi(0)=|0_c,1_m\rangle$. Then the state of the system $\Psi(t)$ evolves according to the Schrödinger equation, i.e.,

$$\Psi(t)=\sum_{n,m}c_{nm}|n_c,m_m\rangle. \tag{4}$$

Noticeably, $|n_c\rangle$ refers to n photon in cavity, while $|m_m\rangle$ refers to m phonons in mechanical oscillator (cavity wall). The average photon number $\langle a^+a\rangle$, the average phonon number $\langle b^+b\rangle$, and the squeezing intensity S_1 and S_2 can be calculated accordingly.

Figure 3(a) shows $\langle a^+a\rangle$ and $\langle b^+b\rangle$, and Fig. 3(b) shows S_1 and S_2. It is obvious from Fig. 3(a) that photons (red curve) and phonons (blue curve) are exchanging with time, in which one phonon can be converted into two photons, and vice versa. When the time starts from 0 to $55/\omega_m$, the average photon number $\langle a^+a\rangle$ increases from 0 to 2. In this process, the cavity field must be in the coherent superposition state of $|0_c\rangle$, $|2_c\rangle$ and $|4_c\rangle$, and there should be squeezing at certain time in principle according to the result of Fig. 1. However, we find S_i $(i=1,2)$ are both positive at any time as shown in Fig. 3(b), in other words, there is no squeezed light anymore.

The reason for this difference is that $\psi_{024}=c_0|0\rangle+c_2|2\rangle+c_4|4\rangle$ of Eq. 2(c) is just the supposition of photonic Fock state, but in DCE $\Psi(t)=\sum_{n,m}c_{nm}(t)|n_c,m_m\rangle$ it is the superposition of direct product states of photons and phonons $|n_c,m_m\rangle$. In the time involved in Figs. 3(a) and 3(b), only the contributions of $|0_c,1_m\rangle$, $|2_c,0_m\rangle$, $|2_c,1_m\rangle$, $|4_c,1_m\rangle$ are included. When calculating the expected values, such as $\langle a\rangle$,

$\langle a^+\rangle$, $\langle a^2\rangle$ and $\langle a^{+2}\rangle$, the two states ψ_{024} and $\Psi(t)$ will get completely different results. Because the photon state and phonon state are entangled in the dynamic Casimir effect, the cavity field does not have squeezing in the free evolution.

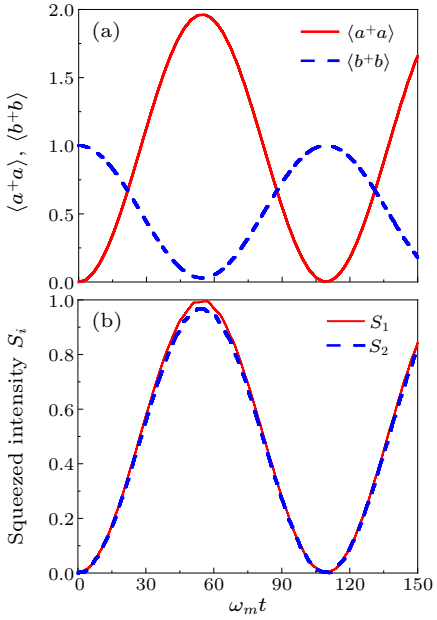


Fig. 3. (a) Evolution of $\langle a^+a\rangle$ and $\langle b^+b\rangle$ in DCE. (b) The corresponding squeezed intensities S_1 and S_2 in DCE. The initial condition is $\Psi(0)=|0_c,1_m\rangle$. The parameters of the system are $\omega_m=2\omega_c$ and $g=0.04\omega_m$.

How can we make the cavity field to be squeezed? The answer is to measure the state of the cavity wall, so that the cavity field will collapse into the superposition state of the Fock state. Suppose that there is a measurement that can measure whether the cavity wall is one, two or three phonons. Each result corresponds to a special cavity field state. For example, the original state is $\Psi(t)=\sum_{n,m}c_{nm}(t)|n_c,m_m\rangle$. At time t, we perform the measurement on phonon. If we get 1 phonon, the cavity field will collapsed into the state

$$\Psi_{|1_m\rangle}(t)=\sum_n d_{1n}|n_c\rangle. \tag{5}$$

Notably, t means that we perform measurement at t. The possibility amplitude d_{1n} of the reduced cavity field reads

$$d_{1n}=\frac{c_{n1}(t)}{\sqrt{\sum_{n'}|c_{n'1}|^2}}. \tag{6}$$

Similarly, if we get m phonon at time t, the cavity field will collapsed into the state

$$\Psi_{|m_m\rangle}(t)=\sum_n d_{mn}|n_c\rangle \tag{7}$$

with

$$d_{mn}=\frac{c_{nm}(t)}{\sqrt{\sum_{n'}|c_{n'm}|^2}}. \tag{8}$$

Therefore, whether there is compression in the cavity field and how strong the compression is depend on when and how many phonons are measured.

With these reduced cavity field state $\Psi_{|m_m\rangle}(t)$, we can calculate the conditional squeezing intensities $S_1(m,t)$ and $S_2(m,t)$ of the cavity field when m phonons are measured at the moment t.

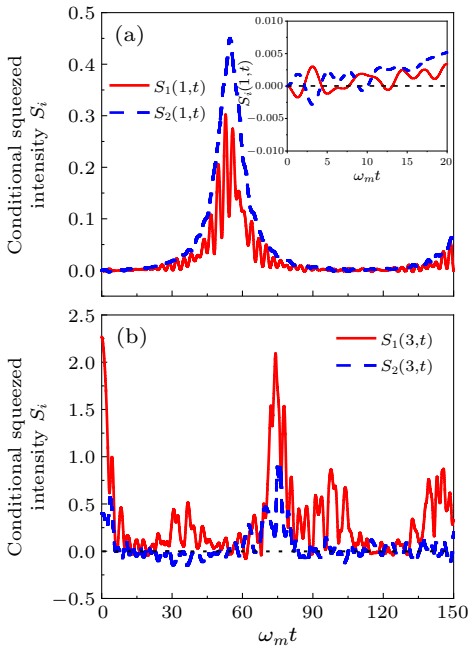


Fig. 4. (a) The conditional squeezing intensity $S_i(1,t)$ when detect 1 phonon at time t. (b) The conditional squeezing intensity $S_i(3,t)$ when detect 3 phonon at time t. The parameters are the same as those in Fig. 3.

The conditional squeezing intensities $S_1(1,t)$ and $S_2(1,t)$ of the cavity field when 1 phonon is detected at the moment t are shown in Fig. 4(a). The inset is the enlarged view of Fig. 4(a). It is shown that when $t<13/\omega_m$, the cavity field possesses squeezing when measuring 1 phonon. In this time interval, whether $S_1(1,t)$ or $S_2(1,t)$ is squeezed depends on when it is measured. The maximum squeezing intensity is about -0.003.

The conditional squeezing intensities $S_1(3,t)$ and $S_2(3,t)$ of the cavity field when 3 phonons are detected at the moment t are shown in Fig. 4(b). The results show that the cavity field is squeezed when 3 phonons are detected in most of the time. Mostly there appears $S_2(3,t)$ squeezed. The maximum squeezing intensity can reach -0.15.

Therefore, in the full quantum representation of DCE, when there is an initial phonon, there is no cavity field squeezing in the free evolution. However, at certain time, we measure the phonons, and the reduced cavity field will be squeezed, depending on the detection results. In this way, we get the squeezed cavity field with very low average photon number.

On the other hand, we can know how many phonons are in the cavity wall according to the squeezed intensity of cavity field, and we can also know whether the cavity field is squeezed according to the number of phonons detected.

4. The semi-classical perspective

In the above section, we have discussed the case in which the system is initially driven by a phonon. Whether the cavity field is squeezed depends on the detection of the number of phonons. Now we discuss the case in which the cavity wall is continuously driven at a fixed frequency $\omega_{\rm m}$. The energy exchange between photons and phonons does not affect the vibration state of the cavity wall, which is simpler in experiment. In this case, the cavity wall vibrator (phonon) can be taken as a constant, and such treatment is called semiclassical approximation. The Hamiltonian of DCE under semiclassical approximation is expressed as (we set $\hbar = 1$)[35]

$$H_0 = \omega(t)\hat{a}^+\hat{a} + \mathrm{i}\chi(t)(\hat{a}^{+2} - \hat{a}^2), \tag{9}$$

where $\omega(t)$ is the instantaneous frequency of the cavity, which is given as $\omega(t) = \omega_{\rm c}[1 + \varepsilon\sin(\omega_{\rm m}t)]$ with ε being the modulation amplitude. The relationship between DCE coefficient $\chi(t)$ and $\omega(t)$ is expressed as $\chi(t) = \omega'(t)/(4\omega(t))$. When $|\varepsilon| \ll 1$, the cavity frequency becomes $\omega(t) \approx \omega_{\rm c}$ and the DCE coefficient can be approximated as $\chi(t) \approx \varepsilon\omega_{\rm m}\cos(\omega_{\rm m}t)/4$ for simplicity.

In order to generate more photons from DCE, resonance conditions $\omega_{\rm m} = 2\omega_{\rm c}$ are generally required.[37] First, we analytically calculate the squeezing characteristics of the cavity field generated in the resonance case.

It is known that the photons generated from DCE grow exponentially with time when $\omega_{\rm m} = 2\omega_{\rm c}$.[38] In order to observe this phenomenon, after applying some deduction,[35] the effective time-independent Hamiltonian is obtained as follows:

$$H_0^{\rm I} \approx \left(\frac{\mathrm{i}\varepsilon\omega_{\rm c}}{4}\right)(\hat{a}^{+2} - \hat{a}^2), \tag{10}$$

whose time evolution operator is

$$U_0^{\rm I} = \exp\left(-\frac{\mathrm{i}}{\hbar}H_0^{\rm I}t\right) = \exp\left(r\frac{a^{+2} - a^2}{2}\right), \tag{11}$$

where r corresponds to dimensionless time, $r = \varepsilon\omega_{\rm c}t$. The time evolution operator generates the following transformation in the field operators

$$a^+(t) = a^+\cosh(r) + a\sinh(r),$$
$$a(t) = a^+\sinh(r) + a\cosh(r). \tag{12}$$

Now the average number of photons in DCE at any time can be calculated as

$$\langle 0|a^+(t)a(t)|0\rangle = \sinh^2(r), \tag{13}$$

and the expected values of $a(t)$, $a^+(t)$, $a^{+2}(t)$ and $a^2(t)$ can be easily acquired correspondingly. Finally, the squeezing intensity of the light field can be obtained according to Eq. (1) as

$$S_1 = \frac{1}{2}\sinh^2(r) + \frac{1}{2}\sinh(r)\cosh(r),$$
$$S_2 = \frac{1}{2}\sinh^2(r) - \frac{1}{2}\sinh(r)\cosh(r). \tag{14}$$

We plot in Figs. 5(a) and 5(b) the evolution of average photon number $\langle a^+a\rangle$ and the squeezed intensity $S_{\rm i}$ $(i = 1, 2)$ according to the above formula, respectively. It is obvious that the average photon number increases exponentially in Fig. 5(a). Notably, the cavity field generated by DCE is squeezed as shown in Fig. 5(b), and the maximum squeezing can reach $S_2 = -0.25$ when $\varepsilon\omega_{\rm c}t > 5$.

As a verification, we repeat the above results using numerical methods. Using the Fock state $|n_{\rm c}\rangle$ to establish Hilbert space, we express Hamiltonian with matrix and wave function with vector composed of probability amplitude of each Fock state. The initial state is set as vacuum state, and then free evolution is carried out. The squeezed intensity calculated by this numerical method is shown in Fig. 5(c). Comparing Figs. 5(c) and 5(b), we can find that the numerical calculation results and the analytical results are completely consistent when $\varepsilon\omega_{\rm c}t < 4.2$. The reason is the truncation of Hilbert space in numerical calculation, and the maximum Fock state in the calculation is $|450_{\rm c}\rangle$. Since the number of photons increases exponentially with time, the size of Hilbert space required also increases exponentially with time. At all events, our results reflect that the DCE will produce squeezed light with an exponential increase in the number of photons at resonance.

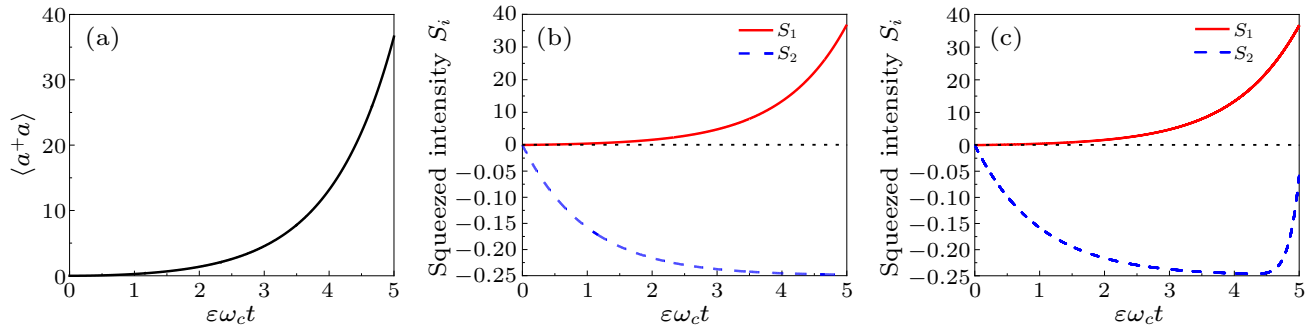


Fig. 5. (a) Evolution of the average number of photons generated by DCE according to the analytical results of Eq. (13). (b) Evolution of squeezing intensities S_1 and S_2 of cavity field according to the analytical results of Eq. (14). (c) The evolution of squeezing intensities S_1 and S_2 of cavity field obtained by numerical calculation. The common parameter is $\omega_{\rm m} = 2\omega_{\rm c}$.

However, squeezed light with macroscopic intensity is not the object of our work. We hope that there is a two-photon process in DCE, and the cavity field maintains a very low intensity at the same time. The result reported in Ref. [35] gives us enlightenment. Ancheyta *et al.* in Ref. [35] pointed out that if the cavity wall vibration frequency $\omega_{\rm m}$ and the cavity frequency $\omega_{\rm c}$ do not meet the resonance condition $\omega_{\rm m}=2\omega_{\rm c}$, the cavity field will not increase exponentially with time.

To explore the dynamics of the system in general off-resonant cases, we take $\omega_{\rm m}=2\omega_{\rm c}+\Delta$, where Δ represents the frequency detuning. After some deduction, the time-independent Hamiltonian is obtained as follows:

$$H_0^{I'}=\left(\frac{{\rm i}\varepsilon\omega_{\rm c}}{4}\right)(\hat{a}^{+2}-\hat{a}^2)-\left(\frac{\Delta}{2}\right)\hat{a}^+\hat{a}. \tag{15}$$

It can be seen from Ref. [35] that the detuning Δ is divided into two ranges, i.e., $\Delta<\varepsilon\omega_{\rm c}$ and $\Delta>\varepsilon\omega_{\rm c}$, and different ranges correspond to different evolution characteristics of the photon number. In detail, when $\Delta<\varepsilon\omega_{\rm c}$, the average photon number increases exponentially; and when $\Delta>\varepsilon\omega_{\rm c}$, the average photon number oscillates with finite amplitude.

Therefore, we calculate the detuning cases numerically. First, we consider the case of $\Delta<\varepsilon\omega_{\rm c}$ in which the average photon number will increase exponentially with time, and choose $\Delta/(\varepsilon\omega_{\rm c})=0.5$ as an example. The average photon number and squeezed intensity of cavity field are shown in Fig. 6. As shown in Fig. 6(a), the average photon number increases exponentially when $\Delta/(\varepsilon\omega_{\rm c})=0.5$, whereas it grows more slowly than the resonance case, i.e., Fig. 5(a). Accordingly, as shown in Fig. 6(b), while the corresponding S_2 still shows squeezing characteristics, only the squeezing exists in a certain time range $\varepsilon\omega_{\rm c}t\in[0,3]$, and the maximum squeezing is $S_2=-0.167$, which smaller than the case of resonance, i.e., -0.25. It can be concluded that, in the case of $\Delta<\varepsilon\omega_{\rm c}$, with the increase of detuning Δ, the rate of exponential increase will be slower, and the squeezing of cavity field will also be weakened. However, in this case, the number of photons increases exponentially to reach the macroscopic intensity.

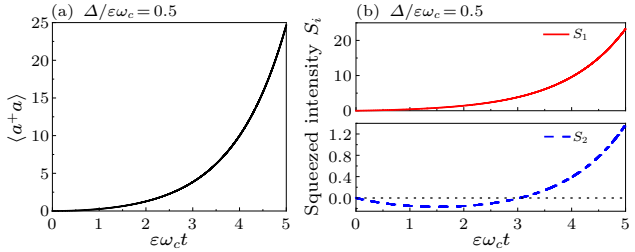


Fig. 6. (a) The evolution of the average number of photons generated by DCE. (b) The evolution of squeezing intensities S_1 and S_2 of cavity field. The parameters $\omega_{\rm m}=2\omega_{\rm c}+\Delta$ and $\Delta/(\varepsilon\omega_{\rm c})=0.5$.

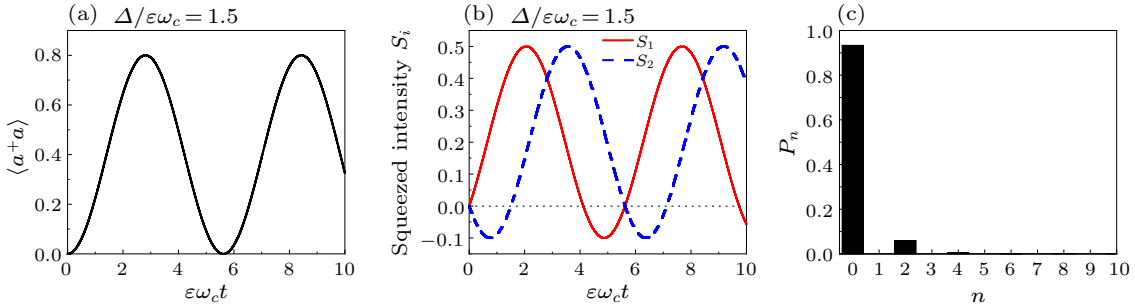


Fig. 7. (a) Evolution of the average number of photons generated by DCE. (b) Evolution of squeezing intensitis S_1 and S_2 of cavity field. (c) Probability decomposition of the cavity field state in Fock states $|n_{\rm c}\rangle$ at $\varepsilon\omega_{\rm c}t=0.79$. The parameters are $\omega_{\rm m}=2\omega_{\rm c}+\Delta$ and $\Delta/(\varepsilon\omega_{\rm c})=1.5$.

Now, we check the case of $\Delta>\varepsilon\omega_{\rm c}$ and take $\Delta/(\varepsilon\omega_{\rm c})=1.5$ as an example. The corresponding average photon number and squeezed intensity are shown in Figs. 7(a) and 7(b), respectively. It is clear from Fig. 7(a) that the average photon number of DCE oscillates periodically with time, and the maximum average photon number is just 0.8. It can be seen that the increase of detuning Δ changes the evolution of the photon number, which changes from an exponential increase in the resonant case to a periodic oscillation. It reflects that photons are exchanging with phonons on the cavity wall. However, the phonon here is a classical quantity that does not change with photons. The two-photon process in Hamiltonian (15) guarantees that the cavity field is a coherent superposition of a series of even photon states. Therefore, there must be squeezed states in the evolution process. Figure 7(b) reveals that S_1 and S_2 of the cavity field will periodically squeezing alternately. By comparing Figs. 7(a) and 7(b), we find that there is no squeezing when the average photon number is maximum. For example, at $\varepsilon\omega_{\rm c}t=2.8$, the average number is maximal with 0.8, but the corresponding S_1 and S_2 are the same and both larger than 0, i.e., 0.4. When the time is $\varepsilon\omega_{\rm c}t=0.79+5.6n$ ($n=0,1,2,3,\ldots$), S_2 reaches the maximum squeezing of -0.11, meanwhile the average photon number is just 0.1. When the time is $\varepsilon\omega_{\rm c}t=4.9+5.6n$ ($n=0,1,2,3,\ldots$), S_1 reaches the maximum squeezing of -0.11, meanwhile the average photon number is still 0.1. Therefore, as time evolves, S_1 and S_2 alternately reach the same maximum squeezing and have the same average photon number. To further verify this conclusion, we decompose the cavity field state at $\varepsilon\omega_{\rm c}t=0.79$ when S_2 reaches the maximum squeezing, and give its probability

distribution in different Fock states in Fig. 7(c). It is found that the probability in $|0_c\rangle$, $|2_c\rangle$, and $|4_c\rangle$ are 0.933, 0.059, and 0.006 respectively, which is consistent with the result in Fig. 1(d). This just proves that under the semiclassical theory, stable low average photon number squeezed states can be generated through the dynamic Casimir effect when frequency shift $\Delta/(\varepsilon\omega_c) > 1$.

Comparing Figs. 5–7, we find that the average photon number of the cavity field is the largest under the condition of resonance, and the average photon number gradually decreases with the increase of frequency detuning Δ. Accordingly, the squeezing intensity of the cavity field weakens with the increase of the frequency detuning. This seems to imply that there is a contradiction between the maximum squeezing and the minimum average photon number in the DCE process. Reducing the average photon number will also weaken the squeezing. However, we can still find a balance between the low average photon number and the squeezed intensity.

5. Conclusion

Different from previous work, we mainly discuss the scheme of generating extremely weak squeezed light from DCE process. We first discuss the squeezing properties of the superposition state composed of $|0_c\rangle$, $|2_c\rangle$ and $|4_c\rangle$, and find that the $|0_c\rangle$ is very important for squeezing.

Then we analyze the possibility of producing extremely weak DCE squeezed light from both the full quantum scheme and the semiclassical scheme.

In the full quantum scheme of DCE, the cavity wall is treated as a phonon. We find that the cavity field does not exhibit any squeezing in the DCE process starting from a phonon state. Only when the cavity wall is measured, the collapsed cavity field states will have different squeezing properties according to the measured phonon states. These collapsed cavity field states have much lower average photon numbers.

The semiclassical scheme of DCE is to drive the cavity wall stably and continuously at a certain frequency. The vibration of the cavity wall is regarded as a classical quantity and is not affected by the photon state. In this scheme, we find that the detuning between $2\omega_c$ and ω_m is very important, and it was attempt to avoid it in the previous DCE research. Under proper detuning, we can periodically obtain squeezed light with very low average photon number.

It should be noted that the verification of our theoretical work in experiments is still a great challenge. However, the current experimental progress still gives us hope. Several generalized DCE schemes have been realized in experiments.[30–33] For example, in Refs. [32,33], the equivalent action of a quickly moving mirror is mimicked by an inductance variation of a SQUID controlled by a quickly oscillating magnetic flux, and detected the generated microwave photons. Furthermore, the preparation and manipulation of single phonon have been experimentally achieved in Ref. [39]. Therefore, the dynamic Casimir effect would be observed through superconducting circuits system which has been realized in experiment.[32] Then, based on the manipulation technique of single phonons in Ref. [39], phonon state can be detected. After detecting phonons, the cavity field can automatically collapse to squeezed state, thereby achieving the preparation of a squeezed state.

Although dissipation and decoherence have a certain impact on the loss and absorption of photons, the possible experimental implementation in our system is usually in the microwave band, and the impact of loss is very small and can be ignored. In the future, when DCE can be observed in the visible or infrared band, the influence of photon loss must be considered.

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References

[1] Walls D F 1983 *Nature* **306** 141
[2] Braunstein S L and Loock P V 2005 *Rev. Mod. Phys.* **77** 513
[3] Loudon R 1987 *J. Mod. Opt.* **34** 709
[4] Su X L, Zhao Y P, Hao S H, Jia X J, Xie C D and Peng K C 2012 *Opt. Lett.* **37** 5178
[5] Zhao Y P, Hao S H, Su X L and Xie C D 2012 *Acta Opt. Sin.* **32** 0627002 (in Chinese)
[6] Slavı'k R, Parmigiani F, Kakande J, Lundström C, Sjödin M, Andreksson P A, Weerasuriya R, Sygletos S, Ellis A D, Nielsen L G, Jakobsen D, Herstrøm S, Phelan R, O'Gorman J, Bogris A, Syvridis D, Dasgupta S, Petropoulos P and Richardson D J 2010 *Nat. Photon.* **4** 690
[7] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[8] Schnabel R, Mavalvala N, McClelland D E and Lam P K 2010 *Nat. Commun.* **1** 121
[9] Aasi J, Abadie J, Abbott B P, *et al.* 2013 *Nat. Photon.* **7** 613
[10] Sun H X, Liu Z L, Liu K, Yang R G, Zhang J X and Gao J R 2014 *Chin. Phys. Lett.* **31** 084202
[11] Wang H, Zhang Y, Pan Q, Su H, Porzio A, Xie C D and Peng K C 1999 *Phys. Rev. Lett.* **82** 1414
[12] Slusher R E, Hollberg L W, Yurke B, Mertz J C and Valley J F 1985 *Phys. Rev. Lett.* **55** 2409
[13] Wu L A, Kimble H J, Hall J L and Wu H F 1986 *Phys. Rev. Lett* **57** 2520
[14] Suzuki S, Yonezawa H, Kannari F, Sasaki M and Furusawa A 2006 *Appl. Phys. Lett.* **89** 061116
[15] Eberle T, Steinlechner S, Bauchrowitz J, Händchen V, Vahlbruch H, Mehmet M, Ebhardt H M and Schnabel R 2010 *Phys. Rev. Lett.* **104** 251102
[16] Mehmet M, Ast S, Eberle T, Steinlechner S, Vahlbruch H and Schnabe R 2011 *Opt. Express* **19** 25763
[17] TSerikawa T, Yoshikawa J I, Makino K and Frusawa A 2016 *Opt. Express* **24** 28383
[18] Zuo G H, Yang C, Zhao J X, Tian Z Z and Zhang T C 2020 *Acta Phys. Sin.* **69** 014207 (in Chinese)
[19] Tian J F, Zuo G H, Zhang Y C, Li G, Zhang P F and Zhang T C 2017 *Chin. Phys. B* **26** 124206
[20] Casimir H B G 1948 *Proc. Kon. Ned. Akad. Wet. B* **793** 51
[21] Milonni P W 1994 *The Quantum Vacuum* (Boston: Academic Press)

[22] Milonni P W 2003 *Phys. Today* **56** 49
[23] Bordag, M, Klimchitskaya G L, Mohideen U and Mostepanenko V M 2009 *Advances in the Casimir Effect* (New York: Oxford University Press) p. 145
[24] Schwinger J 1951 *Phys. Rev.* **82** 664
[25] Fulling S A and Davies P C W 1976 *Proc. R. Soc. A* **348** 393
[26] Yablonovitch E 1989 *Phys. Rev. Lett.* **62** 1742
[27] Schwinger J 1993 *Proc. Natl. Acad. Sci. USA* **90** 2105
[28] Moore G T 1970 *J. Math. Phys.* **11** 2679
[29] Braggio C, Bressi G, Carugno G, Noce C D, Galeazzi G, Lombardi A, Palmieri A, Ruoso G and Zanello D 2005 *Europhys. Lett.* **70** 754
[30] Johansson J R, Johansson G, Wilson C M and Nori F 2009 *Phys. Rev. Lett.* **103** 147003
[31] Wilson C M, Duty T, Sandberg M, Persson F, Shumeiko V and Delsing P 2010 *Phys. Rev. Lett.* **105** 233907
[32] Wilson C M, Johansson G, Pourkabirian A, Simoen M, Johansson J R, Duty T, Nori F and Delsing P 2011 *Nature* **479** 376
[33] Lähteenmäki P, Paraoanu G S, Hassel J and Hakonen P J 2013 *Proc. Natl. Acad. Sci. USA* **110** 4234
[34] Lambrecht A, Jaekel M T and Reynaud S 1996 *Phys. Rev. Lett.* **77** 615
[35] Ancheyta R R, Prieto I R, Leija A P, Busch K and Montiel R J L 2017 *Phys. Rev. A* **96** 032501
[36] Macrì V, Ridolfo A, Stefano O D, Kockum A F, Nori F and Savasta S 2018 *Phys. Rev. X* **8** 011031
[37] Dodonov V V and Klimov A B 1996 *Phys. Rev. A* **53** 2664
[38] Dodonov V 1995 *Phys. Lett. A* **207** 126
[39] Qiao H, Dumur É, Andersson G, Yan H, Chou M H, Grebel J, Conner C R, Joshi Y J, Miller J M, Povey R G, Wu X and Cleland A N 2023 *Science* **380** 6649